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NUMERICAL APPROACH OF HIGH-ORDER LINEAR DELAY DIFFERENCE EQUATIONS WITH VARIABLE COEFFICIENTS IN TERMS OF LAGUERRE POLYNOMIALS

Burcu Gürbüz[1], Mustafa Gülsu[2],Mehmet Sezer[3]
[1,2,3]Department of Mathematics
Mugla University,Mugla, Turkey
[1]burcugrbz@gmail.com
[2]mgulsu@mu.edu.tr, [3]msezer@mu.edu.tr

Abstract- This paper presents a numerical method for the approximate solution of mth-order linear delay difference equations with variable coefficients under the mixed conditions in terms of Laguerre polynomials. The aim of this article is to present an efficient numerical procedure for solving mth-order linear delay difference equations with variable coefficients. Our method depends mainly on a Laguerre series expansion approach. This method transforms linear delay difference equations and the given conditions into matrix equation which corresponds to a system of linear algebraic equation. The reliability and efficiency of the proposed scheme are demonstrated by some numerical experiments and performed on the computer algebraic system Maple.

Key Words- Laguerre Polynomials and Series, Delay Difference Equations, Laguerre Collocation Method

1.INTRODUCTION

Orthogonal polynomials occur often as solutions of mathematical and physical problems. They play an important role in the study of wave mechanics, heat conduction, electromagnetic theory, quantum mechanics and mathematical statistics. They provide a natural way to solve, expand, and interpret solutions to many types of important delay difference equations. Representation of a smooth function in terms of a series expansion using orthogonal polynomials is a fundamental concept in approximation theory, and forms the basis of spectral methods of solution of delay difference equations. Laguerre polynomials $L_n(x)$ constitute complete orthogonal sets of functions on the semi-infinite interval $[0,\infty)$. In this paper, we are concerned with the use of Laguerre polynomials to solve delay difference equations. In recent years, the studies of difference equations, i.e. equations containing shifts of the unknown function are developed very rapidly and intensively. It is well known that linear delay difference equations have been considered by many authors[1-11]. The past couple decades have seen a dramatic increase in the application of delay models to problems in biology, physics and engineering[12-15]. In the field of delay difference equation the computation of its solution has been a great challenge and has been of great importance due to the versatility of such equations in the mathematical modeling of processes in various application fields, where they provide the best simulation of observed phenomena and hence the numerical approximation of such equations has been growing more and more. Based on the obtained method, we shall give sufficient approximate solution of the linear delay difference Eq.(1). The results can extend and improve the recent works. An example is given to demonstrate the effectiveness of the results.In recent years, Taylor and Chebyshev approximation methods have been given to find polynomial solutions of differential equations by Sezer et al. [16-22].

In this study, the basic ideas of the above studies are developed and applied to the mth-order linear delay difference equation (which contains only positive shift in the unknown function) with variable coefficients[23,p.228,p.229]

$$\sum_{k=0}^{m} P_k(t)y(t+k) = f(t), \quad k \geq 0, k \in N \tag{1}$$

under the initial conditions

$$\sum_{k=0}^{m-1} a_{rk}\, y(0) + b_{rk}\, y(b) = \lambda_r, \quad 0 \leq t \leq b \leq \infty,\ r = 0,1,\ldots,m-1 \tag{2}$$

where $P_k(t)$ and $f(t)$ are analytical functions; a_{rk}, b_{rk} *and* λ_i are real or complex constants. The aim of this study is to get solution as truncated Laguerre series defined by

$$y(t) = \sum_{n=0}^{N} a_n L_n(t),\ L_n(t) = \sum_{r=0}^{n} \frac{(-1)^r}{r!}\binom{n}{r} t^r \tag{3}$$

where $L_n(t)$ denotes the Laguerre polynomials, a_n $(0 \leq n \leq N)$ are unknown Laguerre polynomial coefficients, and N is chosen any positive integer such that $N \geq m$.

The rest of this paper is organized as follows. We describe the formulation of Laguerre polynomials required for our subsequent development in section 2. Higher-order linear delay difference equation with variable coefficients and fundamental relations are presented in Section 3. The new scheme are based on Laguerre collocation method. The method of finding approximate solution is described in Section 4. To support our findings, we present result of numerical experiments in Section 5. Section 6 concludes this article with a brief summary. Finally some references are introduced at the end.

