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IN-PLANE VIBRATIONS OF CIRCULAR CURVED BEAMS WITH A TRANSVERSE OPEN CRACK

H. R. Öz and M. T. Daş
Department of Mechanical Engineering, University of Gaziantep, 27310, Gaziantep, Turkey
hroz@gantep.edu.tr, tdas@gantep.edu.tr

Abstract- In this study, the in plane vibrations of cracked circular curved beams is investigated. The beam is an Euler-Bernoulli beam. Only bending and extension effects are included. The curvature is in a single plane. In plane vibrations is analyzed using FEM. In the analysis, elongation, bending and rotary inertia effects are included. Four degrees of freedom for in-plane vibrations is assumed. Natural frequencies of the beam with a crack in different locations and depths are calculated using FEM. Comparisons are made for different angles.

Keywords- Curved beam, crack, vibration

1. INTRODUCTION

Vibrations of curved beams were investigated by many scientists. Curved beams are used in gears, electric machinery, pumps and turbines, ships, bridges etc. The governing equations of motion and solutions were given in the book of Love [1]. Yamada and Takahashi [2] analyzed steady state response of a Timoshenko beam for structural damping case. Ibrahimbegović [3] considered an arbitrary shape beam and included forcing effect. Khdeir and Reddy [4] presented a general model for dynamic response of a curved beam under arbitrary boundary condition and loading. Khan and Pise [5] studied buried curved piles. Kang and Bert [6] applied DQM and included bending moment, radial loading and warping. Bozhevolnaya and Kildegaard [7] performed experiments for uniform loading case. Walsh and White [8] investigated wave propagation of a constant curvature beam by combining flexure and extension effects. Kashimoto *et al.* [9] studied dynamic stress concentration using transfer matrix method. Tong *et al.* [10] investigated free and forced vibrations of circular curved beam for inextensional case. Krishnan *et al.* [11] studied free vibrations for different boundary conditions and subtended angles. Extensional effects were included by considering tangential and normal loadings [12]. Natural frequencies were calculated for classical boundary conditions including shear stress and rotary inertia [13]. Exact solutions of free in plane vibrations including extension, shear and rotary inertia were obtained by Tüfekçi and Arpacı [14]. Inextensional vibrations were investigated by using another version of DQM [15]. FEM was applied by combining polynomial displacement field [16]. Frequencies and mode shapes were obtained for different curves, cross sections and boundary conditions [17].

Crack problems in straight beams were investigated widely. Continuous crack theory was developed [18], finite elements and component mode synthesis methods were combined [19], simple methods for frequencies were proposed [20], dynamic characteristics were studied analytically for a closing crack [21], frequencies of multiple cracked beam were calculated by modeling the cracks as a rotary springs [22], dynamic behaviors were presented using strain energies given by linear fracture mechanics theory [23], and FEM was used to calculate natural frequencies[24].

There are very few studies on the cracked curved beams. Krawczuk and Ostachowicz [25] calculated natural frequencies of a clamped-clamped arch with an open transverse crack. Cerria and Rutab [26] detected localized damage by frequency data. They modeled the crack by a rotary spring attached to both ends and assumed flexural rigidity as a decreasing function. Müller *et al*. [27] obtained stress intensity factors and strain release rate, and applied to the circular curved beams. Nobile [28] studied crack propagation in curved beams using S-theory for mixed mode crack problem and obtained approximate stress intensity factor and compared with that of reference [27].

In this study, the in plane vibrations of a circular curved beam with a Mode 1 open transverse crack is investigated. Extension of neutral axis, bending, and rotary inertia effects are included. FEM is used to calculate natural frequencies for different crack depths and locations and for different boundary conditions.

2. FINITE ELEMENT METHOD

In this section, finite element method [29] will be used to obtain natural frequencies of a circular curved beam with a transverse crack as shown in Figure 1. s is arc length, R is curvature, γ arc angle, E is modulus of elasticity I is mass moment of inertia, ρ is density and A (b*h) is the cross sectional area. u and v are tangential and transverse displacements, respectively. The crack location and the depth are c and a, respectively.

