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DYNAMIC OPTIMIZATION ALGORITHM FOR VERTICAL ALIGNMENT OF HIGHWAYS

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Abstract- In all design criteria, earthwork optimization is essential for the determination of optimal vertical alignment of a highway project. Actually, there can be numerous grade line alternatives for given ground elevations; thus, the objective is to find out the optimal vertical alignment causing the minimum cost considering several design constraints. Basically, this is a constrained nonlinear optimization problem, which can be solved by a multistage optimization technique. This article introduces a methodology for the vertical alignment optimization of a highway using dynamic programming approach. Results indicated that the introduced algorithm can be used successfully in the optimization of vertical highway alignment.

Keywords- vertical alignment, earthwork, least-square, dynamic programming.

1. INTRODUCTION

Basically, highway alignment is a three-dimensional optimization problem subject to nonlinear design constraints and multivariate objective function. The complex behavior of three-dimensional highway alignment requires the solution of a multistage optimization problem. However, considering the variation of design alternatives and the number of design parameters, it is difficult to solve this problem in three-dimensional manner. For this reason, highway alignment studies are generally considered in two parts, in which horizontal alignment is first determined and vertical alignment is adjusted by changing design controls [1].

In the literature, there are numerous studies focusing on the solution of highway alignment optimization problem. Initially, Turner and Miles [2] developed an alignment model that searches the shortest paths on calculated grid networks characterizing relative costs. After this earlier study, Shaw and Howard [3] and Trietsch [4] implemented different horizontal highway alignment models. However, the majority of researchers focused on the development of vertical alignment models for predetermined horizontal alignments. In this context, several optimization methodologies, such as linear programming, dynamic programming, state parameterization, and genetic algorithm, were employed to compute optimal vertical alignment of highway [5-12]. Apart from these, hypothetical weighted ground elevation concept recently suggested to determine optimum grades practically for hand and computer calculations [13,14].

Previously implemented dynamic programming based vertical alignment algorithms used an objective function, in which earthwork amount is considered in terms of elevational differences at two points of each road segments [8,11]. In this study, least-square based objective function was utilized in dynamic programming based optimization algorithm, in which relative differences between grade and ground elevations were calculated at multiple locations to obtain precise alignments. Furthermore, comprehensive geometric design constraints were also adapted into implemented algorithm with accordance to AASHTO-2001 specifications [15]. In order to evaluate the success of implemented algorithm, it was supported by a numerical example. Outcomes of the numerical study indicated that introduced algorithm is precise and successful for the solution of the optimization problem.

2. DYNAMIC PROGRAMMING BASICS

In essence, dynamic programming is a multistage constrained optimization algorithm, in which the problem is divided into several phases. In this method, each sub-problem involving single variable is iteratively solved in these successive phases. It should be noted that, the outcome of a single sub-problem is the input of the next sub-problem; thereby, the objective is satisfied when the last sub-problem is solved. There can be two possible calculation directions in dynamic programming algorithm, namely forward and backward. As the name implies, backward computation is performed through the start point from the end point. In Fig.1, processing scheme of a backward dynamic programming model is depicted. As can be derived from Fig.1, u_i defines the decision variables that are changed iteratively to minimize the cumulative cost (Ti). It should be added that, system calculates the output variable f_{i-1} for the i^{th} stage using output variable f_i that is computed at the previous stage. Therefore, cost values of each stage (C_i) is then calculated to evaluate the measure of success. Consequently, the optimum cost is obtained after the final stage is performed.

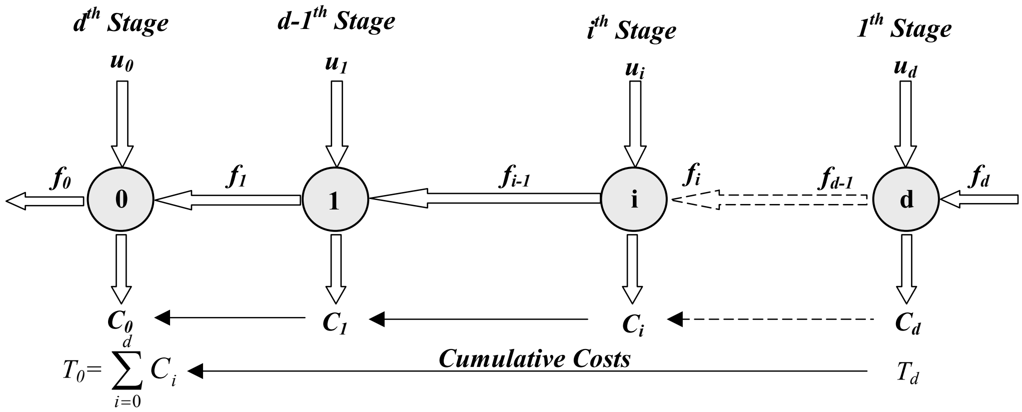

Figure 1. Illustration of backward dynamic programming processing scheme

Mathematically, the general formulation of this cumulative cost at the i^{th} stage is given as follows:

$$T_i(f_{i-1}) = \min_{u_i}\left[C_i(u_i, f_i) + T_{i+1}(f_i)\right] \qquad i = d-1, d-2, \ldots\ldots\ldots, 1 \tag{1}$$

where, T_i represents cumulative cost calculated at the i^{th} step of optimization process, and d is the number of stages. Virtually, decision variables (u_i) are iteratively changed to minimize the stage costs (C_i) until the principal of optimality is satisfied. Successively, optimal u_i and C_i values are stored for stage by stage, and next stage is processed using previous stages output values (f_i). Additional knowledge on dynamic programming can be found elsewhere [16,17].

3. IMPLEMENTED VERTICAL ALIGNMENT ALGORITHM

In order to adapt the discrete dynamic programming approach to vertical alignment optimization problem, it is necessary to partition the problem area by predetermined by vertical and horizontal boundaries into finite number of elements. In this context, horizontal and vertical grid lines are plotted to constitute grid elements that will be necessary for iterative adjustment process. Therefore, each interval are considered to be the finite stages of the multistage optimization problem, in which objective is to minimize the cost of each individual stage. In Fig.2, L x H dimensional calculation grid and computed vertical alignment is illustrated.

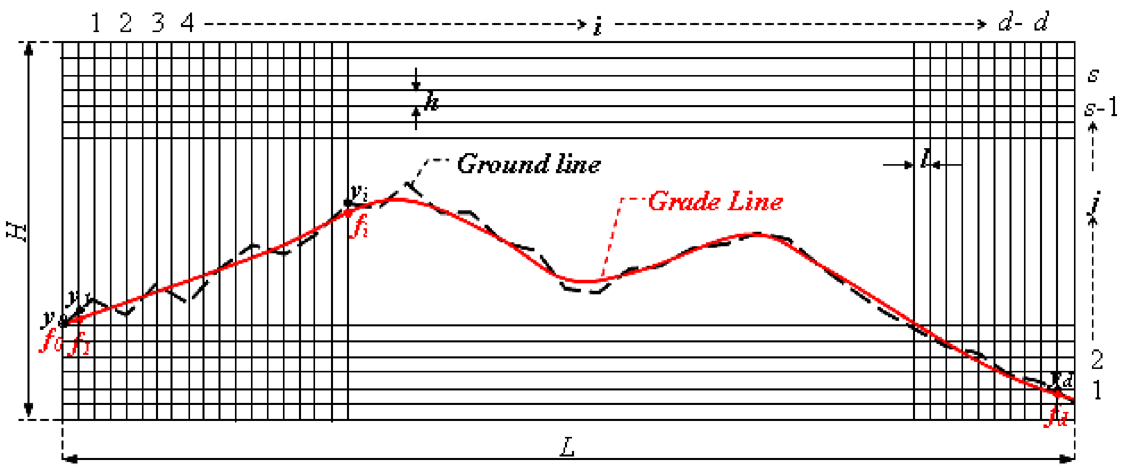

Figure 2. Profile for the illustration of dynamic programming model

Obviously, the row height (h) and column width (l) of each grid (finite element) can be calculated as follows:

$$l = \frac{L}{d+1} \quad and \quad h = \frac{H}{s+1} \tag{2}$$

where, d and s are the numbers of horizontal and vertical grid lines, respectively. Referring to Fig.2 again, ground line is indicated by y_i as well as f_i denotes the grade line. It is essential that, the value of l and h, thus the size of the grid is crucial for the precision and the computational complexity of the algorithm.