2. PROPERTIES OF THE LAGUERRE POLYNOMIALS

A total orthonormal sequence in $L^2(-\infty,b]$ or $L^2[a,+\infty)$ can be obtained from such a sequence in $L^2[0,+\infty)$ by transformations t=b-s and t=s+a, respectively. We consider $L^2[0,+\infty)$. Applying the Gram-Schmidt process to the sequence defined by

$$e^{-t/2}, \quad te^{-t/2}, \quad t^2 e^{-t/2}, \ldots$$

We can obtain an orthonormal sequence (e_n). It can be shown that (e_n) is total in $L^2[0,+\infty)$ and is given by

$$e_n(t) = e^{-t/2} L_n(t), \text{ n=0,1,2,} \ldots$$

where the Laguerre polynomial of order n is defined by

$$L_0(t) = 1,\ L_n(t) = \frac{e^t}{n!}\frac{d^n}{dt^n}(t^n e^{-t}), \text{ n=1,2,3,} \ldots \tag{4}$$

That is

$$L_n(t)=\sum_{r=0}^{n}\frac{(-1)^r}{r!}\binom{n}{r}t^r \tag{5}$$

Explicit expressions for the first few Laguerre polynomials are

$$L_0(t)=1,\ \ L_1(t)=1-t,\ L_2(t)=1-2t+\frac{1}{2}t^2,\ L_3(t)=1-3t+\frac{3}{2}t^2-\frac{1}{6}t^3$$

The Laguerre polynomials $L_n(t)$ are solutions of the Laguerre differential equation

$$tL_n''(t)+(1-t)L_n'(t)+nL_n(t)=0\ [24] \tag{6}$$

In the present application, an approximate solution in terms of linear combination of Laguerre polynomial is assumed of the following form:

$$y(t)=\sum_{n=0}^{N}a_nL_n(t)\,,\quad 0\le n\le N$$

3. FUNDAMENTAL RELATIONS

3.1. General fundamentals

Let us consider the mth-order linear delay difference equation with variable coefficients (1) and find the matrix forms of each term in the equation. First we can convert the solution $y(t)$ defined by a truncated Laguerre series (3) to matrix forms

$$y(t)=\mathbf{L}(t)\mathbf{A},\ y(t+k)=\mathbf{L}(t+k)\mathbf{A} \tag{7}$$

where

$$\mathbf{L}(t)=\begin{bmatrix}L_0(t) & L_1(t) & L_2(t) & \dots & L_N(t)\end{bmatrix} \tag{8}$$

$$\mathbf{A}=[a_0\ a_1\ \dots\ a_N]^T \tag{9}$$

By using the expression (8) we find the corresponding matrix relation as follows

$$\mathbf{L}^T(t)=\mathbf{H}\,\mathbf{X}^T(t)\quad \text{and}\quad \mathbf{L}(t)=\mathbf{X}(t)\mathbf{H}^T \tag{10}$$

where

$$\mathbf{X}(t)=[1\ \ t\ \dots\ t^N] \tag{11}$$

$$\mathbf{H}=\begin{bmatrix}\frac{(-1)^0}{0!}\binom{0}{0} & 0 & 0 & \cdots & 0\\ \frac{(-1)^0}{0!}\binom{1}{0} & \frac{(-1)^1}{1!}\binom{1}{1} & 0 & \cdots & 0\\ \frac{(-1)^0}{0!}\binom{2}{0} & \frac{(-1)^1}{1!}\binom{2}{1} & \frac{(-1)^2}{2!}\binom{2}{2} & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \frac{(-1)^0}{0!}\binom{N}{0} & \frac{(-1)^1}{1!}\binom{N}{1} & \frac{(-1)^2}{2!}\binom{N}{2} & \cdots & \frac{(-1)^N}{N!}\binom{N}{N}\end{bmatrix} \tag{12}$$

