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COMPARISON OF PRESSURE DISTRIBUTION IN INCLINED AND PARABOLIC SLIDER BEARINGS

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Abstract-An infinitely wide lubricated slider bearing consisting of connected surfaces with thirtd grade fluid as lubricant is analyzed in the present study. Under the assumptions of the order of magnitudes of the variables, it is seen that only viscous and non-Newtonian terms have significant contributions, whereas inertia terms are negligible. Choosing non-Newtonian effects to be smaller than the viscous effects, analytical and numerical solutions are constructed. To illustrate the mathematical model, the set of equations is to used calculate the pressure for two special forms of the slider bearings, namely inclined and parabolic slider bearings. In these two cases, the variation of pressure, for a range of fluid and bearing parameters is presented. The results are presented both analytically and graphically.

Key words- slider bearing, third grade fluids, perturbation analysis

1. INTRODUCTION

Bearings find application in many mechanical components in order to reduce the frictional losses between two rotating or sliding mechanical parts. The velocity and pressure distribution in the bearing should be known for a proper functioning. Much work has been done on the Newtonian type of lubrication. However, additives are frequently uses in lubricating fluids, which makes the flow non-Newtonian. In practice almost all lubricants exhibit non-Newtonian behaviour under certain circumstances. Moreover, the use of non-Newtonian fluids as lubricants has become more important with development of modern industrial materials. The lubricant used in the bearing systems is usually non-Newtonian, such as powdered graphite, and the carrier fluid is ethylene glycol. In this case, the hydrodynamic solution of the flow system in the bearing requires extensive computational effort, since the governing equations of flow are coupled due to the different species that exist in the system. However, the assumption of uniform fluid with non-Newtonian behaviour enables the solution of flow equations analytically. Various models are developed to account for the non-Newtonian behaviour of the fluid flow. One group of models includes differential type fluids. Third-grade fluid is a special model of differential type of fluid which has received attention in recent years.

Considerable research studies have been carried out to explain the flow field in bearing systems. Wang presented the approximate analytical solution for Reynolds equation of a slider bearing with a smooth surface [1]. In the study, it is shown that, the lubricant rheological behaviour and surface roughness had an important influence on the load capacity and friction drag at the surface of the bearing. Non-Newtonian effects on the static characteristics of one-dimensional slider bearings in the internal flow regime were investigated by Hashimoto [2]. The modified Reynolds equation to one-dimensional slider bearings is applied and the resulting equation is solved analytically using the perturbation technique. The non-Newtonian effects of powder-lubricant slurries in hydrostatic and squeeze film bearings were studied by Wu [3]. They showed that the damping factor was increased with the addition of powdered graphite into the carrier fluid. Non-Newtonian temperature and pressure effects of graphite powder lubricant when added to a Newtonian carrier fluid and applied in a rotating hydrostatic step bearing was studied by Peterson *et al* [4]. They showed that temperature increased with bearing rotational speed and compared favorably with the mathematical predictions. The transient response of a two-lobe journal bearing with non- Newtonian lubricant was studied by Sinhasan and Goyal [5]. They showed from the nonlinear trajectories that the two-lobe journal bearing system became unstable when the critical journal mass was less than that obtained from Routh's criterion. The model study of double layered porous Rayleigh-step bearings with second-order fluid as lubricant was presented by Naduvinamani [6]. He indicated that maximum dimensionless load-carrying capacity occurred at a slightly larger step ratio as compared with the conventional porous Rayleigh-step bearings. The lubrication of slider bearings with a special third-grade fluid was considered by Yurusoy & Pakdemirli [7]. They used a perturbation method to obtain approximately velocity and pressure fields in the bearings. The theoretical study concerning the effect of nonlinear behaviour of the lubricant on the performance of a slot-entry journal bearing was carried out by Sharma *et al* [8]. He showed that the combined effect of nonlinearity factor and beating flexibility affected the performance characteristics of slot-entry journal bearing significantly. A slider bearing with second- and third-grade fluids as lubricant was analyzed by Yurusoy [9]. He presented the pressure distribution in the bearing analytically. The thermodynamic analysis of the misaligned conical-cylinderical bearing with non-Newtonian lubricants was carried out by Yang & Jeng [10]. He indicated that the normal load carrying capacity was enhanced by higher values of flow behaviour index, higher eccentricity ratios, and larger misalignment factors.

