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# EFFECTS OF JOINT REPLENISHMENT POLICY ON COMPANY COST UNDER PERMISSIBLE DELAY IN PAYMENTS

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**Abstract -** In today's severely competitive business environment, reducing replenishment costs has become one of the most important objectives for companies. This study deals with the replenishment problem under the condition of permissible delay in payments. To better reflect real-world business situations, we extend the traditional EOQ model by considering the situations of permissible delay in payments and multi-item replenishment. This study presents both single-item and joint multi-item replenishment models, and develop theorems to solve the problems. The objective of the single-item replenishment policy is to determine the optimal replenishment cycle time for each item while minimizing the total cost. The objective of the joint multi-item replenishment policy is to determine a common optimal replenishment cycle time for all items. Using computational analysis, we illustrate the solution procedures and draw conclusions. The results of this study can serve as a reference for business managers or administrators.
**Key Words -** EOQ, delay in payments, multi-item, joint replenishment

## 1. INTRODUCTION

In practice, suppliers often provide forward financing to retailers. In this situation, the supplier allows the retailer a credit period in which to settle the amount owed for goods already supplied. Since the publication of Goyal's [1] paper almost 25 years ago, over 50 papers have appeared in the literature dealing with variety of trade credit situations including pricing-dependent demand (Abad and Jaggi [2], Sheen and Tsao [3], Tsao and Sheen [4]), shortages allowed (Jamal *et al.* [5], Ouyang *et al.* [6]), partial backlogging and deterioration (Aggarwal and Jaggi [7], Hwang and Shinn [8]), and variable cost (Tsao and Sheen [9]) etc. These studies indicate that the issue of trade credit is a very popular field of research. It is essential to consider trade credit when formulating a decision-making model.

Joint multi-item replenishment strategies are already widely applied in the real world. Examples of this type of strategy include the supplying of parts for automotive assembly (Hahm and Yano [10]) and refrigerated goods to supermarkets (Hammer [11]). In the automotive industry, a supplier normally produces several different items for a single customer and puts together a combined shipment for that customer. In the grocery supply industry, different types of refrigerated goods (e.g., General Mills yogurt and Land O'Lakes butter) can be shipped in the same truck to the same supermarket (Hammer [11]). Other researches such as Goyal [12], Kao [13], Graves [14], Ben-Khedher and Yano [15], van Eijs [16], Rempala [17], and Chen and Chen [18] have proposed models and algorithms for solving multi-item replenishment problems for different situations.

However, the trade credit papers above only consider single-item problems, and ignore the effect of joint multi-item replenishment. In practice, trade credit and multi-item replenishment coexist. Therefore, none of these studies can be an appropriate reference. To address this problem, this paper formulates a model that combines the credit period with the joint multi-item replenishment policy. As a result, this is the first study to consider the joint replenishment problem in a trade credit situation. The objective of this study is to determine the optimal replenishment policy while still minimizing total cost. We present both single-item and joint multi-item replenishment models and develop theorems to solve these reploenishment problems. This study also compares, for the first time, the performance of these two policies under delay in payments. Using computational analysis, we illustrate the solution procedures and draw conclusions. Results show that the joint multi-item replenishment policy is better than the single-item replenishment policy. We also provide useful references for managerial decision-making and administration based on mathematical modeling.

This study uses the following notations. $I_P$ is the annual interest charged per dollar, $I_e$ is the annual interest earned per dollar, $M$ is the credit period, $T_i$ is the replenishment cycle time for item *i* in the single-item replenishment policy, $T$ is the replenishment cycle time in the joint replenishment policy, $A$ is the major ordering cost per order, $a_i$ is the minor ordering cost for item *i*, $p_i$ is the selling price per unit for item *i*, $c_i$ is the purchasing price per unit for item *i*, $d_i$ is the demand rate for item *i*, $h_i$ is the inventory holding cost per unit for item *I*, $k$ is the number of items, $\theta$ is the set of units whose replenishment cycle is longer or equal to the credit period, and $\phi$ is the set of units whose replenishment cycle is shorter than the credit period.

