Mathematical and Computational Applications, Vol. 10, No. 3, pp. 351-358, 2005.

WEYL MANIFOLDS WITH SEMI-SYMMETRIC CONNECTION

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Abstract- We define a semi-symmetric connection on a Weyl manifold and study projective curvature tensor and conformal curvature tensor after giving some properties of the curvature tensor with respect to semi-symmetric connection.
Keywords- Weyl manifold, semi-symmetric connection, curvature tensor, group manifold.

1. INTRODUCTION

Hayden [1] introduced semi-symmetric metric connection on a Riemannian manifold and this definition was developed by Yano [2] and Imai [3-4].

In this paper, we define a semi-symmetric connection on a Weyl manifold and define the curvature tensor with respect to semi-symmetric connection. We give some theorems by means of a relation between curvature tensors with respect to semi-symmetric connection and symmetric connection. After defining projective curvature tensor and conformal curvature tensor with respect to semi-symmetric connection,we obtain some theorems by using properties of these tensors. In the last section of the paper, with the help of [5] , we examine Weyl group manifolds.

2. THE CURVATURE TENSOR WITH RESPECT TO SEMI-SYMMETRIC CONNECTION

An n-dimensional manifold which has a symmetric connection and a conformal metric tensor g_{ij} is said to be Weyl manifold , if the compatible condition is in the form of $\nabla_k g_{ij} - 2g_{ij}T_k = 0$. In this case, Weyl manifold is denoted by $W_n(g_{ij}, T_k)$. T_k, which is a covariant vector, is called complementary vector of the manifold. If $T_k = 0$ or T_k is gradient, then Riemannian manifold is obtained.

T_k changes by $\hat{T}_k = T_k + \partial_k(\ln\lambda)$, under the transformation of the metric tensor g_{ij} in the form of $\hat{g}_{ij} = \lambda^2 g_{ij}$ with λ is a point function. According to this transformation, if the quantity A changes by $\hat{A} = \lambda^k A$, then the quantity A is called a satellite of g_{ij} with the weight of $\{k\}$ and the quantity $\dot{\nabla}_s A$, which is defined by $\dot{\nabla}_s A = \nabla_s A - kT_s A$, is called generalized covariant derivative of A.In this definition, $\nabla_s A$ denotes ordinary covariant derivative of A.

A generalized connection on a Weyl manifold is given by [6]

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + a_{jkh} g^{hi}, \text{ where } a_{jkh} = g_{jl}\Omega^l_{kh} + g_{lk}\Omega^l_{jh} + g_{lh}\Omega^l_{jk} \tag{2.1}$$

If Ω^i_{jk} is chosen by $\Omega^i_{jk} = \delta^i_j a_k - \delta^i_k a_j$ in (2.1), then a semi-symmetric connection on a Weyl manifold is defined by

$$\overline{\Gamma}^{i}_{jk} = \Gamma^{i}_{jk} + \delta^{i}_{k} S_{j} - g_{jk} S^{i}, \qquad \text{where } S_{i} = -2a_{i} \tag{2.2}$$

The torsion tensor T^{i}_{jk} with respect to semi-symmetric connection is defined by

$$T^{i}_{jk} = \delta^{i}_{k} S_{j} - \delta^{i}_{j} S_{k} \tag{2.3}$$

Analogous to the definition of the curvature tensor with respect to symmetric connection, we define the curvature tensor with respect to semi-symmetric connection by

$$\overline{R}^{h}_{ijk} = \partial_{j} \overline{\Gamma}^{h}_{ik} - \partial_{k} \overline{\Gamma}^{h}_{ij} + \overline{\Gamma}^{h}_{sj} \overline{\Gamma}^{s}_{ik} - \overline{\Gamma}^{h}_{sk} \overline{\Gamma}^{s}_{ij} \tag{2.4}$$

Remembering the definition of the curvature tensor with respect to symmetric connection and using (2.2) and (2.4), we have:

$$\overline{R}^{h}_{ijk} = R^{h}_{ijk} + \delta^{h}_{k} S_{ij} - \delta^{h}_{j} S_{ik} + g_{ij} g^{hl} S_{lk} - g_{ik} g^{hl} S_{lj} \tag{2.5}$$

The tensor S_{ij} in (2.5) is defined by

$$S_{ij} = \nabla_{j} S_{i} - S_{i} S_{j} + \frac{1}{2} g_{ij} g^{kl} S_{k} S_{l} \tag{2.6}$$

where $\nabla_{j} S_{i}$ denotes covariant derivative with respect to symmetric connection.

