SENSOR FAULT DETECTION AND DIAGNOSIS OF A PROCESS USING UNKNOWN INPUT OBSERVER

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Abstract- In this paper, a robust sensor fault detection and isolation (FDI) method based on the unknown input observer (UIO) approach is presented. The basic principle of unknown input observers is to decouple disturbances from the state estimation error. A single full-order observer is designed to detect sensor faults in the presence of unknown inputs (disturbances). By doing so, we generate a residual, a weighted output of the state estimation error, decoupled from disturbances. The resulting robust (in the sense of disturbances) residual can be used for fault detection. Although this scheme has successful fault detection, using one observer is not successful in fault isolation. Therefore, a robust sensor fault isolation observer scheme is proposed. In order to evaluate its ability, the presented method is adopted to detect and isolate sensor faults of a highly nonlinear dynamic system. The faulty behavior of output sensors in a jacketed continuous stirred tank reactor (CSTR), around operating point, is investigated. Simulation results show that model uncertainties and disturbances can be distinguished from a response to a sensor fault.

Key Words- Robust Fault Detection and Isolation, Unknown Input Observers, Disturbance decoupling

1. INTRODUCTION

In the real world, no system can work perfectly at all time under all conditions. In chemical plants faulty sensors may cause process performance degradation (e.g., lower product quality) or fatal accidents (e.g., temperature run away) [1]. A report estimates that the loss to petrochemical industries in the U.S. alone is $20 billion/year [2]. While, petrochemical plants are becoming larger, loss and maintenance costs will increase. Besides the economic loss, irreparable damage to human operators should be considered. Therefore, it is essential that a fault detection scheme can be developed so as to be able to detect and identify possible faults in the system as early as possible [3]. Then, the system can be maintained and kept reliable by means of this early warning enabling repair or replacement to take place at the earliest or most convenient time, with the minimum of loss of time or productivity. Today, fault detection and diagnosis have become inseparable parts of modern complex systems.

Existing fault detection approaches can be roughly classified into model-free approaches, i.e., approaches based on statistical analysis, neural networks, and/or expert systems; and approaches based on the analytical redundancy, resorting to the available mathematical model of the process [4]. Statistical techniques do not require a model of the system but only a good database of historical data regarding normal operating conditions is needed, since statistical tests on the measured data are used to detect any abnormal behavior. This is, of course, the main disadvantage of these methods in that
they need a large amount of plant data that are collected along a quite large window of operating time and are used to construct a statistical model of the process [5].

Model-based fault detection and isolation (FDI) techniques use mathematical models of the monitored process and extract features from measured signals, to generate a fault indicating signal which is called a residual. Whereas mathematical models are necessary for control purposes, model-based FDI technology has attracted remarkable attention in modern complex systems during the last three decades [3].

There are a great variety of FDI methods in the literature [3, 4, 6-9]. Among model-based analytical redundancy approaches, observer-based schemes have been successfully adopted in a variety of application fields. The Extended Kalman filter (EKF) is one of the most popular model-based techniques used for fault detection and diagnosis in chemical processes [10-13]. Although successful applications of this tool have been reported in the literature for fault detection and diagnosis in chemical processes, the EKF contains several flaws that may seriously affect its performance. Therefore, practical applications of EKF are still very limited [14]. Some of EKF’s inconvenience can be overwhelmed using the unscented Kalman filter (UKF) [15]. However, model-based FDI is built upon a number of idealized assumptions, one of which is that the mathematical model used is a faithful replica of the plant dynamics [6]. There are, of course, disturbances and model uncertainties unavoidable for any practical system. Therefore, it is essential in the design of any fault diagnosis system to take these effects into consideration, so that fault diagnosis can be done reliably and robustly.

The goal of a robust FDI is to discriminate between the fault effects and the effects of uncertain signals and perturbations. Indeed, one of the known successful robust fault diagnosis approaches is the use of the disturbance decoupling principle [6], in which the residual is designed to be insensitive to unknown disturbances, whilst sensitive to faults. For this purpose, Frank and Ding developed the unknown input fault detection observer for linear systems which can be designed by use of the Kronecker canonical form [7, 8].

