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ELASTO-PLASTIC STRESS ANALYSIS IN SILICON CARBIDE FIBER REINFORCED MAGNESIUM METAL MATRIX COMPOSITE WITH A SQUARE HOLE

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abstract: In the present study an elastic-plastic stress analysis is carried out for long silicon carbide fiber reinforced magnesium metal matrix composite with a square hole by using finite element technique. Composite material is manufactured by using molds under a pressure of 35 MPa and heating up to 550 ^{0}C by electrical resistance. The laminated plates with square hole consist of four layers and bonded symmetrically. Each layer with constant thickness is meshed into 48 elements and 228 nodes with simply supported. The expansion of plastic zone and residual stress are determined in the symmetric cross-ply and angle-ply laminated plates for small deformations. The load is increased by 0.01 MPa at each step . Iteration numbers are chosen as 150, 300 and 600. The resılts show that the intensity of the residual stresses is maximum near the open square hole and the yield points in symmetric laminates are higher than those in antisymmetric laminates.The plastic regions at the plate edges expand in the direction of the fiber, but at the border of the hole expand toward the diagonal of the hole.

Key words: Elasto-plastic stress, residual stress, metal matrix plates, finite element analysis.

1.INTRODUCTION

Metal-matrix composites consist of several layers of unidirectionally reinforced fibrous composite laminate which have different in-plane orientations. When the matrix that contains the fibers is made of a soft metal such as magnesium, the laminated plate may experience plastic deformation in the course of loading. The reinforcing fibers usually have high stiffness and strength and are elastic until failure. Plastic straining in metal-matrix laminates may start at relatively low applied load levels and it may affect the mechanical response in a large part in the strength range of the composite material.
Metal-matrix composites provide a high ratio of stiffness to weight, strength, thermal Properties wear resistance. Therefore in recent years, metal-matrix composites have been developed rapidly. Plastic deformations and residual stresses occur in laminated plates when the yield point of the laminate is exceeded. These residual stresses can be used to raise the yield points of the composite plates. Therefore, the investigation of plastic deformations and residual stresses is very important in composite plates. Many studies on stability of elastic, isotropic plates have been made, but very little work has been done on the anisotropic plates with holes. Vanden Brink and Kamot [1] presented a finite element analysis for the isotropic and laminated composite square plates with circular holes. Applying finite element and perturbation methods, Larsson [2] investigated buckling

of orthotropic compressed plates with circular holes. Using the finite element method for computing in-plane stress, Uenaya and Rewood [3] determined the shear buckling of square plates with holes by the aid of the Rayleigh-Ritz method.Chou et al. [4] reviewed the work on fibre-reinforced metal-matrix composites involving fabrication methods, mechanical properties, secondary working techniques and interfaces. Ananth et al. [5] investigated the application of push out test to characterize the mechanical behavior of interfaces in metallic and intermetallic matrix composites by using the finite element method and also studied the effect of different material and testing variables on the experimental data. Pitchumani et al. [6] used an eddy current technique for the measurement of constituent volume fractions in a three phase metal-matrix composite. The volume fractions were calculated using eddy current measurements made on a wide range of silicon carbide particulate reinforced aluminum matrix composite extrusions.Pomies and Carlsson [7] carried out micromechanical finite element analysis of rigidity and strength of transverse tension loaded continous fiber composites and compared their results with experimental data for dry and wet glass-epoxy, carbon-epoxy composites. Aktaş [8] carried out an elastic-plastic stress analysis aluminum metal – matrix laminated plates with square hole. Zocher [9] presented numerically generated failure envolopes for several three-dimesional failure criteria and compared them with experimental data. Lin and Kuo [10] studied the effect on critical load of materials of epoxy-glass and Boron-epoxy and some comparsons are also made with those in the literature. Sayman et al. [11] investigated elasto-plastic stress analysis of aluminum metal matrix composite laminated plates under in-plane loading. In this study, a silicon carbide fiber reinforced magnesium metal-matrix symmetric laminated composite plates having square holes are loaded. Loading is gradually increased from the yield point of the plate by 0.01 MPa at each load step. The first-order shear deformation theory is employed in the mathematical formulation and the finite element technique is used . Load step numbers are chosen as 150, 300 and 600. Results are given in the tables and figures.