The mathematical model is developed under the following assumptions: 1. The demand rates for items are constant with time. 2. Replenishments occur instantaneously. 3. Shortages are not allowed. 4. The selling price is higher than the purchasing price. 5. The unit retail price of the products sold during the credit period is deposited in an interest bearing account with rate $I_e$. At the end of this period, the credit is settled and the retailer starts paying interest charges for the items in stock with rate $I_P$ ($I_P > I_e$).

## 2. THE SINGLE-ITEM REPLINISHMENT MODEL

In the single-item replenishment model, the total annual cost $TVC_1(T_i)$ has two different functions as follows:

$$TVC_{11}(T_i) = \sum_{i\in\theta}\left[\frac{A+a_i}{T_i} + \frac{h_i d_i T_i}{2} + \frac{c_i I_p d_i (T_i - M)^2}{2T_i} - \frac{p_i I_e d_i M^2}{2T_i}\right] \quad if \quad T_i \geq M, \tag{1}$$

$$TVC_{12}(T_i) = \sum_{i\in\phi}\left[\frac{A+a_i}{T_i} + \frac{h_i d_i T_i}{2} - p_i I_e d_i \left(M - \frac{T_i}{2}\right)\right] \quad if \quad T_i < M, \tag{2}$$

$$TVC_1(T_i) = TVC_{11}(T_i) + TVC_{12}(T_i), \text{ and } \theta + \phi = \{1,2,3,......,k\}. \tag{3}$$

**Case 1**: When $T_i \geq M$, in which *i* belongs to $\theta$, the first and second-order derivatives of $TVC_{11}(T_i)$ with respect to $T_i$ are

$$TVC'_{11}(T_i) = \frac{-\left[2A + 2a_i + d_i M^2 \left(c_i I_p - p_i I_e\right)\right]}{2T_i^2} + \frac{d_i\left(h_i T_i^2 + I_p c_i T_i^2\right)}{2T_i^2}, \tag{4}$$

$$TVC''_{11}(T_i) = \frac{2A + 2a_i + d_i M^2 \left(c_i I_p - p_i I_e\right)}{T_i^3}. \tag{5}$$

**Case 2**: When $T_i < M$, in which *i* belongs to $\phi$, the first and second-order derivatives of $TVC_{12}(T_i)$ with respect to $T_i$ are

$$TVC'_{12}(T_i) = \frac{d_i\left(h_i + p_i I_e\right)}{2} - \frac{\left(A + a_i\right)}{T_i^2}, \tag{6}$$

$$TVC''_{12}(T_i) = \frac{2\left(A + a_i\right)}{T_i^3} > 0. \tag{7}$$

Equation (7) implies that $TVC_{12}(T_i)$ is convex on $T_i > 0$. Let $\Delta_1 = 2A + 2a_i + d_i M^2 \left(c_i I_p - p_i I_e\right)$, then Equation (5) implies that $TVC_{11}(T_i)$ is convex on $T_i > 0$ if $\Delta_1 > 0$. Furthermore, Equations (1) and (2) imply that $TVC_1(T_i)$ is convex on $T_i > 0$ if $\Delta_1 > 0$. At $T_i = M$, we find $TVC_{11}(M) = TVC_{12}(M)$. Hence, $TVC_1(T_i)$ is continuous and well-defined.

To minimize $TVC_1(T_i)$, solve Equations $(6)=0$ to obtain the optimal replenishment cycle time for item *i* in $T_i < M$,

$$T_{12}^* = \sqrt{\frac{2(A+a_i)}{d_i(h_i + I_e p_i)}} , \tag{8}$$

the optimal order quantity for item *i* in $T_i < M$ is $Q(T_{12}^*) = T_{12}^* d_i = \sqrt{\frac{2d_i(A+a_i)}{h_i + I_e p_i}}$.

If $\Delta_1 > 0$, Equation $(5)$ implies that $TVC_{11}(T_i)$ is convex on $T_i > 0$. Solve Equation $(4)=0$ to obtain the optimal replenishment cycle time for item *i* in $T_i \ge M$,

$$T_{11}^* = \sqrt{\frac{2A + 2a_i + d_i M^2 (c_i I_p - p_i I_e)}{d_i(h_i + I_p c_i)}} , \tag{9}$$

and the optimal order quantity for item *i* in $T_i \ge M$ is $Q(T_{11}^*) = T_{11}^* d_i = \sqrt{\frac{d_i\left[2A + 2a_i + d_i M^2 (c_i I_p - p_i I_e)\right]}{h_i + I_p c_i}}$.