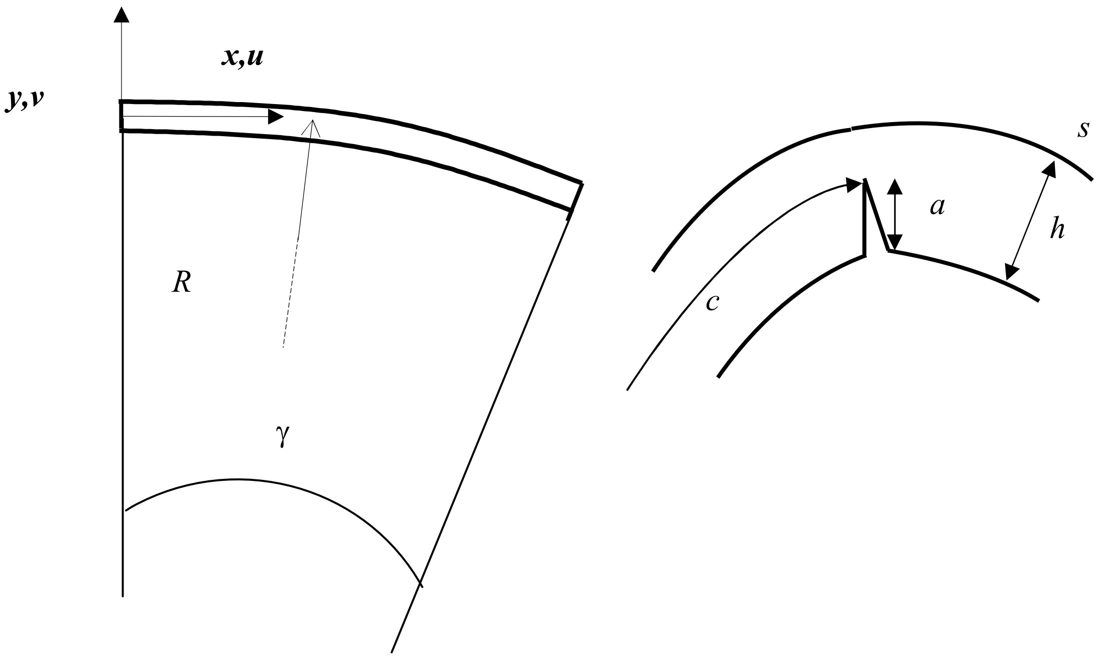

Figure 1. A circular curved beam with a transverse crack with depth *a* at location *c* and cross section height *h*

Cubic interpolation functions are used for tangential and transverse displacements [30]. Kinetic and elastic energies can be written as follows

$$U = \frac{1}{2} E \int_0^s \left[A\varepsilon^2 + I\kappa^2 \right] dx \, , \; T = \frac{1}{2} \rho \int_0^s \left[A(\dot{u}^2 + \dot{v}^2) + I\dot{\beta}^2 \right] dx \tag{1}$$

here $(\dot{\;})$ denotes differentiation with respect to time (t). In plane strain, net cross sectional rotation and curvature change in equation (1) are given as follows

$$\varepsilon = \frac{\partial u}{\partial x} + \frac{v}{R} \, , \; \beta = \frac{\partial v}{\partial \mathrm{x}} - \frac{u}{R} \, , \; \kappa = \frac{\partial \beta}{\partial x} = \frac{\partial^2 v}{\partial x^2} - \frac{1}{R}\frac{\partial u}{\partial x} \tag{2}$$

For rectangular cross sectional beams ($b*h$), equivalent flexural rigidity for Mode 1 crack case is given as follows [31, 32].

$$EI_c = \frac{EI}{1+\dfrac{EIR(a,c)}{1+\left(\dfrac{(s-c)}{k(a)a}\right)^2}} \tag{3}$$

where

$$R(a,c)=\frac{2D(a)}{k(a)a\left[\arctan\left(\dfrac{s-c}{k(a)a}\right)+\arctan\left(\dfrac{c}{k(a)a}\right)\right]},\ k(a)=\frac{3\pi(F(a))^2(h-a)^3a}{\left(h^3-(h-a)^3\right)h}$$

$$D(a)=\frac{18\pi(F(a))^2a^2}{Ebh^4},\ F(a)=1.12-1.4\left(\frac{a}{h}\right)+7.33\left(\frac{a}{h}\right)-13.8\left(\frac{a}{h}\right)^3+14\left(\frac{a}{h}\right)^4 \tag{4}$$

These equations are valid for straight beams and obtained using energy method. But we now assume that the equations above represent flexural rigidity variation with tangential coordinate for small arc angles.

3. NUMERICAL SOLUTIONS

In this section, natural frequency variation with respect to crack location and depth will be presented. Clamped-clamped, simple-simple and clamped-free boundary conditions are considered. Crack is assumed at different locations and depths.