2. MATHEMATICAL FORMULATION

The metal-matrix laminated plate of constant thickness of 2mm. is composed of orthotropic layers bonded symmetrically about the middle surface.
Composite plate geometry and loading configuration is illustrated in Figure 1.The solution of laminated plate elements includes transverse shear deformations need for the solution of advanced composites with high modulus ratios. Therefore, the constitutive relations for an orthotropic layer can be written as,

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{11}} & \overline{Q_{12}} & \overline{Q_{16}} \\ \overline{Q_{21}} & \overline{Q_{22}} & \overline{Q_{26}} \\ \overline{Q_{16}} & \overline{Q_{26}} & \overline{Q_{66}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} \overline{Q_{44}} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q_{55}} \end{bmatrix} \begin{Bmatrix} \gamma_{YZ} \\ \gamma_{XZ} \end{Bmatrix}$$

Where the transformed reduced stifness, Q_{ij} are given in terms of engineering constants of the material. According to first-order shear deformation theory, the particles of the plate, originally on a line that is normal to the undeformed middle surface, remain on a straight line during deformation, but this line is not necessarily normal to the deformed middle surface. Thus, the displacement components of a point of coordinates x, y, z for small deformation can be written as follows,

$$\begin{aligned} u(x,y,z) &= u_O(x,y) + Z\Psi_x(x,y) \\ v(x,y,z) &= v_O(x,y) + Z\,\Psi_y(x,y) \\ w(x,y,z) &= w(x,y) \end{aligned} \quad (2)$$

where u_O, v_O and w are the displacements of a point on the middle surface, and Ψ_x, Ψ_y are the rotation angles of normals to the y and x axes, respectively. By using linear strain-displacement relations, bending strains are found to vary linearly through the plate thickness, whereas shear strains are assumed to be constant throughtout the thickness as,

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_Y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix} + \begin{Bmatrix} \dfrac{\partial \psi_x}{\partial x} \\ -\dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} - \dfrac{\partial \psi_y}{\partial x} \end{Bmatrix} \quad (3)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{vmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} + \psi_x \end{vmatrix}$$

In order to obtain the element equilibrium equations, the total energy of a laminated plate under static loading is given as follows:

$$\Pi = U_b + U_s + V \quad (4)$$

Where U_b is the strain energy of bending, U_s is the strain energy of shear and V denotes the potential energy of external in-plane loadings (N_1, N_2, N_{12}).They are defined as,

$$\Pi = \frac{1}{2}\int_{-h/2}^{h/2} [\int_A (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})\, dA\,]\, dz + \frac{1}{2} \quad (5)$$

$$\Pi \int_{-h/2}^{h/2} [\int_A (\tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz})\, dA\,]\, dz - \int_A Wp\, dA - \int_{\partial R} (N_n^b u_n^0 + N_s^b u_s^0) ds$$

Where dA = dxdy and R is the region of a rectangle excluding hole, and N_n^b, N_s^b are the in-plane loads applied on the boundary ∂R.The resultant forces N_x ,N_y, N_{xy}, are not constant but are functions of x and y.The forces (N_x ,N_y, N_{xy}), moments M_x, M_y and M_{xy} and shearing forces Q_x and Q_y per unit length of the cross section of the laminated plate are given as,

$$\begin{vmatrix} N_x & M_x \\ N_y , & M_y \\ N_{xy} & M_{xy} \end{vmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_Y \\ \tau_{xy} \end{Bmatrix} (1, z)\, dz \qquad (6)$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz$$

For equilibrium, the potential energy Π must be stationary. It is obtained so that $\delta\Pi = 0$
which may be regarded as the principle of virtual displacement for the plate element [10].

Table 1. The mechanical properties of a composite layer.

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}	Axial strength, X (MPa)	Transverse Strength, Y (MPa)	Shear strength, S (MPa)
82	76	34	0.30	217.8	25.7	46.3

Table 2. The yielding loads of the symmetric composite plates.

	$[0° / 90°]_2$	$[30° / -30°]_2$
S (MPa)	0.0685	0.0636

Table 3. Residual stresses for symmetric cross-ply,($[0° / 90°]_2$), at the corners of the square hole for 150, 300 and 600 iterations.