Based on the equations above, we derive and deduce Theorem 1 and Theorem 2 to determine the optimal replenishment cycle time for single-item replenishment when $\Delta_1 < 0$ and $\Delta_1 \ge 0$ respectively.

**Theorem 1:** If $\Delta_1 < 0$, then $TVC_1(T_i)$ has the minimum value $T_1^* = T_{12}^*$.

**Proof:** Because $TVC_{12}(T_i)$ is convex on $T_i > 0$ and $T_{12}^* < M$. Therefore, $TVC_{12}(T_i)$ is decreasing on $\left(0, T_{12}^*\right]$ and increasing on $\left[T_{12}^*, M\right]$. As a result, $TVC_{12}(T_i)$ has a minimum value at $T_{12}^*$ on $(0, M]$. On the other hand, if $\Delta_1 < 0$, Equation $(4)$ implies that $TVC_{11}'(T_i) \ge 0$ and $TVC_{11}(T_i)$ is increasing on $T_i > 0$. Thus, $TVC_{11}(T_i)$ is increasing on $[M, \infty)$. So $TVC_{11}(T_i)$ has a minimum value at $M$. From $TVC_{11}(M) = TVC_{12}(M)$, Equations $(1)$ and $(2)$ imply that $TVC_{11}(T_i)$ has the minimum value at $T_{12}^*$ on $T_i > 0$. Therefore, $T_1^* = T_{12}^*$. □

**Theorem 2:** If $\Delta_1 \ge 0$, let $\Delta_2 = 2A + 2a_i - d_i M^2 (h_i + p_i I_e)$, then

(a) When $\Delta_2 > 0$, the optimal replenishment cycle time is $T_1^* = T_{11}^*$.

(b) When $\Delta_2 < 0$, the optimal replenishment cycle time is $T_1^* = T_{12}^*$.

(c) When $\Delta_2 = 0$, the optimal replenishment interval is $T_1^* = T_{11}{}^* = T_{12}{}^* = M$.

**Proof:** (a) If $\Delta_2 > 0$, Equations (9) and (8) imply that $T_{11}^* > M$ and $T_{12}^* > M$. According to the convexities and the definitions of $TVC_{11}(T_i)$ for Case 1 and $TVC_{12}(T_i)$ for Case 2, we find that $TVC_{11}(T_i)$ is decreasing on $[M, T_{11}^*]$ and $TVC_{12}(T_i)$ is decreasing on $(0, M]$. This means that $TVC_{11}(T_i)$ has the minimum value at $T_{11}{}^*$ and $TVC_{12}(T_i)$ has the minimum value at $M$. Therefore, from $TVC_{12}(M) = TVC_{11}(M) \geq TVC_{11}(T_{11}^*)$, we know that $TVC_1(T_i)$ has the minimum value at $T_1^* = T_{12}{}^*$. The proofs in (b) and (c) are similar to that in (a). □

## 3. THE JOINT MULTI-ITEM REPLENISHMENT MODEL

In the joint multi-item replenishment model, the total annual cost $TVC_2(T_i)$ has two different functions as follows:

$$TVC_2(T) = \begin{cases} TVC_{21}(T) = \dfrac{A}{T} + \displaystyle\sum_{i=1}^{k}\left[\dfrac{a_i}{T} + \dfrac{h_i d_i T}{2} + \dfrac{c_i I_p d_i (T-M)^2}{2T} - \dfrac{p_i I_e d_i M^2}{2T}\right] \quad if \quad T \geq M, & (10) \\ TVC_{22}(T) = \dfrac{A}{T} + \displaystyle\sum_{i=1}^{k}\left[\dfrac{a_i}{T} + \dfrac{h_i d_i T}{2} - p_i d_i I_e\left(M - \dfrac{T}{2}\right)\right] \quad if \quad T < M. & (11) \end{cases}$$