Multiplying (2.5) by g_{mh},

$$\overline{R}_{mijk} = R_{mijk} + g_{mk} S_{ij} - g_{mj} S_{ik} + g_{ij} S_{mk} - g_{ik} S_{mj} \tag{2.7}$$

Multiplying (2.7) by g^{mk},

$$\overline{R}_{ij} = R_{ij} + (n-2) S_{ij} + S g_{ij}, \text{ where } S = g^{mk} S_{mk} \tag{2.8}$$

By using the definitions of $\overline{R}$ and R,

$$\overline{R} = R + 2\,(n-1)\,S \tag{2.9}$$

Lemma 2.1: The tensor S_{ij} is symmetric if and only if S_{i} is gradient.

Proof: It is shown easily by using (2.6).

Theorem 2.1: The curvature tensor with respect to semi-symmetric connection has the following properties:

(i) $\overline{R}_{mijk} + \overline{R}_{mikj} = 0.$

(ii) $\overline{R}_{mijk} + \overline{R}_{imjk} = 2g_{mi} \, (T_{j,k} - T_{k,j})$

(iii) $\overline{R}^{h}_{hjk} = R^{h}_{hjk} = 2R_{[kj]}$

(iv) $\overline{R}^{h}_{ijk} + \overline{R}^{h}_{jki} + \overline{R}^{h}_{kij} = 2\,(\delta^{h}_{i} \nabla_{[k} S_{j]} + \delta^{h}_{j} \nabla_{[i} S_{k]} + \delta^{h}_{k} \nabla_{[j} S_{i]}$

Proof:

(i) It is obtained by adding (2.7) and the equation by changing the last two indices in (2.7).

(ii) By using (2.7) and remembering the same relation with respect to symmetric connection , the required result is obtained.

(iii) It is easily seen by making contraction with respect to indices h and I in (2.5).
(iv) It is shown by changing indices i,j and k cyclically in (2.5).

Corollary 2.1: If S_i is gradient on a Weyl manifold with semi-symmetric connection, then the followings hold:

(i) $\overline{R}^{\,h}_{ijk} + \overline{R}^{\,h}_{jki} + \overline{R}^{\,h}_{kij} = 0.$

(ii) $\dot{\nabla}_l T^k_{ij} + \dot{\nabla}_i T^k_{jl} + \dot{\nabla}_j T^k_{li} = 0.$

Theorem 2.2: If the curvature tensors with respect to semi-symmetric and symmetric connection coincide, then S_i is gradient.

Proof: Let $\overline{R}_{mijk} = R_{mijk}$. From (2.7),

$$g_{mk} S_{ij} - g_{mj} S_{ik} + g_{ij} S_{mk} - g_{ik} S_{mj} = 0$$

Multiplying both sides of (2.10) by g^{mk},

$$(n\text{-}2)S_{ij} + Sg_{ij} = 0 \qquad (2.11)$$

If (2.11) is multiplied by g^{ij},

$$S = 0 \qquad (2.12)$$

By using (2.12) in (2.11), it is found that $S_{ij} = 0$. This shows that S_i is gradient.

Definition 2.1: A semi-symmetric connection is said to be local-flat, if the curvature tensor with respect to semi-symmetric connection.

Theorem 2.3: If the semi-symmetric connection defined on a Weyl manifold is local-flat, then T_k is gradient.

Proof: From (2.7) and Definition 2.1 ,

$$R_{mijk} = g_{mj} S_{ik} - g_{mk} S_{ij} + g_{ik} S_{mj} - g_{ij} S_{mk} \qquad (2.13)$$

Multiplying (2.13) by g^{mi},

$$n(T_{j,k} - T_{k,j}) = 0 \qquad (2.14)$$

(2.14) shows that T_k is gradient.