Then, by taking into account(10) we obtain

$$\mathbf{X}(t) = \mathbf{L}(t)(\mathbf{H}(t))^{-1} \tag{13}$$

To obtain the matrix $\mathbf{X}(t+k)$ in terms of the matrix $\mathbf{X}(t)$, we can use the following relation:

$$\mathbf{X}(t) = [1 \;\; t \;\; t^2 \ldots t^N], \;\; \mathbf{X}(t+k) = [1 \;\; t+k \;\; (t+k)^2 \; \ldots \; (t+k)^N] \tag{14}$$

$$\mathbf{X}(t+k) = \mathbf{X}(t)\,\mathbf{B}_k^T \tag{15}$$

where

$$\mathbf{B}_k^T = \begin{bmatrix} \binom{0}{0}(k)^0 & \binom{1}{0}(k)^1 & \binom{2}{0}(k)^2 & \cdots & \binom{N}{0}(k)^N \\ 0 & \binom{1}{1}(k)^0 & \binom{2}{1}(k)^1 & \cdots & \binom{N}{1}(k)^{N-1} \\ 0 & 0 & \binom{2}{2}(k)^0 & \cdots & \binom{N}{2}(k)^{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \binom{N}{N}(k)^0 \end{bmatrix} \tag{16}$$

Consequently, by substituting the matrix form (10) into (7), we have the matrix relation of solution

$$y(t+k) = \mathbf{L}(t+k)\mathbf{A} = \mathbf{X}(t+k)\mathbf{H}^T\mathbf{A} \tag{17}$$

and by means of (15), the matrix relation is

$$y(t+k) = \mathbf{L}(t+k)\mathbf{A} = \mathbf{X}(t)\,\mathbf{B}_k^T\;\mathbf{H}^T\mathbf{A} \tag{18}$$

3.2. Method of solutions

In this section, we consider high order linear delay difference equation in(1) and approximate to solution by means of finite Laguerre series defined in (3).The aim is to find Laguerre coefficients, that is the matrix **A**. For this purpose, substituting the matrix relations (18) into Eq.(1) and then simplifying, we obtain the fundamental matrix equation

$$\sum_{k=0}^{m} P_k(t)\,\mathbf{X}(t)\,\mathbf{B}_k^T\;\mathbf{H}^T\mathbf{A} = f(t) \tag{19}$$

By using in Eq. (19) collocation points t_i defined by

$$t_i = \frac{b}{N} i,\; i = 0,1,\ldots,N \tag{20}$$

we get the system of matrix equations

$$\sum_{k=0}^{m} P_k(t_i)\,\mathbf{X}(t_i)\,\mathbf{B}_k^T\;\mathbf{H}^T\mathbf{A} = f(t_i),\; i = 0,1,\ldots N \tag{21}$$

or briefly the fundamental matrix equation

$$\sum_{k=0}^{m} \mathbf{P}_k\mathbf{X}\,\mathbf{B}_k^T\;\mathbf{H}^T\mathbf{A} = \mathbf{F} \tag{22}$$

where

$$\mathrm{P}_k=\begin{bmatrix} P_k(t_0) & 0 & \cdots & \cdots & 0 \\ 0 & P_k(t_1) & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \cdots & P_k(t_N) \end{bmatrix}\quad \mathbf{F}=\begin{bmatrix} f(t_0) \\ f(t_1) \\ . \\ . \\ . \\ f(t_N) \end{bmatrix}\quad \mathbf{X}=\begin{bmatrix} X(t_0) \\ X(t_1) \\ . \\ . \\ . \\ X(t_N) \end{bmatrix}=\begin{bmatrix} 1 & t_0 & t_0^{\ 2} & . & . & . & t_0^{\ N} \\ 1 & t_1 & t_1^{\ 2} & . & . & . & t_1^{\ N} \\ . & . & . & . & & & . \\ . & . & . & & . & & . \\ . & . & . & & & . & . \\ 1 & t_N & t_N^{\ 2} & . & . & . & t_N^{\ N} \end{bmatrix}$$

Hence, the fundamental matrix equation (22) corresponding to Eq. (1) can be written in the form

$$\mathbf{WA}=\mathbf{F} \ \text{ or } \ [\mathbf{W};\mathbf{F}],\ \ \mathbf{W}=[w_{i,j}],\ i,j=0,1,...,N \tag{23}$$

where

$$\mathbf{W}=\sum_{k=0}^{m}\mathbf{P}_k\mathbf{X}\,\mathbf{B}_k^T\ \mathbf{H}^T \tag{24}$$