As discussed earlier, in the EKF-based method, the bias in the residuals resulted from modeling uncertainties and disturbances can be misinterpreted as a response to a fault in the sensor. In spite of the importance of robust FDI, the issue of robustness has not been sufficiently addressed for chemical processes fault detection and isolation in the literature [1, 5]. Sotomayor & Odloak, designed a robust FDI scheme based on UIO for tow chemical processes, but in their work robustness to disturbance and uncertainty has not been shown [5]. The objective of this paper is to design a robust sensor fault detection and isolation scheme for a chemical process. An unknown input observer is designed for a linearized model of jacketed CSTR. The ability and performance of the UIO is investigated for abrupt fault detection and isolation in a highly nonlinear chemical process. It is shown that this method can discriminate disturbance and a certain degree of model uncertainty from faulty sensors.

2. FAULT DETECTION OBSERVER
Consider a system with additive disturbances described by the following equations:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + E \ d(t) \\
y(t) &= Cx(t)
\end{align*}
\]
where \( x(t) \in \mathbb{R}^n \) is the state vector, \( y(t) \in \mathbb{R}^m \) is the output vector, \( u(t) \in \mathbb{R}^r \) is the input vector, and \( d(t) \in \mathbb{R}^r \) is an unknown scalar function representing the disturbance. Note that matrices \( A, B, \) and \( C \) correspond to the state-space description of the linear, time-invariant system.

Consider an observer of the form:

\[
\dot{\hat{x}}(t) = F \hat{x}(t) + TBu(t) + Ky(t) \\
\hat{x}(t) = z(t) + Hy(t)
\]

where \( \hat{x}(t) \in \mathbb{R}^n \) is the estimated state vector and \( z(t) \in \mathbb{R}^n \) is the state of the observer. The observer state-space matrices \( F, T, K, \) and \( H \) will be designed to decouple the disturbance from the state estimation error, \( e(t) = x(t) - \hat{x}(t) \). The state estimation error is governed by the following equation:

\[
\dot{e}(t) = (A - HCA - K_1C)e(t) + [F - (A - HCA - K_1C)]z(t) \\
+ [K_2 - (A - HCA - K_1C)H]y(t) \\
+ [T - (I - HC)]Bu(t) + (HC - I)Ed(t)
\]

where \( K = K_1 + K_2 \).

By definition, an observer is defined as UIO for the system defined by (1) if its state estimation error vector \( e(t) \) approaches zero asymptotically, regardless of the presence of the unknown inputs (disturbances) in the system [6].

To synthesize UIO, the following relationships must hold for the observer matrices \( F, T, K, \) and \( H \):

\[
(HC - I)E = 0  \\
T = I - HC  \\
F = A - HCA - K_1C  \\
K_2 = FH
\]

Given these relationships, the state estimation error reduces to:

\[
\dot{e}(t) = F e(t)
\]

By selecting a stable \( F \), \( e(t) \) will be made to approach zero asymptotically. Therefore, according to the definition, an unknown input observer is designed by first selecting a stable \( F \) and then solving equations (4) - (5).

**Theorem 1:** Necessary and sufficient conditions for the observer (2) to be a UIO for defined system in (1) are [6]:

(i) \( \text{rank}(CE) = \text{rank}(E) \)

(ii) \( (C, A_1) \) is a detectable pair,

where \( A_1 = A - E \left[ (CE)^T CE \right]^{-1} (CE)^T CA \).
Note that in equation (4), the matrix $K_1$ (that stabilizes matrix $F$) is not unique. This design freedom can be used to generate a directional residual for fault isolation.

The observer described by equation (2) is illustrated in Fig. 1. A flow chart that describes the UIO design procedure is depicted in Fig. 2.

![Fig. 1: Structure of a full-order UIO.](image)

### 2.1. Robust sensor fault detection and isolation scheme [6]

Fig. 3 depicts a robust sensor fault detection and isolation scheme which also includes a general unknown input observer (GUIO). The GUIO generates the residuals that are used to detect faults in sensors.

Assuming that all actuators are fault-free, the system subject to sensor faults can be expressed as:

$$
\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\
y(t) = Cx(t) + f_s
$$

where $f_s \in \mathbb{R}^m$ is an immeasurable vector considered as an additive bias resulting from sensor faults. Then the following vectors can be defined as:

$$
y^j = C^j x + f^j_s \\
y_j = c_j x + f^j_{sj}
$$

where $c_j \in \mathbb{R}^{l \times m}$ is the $j$th row of the matrix $C$, $C^j \in \mathbb{R}^{(m-1) \times n}$ is obtained from $C$ by deleting the $j^{th}$ row $c_j$, $y_j$ is $j$th component of $y$ and $y^j \in \mathbb{R}^{m-1}$ is obtained from the vector $y$ by deleting the $j^{th}$ component $y_j$. Then, $m$ UIO-based residual generator is constructed as:

$$
\dot{z}^j(t) = F^j z^j(t) + T^j Bu(t) + K^j y^j(t) \\
r^j = (I - C^j H^j) y^j + C^j z^j
$$