Firstly, let's consider flexural rigidity ratio of the cracked and uncracked beam, EI_c/EI. In Figure 2, the variation is given for a beam of area (h*b) 0.02*0.01 m^2, length 1 m and E=2*10^{11} Pa. Crack depths are selected h/10, 3h/10 and 5h/10 respectively. The location of the crack is c=0.5s. The ratio decreases toward the crack and it becomes minimum at the crack. The larger the depth, the smaller the ratio. The ratio approaches to unity away from the crack which is uncracked case.

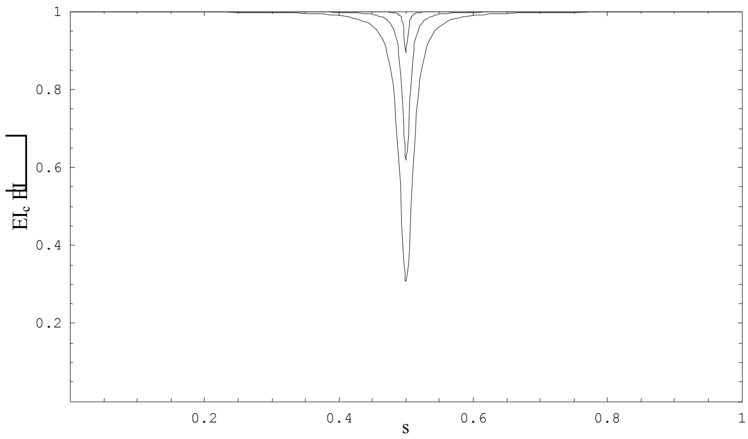

Figure 2. The variation of flexural rigidity ratio with the crack location and depth. Crack depths are *h*/10, 3*h*/10, 5*h*/10 and location is *c*=0.5*s*.

Analytical solutions were presented in reference [26] for a crack modeled by a rotary spring. Also it was explained that the model can be used for a curved beam. In this study, we assume that the flexural rigidity varies according to equations (3) and (4). Also we consider arc angle changes from 0^{o} to 20^{o} while keeping the length constant (1 m).

In Figures 3 and 4, the natural frequency variation of a clamped-free curved beam is presented for the first four modes. Arc angle is 20^{o}, crack depth is a=h/10 a=2h/10 and a=3h/10, respectively, b=0.02, h=0.01, s=1m. Cracked cases are denoted by solid lines and uncracked cases are denoted by dashed lines. For the ucracked case, the natural frequencies are 51.522036 rad/s, 317.61959 rad/s, 896.54848 rad/s, and 1760.7971 rad/s respectively for the first four modes for clamped-free boundary condition. The frequencies are decreasing very much when the crack is around the clamped end. Because the moment is maximum at the clamped end and the stress concentration is high. If the moment is maximum at some other locations, the frequencies again will be very much lower than that of the uncracked one, e.g. 0.5 m< c <0.6 m in the second mode (Figure 3, right), c ≅ 0.3 and 0.7 m in the third mode (Figure 4, left), and c ≅ 0.2, 0.5, and 0.8 m in the fourth mode (Figure 4, right). As the crack is located at the other places the moment decreases and at the free end it becomes zero. That means that the natural frequencies of the cracked beam approache to that of uncracked beam as the crack is moved to the free end. Also when the crack is located around the nodes of uncracked beam, the effect of the crack is decreasing and the frequencies of cracked and uncracked beams are close to each other at these locations. The deeper the crack the smaller the frequencies.