**Case 1**: When $T \geq M$, the first and second-order derivatives of $TVC_{21}(T_i)$ with respect to $T_i$ are

$$TVC_{21}'(T) = \frac{-\left[2A + \sum_{i=1}^{k} 2a_i + \sum_{i=1}^{k} d_i M^2 \left(c_i I_p - p_i I_e\right)\right]}{2T^2} + \frac{\sum_{i=1}^{k} c_i I_p d_i + \sum_{I=1}^{K} h_i d_i}{2}, \tag{12}$$

$$TVC_{21}''(T) = \frac{2A + \sum_{i=1}^{k} 2a_i + \sum_{i=1}^{k} d_i M^2 \left(c_i I_p - p_i I_e\right)}{T^3}. \tag{13}$$

**Case 2**: When $T < M$, the first and second-order derivatives of $TVC_{22}(T_i)$ with respect to $T_i$ are

$$TVC_{22}'(T) = -\frac{A}{T^2} + \sum_{i=1}^{k}\left(-\frac{a_i}{T^2} + \frac{h_i d_i}{2} + \frac{p_i I_e d_i}{2}\right), \tag{14}$$

$$TVC_{22}''(T) = \frac{2A}{T^3} + \sum_{i=1}^{k}\frac{2a_i}{T^3} > 0. \tag{15}$$

Equation $(15)$ implies that $TVC_{22}(T)$ is convex on $T>0$. Let $\Delta_3 = 2A+\sum_{i=1}^{k}2a_i+\sum_{i=1}^{k}d_iM^2\left(c_iI_p-p_iI_e\right)$, Equation $(13)$ implies that $TVC_{21}(T)$ is convex on $T>0$ when $\Delta_3>0$. Furthermore, Equations $(10)$ and $(11)$ imply that $TVC_2(T)$ is convex on $T>0$ when $\Delta_3>0$. At $T=M$, we find $TVC_{21}(M)=TVC_{22}(M)$. Hence, $TVC_2(T)$ is continuous and well-defined.

To minimize $TVC_2(T)$, solve Equations $(14)=0$ to obtain the optimal replenishment cycle time in $T<M$,

$$T_{22}^{\ *}=\sqrt{\frac{2A+\sum_{i=1}^{k}2a_i}{\sum_{i=1}^{k}h_id_i+\sum_{i=1}^{k}I_ep_id_i}}\ , \tag{16}$$

the optimal order quantity in $T<M$ is $Q\left(T_{22}^{\ *}\right)=T_{22}^{\ *}d_i=\sqrt{\dfrac{d_i(2A+\sum_{i=1}^{k}2a_i)}{\sum_{i=1}^{k}h_i+\sum_{i=1}^{k}I_ep_i}}$.

If $\Delta_3>0$, Equation $(13)$ implies that $TVC_{21}(T)$ is convex on $T>0$. Solve Equation $(12)=0$ to obtain the optimal replenishment cycle time in $T>M$,

$$T_{21}^{\ *}=\sqrt{\frac{2A+\sum_{i=1}^{k}2a_i+M^2\left(I_p\sum_{i=1}^{k}c_id_i-I_e\sum_{i=1}^{k}p_id_i\right)}{I_p\sum_{i=1}^{k}c_id_i+\sum_{i=1}^{k}h_id_i}}, \tag{17}$$

and the optimal order quantity in $T\ge M$ is

$$Q\left(T_{21}^{\ *}\right)=T_{21}^{\ *}d_i=\sqrt{\frac{d_i[2A+2\sum_{i=1}^{k}a_i+M^2(I_p\sum_{i=1}^{k}c_id_i-I_e\sum_{i=1}^{k}p_id_i)]}{I_p\sum_{i=1}^{k}c_i+\sum_{i=1}^{k}h_i}}.$$

Based on the equations above, we derive and deduce Theorem 3 and Theorem 4 to determine the optimal replenishment cycle time for joint multi-item replenishment when $\Delta_3<0$ and $\Delta_3\ge 0$ respectively.

