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VIBRATION ANALYSIS OF ASYMMETRIC-PLAN FRAME BUILDINGS USING TRANSFER MATRIX METHOD

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Abstract- A method for vibration analysis of proportional asymmetric plan frame buildings is presented in this paper. The whole structure is idealized as an equivalent shear-torsion beam in this method. The governing differential equations of equivalent shear-torsion beam are formulated using continuum approach and posed in the form of simple storey transfer matrix. By using the storey transfer matrices and point transfer matrices which consider the inertial forces, system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. The structural properties of building may change in the proposed method. A numerical example has been solved at the end of study by a program written in MATLAB to verify the presented method. The results of this example display the agreement between the proposed method and the other valid method given in literature.

Key Words- Vibration, Asymmetric, Frame, Transfer matrix.

1.INTRODUCTION

Number of methods, such as finite element method, has been developed for analyses of buildings. The continuum model is very simple and efficient method used in static and dynamic analysis of buildings. There are numerous studies [1-12] on asymmetric structures in the literature regarding continuum method. Kuang and Ng [4] considered the problem of doubly asymmetric structures; in which the motion is dominated by shear walls. For the analysis, the structure was replaced by an equivalent uniform cantilever whose deformation was coupled in flexure and warping torsion. Rafezy and Howson [8] proposed a global approach to the calculation of natural frequencies of doubly asymmetric, three dimensional, multi bay, and multi storey frame structures. It was assumed that the primary frames of the original structure ran in two original directions and that their properties may have varied in a step-wise fashion at one or more storey levels. The structure therefore divided naturally into uniform segments according to changes in section properties. A typical segment was then replaced by an equivalent shear- torsion coupled beam; whose governing differential equations were formulated by using continuum approach and posed in the form of a dynamic member stiffness matrix. Kuang and Ng [11] derived the governing equation and the corresponding eigenvalue problem of asymmetric frame structures using continuum assumption. A theoretical method of solution was proposed and a general solution to the eigenvalue equation of the problem was presented for determining the coupled natural frequencies and associated mode shapes based on the theory of differential equations. Step changes of properties along the height of the structure were not allowed in any of the studies with the exception of Rafezy and Howson's. A method for vibration analysis of proportional asymmetric plan frame structures is suggested in this study. The following assumptions are made in this study; the behavior of the material is linear elastic, small displacement theory is valid, P-delta effects are negligible, the shear center and geometric center at each floor is assumed to lie on a vertical line through the height of structures, the axial deformations of columns and beams are negligible, the storey mass acts on the storey (floor) level, the frames are orthogonal and the floor system is rigid in its plane.

2.ANALYSIS

2.1. Physical Model

Figure 1 shows a typical floor plan of asymmetric, three dimensional frame structures [11]. Frame structures ignoring axial deformations, demonstrate Shear torsional beam behavior.

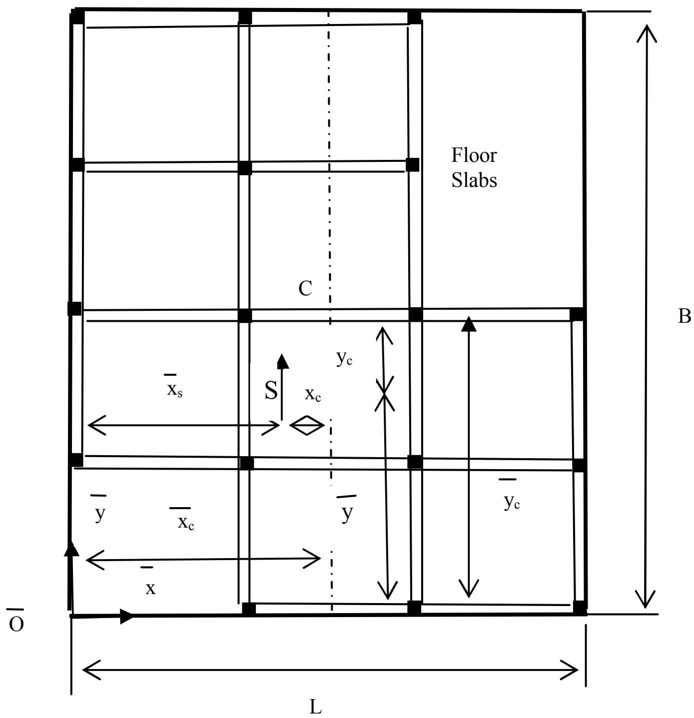

Figure 1 Typical flor plan of asymmetric frame structures [11]

2.2. Storey Transfer Matrices

Under the lateral loads which acting on the storey levels equations of i.th storey can be written as ,

$$(GA)_{xi}\frac{d^2u_i}{dz_i^2}=0 \qquad (GA)_{yi}\frac{d^2v_i}{dz_i^2}=0 \qquad (GJ)_i\frac{d^2\theta_i}{dz_i^2}=0 \qquad (1, 2, 3)$$

where u_i and v_i are the lateral deflections of the shear center, respectively, θ_i is the torsional rotation of the floor plan about shear center at the given height, and z_i is the vertical axis of each storey.