3. THE PROJECTIVE CURVATURE TENSOR WITH RESPECT TO SEMI-SYMMETRIC CONNECTION

A generalized connection on a non-Riemannian manifold is denoted by $L^i_{jk} = \widetilde{\Gamma}^i_{jk} + \widetilde{\Omega}^i_{jk}$, where $\widetilde{\Gamma}^i_{jk}$ is symmetric part and $\widetilde{\Omega}^i_{jk}$ is anti-symmetric part , and the curvature tensor with respect to this connection is given by $L^i_{jkl} = B^i_{jkl} + \Omega^i_{jkl}$, where B^i_{jkl} denotes the terms with respect to symmetric part of the generalized connection. B^i_{jkl} is given with the following definition:

$$B^i_{jkl} = \partial_k \widetilde{\Gamma}^i_{jl} - \partial_l \widetilde{\Gamma}^i_{jk} + \widetilde{\Gamma}^i_{sk} \widetilde{\Gamma}^s_{jl} - \widetilde{\Gamma}^i_{sl} \widetilde{\Gamma}^s_{jk}$$

In this case the projective curvature tensor with respect to a generalized connection on a non-Riemannian manifold is defined in [7] by

$$\mathrm{W}^{i}_{jkl} = \mathrm{B}^{i}_{jkl} + \frac{\delta^i_j}{n+1}\left(B_{kl} - B_{lk}\right) + \frac{1}{n^2-1}\left(\delta^i_k B_{lj} - \delta^i_l B_{kj}\right) + \frac{n}{n^2-1}\left(\delta^i_k B_{jl} - \delta^i_l B_{jk}\right) \quad (3.1)$$

where $\mathrm{B}_{jk} = \mathrm{B}^{i}_{jki}$.

Since $\widetilde{\Gamma}^{i}_{jk} = \Gamma^{i}_{jk} + \left(\delta^i_k S_j + \delta^i_j S_k\right)$ and $\widetilde{\Omega}^{i}_{jk} = \frac{1}{2}\left(\delta^i_k S_j - \delta^i_j S_k\right)$ for the semi-symmetric connection on a Weyl manifold, we define B^{i}_{jkl} with respect to semi-symmetric connection as follows:

$$\mathrm{B}^{i}_{jkl} = \overline{R}^{i}_{jkl} + \delta^i_j \nabla_{[k} \mathrm{S}_{l]} + \delta^i_k \overline{S}_{jl} - \delta^i_l \overline{S}_{jk} \quad (3.2)$$

where $\overline{S}_{jk} = \frac{1}{2}\left(S_{jk} - \frac{1}{2} S_k S_j + \frac{1}{2} g_{jk} g^{sr} S_s S_r\right)$

By making contraction on the indices i and l in (3.2), we get:

$$\mathrm{B}_{jk} = \overline{R}_{jk} + \nabla_{[k} S_{j]} - (n-1)\overline{S}_{jk} \quad (3.3)$$

The equation by changing indices j and k in (3.3) is subtracted from (3.3),

$$B_{jk} - B_{kj} = (\overline{R}_{jk} - \overline{R}_{kj}) - (\text{n-3})\nabla_{[k} S_{j]} \quad (3.4)$$

By using (3.2), (3.3) and (3.4) are used in (3.1), we obtain projective curvature tensor with respect to semi-symmetric connection on a Weyl manifold as follows:

$$\overline{W}^{i}_{jkl} = \overline{R}^{i}_{jkl} + \frac{\delta^i_j}{n+1}\left\{\left(\overline{R}_{kl} - \overline{R}_{lk}\right) + 2(n-1)\nabla_{[k} S_{l]}\right\} + \frac{1}{n^2-1}\left(\delta^i_k \overline{H}_{jl} - \delta^i_l \overline{H}_{jk}\right) \quad (3.5)$$

where $\overline{H}_{jl} = n\overline{R}_{jl} + \overline{R}_{lj} + 2(n-1)\nabla_{[l} S_{j]}$.

Theorem 3.1: Projective curvature tensor with respect to semi-symmetric connection has the following properties:

(i) $\overline{W}^{i}_{jkl} + \overline{W}^{i}_{jlk} = 0$

(ii) $\overline{W}^{i}_{ikl} = 0$

(iii) $\overline{W}_{jk} = \frac{2(n-1)}{n+1}\nabla_{[k} S_{j]}$

(iv) $\overline{W}^{i}_{jkl} + \overline{W}^{i}_{klj} + \overline{W}^{i}_{ljk} = 0$

Proof:

(i) It is obtained by adding (3.5) and the relation by changing the indices j and k in (3.5).

(ii) By contraction on the indices i and j in (3.5), the required result is obtained.

(iii) It is easily seen by contraction on the indices i and l in (3.5).

(iv) I.Bianchi identity for projective curvature tensor is shown by changing the indices i,j and k cyclically and using (3.5).

Corallary 3.1: $\overline{W}_{jk} = 0$ if and only if S_k is gradient.