Here, Eq. (23) corresponds to a system of $(N+1)$ linear algebraic equations with unknown Laguerre coefficients $a_0,a_1,...,a_N$. We can obtain the corresponding matrix forms for the conditions (2), by means of the relation (7),

$$\sum_{k=0}^{m-1}\left[a_{rk}\mathbf{X}(0)+b_{rk}\mathbf{X}(b)\right]\mathbf{H}^T\mathbf{A}=[\lambda_r],\quad r=0,1,\ldots,m-1 \tag{25}$$

On the other hand, the matrix form for conditions can be written as

$$\mathbf{U}_r\mathbf{A}=[\lambda_r] \ \text{ or } \ [\mathbf{U}_r;\lambda_r],\quad r=0,1,2,...,m-1 \tag{26}$$

where

$$\mathbf{U}_r=\sum_{k=0}^{m-1}\left[a_{rk}\mathbf{X}(0)+b_{rk}\mathbf{X}(b)\right]\mathbf{H}^T,\quad r=0,1,\ldots,m-1 \tag{27}$$

and

$$\mathbf{U}_r=[u_{r0}\ u_{r1}\ u_{r2}\ldots u_{rN}],\ r=0,1,2,...m-1 \tag{28}$$

To obtain the solution of Eq. (1) under conditions (2), by replacing the row matrices (26) by the last m rows of the matrix (23), we have the new augmented matrix,

$$[\widetilde{\mathbf{W}};\widetilde{\mathbf{F}}]=\begin{bmatrix} w_{00} & w_{01} & . & . & . & w_{0N} & ; & f(t_0) \\ w_{10} & w_{11} & . & . & . & w_{1N} & ; & f(t_1) \\ \vdots & \vdots & & \ddots & & \vdots & ; & \vdots \\ w_{N-m,0} & w_{N-m,1} & . & . & . & w_{N-m,N} & ; & f(t_{N-m}) \\ u_{00} & u_{01} & . & . & . & u_{0N} & ; & \lambda_0 \\ u_{10} & u_{11} & . & . & . & u_{1N} & ; & \lambda_1 \\ \vdots & \vdots & & \ddots & & \vdots & ; & \vdots \\ u_{m-1,0} & u_{m-1,1} & . & . & . & u_{m-1,N} & ; & \lambda_{m-1} \end{bmatrix} \tag{29}$$

If $rank\widetilde{\mathbf{W}}=rank[\widetilde{\mathbf{W}};\widetilde{\mathbf{F}}]=N+1$, then we can write

$$\mathbf{A} = (\widetilde{\mathbf{W}})^{-1}\widetilde{\mathbf{F}} \tag{30}$$

Thus the matrix $\mathbf{A}$ (thereby the coefficients $a_0, a_1, \ldots, a_N$) is uniquely determined. Also the Eq.(1) with conditions (2) has a unique solution. This solution is given by truncated Laguerre series (3). We use the relative error to measure the difference between the numerical and analytic solutions.

We can easily check the accuracy of the method. Since the truncated Laguerre series (3) is an approximate solution of Eq.(1), when the solution $y_N(t)$ and its derivatives are substituted in Eq.(1), the resulting equation must be satisfied approximately; that is, for $t = t_q \in [0,b],\ q = 0,1,2,...$

$$E(t_q) = \left|\sum_{k=0}^{m} P_k(t)y_N(t+k) - f(t)\right| \cong 0 \tag{31}$$

and $E(t_q) \le 10^{-k_q}$ (k_q positive integer). If max $10^{-k_q} = 10^{-k}$ (k positive integer) is prescribed, then the truncation limit N is increased until the difference $E(t_q)$ at each of the points becomes smaller than the prescribed 10^{-k}. On the other hand, the error can be estimated by the function

$$E_N(t) = \sum_{k=0}^{m} P_k(t)y_N(t+k) - f(t) \tag{32}$$

If $E_N(t) \to 0$, when N is sufficiently large enough, then the error decreases.