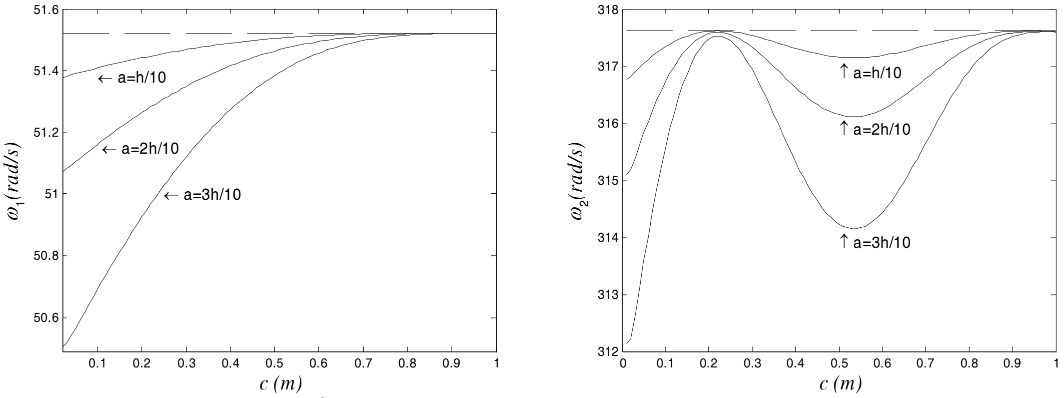

Figure 3. The 1st and 2nd mode natural frequency variations of a clamped-free curved beam, cracked and uncracked cases, angle=20^{o}, *b*=0.02, *h*=0.01, *s*=1m

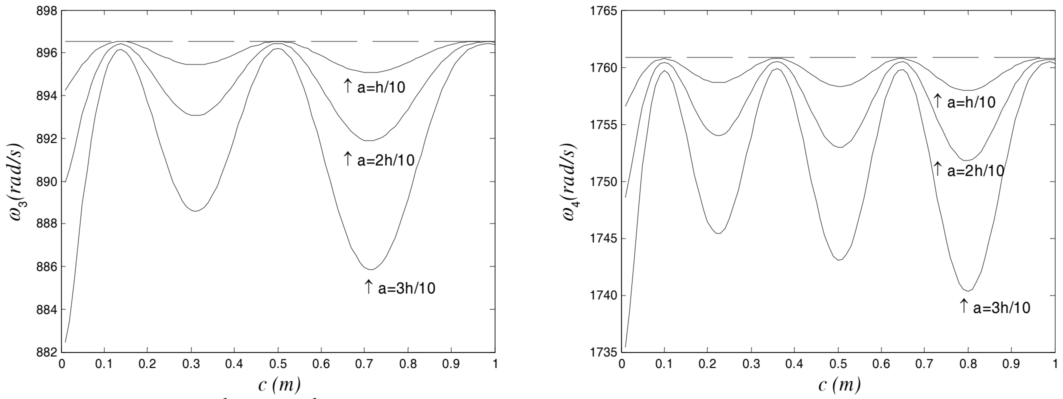

Figure 4. The 3rd and 4th mode natural frequency variations of a clamped-free curved beam, cracked and uncracked cases, angle=20^{o}, *b*=0.02, *h*=0.01, *s*=1m

In Figures 5 and 6, frequency variation is given for simple-simple end conditions. The uncracked frequencies are 572.55663 rad/s, 1164.80173 rad/s, 1766.80529 rad/s, and 2302.33641 rad/s for the first four modes respectively.

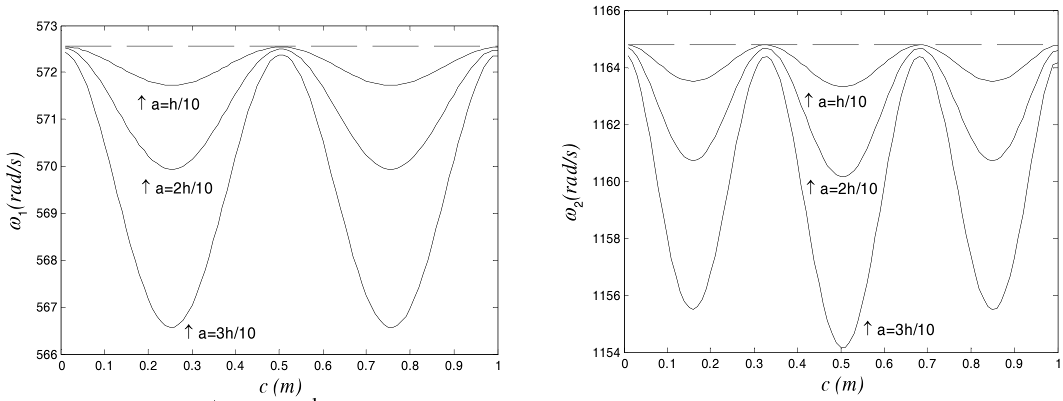

Figure 5. The 1st and 2nd mode natural frequency variations of a simple-simple curved beam, cracked and uncracked cases, angle=20^{o}, *b*=0.02, *h*=0.01, *s*=1m