$(GA)_{xi}$ and $(GA)_{yi}$ are the equivalent shear rigidity of the storey for framework in x and y directions. For frame elements which consists of n columns and n-1 beams, GA can be calculated as follows [13,14];

$$(GA)_i=\frac{12E}{h_i[1/\sum_1^n I_c/h_i+1/\sum_1^{n-1} I_g/l)} \qquad (4)$$

where $\sum I_c/h_i$ represents the sum of moments of inertia of the columns per unit height in i.th storey of frame j, and $\sum I_g/l$ represents the sum of moments of inertia of each beam per unit span across one floor of frame j .

$(GJ)_i$ are the St. Venant torsion stiffness of i.th storey and can be calculates as follows [11];

$$(GJ)_i=\sum_j[(\bar{y}_j-\bar{y}_s)^2(GA)_{xj}+(\bar{x}_j-\bar{x}_s)^2(GA)_{yj}] \qquad (5)$$

where $\bar{y}_j$ and $\bar{x}_j$ are the coordinates at the location of the center of shear of the j-th bent at i-th storey in coordinate system ($\bar{y}_j$, $\bar{x}_j$).

$\bar{y}_s$ and $\bar{x}_s$ are the coordinates of shear center and can be calculated as follows [11];

$$\bar{y}_s=\frac{\sum_j \bar{y}_j(GA)_{xj}}{\sum_j (GA)_{xj}} \qquad \bar{x}_s=\frac{\sum_j \bar{x}_j(GA)_{yj}}{\sum_j (GA)_{yj}} \qquad (6, 7)$$

When equations (1) ,(2) and (3) are solved with respect to the z_i, $u_i(z_i)$ and $v_i(z_i)$ and $\theta_i(z_i)$ can be obtained as follows;

$$u_i(z_i)=c_1+c_2z_i \qquad v_i(z_i)=c_3+c_4z_i \qquad \theta_i(z_i)=c_5+c_6z_i \qquad (8, 9, 10)$$

where c_1, c_2, c_3, c_4, c_5, and c_6 are integral constants.

By using equations (8), (9) and (10), the shear force in x and y direction, and torsion moment can be obtained as follows;

$$V_{xi} = (GA)_{xi}\frac{du_i}{dz_i} = (GA)_{xi}c_2 \qquad V_{yi} = (GA)_{yi}\frac{dv_i}{dz_i} = (GA)_{yi}c_4 \qquad M_{ti} = (GJ)_i\frac{d\theta_i}{dz_i} = (GJ)_i c_6 \tag{11, 12, 13}$$

Equation (14) shows the matrix form of equations (8), (9), (10), (11), (12) and (13):

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ M_{ti}(z_i) \end{bmatrix} = \begin{bmatrix} 1 & z_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & z_i & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & z_i \\ 0 & (GA)_{xi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (GA)_{yi} & 0 & 0) \\ 0 & 0 & 0 & 0 & 0 & (GJ)_i \end{bmatrix} * \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{14}$$

At the initial point of the storey for $z_i=0$, equation (14) can be written as;

$$\begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ M_{ti}(0) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & (GA)_{xi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (GA)_{yi} & 0 & 0) \\ 0 & 0 & 0 & 0 & 0 & (GJ)_i \end{bmatrix} * \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{bmatrix} \tag{15}$$

The vector in right-hand side of equation (15) can be shown as follows;

$$c = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{bmatrix}^t \tag{16}$$

When vector c is solved by implementing equation (15) and substituted in equation (14), then equation (17) would be obtained.

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ M_{ti}(z_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & z_i/(GA)_{xi} & 0 & 0 \\ 0 & 1 & 0 & 0 & z_i/(GA)_{yi} & 0 \\ 0 & 0 & 1 & 0 & 0 & z_i/(GJ)_i \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ M_{ti}(0) \end{bmatrix} \tag{17}$$

$$\begin{bmatrix} u_i(z_i) \\ v_i(z_i) \\ \theta_i(z_i) \\ V_{xi}(z_i) \\ V_{yi}(z_i) \\ M_{ti}(z_i) \end{bmatrix} = T_i * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ M_{ti}(0) \end{bmatrix} \tag{18}$$

T_i represents the storey transfer matrix for $z=h_i$ in equation (18).