Iteration Numbers	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)
150	0.151	-1.231	-0.227	-0.005	-0.021
300	0.044	-4.861	-0.612	-0.041	-0.098
600	0.463	-9.012	-1.124	-0.062	-0.154

Table 4. Residual stresses in the symmetric angle-ply laminated plates at the corners of the square hole for 600 iterations.

corners	σ_x (MPa)	σ_y (MPa)	τ_{xy} (MPa)	τ_{yz} (MPa)	τ_{xz} (MPa)	
$[30° / -30°]_2$ 0.275	B,C	-7.576	-20.154	6.186	0.401	-
0.071	A,D	0.573	0.4164	0.504	0.039	

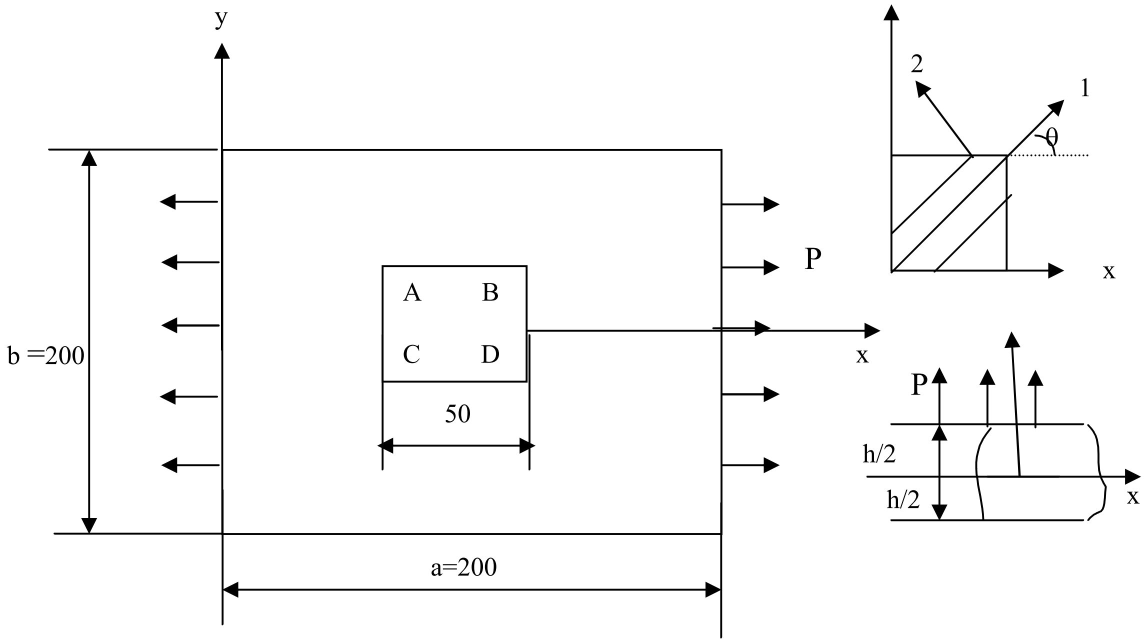

Figure 1.Notch geometry and loading configuration.

3. FINITE ELEMENT MODEL

The symmetric laminated plate is composed of four layers. Nine node square finite elements were used in the laminated plates to obtain the yield points of the laminated composite plates, the expansion of plastic regions, and residual stresses. The plate is meshed to 48 elements and 228 nodes with clamped boundary conditions .The stiffness matrix of the plate is obtained using the minimum potential energy principle.Bending, shear and geometric stiffness matrix are,

$$\left| K_b \right| = \int_A \left| B_b \right|^T \left| D_b \right| \left| B_b \right| dA \tag{7}$$

$$\left| K_s \right| = \int_A \left| B_s \right|^T \left| D_s \right| \left| B_s \right| dA$$

where,

$$\left| D_b \right| = \begin{vmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{vmatrix} \tag{8}$$

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij} \, (1, z, z^2) \, dz ,$$

$(i, j = 1, 2, 6)$

$$(A_{44}, A_{55}) = \int_{-h/2}^{h/2} (Q_{44,} Q_{55}) \, dz$$

D_b, D_s and are the denoted stiffness matrix, bending coupling matrix and ending Stiffness matrix respectively. The term A_{45} has been neglected. Because , A_{45} is negligible in comparison with A_{44} and A_{55} . The shear correction factors are given by,

$k_1^2 = k_2^2 = 5/6$ [10].