Theorem 3.2: Projective curvature tensors with respect to semi-symmetric connection and symmetric connection are related with the following relation:

$$\overline{W}^{i}_{jkl} = W^{i}_{jkl} + 2\nabla_{[k}S_{l]} + \frac{1}{n^2-1}\left(\delta^i_l K_{jk} - \delta^i_k K_{jl}\right) + g_{jk}g^{ir}S_{rl} - g_{jl}g^{ir}S_{rk} \tag{3.6}$$

where $K_{jl} = nS_{jl} + S_{lj} + (n+1)Sg_{jl}$

Proof: By choosing $S_k = 0$ in (3.5), the projective curvature tensor with respect to symmetric connection is obtained by

$$W^{i}_{jkl} = R^{i}_{jkl} + \frac{\delta^i_j}{n+1}\left(R_{kl} - R_{lk}\right) + \frac{1}{n^2-1}\left(\delta^i_k H_{jl} - \delta^i_l H_{jk}\right) \tag{3.7}$$

where $H_{jl} = nR_{jl} + R_{lj}$.

By virtue of (2.5),(3.5) and (3.7), (3.6) is obtained.

Definition 3.1: A semi-symmetric connection is said to be projectively-flat, if the projective curvature tensor with respect to semi-symmetric connection vanishes.

Theorem 3.3: If a semi-symmetric connection is local-flat and S_k is gradient, then the connection is also projectively flat.

Proof: By using Definition 2.1 and $\nabla_{[k}S_{l]} = 0$, the required result is found.

4. THE CONFORMAL CURVATURE TENSOR WITH RESPECT TO SEMI-SYMMETRIC CONNECTION

Under conformal transformation, the metric tensor g_{ij} and the semi-symmetric connection $\overline{\Gamma}^i_{jk}$ on a Weyl manifold are transformed as follows:

$$g^*_{ij} = g_{ij} \tag{4.1}$$

$$\overline{\Gamma}^{i\,*}_{jk} = \overline{\Gamma}^i_{jk} + \delta^i_j P_k + \delta^i_k (P_j - Q_j) - g_{jk}(P^i - Q^i) \tag{4.2}$$

where $P_k - P^*_k = T_k$ and $S_k - S^*_k = Q_k$.

With the help of (4.1) and (4.2), we define the change of curvature tensor under conformal transformation with respect to semi-symmetric connection by

$$\overline{R}^{h\,*}_{ijk} = \overline{R}^h_{ijk} + 2\delta^h_i\left(\nabla_{[j}P_{k]} + P_{[j}S_{k]}\right) + \delta^h_k W_{ij} - \delta^h_j W_{ik} + g_{ij}g^{hl}W_{lk} - g_{ik}g^{hl}W_{lj} + 2g^{sl}P_s Q_l G^h_{ijk}$$

where $W_{ij} = \underline{P}_{ij} - \underline{Q}_{ij} + 2P_{(i}Q_{j)}$, $\underline{P}_{ij} = P_{ij} - P_i S_j$, $\underline{Q}_{ij} = Q_{ij} - Q_i S_j$ (4.3)

Multiplying (4.3) by $g_{mh}{}^* = g_{mh}$, we have the equality numbered by (4.4):

$$\overline{R}^*_{mijk} = \overline{R}_{mijk} + 2g_{mi}\left(\nabla_{[j}P_{k]} + P_{[j}S_{k]}\right) + g_{mk}W_{ij} - g_{mj}W_{ik} + g_{ij}W_{mk} - g_{ik}W_{mj} + 2g^{sl}P_s Q_l G_{mijk}$$

Multiplying (4.4) by $g^{mk\,*} = g^{mk}$,

$$\overline{R_{ij}}^{\,*} = \overline{R}_{ij} + 2\left(\nabla_{[j}P_{i]} + P_{[j}S_{i]}\right) + (n-2)W_{ij} + g_{ij}W^m_m - 2(n-1)g_{ij}g^{sl}P_s Q_l \tag{4.5}$$

Consequently from (4.5),

$$\overline{R}^* = \overline{R} + 2(n-1)(W^m_m - ng^{sl}P_s Q_l) \tag{4.6}$$

where $W^m_m = g^{mk}W_{mk}$.