4. ILLUSTRATIVE EXAMPLE

In this section, several numerical examples are given to illustrate the accuracy and effectiveness properties of the method and all of them were performed on the computer using a program written in Maple9. The absolute errors in Tables are the values of $|y(x) - y_N(x)|$ at selected points.

Example1.

Let us first consider the second order linear delay difference equation with variable coefficients

$$(t-1)y(t+2) + (2-3t)y(t+1) + 2ty(t) = 1,$$

with

$$y(0) = 2,\ y(1) = 2,\ t \in [0,1]$$

and seek the solution $y(t)$ as a truncated Laguerre series

$$y(t) = \sum_{n=0}^{N} a_n L_n(t)$$

So that $P_0(t) = 2t,\ P_1(t) = (2-3t),\ P_2(t) = (t-1),\ f(t) = 1$. Then, for N=5, the collocation points are $t_0 = 0, t_1 = \frac{1}{5}, t_2 = \frac{2}{5}, t_3 = \frac{3}{5}, t_4 = \frac{4}{5}, t_5 = 1$ and the fundamental matrix equation of the problem is defined by

$$\mathbf{W} = \mathbf{P}_0\mathbf{X}\mathbf{H}^T + \mathbf{P}_1\mathbf{X}\mathbf{B}_1\mathbf{H}^T + \mathbf{P}_2\mathbf{X}\mathbf{B}_2\mathbf{H}^T$$

where the matrices are defined by [24]. If these matrices are substituted in (22), it is obtained linear algebraic system.This system yields the approximate solution of the problem. We display a plot of absolute difference exact and approximate solutions in Fig.1 and error functions for various N is shown in Fig.2. Table1 shows solution of the problem for various N. The exact solution of this problem is $y = 2^t - t + 1$.

Table1
Error analysis of Example 1 for the t value

t	Exact Solution	Present Method					
		N=7	N_e=7	N=8	N_e=8	N=9	N_e=9
0.0	2.000000	1.999999	0.10000E-5	1.999999	0.10000E-5	2.000001	0.10000E-5
0.1	1.971773	1.971824	0.51000E-4	1.971629	0.14400E-3	1.971690	0.14400E-3
0.2	1.948698	1.948787	0.89000E-4	1.948519	0.17900E-3	1.948577	0.17900E-3
0.3	1.931144	1.931253	0.10900E-3	1.930986	0.15800E-3	1.931018	0.15800E-3
0.4	1.919508	1.919619	0.11100E-3	1.919390	0.11800E-3	1.919396	0.11800E-3
0.5	1.914214	1.914313	0.99000E-4	1.914137	0.77000E-4	1.914125	0.76000E-4
0.6	1.915717	1.915795	0.78000E-4	1.915673	0.44000E-4	1.915654	0.43000E-4
0.7	1.924505	1.924558	0.53000E-4	1.924484	0.21000E-4	1.924465	0.20000E-4
0.8	1.941101	1.941129	0.28000E-4	1.941093	0.80000E-5	1.941079	0.60000E-5
0.9	1.966066	1.966071	0.50000E-5	1.966065	0.10000E-5	1.966055	0.10000E-5
1.0	2.000000	1.999988	0.12000E-4	2.000004	0.40000E-5	1.999996	0.60000E-5

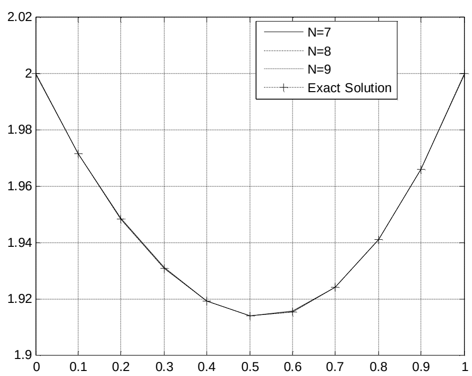

Fig.1.Numerical and exact solution of the Example1 for N=7,8,9

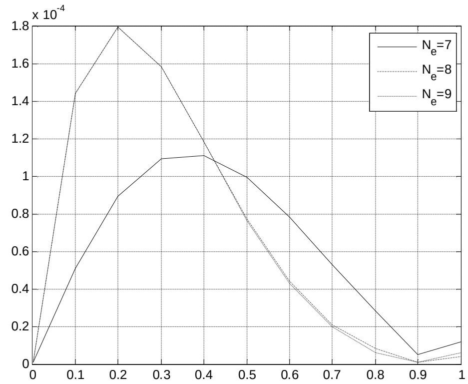

Fig.2.Error function of Example1 for various N.