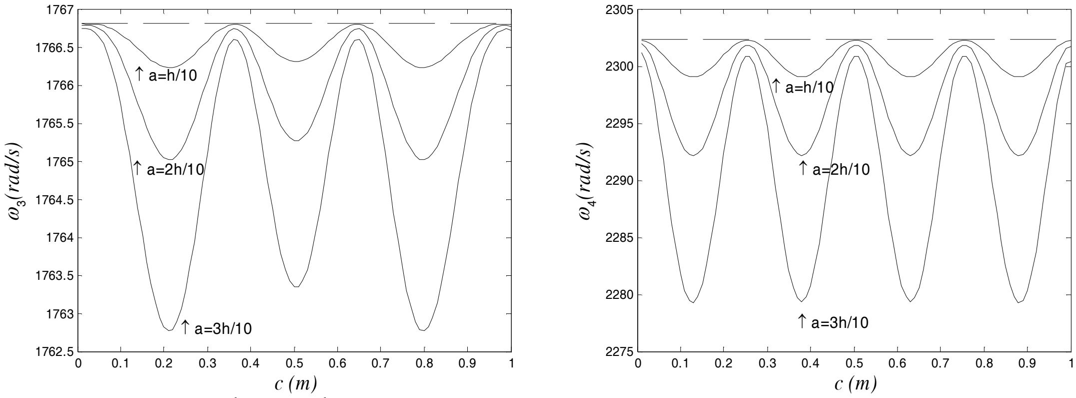

Figure 6. The 3rd and 4th mode natural frequency variations of a simple-simple curved beam, cracked and uncracked cases, angle=20^o, *b*=0.02, *h*=0.01, *s*=1m

Similar conclusions can be drawn for this end condition. Since the ends are free to rotate, the moments are zero and the effect of the crack near the ends is less. The smaller the depth of the crack, the closer the frequencies to the uncracked one.

In Figures 7 and 8, frequency variation is given for clamped-clamped end conditions. The uncracked frequencies are 896.69882 rad/s, 1320.66377 rad/s, 2002.56499 rad/s, 2913.96424 rad/s for the first four modes, respectively. Again similar conclusions can be drawn for this end condition. Also, since the ends are clamped, the moments are maximum and the decrease in natural frequencies is obvious. The smaller the depth of the crack, the closer the frequencies to the uncracked one.

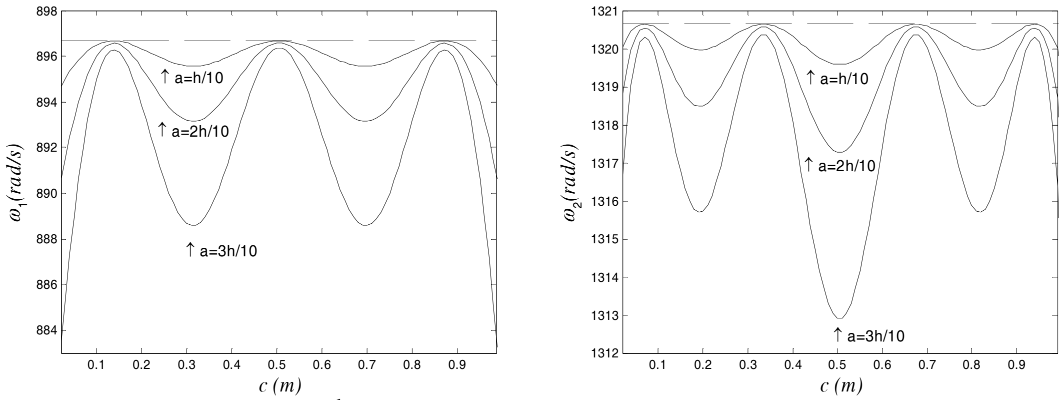

Figure 7. The 1st and 2nd mode natural frequency variations of a clamped-clamped curved beam, cracked and uncracked cases, angle=20^o, *b*=0.02, *h*=0.01, *s*=1m