In this work, third grade fluid is considered as a lubricant in a slider bearing. A generalized Reynolds equation for a finite bearing with the mentioned lubricants by using perturbation technique has been obtained. To illustrate the mathematical model, the set of equations is used for the calculation of the pressure for two special forms of the slider bearings, namely, inclined and parabolic slider bearings. For these forms, both numerical and analytical solutions are presented. Numerical solutions of the outcoming nonlinear differential equations are found by using a combination of a Runge-Kutta algorithm and shooting technique. By employing perturbation analysis, the analytical solutions of equations are calculated. The variation of pressure for both inclined and parabolic slider bearings, for a range of fluid and bearing parameters is presented. For the special forms, perturbation solutions are contrasted with numerical solutions.

2. EQUATION OF MOTION AND VELOCITY PROFILE

The slider bearing is shown in Figure 1. The dimensionless quantities are defined from the dimensional parameters (denoted by asterisk) as follows:

$$x=\frac{x^*}{L},\ y=\frac{y^*}{b},\ r=\frac{b_2}{b_1},\ u=\frac{u^*}{U},\ v=\frac{Lv^*}{bU},\ p=\frac{p^*}{\left(\mu UL/b_1^2\right)},b=\frac{b^*}{b_1} \tag{1}$$

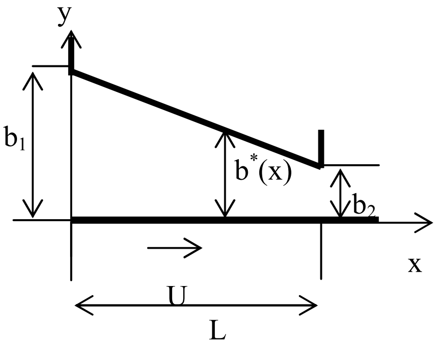

Fig. 1. Schematic view of slider bearing.

A tedious but straightforward algebra yields the non-dimensional form of the equations of motion of a third-grade fluid in a slider bearing (see reference [7] for details of how to calculate the equations of motion with boundary conditions).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\frac{\partial^2 u}{\partial y^2}+6k\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}=\frac{dp}{dx} \tag{3}$$

$$u(x,0)=1,\quad u(x,b)=0,\quad v(x,0)=0,\quad v(x,b)=0 \tag{4}$$

Taking the approximate velocity profiles from Yurusoy and Pakdemirli [7] and expressing them in terms of dimensionless quantities yield

$$\begin{aligned}u=b^2\left(\frac{y^2}{2}-\frac{y}{2}\right)\frac{dp}{dx}+1-y-k\Bigg\{b^4\left(\frac{y^4}{2}-y^3+\frac{3y^2}{4}-\frac{y}{4}\right)\left(\frac{dp}{dx}\right)^3\\+b^2\left(-2y^3+3y^2-y\right)\left(\frac{dp}{dx}\right)^2+3\left(y^2-y\right)\frac{dp}{dx}\Bigg\}+\ldots\end{aligned} \tag{5}$$

The pressure distribution p(x) remains unknown in the above equation. For both inclined and parabolic slider bearings, the goal would then be to determine the pressure distribution approximately.

3. PRESSURE DISTRIBUTION

Using continuity equation together with the derived velocity profile, one may find the ordinary differential equation for the velocity profile.

Integrating the continuity equation along the y coordinate with the boundary conditions v(0)=v(b)=0. Function b was written as dimensionless parameter in equation (1).

$$\int_0^b \frac{\partial u}{\partial x} dy = -\int_0^b \frac{\partial v}{\partial y} dy = -v(b) + v(0) = 0 \tag{6}$$

one has

$$\int_0^b \frac{\partial u}{\partial x} dy = 0 \tag{7}$$

Substituting equation (5) into equation (7) and arraying yields

$$\frac{d}{dx}\left(b^3 \frac{dp}{dx}\right) = 6\frac{db}{dx} + k\left[\frac{3}{10}\frac{d}{dx}\left(b^5\left(\frac{dp}{dx}\right)^3\right) + 6\frac{d}{dx}\left(b\frac{dp}{dx}\right)\right] \tag{8}$$

An approximate solution will be searched for the above equation since it is variable coefficient and highly nonlinear. The associated boundary conditions are

$$p(0)=p(1)=0 \tag{9}$$

To interpret the approximate analytical solution for equation (8) and observe variation of pressure distribution with third grade of lubricants, b(x) is defined in the following two cases;