3. DYNAMIC ANALYSIS

The storey transfer matrices obtained from equation (17) can be used for the dynamic analysis of asymmetric- plane frame. Therefore, when considering the inertial forces in the storey levels, the relationship between the ith and the (i+1)th stories can be shown by the following matrix equation;

$$\begin{bmatrix} u_i(h_i) \\ v_i(h_i) \\ \theta_i(h_i) \\ V_{xi}(h_i) \\ V_{yi}(h_i) \\ M_{ti}(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\omega^2 * m(j) & 0 & \omega^2 * m(j) * y_c & 1 & 0 & 0 \\ 0 & -\omega^2 * m(j) & -\omega^2 * m(j) * x_c & 0 & 1 & 0 \\ \omega^2 * m(j) * y_c & -\omega^2 * m(j) * x_c & -\omega^2 * m(j) * r_m^2 & 0 & 0 & 1 \end{bmatrix} * T_i * \begin{bmatrix} u_i(0) \\ v_i(0) \\ \theta_i(0) \\ V_{xi}(0) \\ V_{yi}(0) \\ M_{ti}(0) \end{bmatrix} \tag{19}$$

where, m_i is the mass of the ith storey and ω are the natural frequencies of the system and r_m^2 is the inertial radius of gyration; and can be calculated as [8,11];

$$r_m^2 = \frac{L^2 + B^2}{12} + y_c^2 + x_c^2 \tag{20}$$

y_c and x_c are the dimensions of the location of the geometric center in the coordinate system xSy and can be calculated as follows [8,11];

$$y_c = \bar{y}_C - \bar{y}_S \qquad\qquad x_c = \bar{x}_C - \bar{x}_S \tag{21, 22}$$

where the coordinate ($\bar{y}_C$, $\bar{x}_c$) is the location of the geometric center C in the coordinate system ($\bar{y}$, $\bar{x}$).

Dynamic transfer matrix can be shown as T_{di} .

$$T_{di}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -\omega^2 * m(j) & 0 & \omega^2 * m(j) * y_c & 1 & 0 & 0 \\ 0 & -\omega^2 * m(j) & -\omega^2 * m(j) * x_c & 0 & 1 & 0 \\ \omega^2 * m(j) * y_c & -\omega^2 * m(j) * x_c & -\omega^2 * m(j) * r_m^2 & 0 & 0 & 1 \end{bmatrix} * T_i \tag{23}$$

The displacements - internal forces relationship between the base and the top of the structure can be found as follows;

$$\begin{bmatrix} u_{top} \\ v_{top} \\ \theta_{top} \\ V_{xtop} \\ V_{ytop} \\ M_{ttop} \end{bmatrix} = T_{dn} * T_{d(n-1)} * \ldots T_{di} * \ldots\ldots T_{d2} * T_{d1} \begin{bmatrix} u_{base} \\ v_{base} \\ \theta_{base} \\ V_{xbase} \\ V_{ybase} \\ M_{tbase} \end{bmatrix} \tag{24}$$

The boundary conditions of the shear torsion beam are;

1) $u_{base}=0$ 2) $v_{base}=0$ 3) $\theta_{base}=0$ 4) $V_{xtop}=0$ 5) $V_{ytop}=0$ 6) $M_{ttop}=0$

When boundary conditions are considered for equation (24) for the nontrivial solution of $\mathbf{t_d} = \mathbf{T_{dn}} \mathbf{T_{dn-1}} \mathbf{T_{dn-2}} \ldots\ldots \mathbf{T_{d1}}$, equation (25) can be attained;

$$\mathrm{f} = \begin{bmatrix} t_{44} & t_{45} & t_{46} \\ t_{54} & t_{55} & t_{56} \\ t_{64} & t_{65} & t_{66} \end{bmatrix} \tag{25}$$

The values of ω, which set the determinant to zero, are natural frequencies of the asymmetric plan-frame building.

4. PROCEDURE OF COMPUTATION

A program that considers the method presented in this study as basis, has been prepared in Matlab and the operation stages are presented below:

1) The equivalent rigidities of each storey are calculated by using the geometric and material properties of the structure.
2) Transfer matrices are calculated for each storey by using equivalent rigidities.
3) System transfer matrix (equation 24) is obtained with the help of storey transfer matrices and inertia forces effecting to the storey levels with the procedure told in section 3.
4) The nontrivial equation is obtained by using equation (25) as a result of the application of the boundary conditions.

5) The angular frequencies and relevant periods are found with the help of a method obtained from numerical analysis.
6) The modes are found with the help of angular frequency and equation (19).
7) The effective mass ratio and participation factor is found by using the modes.
8) With the help of the acceleration and displacement spectrums, obtained from an earthquake record or design spectrum from codes, the displacement and internal forces are found by using effective mass and participation factor.

5. A NUMERICAL EXAMPLE

In this part of the study a numerical example was solved by a program written in MATLAB to validate the presented method. The results are compared with those given in the literature.