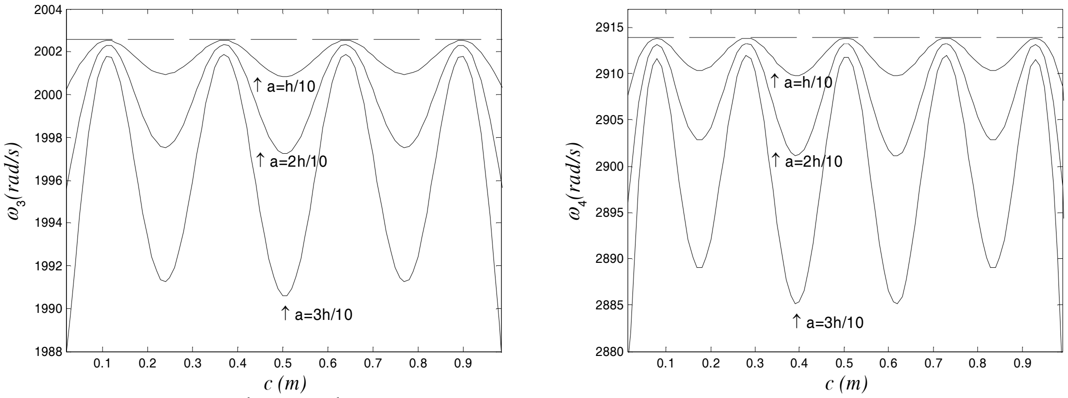

Figure 8. The 3rd and 4th mode natural frequency variations of a clamped-clamped curved beam, cracked and uncracked cases, angle=20°, b=0.02, h=0.01, s=1m

In Figure 9, natural frequency variation is plotted for different arc angles for clamped-free end conditions. Crack location is c=0.5s, depth a=3h/10, b=0.02, h=0.01, s=1m. The frequencies of the cracked beam are always lower than that of the uncracked beam. Because the crack decreases the flexural rigidity of the beam.

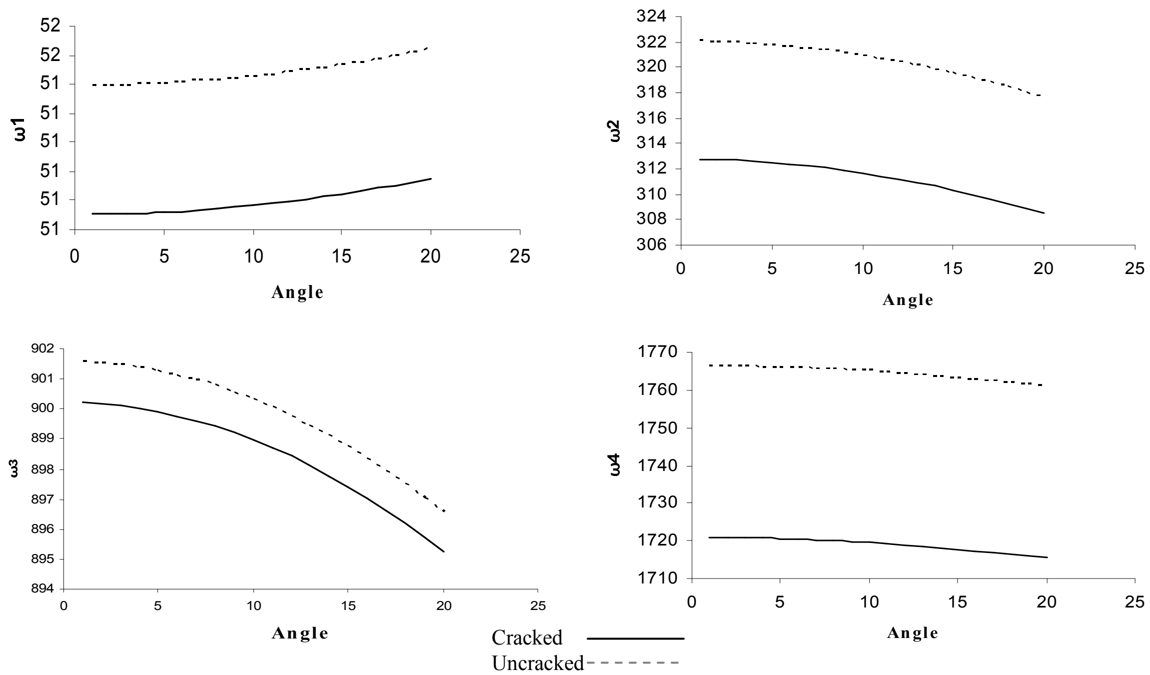

Figure 9. The natural frequency variation of clamped-free beam c=0.5s, a=3h/10, b=0.02, h=0.01, s=1m

4. CONCLUSIONS

In this study, the in plane vibrations of circular curved beams with a mode 1 transverse crack is investigated. Euler-Bernoulli type beam is considered. Bending, extension and rotation effects are included, shear effect is excluded. FEM is used to calculate natural frequencies for different crack locations and depths. Comparisons are made for different arc angle and for different end conditions. Increasing the crack depth decreases the frequencies.

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