Case 1: Inclined slider bearing:

$$b = (1-(1-r)x) \tag{10}$$

where $r=b_2/b_1$. Taking the approximate pressure profile for inclined slider bearing from Yurusoy and Pakdemirli [7] and expressing them in terms of dimensionless quantities yield

$$\begin{aligned} p = {} & \frac{6x(1-x)(1-r)}{(1+r)b^2} + k\left\{\frac{6}{25(1-r)(1+r)^3 b^6}\right. \\ & \left(648r^2 b(1+r) + 140(1+r)^3 b^3 - 360r^3 - 480rb^2(1+r)^2\right) \\ & \left. + \frac{24(1-r)(13+13r^2-r)}{25r(1+r)^3 b^2} - \frac{24(13+13r^2+8r)}{25r(1-r)(1+r)^3}\right\} + \ldots \end{aligned} \tag{11}$$

Case 2: Parabolic slider bearing:

$$b = (1 + a(x^2 - 2x)) \tag{12}$$

where $a = (1-r)$, $r=b_2/b_1$. A tedious but straightforward algebra yields the approximate pressure profile for parabolic slider bearing (see Ref[7] for details of how to calculate the pressure distribution for parabolic slider bearing)

$$
\begin{aligned}
p = {} & \frac{(x-1)(6-c)}{4(a-1)b^2} + \frac{3(x-1)(2-8a+c)}{8(a-1)^2 b} + \frac{3(2-8a+c)\arctan\left(\sqrt{\frac{a}{(1-a)}}\,(x-1)\right)}{8(a-1)^2\sqrt{a(1-a)}} \\
& + k\Bigg\{ -\frac{(c-6)^3(x-1)}{40(a-1)b^6} - \frac{(-150+216a-11c)(c-6)^2(x-1)}{400(a-1)^2 b^5} \\
& - \frac{3(c-6)(x-1)(2420+5120a^2+252c+33c^2-8a(794+81c))}{3200(a-1)^3 b^4} \\
& + \left(-840+15360a^3-908c-54c^2-33c^3-5120a^2(c+3)+8a(996+308c+81c^2)\right) \\
& \left(-\frac{7(x-1)}{6400(a-1)^4 b^3} + \frac{7(x-1)}{5120(a-1)^5 b^2} - \frac{21(x-1)}{10240(a-1)^6 b} - \frac{21\arctan\left(\sqrt{\frac{a}{(1-a)}}\,(x-1)\right)}{10240(a-1)^6\sqrt{a(1-a)}}\right) \\
& - d\left(\frac{(x-1)}{4(a-1)b^2} - \frac{3(x-1)}{8(a-1)^2 b} - \frac{3\arctan\left(\sqrt{\frac{a}{(1-a)}}\,(x-1)\right)}{8(a-1)^2\sqrt{a(1-a)}}\right)\Bigg\}
\end{aligned}
\tag{13}
$$

where d

$$
\begin{aligned}
d = {} & \frac{1}{\dfrac{3}{8(a-1)^2} - \dfrac{1}{4a-4} + \dfrac{3\arctan\left(\sqrt{\frac{a}{1-a}}\right)}{8(a-1)^2\sqrt{a(1-a)}}} \left(k\left(\frac{(c-6)^3}{40(a-1)} + \frac{(216a-11c-150)(c-6)^2}{400(a-1)^2}\right.\right. \\
& + \frac{3(c-6)(2420+5120a^2+252c+33c^2-8a(794+81c))}{3200(a-1)^2} + \left(-840+15360a^2-908c^3\right. \\
& \left. -54c^2-33c^3-5120a^2(c+3)+8a(996+308c+81c^2)\right)\left(\frac{21\arctan\left(\sqrt{\frac{a}{1-a}}\right)}{10240(a-1)^6\sqrt{a(1-a)}}\right. \\
& \left.\left.\left. + \frac{21}{10240(a-1)^6} + \frac{7}{6400(a-1)^4} - \frac{7}{5120(a-1)^5}\right)\right)\right)
\end{aligned}
\tag{14}
$$

A plot of x versus p is given in Fig.1 for different k values. k=0 corresponds to the Newtonian solution. As k increases (Non-Newtonian behaviour) the pressure inside the inclined slider bearing increases which means a higher loading capacity for the inclined bearing.