In this analysis , the external forces are applied transversely. Since, the calculated stresses do not coincide with the true stresses in a nonlinear problem, the unbalanced nodal forces and the equivalent nodal forces must be calculated. The equivalent nodal point forces correspond to the element stresses at each iteration can be calculated as follows,

$$\{R\}_{unbalanced} = \{R\}_{applied} - \{R\}_{equivalent} \qquad (9)$$

These unbalanced nodal forces are applied for obtaining increments in the solution and must satisfy the convergence tolerance in a nonlinear analysis. Residual stresses are very important in failure analysis of metal matrix laminated plates. When the yield point of the laminate is exceeded, the residual stresses occur in laminate plates. The obtained residual stresses can be used to raise the yield strength of the composite plates. In this study the Tsai-Hill theory is used as a yield criterion [8]. Four layers have been bonded to form a laminated plate symmetrically by using a pressure of 30 MPa and heating up to 550 ^{0}C.The stress-strain relation in plastic region is given as

$$\sigma = \sigma_0 + k\,\varepsilon^n_p \qquad (10)$$

In this solution, 228 nodes and 48 nine node plate elements are used.

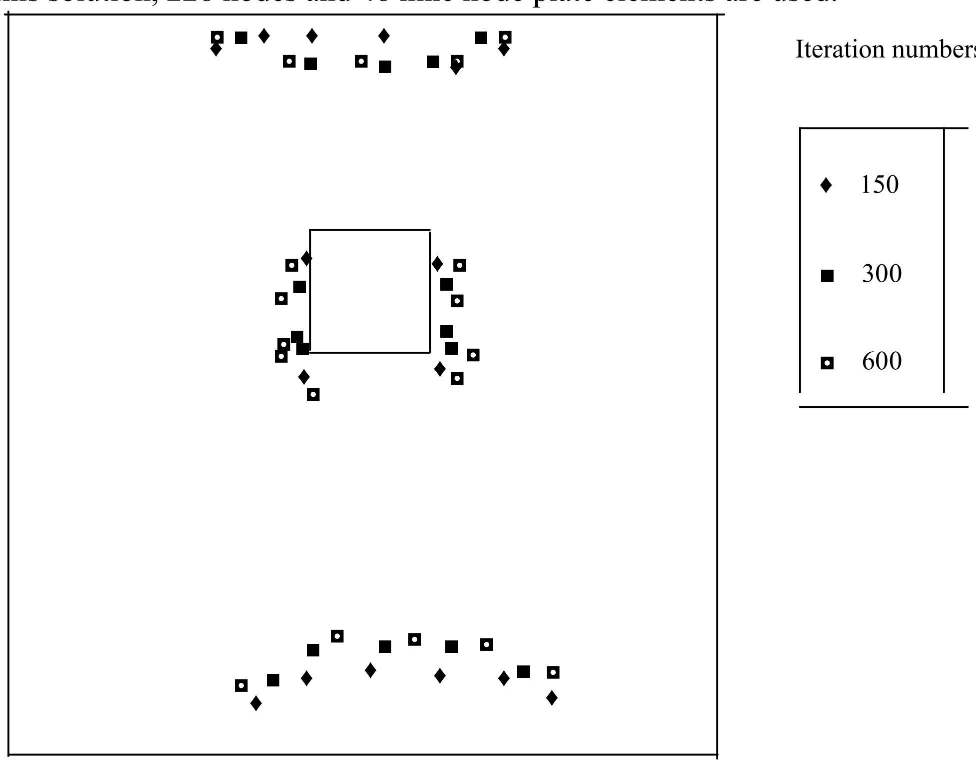

Figure 2. The expansion of the plastic zones on the upper surface for the symmetric Cross-ply, $[0° / 90°]_2$ composite laminated plate.

4. NUMERICAL RESULTS AND DISCUSSIONS

The metal-matrix composite plate with square hole is assumed to be under uniform axial in-plane loads along the square edges and the square hole is unloaded. The composite plates are composed of four orthotropic and generally orthotropic layers bonded symmetric form. Loading is gradually increased up to plastic zone which is not allowed to be very large. In the iterative solution, the overall stiffness matrix of the laminated plate is the same at each loading step. The in-plane load is increased by 0.01 MPa per step. Load steps numbers are chosen as 150, 300, and 600. The laminated plates are formed as cross-ply laminated plate, $[0° / 90°]_2$ and angle-ply laminated plates, $[30° / -30°]_2$, . The dimensions of the laminated plates are 200 x 200 mm^2. There is a square hole with a 50 x 50 mm^2 dimensions at the middle of the plate. The transverse load at the yield points of the symmetric laminated plates are given in table 2. It can be seen from the table that the transverse loads at yield points reach their maximum value at the configuration of the laminated plates .