**Theorem 3:** If $\Delta_3<0$, then $TVC_2(T)$ has the minimum value $T_2^{\ *}=T_{22}^{\ *}$.

**Proof:** $TVC_{22}(T)$ is convex on $T>0$ and $T_{22}^{\ *}<M$. Therefore, $TVC_{22}(T)$ is

decreasing on $\left(0,T_{22}^{\ *}\right]$ and increasing on $\left[T_{22}^{\ *},M\right]$. Thus, $TVC_{22}\left(T\right)$ has a minimum value at $T_{22}^{\ *}$ on $\left(0,M\right]$. On the other hand, if $\Delta_3<0$, Equation $\left(14\right)$ implies that $TVC_{21}'\left(T\right)\geq 0$ and $TVC_{21}\left(T\right)$ is increasing on $T>0$. Therefore, $TVC_{21}\left(T\right)$ is increasing on $\left[M,\infty\right)$. Thus, $TVC_{21}\left(T\right)$ has a minimum value at $M$. From $TVC_{21}\left(M\right)=TVC_{22}\left(M\right)$, Equations $\left(11\right)$ and $\left(12\right)$ imply that $TVC_{2}\left(T\right)$ has the minimum value at $T_{22}^{\ *}$ on $T>0$. Therefore, $T_2^{\ *}=T_{22}^{\ *}$. □

**Theorem 4:** If $\Delta_3\geq 0$, let $\Delta_4=2A+2\sum_{i=1}^{k}a_i-M^2\left(\sum_{i=1}^{k}h_i d_i+I_e\sum_{i=1}^{k}p_i d_i\right)$, then

(a) When $\Delta_4>0$, the optimal replenishment cycle time is $T_2^{\ *}=T_{21}^{\ *}$.

(b) When $\Delta_4<0$, the optimal replenishment cycle time is $T_2^{\ *}=T_{22}^{\ *}$.

(c) When $\Delta_4=0$, the optimal replenishment interval is $T_2^{\ *}=T_{21}^{\ *}=T_{22}^{\ *}=M$.

**Proof:** (a) If $\Delta_4>0$, Equations (17) and (16) imply that $T_{21}^{\ *}\geq M$ and $T_{22}^{\ *}\geq M$. The convexities and the definitions of $TVC_{21}\left(T_i\right)$ for Case 1 and $TVC_{22}\left(T_i\right)$ for Case 2 show that that $TVC_{21}\left(T_i\right)$ is decreasing on $[M,T_{21}^{*}]$ and $TVC_{22}\left(T_i\right)$ is decreasing on $(0,M]$. This means that $TVC_{21}\left(T_i\right)$ has the minimum value at $T_{21}^{\ *}$ and $TVC_{22}\left(T_i\right)$ has the minimum value at $M$. Therefore, from $TVC_{22}(M)=TVC_{21}(M)\geq TVC_{21}(T_{21}^{*})$, we know that $TVC_2\left(T_i\right)$ has the minimum value at $T_2^{\ *}=T_{21}^{\ *}$. The proofs in (b) and (c) are similar to that in (a). □

## 4. COMPUTATIONAL ANALYSIS

This section discusses five items and summarizes the parameter values in Table 1.

Table 1 values of parameters

| | $M$ | $A$ | $di$ | $ci$ | $Ip$ | $pi$ | $Ie$ | $hi$ | $ai$ |
|---|---|---|---|---|---|---|---|---|---|
| Item 1 | $\frac{30}{365}$ | 500 | 1,000 | 25 | 0.15 | 30 | 0.1 | 2 | 3 |
| Item 2 | $\frac{30}{365}$ | 500 | 800 | 125 | 0.15 | 150 | 0.1 | 3 | 3 |
| Item 3 | $\frac{30}{365}$ | 500 | 600 | 170 | 0.15 | 200 | 0.1 | 2 | 3 |
| Item 4 | $\frac{30}{365}$ | 500 | 400 | 200 | 0.15 | 250 | 0.1 | 3 | 3 |
| Item 5 | $\frac{30}{365}$ | 500 | 50 | 350 | 0.15 | 420 | 0.1 | 4 | 3 |