From (4.5), we have W_{ij} defined by the following eguation by (4.7)

$$\frac{(n-1)(\overline{R}_{ij}{}^{*}-\overline{R}_{ij})+(\overline{R}_{ji}{}^{*}-\overline{R}_{ji})+2(n-2)(\nabla_{[i}Q_{j]}+Q_{[i}S_{j]}-ng_{ij}W_m^m+2n(n-1)g_{ij}g^{sl}P_sQ_l}{n(n-2)}$$

From the definition of W_m^m,

$$W_m^m=\frac{\overline{R}^*-\overline{R}}{2(n-1)}+ng^{sl}P_sQ_l \text{ or equivalently } ng^{sl}P_sQ_l=W_m^m-\frac{(\overline{R}^*-\overline{R})}{2(n-1)} \tag{4.8}$$

On the other hand, if (4.4) is multiplied by $g^{mi\,*}=g^{mi}$,

$$\overline{R}^*_{mijk}\,g^{mi\,*}=\overline{R}_{mijk}g^{mi}+2n(\nabla_{[j}P_{k]}+P_{[j}S_{k]}\ ) \tag{4.9}$$

By using (4.9) in (4.7), we obtain:

$$W_{ij}=\frac{n(\overline{R}^*_{ij}-\overline{R}_{ij})-(\overline{R}^*_{mkji}-\overline{R}_{mkji}g^{mk})+(n-2)g_{ij}W_m^m-g_{ij}(\overline{R}^*-\overline{R})}{n(n-2)} \tag{4.10}$$

Putting (4.9) and (4.10) in (4.4); we have:

$$\overline{C}^*_{mijk}=\overline{C}_{mijk} \tag{4.11}$$

where $\overline{C}_{mijk}$ is defined by

$$\overline{C}_{mijk}=\overline{R}_{mijk}-\frac{2}{n}g_{mi}\overline{R}^l_{ljk}+\frac{1}{n-2}\overline{A}_{mijk}-\frac{1}{n(n-2)}\overline{B}_{mijk}-\frac{\overline{R}}{(n-1)(n-2)}G_{mijk} \tag{4.12}$$

where $\overline{A}_{mijk}=g_{mj}\overline{R}_{ik}-g_{mk}\overline{R}_{ij}-g_{ij}\overline{R}_{mk}+g_{ik}\overline{R}_{mj}$

and $\overline{B}_{mijk}=g_{mj}\overline{R}^h_{hki}-g_{mk}\overline{R}^h_{hji}-g_{ij}\overline{R}^h_{hkm}+g_{ik}\overline{R}^h_{hjm}$

Multiplying (4.12) by g^{mh}; we find the conformal curvature tensor with respect to semi-symmetric connection in the following form:

$$\overline{C}^h_{ijk}=\overline{R}^h_{ijk}-\frac{1}{n}\delta^h_i\overline{R}^l_{ljk}+\frac{1}{n-2}\overline{A}^h_{ijk}-\frac{1}{n(n-2)}\overline{B}^h_{ijk}-\frac{\overline{R}}{(n-1)(n-2)}G^h_{ijk} \tag{4.13}$$

where $\overline{A}^h_{ijk}=\delta^h_j\overline{R}_{ik}-\delta^h_k\overline{R}_{ij}-g_{ij}g^{mh}\overline{R}_{mk}+g_{ik}g^{mh}\overline{R}_{mj}$

and $\overline{B}^h_{ijk}=\delta^h_j\overline{R}^l_{lki}-\delta^h_k\overline{R}^l_{lji}-g_{ij}g^{mh}\overline{R}^l_{lkm}+g_{ik}g^{mh}\overline{R}^l_{ljm}$.

Theorem 4.1: The conformal curvature tensor with respect to semi-symmetric connection has the following properties:

(i) $\overline{C}_{mijk}+\overline{C}_{mikj}=0$

(ii) $\overline{C}_{mijk}+\overline{C}_{imjk}=0$

(iii) $\overline{C}^i_{ijk}=0$

(iv) $\overline{C}^i_{jki}=\overline{C}_{jk}=0$

(v) $\overline{C}^h_{ijk}+\overline{C}^h_{jki}+\overline{C}^h_{kij}=0$.

Proof:

(i) With the help of (4.12), it is written the definition of $\overline{C}_{mijk}$. By adding this and (4.12), the required result is obtained.
(ii) If the process in (i) is applied to the first two indices in (ii), the result is found.
(iii) It is found by contraction on the indices h and i in (4.13).
(iv) The same process is applied to the indices h and k in (4.13).
(v) By changing cyclically the indices i,j and k , three equations are written. And by adding these equations, the required result is obtained.