Example2.

Let us find the Laguerre series solution of the following linear delay difference equation

$$y(t+1) - y(t) = \sin(t+1) - \sin(t)$$

with $y(0) = 0$.The exact solution of this problem is $y = \sin(t)$. Using the procedure in Section 3 and taking N=7,8 and 9 the matrices in Eq.(22) are computed. Hence linear algebraic system is gained. This system is approximately solved using the Maple9.

We display a plot of absolute difference exact and approximate solutions in Fig.3 and error functions for various N is shown in Fig.4. The solution of the linear delay difference equation is obtained for N=7,8,9. The difference between the respective solutions is of the order of 10^{-5} and the accuracy increases as the N is increased. For numerical results, see Table 3.

Table2
Error analysis of Example 2 for the t value

t	Exact Solution	Present Method N=7	N_e=7	N=8	N_e=8	N=9	N_e=9
0.0	0.000000	0.200E-6	0.20000E-6	0.530E-6	0.53000E-6	0.122E-6	0.12200E-6
0.1	0.099833	0.099834	0.15100E-5	0.099832	0.12300E-5	0.099833	0.35000E-6
0.2	0.198669	0.198675	0.57000E-5	0.198669	0.10000E-6	0.198672	0.30000E-5
0.3	0.295520	0.295530	0.10400E-4	0.295523	0.37000E-5	0.295528	0.80000E-5
0.4	0.389418	0.389432	0.13700E-4	0.389426	0.83000E-5	0.389431	0.12800E-4
0.5	0.479425	0.479439	0.14400E-4	0.479436	0.11400E-4	0.479440	0.15200E-4
0.6	0.564642	0.564655	0.12500E-4	0.564654	0.12100E-4	0.564657	0.14900E-4
0.7	0.644217	0.644226	0.86000E-5	0.644227	0.10200E-4	0.644229	0.12100E-4
0.8	0.717356	0.717360	0.47000E-5	0.717362	0.64000E-5	0.717363	0.75000E-5
0.9	0.783326	0.783328	0.15000E-5	0.783329	0.21000E-5	0.783330	0.34000E-5
1.0	0.841471	0.841471	0.20000E-6	0.841469	0.17000E-5	0.841471	0.00000000

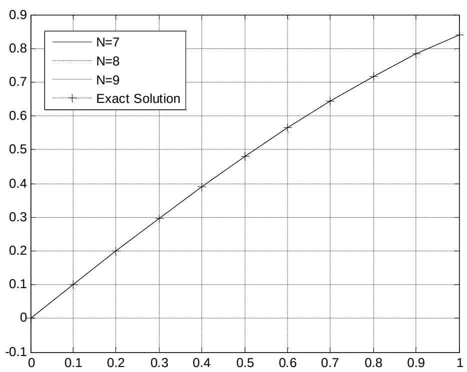

Fig.3.Numerical and exact solution of the Example2 for N=7,8,9

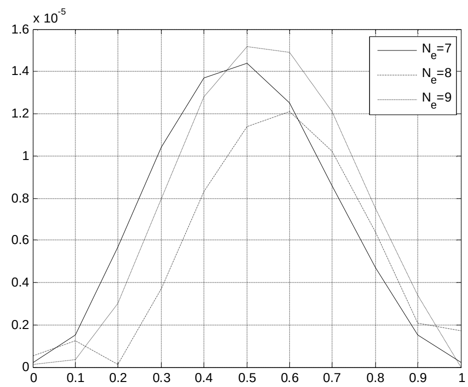

Fig.4.Error function of Example2 for various N.