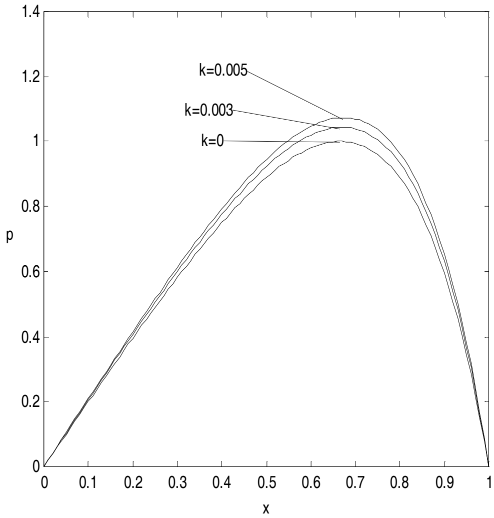

Fig. 2 Dimensionless pressure in the inclined bearing corresponding to different non-Newtonian effect for r=0.5

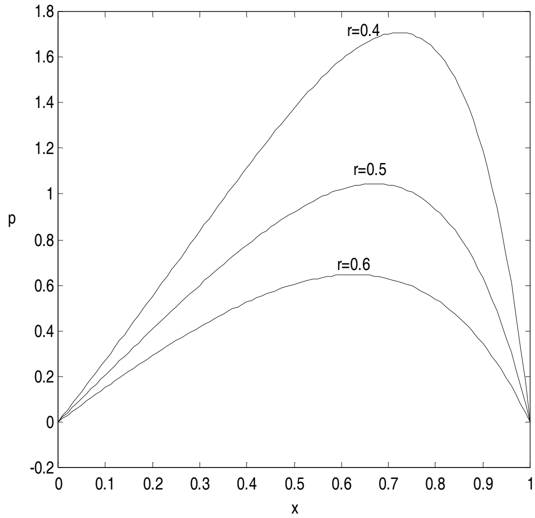

Fig. 3 Dimensionless pressure in the inclined bearing corresponding to different clearance ratios for k=0.003

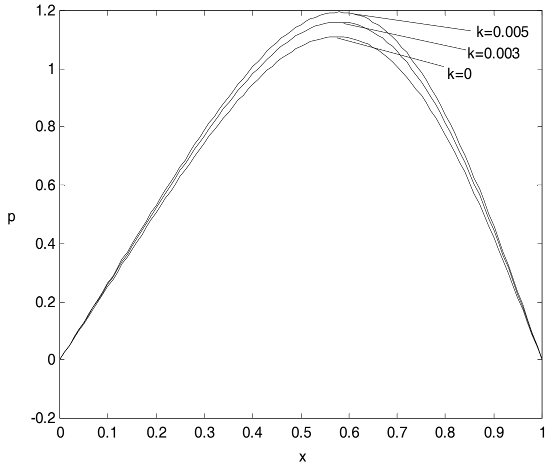

Fig. 4 Dimensionless pressure in the parabolic bearing corresponding to different non-Newtonian effect for r=0.5

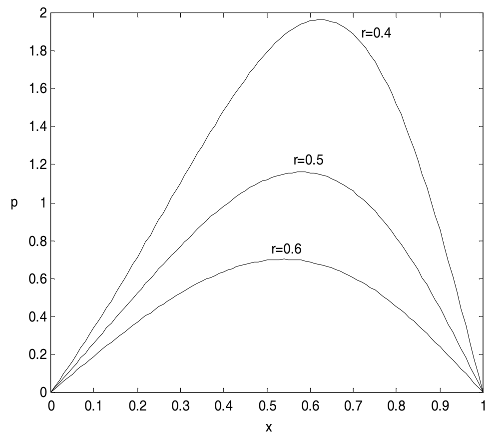

Fig. 5 Dimensionless pressure in the parabolic bearing corresponding to different clearance ratios for k=0.003

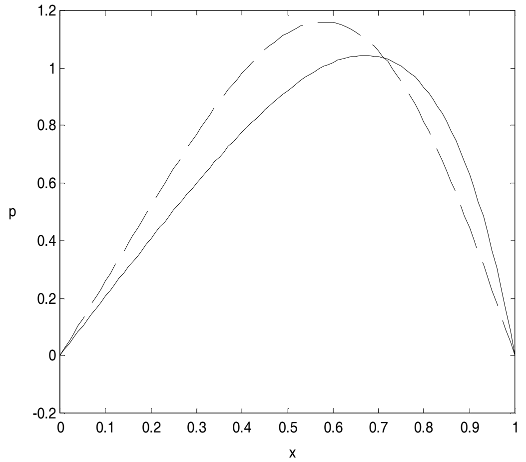

Fig .6 Pressure distribution in the bearing for r=0.5 and k=0.003 for two different cases (parabolic -----, inclined —)