5.1. Example 1

A typical asymmetric frame system (Fig 1) is analyzed as an example. The general multi- bent is considered as asymmetric reinforced concrete frame building (Fig.1). The structure has 20 storeys with total height H=60 m, and floor dimensions L=18 m and B=24 m. The structural properties are given in Table 1. The natural frequencies calculated by this method are compared with the results in the reference [11]. The results are presented in Table 2, Figure 2, Figure 3 and Figure 4.

Table 1 Structural Properties of Asymmetric Frame Structures

Structural Properties of Asymmetric Frame Structures	
$(GA)_x$	274300 kN
$(GA)_y$	297100 kN
(GJ)	27972000 kNm^2
x_c	0.692 m
y_c	0.5 m
m	121.5 $kNsn^2/m$
r_m	8.702 m

Table 2 Comparison of natural frequencies in Example 1

Natural frequencies of the first three modes (s^{-1})									
	Proposed Method			Kuang and Ng [11]			ETABS [11]		
Mode	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3	ω_1	ω_2	ω_3
1	2.090	2.166	2.488	2.140	2.218	2.548	2.078	2.191	2.487
2	6.257	6.485	7.449	6.419	6.654	7..643	6.396	6.542	7.372
3	10.388	10.767	12.367	10.698	11.089	12.737	10.422	10.850	12.345

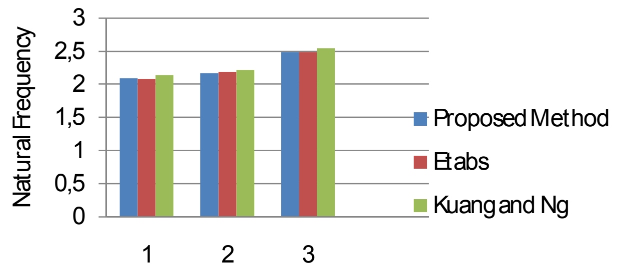

Figure 2 Comparison of Natural Frequencies of the First Mode

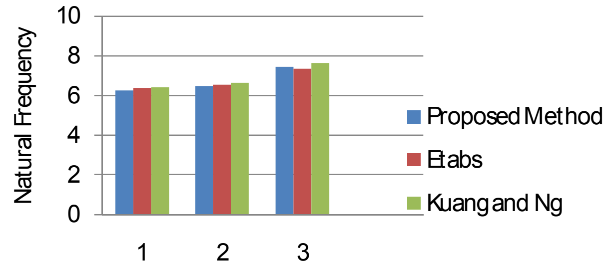

Figure 3 Comparison of Natural Frequencies of the Second Mode

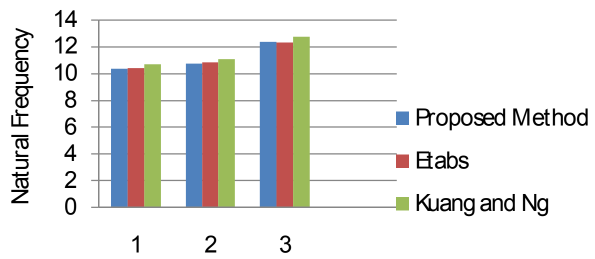

Figure 4 Comparison of Natural Frequencies of the Third Mode

As to investigate the accuracy of the method, natural frequencies which were calculated by using proposed method are compared with the finite element solutions. The differences (Error) are given in Table 3.

Table 3 Differences of natural frequencies of the proposed method and the finite element method (%)

	Differences of natural frequencies (%)		
Mode	1.	2.	3.
1	0.58	-1.14	0.04
2	-2.17	-0.87	1.04
3	-0.33	-0.76	0.18

The natural frequencies without considering the torsion are presented in Table 4. The main parameters which affect the torsion are the eccentricities (x_c and y_c). As the parameters are small, the differences between frequencies with torsion and without torsion are small.

Table 4 Natural frequencies without torsion (s^{-1})

	The frequencies without torsion	
Mode	ω_x	ω_y
1	2.102	2.187
2	6.292	6.548
3	10.446	10.87

6. CONCLUSIONS

This paper presents a method for vibration analysis of proportional asymmetric- plane frame buildings. The whole structure is idealized as an equivalent shear –torsion beam in this method. The governing differential equations of equivalent shear-torsion beam are formulated using continuum approach and posed in the form of simple storey transfer matrix. By using the storey transfer matrices and point transfer matrices which consider the inertial forces, system transfer matrix is obtained. Natural frequencies can be calculated by applying the boundary conditions. Example solved in this study shows that results obtained from the proposed method are in close agreement with the solution which was developed in literature. The error of the proposed method is shown to be less than 5%. The structural properties of building may change in the proposed method and different numerical examples can also be solved. The proposed method is simple and very accurate enough to be used both at the concept design stage and for final analyses.

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