Example3.
Consider another linear delay difference equation

$$(t+3)y(t+2)-2(t+2)y(t+1)+(t+1)y(t)=0$$

We follow the same procedure as in Section 3 to find the solution of delay difference equation with the conditions $y(0)=0, y(1)=1/2$. The exact solution of the problem is given by $y=\dfrac{t}{t+1}$. For numerical results, see Table 3. We display a plot of absolute difference exact and approximate solution in Fig.5 and error functions for various N is shown in Fig.6. This plot clearly indicates that when we increase truncation limit N, we have less error.

Table3
Error analysis of Example 3 for the t value

t	Exact	Present Method					
	Solution	N=7	N_e=7	N=8	N_e=8	N=9	N_e=9
0.0	0.000000	0.500E-6	0.50000E-6	0.320E-5	0.32000E-5	0.100E-6	0.10000E-6
0.1	0.090909	0.912E-1	0.32807E-3	0.915E-1	0.61928E-3	0.914E-1	0.55292E-3
0.2	0.166666	0.167959	0.12928E-2	0.166427	0.23891E-3	0.167577	0.91094E-3
0.3	0.230769	0.232833	0.20638E-2	0.229189	0.15797E-2	0.231696	0.92713E-3
0.4	0.285714	0.288091	0.23775E-2	0.282990	0.27238E-2	0.286408	0.69451E-3
0.5	0.333333	0.335585	0.22523E-2	0.330013	0.33201E-2	0.333696	0.36362E-3
0.6	0.375000	0.376825	0.18252E-2	0.371714	0.32855E-2	0.375061	0.61970E-4
0.7	0.411764	0.413026	0.12621E-2	0.409043	0.27215E-2	0.411631	0.13306E-3
0.8	0.444444	0.445153	0.70886E-3	0.442613	0.18312E-2	0.444249	0.19514E-3
0.9	0.473684	0.473953	0.26909E-3	0.472835	0.84832E-3	0.473546	0.13786E-3
1.0	0.500000	0.499998	0.15700E-5	0.500017	0.17300E-4	0.500000	0.69000E-6

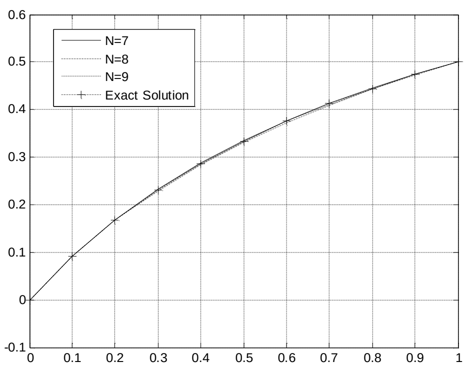

Fig.5.Numerical and exact solution of the Example3 for N=7,8,9

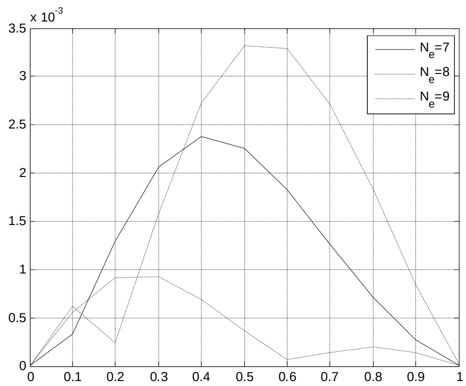

Fig.6.Error function of Example3 for various N.

Example4.

We consider linear delay difference equation to demonstrate that the Laguerre polynomials are powerful to approximate the solution to desired accuracy. The equation we consider is

$$(t+2)y(t+1)-3ty(t)=(t+2)e^{(t+1)}-3te^{t}$$

with the conditions $y(0)=1$. We again use Laguerre polynomials to approximate the solution of problem and compare it with the exact solution given by $y(t)=e^{t}$ and following the procedure given in Section 3. The comparision of the solutions given above with the exact solution of the problem is given in Table4. We plot the approximate solutions by this method and the exact solution in Fig.7 and the error functions in Fig.8.