In the generalized observer scheme (GOS), the following conditions must be satisfied to design UIOs:
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\[ H^j C^j E = E \]
\[ T^j = I - H^j C^j \]
\[ F^j = T^j A - K_1^j C^j \]
\[ K_2^j = F^j H^j \]
\[ K^j = K_1^j + K_2^j \]

Fig. 2: Flowchart of the UIO design.
Each residual generator is driven by all inputs and all outputs except one output. When all actuators are fault-free and a fault occurs in the jth sensor, the residual will satisfy the following isolation logic:

\[
\begin{align*}
\| r^j \| &< T_{\text{SFI}}^j \\
\| r^k \| &\geq T_{\text{SFI}}^k \quad \text{for} \quad k = 1, 2, \cdots, j-1, j+1, \cdots, m
\end{align*}
\]  

(10)

where \( T_{\text{SFI}}^j \)'s are isolation thresholds and \( \| r^j \| \)'s are the Euclidean norms of the residuals.

3. SIMULATION RESULTS OF THE FDI METHODS APPLIED TO A CSTR

In this section, the simulation results of the generalized unknown input observer scheme (GUIOS) will be demonstrated. This method is a commonly accepted robust FDI scheme. In order to evaluate the performance of the proposed method compared to the Leunberger observer, it is applied to a highly nonlinear CSTR model explained in [16].

3.1. Process description

This is a highly nonlinear dynamic system describing the behavior of a non-adiabatic CSTR in which an irreversible highly exothermic chemical reaction \( \text{A} \rightarrow \text{B} \) takes place. The reactor’s wall significantly affects the system dynamics and therefore has also been taken into account.

3.2. Dynamic process model

The corresponding model leads to the following set of ODEs in a normalized and dimensionless form:
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\[
\begin{align*}
\frac{dx}{dt} &= p_1 u_1 + p_1 u_2 x_2 - p_3 e^{-p_1 (1 + x_1)} (1 + x_1) - p_3 x_1 \\
\frac{dx_2}{dt} &= p_3 u_1 + p_3 u_2 x_2 - p_4 x_2 + p_4 x_3 + p_5 e^{-p_1 (1 + x_1)} (1 + x_1) \\
\frac{dx_3}{dt} &= p_6 x_2 - p_6 x_1 - p_7 x_4 \\
\frac{dx_4}{dt} &= p_8 u_1 + p_8 u_4 x_2 - p_9 x_3 - p_9 x_4
\end{align*}
\]

(11)

with the system state vector defined as:

\[
x^T = \begin{bmatrix}
C_A - C_{A,\text{ref}} & T_R - T_{R,\text{ref}} & T_W - T_{W,\text{ref}} & T_J - T_{J,\text{ref}}
\end{bmatrix}
\]

(12)

where \( C_A \) is the concentration of reactant \( A \), \( T_R \) the reactor temperature, \( T_W \) the wall temperature, and \( T_J \) is the jacket temperature. The unit of \( C_A \) is \( \text{mol/m}^3 \) and the unit of \( T_R, T_W \) and \( T_J \) is \( \text{K} \).

The input vector is:

\[
u^T = \begin{bmatrix}
C_{A0} - C_{A,\text{ref}} & F_R - F_{R,\text{ref}} & T_0 - T_{0,\text{ref}} & T_{J0} - T_{J0,\text{ref}} & F_J - F_{J,\text{ref}}
\end{bmatrix}
\]

(13)

where \( C_{A0} \) is the feed concentration of reactant \( A \), \( F_R \) the flow rate, \( T_0 \) the feed temperature, \( T_{J0} \) the inlet jacket temperature, and \( F_J \) is the coolant flow rate. The unit of \( C_{A0} \) is \( \text{mol/m}^3 \) and the unit of \( T_0 \) and \( T_{J0} \) is \( \text{K} \), and the unit of \( F_R \) and \( F_J \) is \( \text{m}^3/\text{s} \). The corresponding reference values are:

\[
C_{A0,\text{ref}} = 3 \text{ mol/m}^3, \quad F_{R,\text{ref}} = 60 \times 10^{-5} \text{ m}^3/\text{s}, \quad F_{J,\text{ref}} = 15 \times 10^{-4} \text{ m}^3/\text{s}, \\
F_{R,\text{ref}} = 60 \times 10^{-5} \text{ m}^3/\text{s}, \quad T_{R,\text{ref}} = T_{W,\text{ref}} = T_{J,\text{ref}} = T_{0,\text{ref}} = T_{J0,\text{ref}} = 298 \text{ K}
\]

The measurement model is assumed to be:

\[
y = Cx \quad \text{with} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

(14)

that is, measurements of the wall temperature \( x_3 \) are not available. Table 1 summarizes the model parameters used in the present work. The model has three steady states which are represented in Table 2.