### 4.1. The Single-item Replenishment Model v.s. The Joint Multi-item Replenishment Model

Using Theorem 2 for item 1, $\Delta_1 = 1011.07$ and $\Delta_2 = 972.223$, the optimal replenishment cycle time is $T_{11}^* = 0.41933$ and the total cost is $2102.93$. Using Theorem 2 for item 2, $\Delta_1 = 1026.27$ and $\Delta_2 = 908.721$, the optimal replenishment cycle time is $T_{11}^* = 0.24286$ and the total cost is $2992.88$. Using Theorem 2 for item 3, $\Delta_1 = 1028.29$ and $\Delta_2 = 916.828$, the optimal replenishment cycle time is $T_{11}^* = 0.249641$ and the total cost is $2861.55$. Using Theorem 2 for item 4, $\Delta_1 = 1019.51$ and $\Delta_2 = 930.339$, the optimal replenishment cycle time is $T_{11}^* = 0.277913$ and the total cost is $2682.15$. Using Theorem 2 for item 5, $\Delta_1 = 1009.55$ and $\Delta_2 = 990.462$, the optimal replenishment cycle time is $T_{11}^* = 0.597797$ and the total cost is $1473.02$. The sum of these items, or total cost, is $12112.5$.

For the joint multi-item replenishment model, we get $\Delta_3$=2094.68, $\Delta_4 = 718.572$. Using Theorem 4, the optimal replenishment cycle time is $T_2^* = T_{22}^* = 0.140221$ and the total cost is $3806.14$. The results above show that the joint multi-item replenishment model is better than the single-item replenishment model in reducing total cost.

### 4.2. Effects of different parameter values

Table 2 presents the effects of $I_p$ and $I_e$ on total cost and decision, showing that when $I_e$ increases, the optimal replenishment cycle time $T^*$ and total cost $TVC$

will decrease. When $I_p$ increases, the optimal replenishment cycle time $T^*$ will decrease, but the total cost $TVC$ will increase.

Table 2 Effects of $I_p$ and $I_e$

| | | *Ie* | | |
|---|---|---|---|---|
| | | 0.08 | 0.1 | 0.12 |
| *Ip* | 0.13 | *TVC*=3900.5<br>*T**=0.149797 | *TVC*=3722.01<br>*T**=0.146168 | *TVC*=3538.97<br>*T**=0.142446 |
| | 0.15 | *TVC*=3992.29<br>*T**=0.143565 | *TVC*=3806.14<br>*T**=0.140221 | *TVC*=3615.44<br>*T**=0.136796 |
| | 0.17 | *TVC*=4071.74<br>*T**=0.138436 | *TVC*=3878.77<br>*T**=0.135331 | *TVC*=3681.27<br>*T**=0.132154 |

Table 3 presents the effects of *T* and *M* on total cost and decision, showing that when $A$ increases, the optimal replenishment cycle time $T^*$ and total cost $TVC$ will increase. When $M$ increases, the optimal replenishment cycle time $T^*$ will increase, but the total cost $TVC$ will decrease.

Table 3 Effects of $M$ and $T$

| | | *A* | | |
|---|---|---|---|---|
| | | 500 | 700 | 1000 |
| *M* | $\frac{15}{365}$ | *TVC*=5631.54<br>*T**=0.137079 | *TVC*=6972.7<br>*T**=0.161168 | *TVC*=8673.02<br>*T**=0.191708 |
| | $\frac{30}{365}$ | *TVC*=3806.14<br>*T**=0.140221 | *TVC*=5121.62<br>*T**=0.163849 | *TVC*=6798.46<br>*T**=0.193968 |
| | $\frac{45}{365}$ | *TVC*=2088.97<br>*T**=0.145308 | *TVC*=3364.77<br>*T**=0.168223 | *TVC*=5004.57<br>*T**=0.197676 |

## 5. CONCLUSION

This paper considers replenishment problems under the permissible delay in payments. We present both single-item and joint multi-item replenishment models, and develop theorems to solve these problems. The objective of this study is to determine the optimal replenishment policy while minimizing the total cost. Using computational examples, we illustrate the solution procedures and show that the joint multi-item replenishment policy is better than the single-item replenishment policy. Numerical analysis revealsthe effects of interest charged, interest earned, ordering cost, and credit period on the total cost and replenishment decision. This study provides a useful reference for managerial decision-making and administration.

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