The results of residual stresses for symmetric cross-ply, $[0° / 90°]_2$, laminated plate with a square hole for 150,300, and 600 iteration numbers are presented in Table 3. The stress values at the corner of the square hole are of the same for cross-ply due to symmetry of the geometry of the composite plate. It can be seen from Table 3. that the effect of the residual stress component for 600 iterations are greater than that of other iteration numbers. For that reason, the residual stress components of symmetric is given for 600 iteration numbers in Table 4 . It can be seen from these tables that the left side corners of the hole (upper and lower) are of the same residual stress components with the right side corners of the hole respectively due to symmetry of the geometry of the composite plate.The expansion of the plastic region for the symmetric cross-ply, $[0° / 90°]_2$, is shown in Figure 2. As seen from this figure, the plastic region start at the hole edges perpendicular to the x-axis, then the regions expand in the directions of the fibers.The expansion of the plastic region for the symmetric clamped angle-ply, $[30° / -30°]_2$, Composite plate is shown in Figure 3. In this case, the plastic regions start at the plate edge parallel to the x-axis and around the corner of the square hole. And then , (similar before case) at plate edge the plastic regions expand in the fiber direction. The expansion of the plastic region on the upper surface for the symmetric angle-ply, laminated plate is given in Figure 4. It can be seen from the figure, the plastic regions start at the midpoint of the edges and the corner of the square hole and then the plastic regions expand in the direction of the fiber.They are seen from the figures that the expand of the plastic zones are symmetric to each other for the upper and lower surfaces.

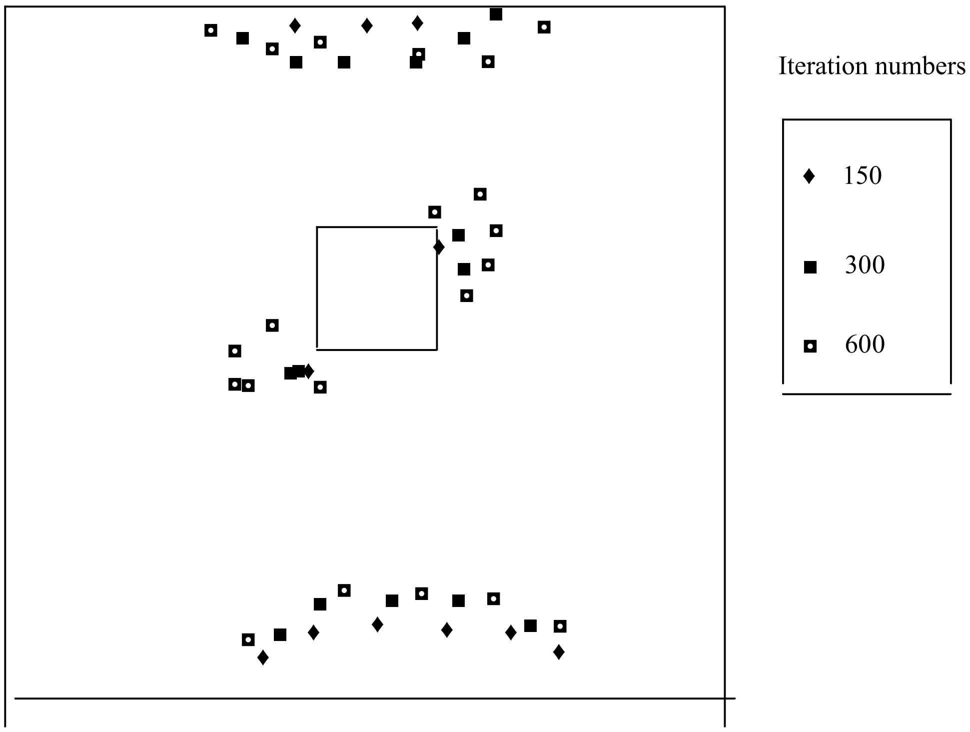

Figure 3. The expansion of the plastic zones on the upper surface for the symmetric angle-ply [30° / -30°]$_2$, composite laminated plate.