3.3. UIO design and simulation results for sensor fault detection

Linear model of the system, which is obtained by linearizing the system around High-temperature stable equilibrium point, is considered for the design procedures. The system matrices are:
This approach is based on the assumption that the disturbance distribution matrix $E$ is known a priori. Therefore, disturbance distribution matrix is assumed as, 

$E = [1 \ 1 \ 1]^T$.

### TABLE 1. Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>EXPRESSION</th>
<th>VALUE</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$</td>
<td>$\frac{F_{ref}}{V_g}$</td>
<td>$3.33 \times 10^{-2}$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$p_2$</td>
<td>$k_e$</td>
<td>$4.08 \times 10^7$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$p_3$</td>
<td>$\frac{E_d}{RT_{ref}}$</td>
<td>$25.347$</td>
<td>-</td>
</tr>
<tr>
<td>$p_4$</td>
<td>$\frac{(hA_i)_f}{V_g (\rho C_p)_f}$</td>
<td>$6.63 \times 10^{-1}$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$p_5$</td>
<td>$-\frac{(\Delta H)<em>f C</em>{sum}}{(\rho C_p)<em>f T</em>{ref}}$</td>
<td>$1.45$</td>
<td>-</td>
</tr>
<tr>
<td>$p_6$</td>
<td>$\frac{(hA_i)_i}{V_g (\rho C_p)_i}$</td>
<td>$5.97$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$p_7$</td>
<td>$\frac{(hA_i)_j}{V_j (\rho C_p)_j}$</td>
<td>$5.97$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$p_8$</td>
<td>$\frac{F_{ref}}{V_f}$</td>
<td>$1.67 \times 10^{-1}$</td>
<td>$s^{-1}$</td>
</tr>
<tr>
<td>$p_9$</td>
<td>$\frac{(hA_i)_e}{V_f (\rho C_p)_e}$</td>
<td>$1.33$</td>
<td>$s^{-1}$</td>
</tr>
</tbody>
</table>

### TABLE 2. Steady states

<table>
<thead>
<tr>
<th></th>
<th>LOW-TEMPERATURE STABLE</th>
<th>UNSTABLE</th>
<th>High-temperature stable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$-0.0140582$</td>
<td>$-0.37748$</td>
<td>$-0.97640$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$0.0068168$</td>
<td>$0.18304$</td>
<td>$0.47345$</td>
</tr>
<tr>
<td>$x_3$</td>
<td>$0.0061321$</td>
<td>$0.16465$</td>
<td>$0.42590$</td>
</tr>
<tr>
<td>$x_4$</td>
<td>$0.0054473$</td>
<td>$0.14627$</td>
<td>$0.37834$</td>
</tr>
</tbody>
</table>

To start the design process, condition (i) of Theorem 1 (Fig. 2), is first verified by observing that $\text{rank}(E) = \text{rank}(CE) = 1$. Then the matrices $H$, $T$ and $A_1$ are computed as,
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\[ H = \begin{bmatrix} 0.3333 & 0.3333 & 0.3333 \\ 0.3333 & 0.3333 & 0.3333 \\ 0.3333 & 0.3333 & 0.3333 \end{bmatrix}, \quad T = \begin{bmatrix} 0.6667 & -0.3333 & 0 & -0.3333 \\ -0.3333 & 0.6667 & 0 & -0.3333 \\ -0.3333 & -0.3333 & 1 & -0.3333 \\ -0.3333 & -0.3333 & 0 & 0.6667 \end{bmatrix} \]

\[ A_i = \begin{bmatrix} -1.5855 & -0.2159 & -0.6643 & 0.4433 \\ 1.7923 & 0.0518 & -0.0013 & 0.4433 \\ -0.2068 & 6.1340 & -12.6043 & 6.4133 \\ -0.2068 & 0.1640 & 0.6657 & -0.8867 \end{bmatrix}. \]