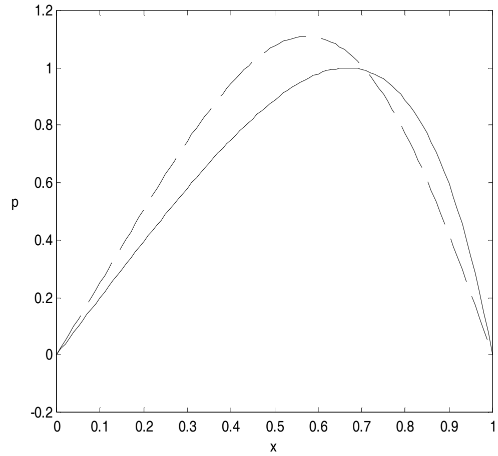

Fig .7 Pressure distribution in the bearing for r=0.5 and k=0 for two different cases (parabolic -----, inclined —)

In Fig. 3 for k=0.003 the dimensionless length versus dimensionless pressure is plotted for different clearance ratios. Similar to the Newtonian behaviour in non-Newtonian case also, pressure builds up in the inclined bearing for lower clearance ratios. Similarly, Fig. 4 and Fig. 5 are plotted for the parabolic slider bearing. Fig. 4 shows pressure distribution along the x axis for three k values, while r=0.5 is kept constant. Pressure distribution along the x axis increases as non-Newtonian parameter increases. In Fig. 5 for k=0.03 x versus dimensionless pressure is plotted for different clearance ratios. Pressure distribution increases as clearance ratios reduces.

For r=0.5, k=0.003 and k=0, the analytical solution in the cases of the parabolic slider bearing and the inclined slider bearing are contrasted in Fig. 6 and Fig. 7. It is observed that the pressure inside the bearing for parabolic slider bearing is higher than inclined slider bearing in the region I ($0 \leq x \leq 0.7$) and in region II ($0.7 \leq x \leq 1$) the pressure for inclined slider bearing is higher than the parabolic slider bearing.

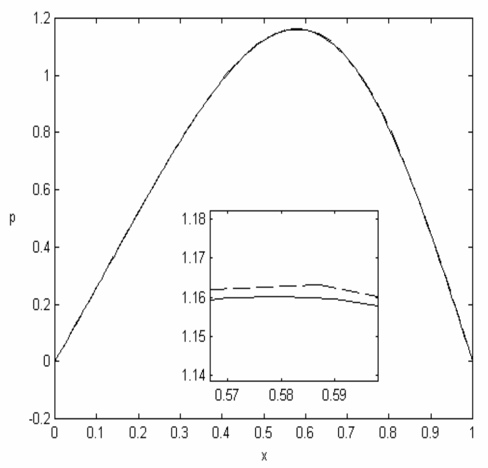

Fig. 8 Comparison of numerical (------), analytical (—) solutions for pressure distribution in the parabolic slider bearing (r=0.5, k=0.003)

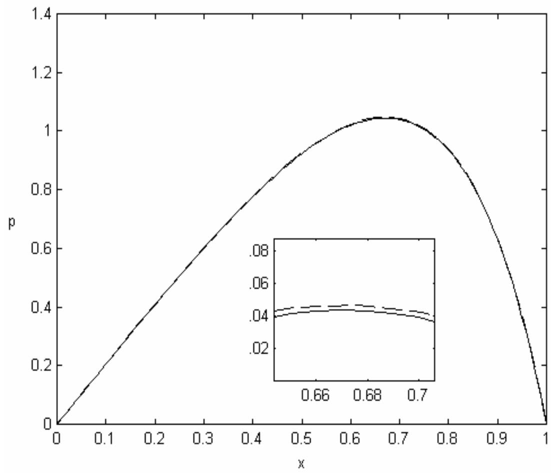

Fig. 9 Comparison of numerical (------), analytical (—) solutions for pressure distribution in the inclined slider bearing (r=0.5, k=0.003)

For two type slider bearings, the analytical solutions (perturbation solutions) are contrasted with numerical solutions. In numerical computations, Matlab package Ode45, which uses a combination of second/third order Runge-Kutta algorithm and shooting technique, is employed. For the parabolic and inclined slider bearings, analytical solution and numerical solution are contrasted in Fig.8 and Fig.9. As can be seen, analytical solutions are in very close match with numerical solutions. However, small difference is evident for the curves as can be seen from the zoomed parts.

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