Theorem 4.2: Conformal curvature tensors with respect to symmetric connection and semi-symmetric connection on a Weyl manifold coincide.
Proof: Conformal curvature tensor with respect to symmetric connection is defined with the help of (4.13) by

$$C^h_{ijk} = R^h_{ijk} - \frac{1}{n}\delta^h_i R^l_{ljk} + \frac{1}{n-2}A^h_{ijk} - \frac{1}{n(n-2)}B^h_{ijk} - \frac{R}{(n-1)(n-2)}G^h_{ijk} \tag{4.14}$$

where $A^h_{ijk} = \delta^h_j R_{ik} - \delta^h_k R_{ij} - g_{ij} g^{mh} R_{mk} + g_{ik} g^{mh} R_{mj}$

and

$$B^h_{ijk} = \delta^h_j R^l_{lki} - \delta^h_k R^l_{lji} - g_{ij} g^{mh} R^l_{lkm} + g_{ik} g^{mh} R^l_{ljm}.$$

Using (2.5) in (4.13) and with the help of (4.14), the required result is obtained.

Definition 4.1: A semi-symmetric connection is called conformal-flat, if the conformal curvature tensor with respect to semi-symmetric connection vanishes.
Definition 4.2: A manifold is called conformal-flat, if the conformal curvature tensor with respect to symmetric connection vanishes.
Corallary 4.1: Semi-symmetric connection defined on a Weyl manifold is conformal-flat if and only if the Weyl manifold is conformal-flat.
Theorem 4.3: If semi-symmetric connection defined on a Weyl manifold is local-flat, then it is also conformal-flat.
Proof: $\overline{R}_{mijk} = 0$ implies $\overline{R}_{ij} = 0$ and $\overline{R} = 0$. If the results are used in (4.13), we find $\overline{C}^h_{ijk} = 0$ meaning the connection is conformal-flat.

5. WEYL GROUP MANIFOLDS

Definition 5.1: If the curvature tensor with respect to semi-symmetric connection and the tensor S_{ij} vanish on a Weyl manifold, then that one is called Weyl group manifold.

By above definition we have the following lemmas:
Lemma 5.1: Weyl group manifold is local-flat.
Lemma 5.2: S_i is gradient on a Weyl group manifold.
Theorem 5.1: Projective curvature tensors with respect to symmetric connection and semi-symmetric connection coincide on a Weyl group manifold.
Proof: By using Definition 5.1 and Lemma 5.1 in (3.7), the required result is obtained.

Corallary 5.1: Semi-symmetric connection defined on a Weyl group manifold is projectively-flat if and only if Weyl group manifold is projectively-flat.

Theorem 5.2: A Weyl group manifold is conformal-flat.
Proof: If the conditions $\overline{R}_{ijk}^{h} = 0,\ S_{ij} = 0,\ R_{ijk}^{h} = 0$ are used in (4.14), the result is found.
Theorem 5.3: A Weyl group manifold is projectively-flat.
Proof: If the same conditions in the proof of Theorem 5.2 are used in (3.8),the required result is obtained.
Theorem 5.4: Conformal curvaure tensor and projective curvature tensor with respect to semi-symmetric connection coincide on a Weyl group manifold.
Proof: Conformal curvature tensor, C_{ijk}^{h} , is defined in terms of the projective curvature tensor with respect to symmetric connection by the following relation:

$$W_{ijk}^{h} + 2D_{ijk}^{h} + \frac{1}{(n-1)(n-2)}\left[\delta_j^h R_{ik} - \delta_k^h R_{ij} + (n-1)(g_{ik} g^{hm} R_{mj} - g_{ij} g^{hm} R_{mk}) - RG_{ijk}^{h}\right]$$

In this relation D_{ijk}^{h} is defined as follows:

$$\frac{\delta_i^h}{n(n+1)} R_{[jk]} - \frac{1}{n(n-2)}(\delta_j^h R_{[ik]} - \delta_k^h R_{[ij]} + g_{ik} g^{hm} R_{[mj]} - g_{ij} g^{hm} R_{[mk]}) + \frac{1}{n^2-1}(\delta_j^h R_{[ik]} - \delta_k^h R_{[ij]})$$

.

On the other hand by using the facts that $\overline{C}_{ijk}^{h} = C_{ijk}^{h}$ (always) and $\overline{W}_{ijk}^{h} = W_{ijk}^{h}$ for Weyl group manifolds, the equation of $\overline{C}_{ijk}^{h} = \overline{W}_{ijk}^{h}$ is obtained.
Corollary 5.2: Semi-symmetric connection defined on a Weyl group manifold is conformal flat if and only if it is projectively-flat.

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