Condition (ii) of Theorem 1 is also satisfied because the observability matrix is full rank, thus, the pair \((C,A)\) is observable. Therefore, the UIO can be constructed by using the following values of \(F\) and \(K\).

\[ F = \begin{bmatrix} -2.1854 & -0.6495 & -0.6643 & 0.2571 \\ 1.3587 & -0.9045 & -0.0013 & 0.0454 \\ -0.1880 & 6.1162 & -12.6043 & 6.3677 \\ -0.3930 & -0.2339 & 0.6657 & -1.6421 \end{bmatrix}, \quad K = \begin{bmatrix} -0.4808 & -0.6471 & -0.8945 \\ 0.5997 & 1.1224 & 0.5640 \\ -0.1216 & -0.0850 & -0.0572 \\ -0.3482 & -0.1365 & 0.2210 \end{bmatrix}. \]

Three different scenarios will be considered. In the first case, it is assumed that the sensor measuring the concentration of reactant A \((C_A)\) is damaged (case A). The measured value is is suddenly deviated \(+10\%\) from the normal measurement after 50 seconds elapsed from running of the process. In the second case, a similar fault is occurred in the second output which is measuring the reactor temperature \((T_R)\), (case B). In third case, similar fault is occurred in the sensor measuring jacket temperature \((T_j)\), (case C).

All cases have been simulated and residuals have been achieved. The simulation result of case A is illustrated in Fig. 4. For the sake of space, we omit the respective graphics of cases B and C as the residuals behave similarly. As it can be seen in this figure, successful fault detection has been achieved. However, this scheme is not successful in fault isolation. Therefore, as discussed earlier, a robust sensor fault isolation scheme is proposed for fault isolation.

![Fig. 4: Successful fault detection in sensor C_A.](image-url)
Fig. 5: Successful fault isolation in sensor measuring $C_A$.

Fig. 6: Successful fault isolation in sensor measuring $T_R$.

Fig. 7: Successful fault isolation in sensor measuring $T_J$.

Fig. 8: Successful isolation of the fault in sensor measuring $C_A$ in the presence of model uncertainty.
3.4. UIO design and simulation results of the sensor fault detection and isolation

The fault isolation problem is to locate the fault or to determine which sensor has failed. In this study, a structured residual set in which each residual is sensitive to certain group of faults and insensitive to others, is designed for fault isolation. The Generalized Observer Scheme (GOS) method allows one to detect the faulty sensor by checking if the residuals have exceeded the predefined thresholds.

To start the design process, the conditions of Theorem 1 are checked. Since the rank condition \( \text{rank} (C_j E) = \text{rank} (E) \) for \( j=1,2,3 \), is satisfied, then the full set of UIOs exist. Matrices \( H_j, T_j \) for \( j=1,2,3 \), and \( A_1 \) are computed. For all observers the pairs \( (C_j, A_1) \) are observable. Therefore, \( F_j \) and \( K_j \) \( j=1,2,3 \) are computed. Simulations show that this scheme is successful in both fault detection and isolation. Figures 5 to 7 show the residuals.

1.1. Robustness evaluation

From the above simulation results, one may see that the fault detection and isolation scheme is robust to nonlinearity in \( d(t) \). In the following, robustness with respect to parameter variations is investigated. The robustness of UIOs to process parameter variations can be evaluated by the simulation in which matrix \( A \) is changed to:

\[
A' = A + \Delta A = \begin{bmatrix}
-1.5166 & -0.4179 & 0 & 0 \\
2.1990 & -0.1234 & 0.7293 & 0 \\
0 & 6.5670 & -13.1340 & 6.5670 \\
0 & 0 & 1.4630 & -1.4630
\end{bmatrix}
\]

The residuals for all cases have been obtained. For the sake of space, we only show simulation result of case A in Fig. 8. From these results one can conclude that the robust FDI scheme can reliably detect and isolate faulty sensors even in the presence of process parameter mismatch. In other words, model uncertainties can be distinguished from a response to a fault in the sensor.

1. CONCLUSIONS

In this paper robust sensor fault detection and isolation was presented. Robustness of the observers with respect to external disturbances is ensured using unknown input observers. The presented method is applied to a continuous stirred tank reactor (CSTR). Several simulations in the presence of external disturbances and different classes of faults have been performed. The simulation results confirm the robustness and effectiveness of the proposed scheme for fault detection in the presence of external disturbances. It is also shown that a certain degree of model uncertainties can be distinguished from a response to a fault in the sensor.

ACKNOWLEDGEMENTS

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2. REFERENCES