Table4
Error analysis of Example 4 for the t value

t	Exact Solution	Present Method N=7	N_e=7	N=8	N_e=8	N=9	N_e=9
0.0	1.000000	1.000002	0.20000E-5	0.999999	0.11000E-5	1.000005	0.50000E-5
0.1	1.105171	1.104466	0.70500E-3	1.105296	0.12500E-3	1.105514	0.34300E-3
0.2	1.221403	1.220270	0.11330E-2	1.221315	0.88000E-3	1.221284	0.11900E-3
0.3	1.349859	1.348573	0.12860E-2	1.349479	0.38000E-3	1.349102	0.75700E-3
0.4	1.491825	1.490613	0.12120E-2	1.491226	0.59900E-3	1.490590	0.12350E-2
0.5	1.648721	1.647740	0.98100E-3	1.648035	0.68600E-3	1.647304	0.14170E-2
0.6	1.822119	1.821443	0.67600E-3	1.821483	0.63600E-3	1.820809	0.13100E-2
0.7	2.013753	2.013379	0.37400E-3	2.013263	0.49000E-3	2.012755	0.99800E-3
0.8	2.225541	2.225404	0.13700E-3	2.225241	0.30000E-3	2.224941	0.60000E-3
0.9	2.459603	2.459595	0.80000E-5	2.459581	0.12200E-3	2.459366	0.23700E-3
1.0	2.718282	2.718283	0.10000E-5	2.718280	0.20000E-5	2.718282	0.00000000

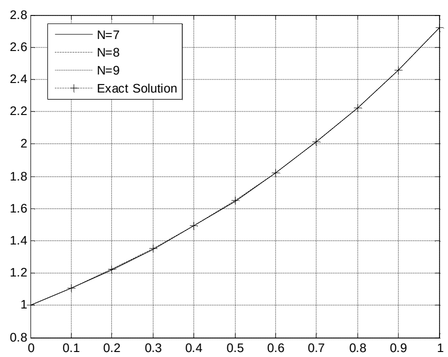

Fig.7.Numerical and exact solution of the Example4 for N=7,8,9

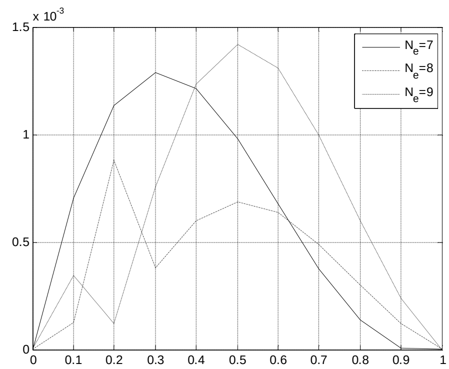

Fig.8.Error function of Example4 for various N.

Example5.
Let us find the Laguerre series solution of fourth order linear delay difference equation

$$y(t+4) - y(t+2) + y(t) = t + 2$$

with conditions

$$y(0) = 0, y(1/3) = 1/3, y(2/3) = 2/3, y(1) = 1$$

and the exact solution $y(t) = t$. Using the procedure in Section 4, we find the approximate solution of this equation which is the same with the exact solution.

5. CONCLUSION

In recent years, the studies of high order linear delay difference equation have attracted the attention of many mathematicians and physicists. The Laguerre collocation methods are used to solve the high order linear delay difference equation numerically. A considerable advantage of the method is that the Laguerre polynomial coefficients of the solution are found very easily by using computer programs. Shorter computation time and lower operation count results in reduction of cumulative truncation errors and improvement of overall accuracy. For this reason, this process is much faster than the other methods. Illustrative examples are included to demonstrate the validity and applicability of the technique, and performed on the computer using a program written in Maple9. To get the best approximating solution of the equation, we take more forms from the Laguerre expansion of functions, that is , the truncation limit N must be chosen large enough. In addition, an interesting feature of this method is to find the analytical solutions if the equation has an exact solution that is a polynomial functions. Illustrative examples with the satisfactory results are used to demonstrate the application of this method. Suggested approximations make this method very attractive and contributed to the good agreement between approximate and exact values in the numerical example.

As a result, the power of the employed method is confirmed. We assured the correctness of the obtained solutions by putting them back into the original equation with the aid of Maple, it provides an extra measure of confidence in the results. We predict that the Laguerre expansion method will be a promising method for investigating exact analytic solutions to linear delay difference equations.The method can also be extended to the system of linear delay difference equations with variable coefficients, but some modifications are required.

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