5. PRODUCTION OF LAMINATED PLATE

The composite layer consists of silicon carbide fibers and an magnesium matrix. For the production of the composite layer, moulds heated by electrical resistance are employed. The moulds and materials are insulated by glass-fibers. The thickness of each mould is 1 mm, thus the thickness of each layer is obtained as 2 mm. A pressure of 35 MPa is applied to the upper mould by a hydraulic press at 550 °C. Under these conditions, the yield strength of magnesium is exceeded and a good bonding between the matrix and fiber is realized. The layer is loaded in principal material directions by lnstron tensile machine; thus, the yield points in principal material directions and shear are found experimentally. Mechanical properties of the layer are obtained experimentally by using strain gauges. The laminated plate is manufactured by using two layers or more than two layers. In this process again, a pressure of 35 MPa is applied to the upper moulds and all the moulds and layers are heated. Mechanical properties and yield points of a layer is given in table 1.

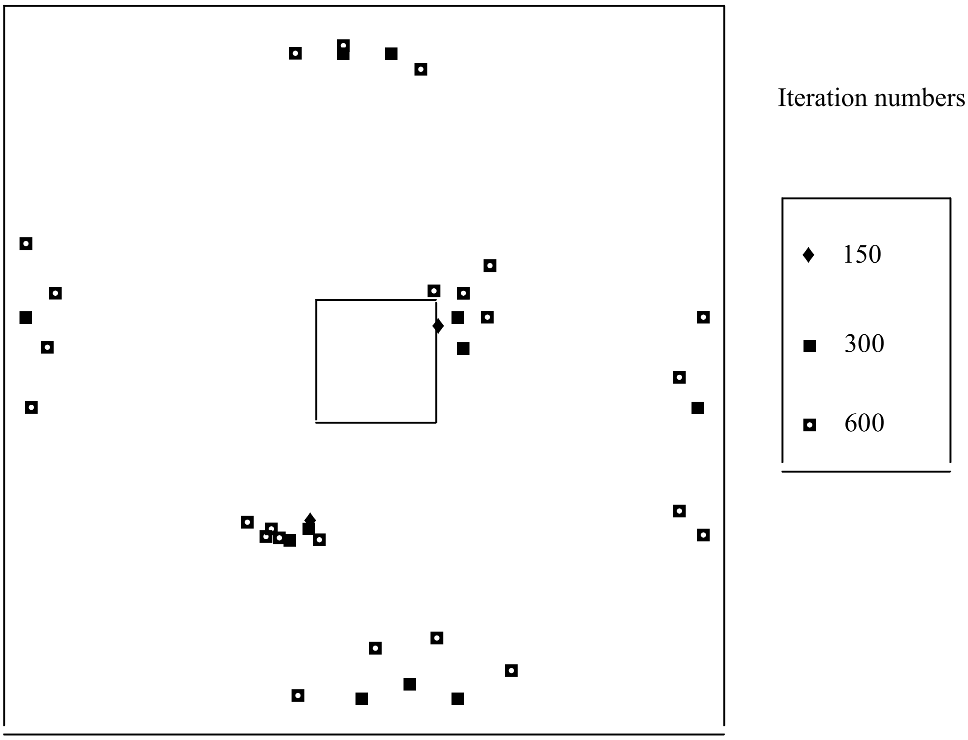

Figure 4. The expansion of the plastic zones on the upper surface for the symmetric angle-ply $[45° / -45°]_2$, composite laminated plate.

6. CONCLUSIONS

The following results of elastic-plastic analysis of the silicon carbide fiber reinforced magnesium metal-matrix composite laminates with square hole are obtained.

1. Metal fiber reinforced metal matrix composite plate gives the higher improved mechanical properties.
2. The expansion of the plastic zone and residual stress components are the same at the upper and lower surface of the symmetric composite laminated plates.
3. The yield points in symmetric laminated plates are higher than those in antisymmetric laminated plates.
4. The expands of the plastic zones are symmetric to each other for the upper and lower surfaces.
5 .The values of residual stresses are highest at the corner of the square hole due to stress concentration.
6.The strength of the composite plates can be increased by residual stresses.

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