In the first step of the vertical alignment algorithm presented here, the STAs (distances from the beginning) intersecting with vertical grid lines are added to the ground line data, and related ground elevations are computed. The flow chart of the first procedure used in this algorithm is illustrated in Fig.3. As can be derived from the figure, $\mathbf{G}(n,2)$ is two-dimensional ground line matrix, in which $\mathbf{G}(n,1)$ represents y coordinate (elevation) and $\mathbf{G}(n,2)$ exhibits x coordinate (STA). In addition, n' is the total number of STAs modified in this procedure, q is the number of vertical grid lines, u is the number of horizontal grid lines, and $\mathbf{p}(q)$ is the vector storing the indices of intersections. It should be added that, the elevation of an intersecting STA is calculated by following routine:

$$\mathbf{G}(j,1) = \mathbf{G}(j-1,1) + \frac{[\mathbf{G}(j+1,1) - \mathbf{G}(j-1,1)]}{[\mathbf{G}(j+1,2) - \mathbf{G}(j-1,2)]} \times [j \times l - \mathbf{G}(j-1,2)] \tag{3}$$

where, j is the index number and l represents the spacing between two vertical grid lines.

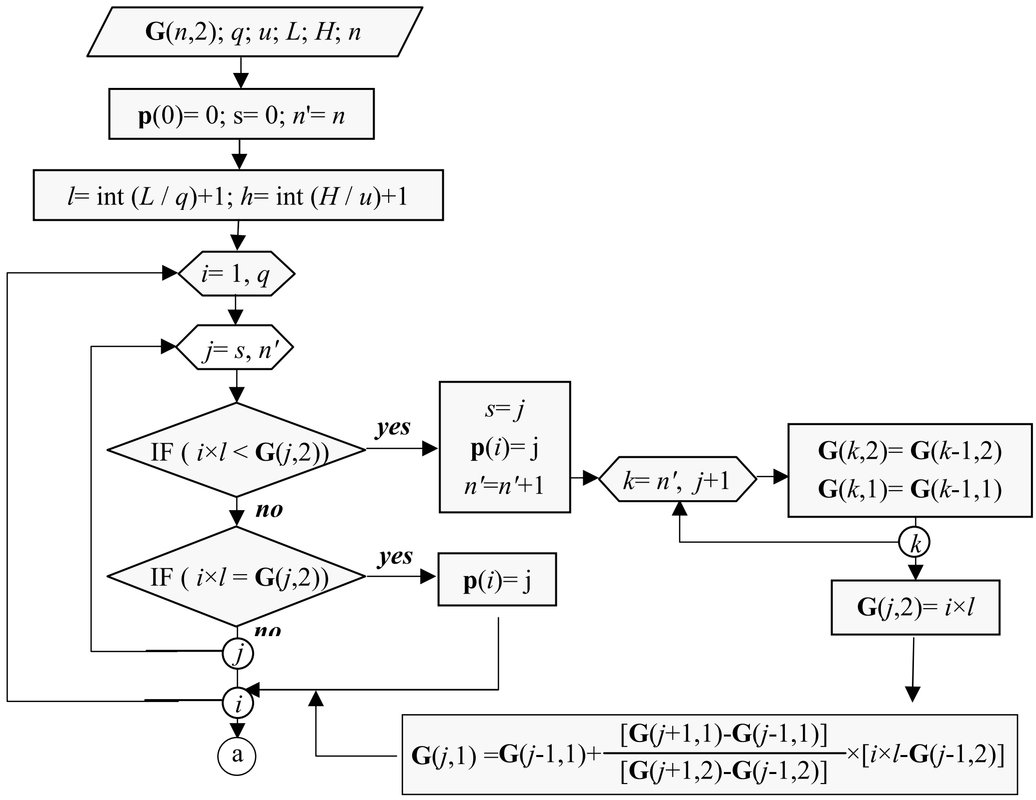

Figure 3. The procedure for the calculation of intersected STAs and related elevations

As mentioned before, Goh and Chew (1988) was first to employ dynamic programming approach for the solution of vertical alignment optimization problem. In this earlier attempt, two centerline elevations (at the beginning and at the end of each alignment segment) and four different objective functions were used for the calculation of earthwork cost. However, this approach may be misleading with respect to the vertical grid spacing and the variation of ground elevations. For this reason, the least-square based objective function, which considers each grade elevation comprised within the line segment, is employed in this study. The formulation of the objective function $C(u_i)$ is given by:

$$\mathbf{C}(i)=\left[\mathbf{R}(\mathbf{p}(i-1),1)+\frac{\mathbf{R}(\mathbf{p}(i),1)-\mathbf{R}(\mathbf{p}(i-1),1)}{\mathbf{G}(\mathbf{p}(i),2)-\mathbf{G}(\mathbf{p}(i-1),2)}\times\left(\mathbf{G}(\mathbf{p}(i-1)+j,2)-\mathbf{G}(\mathbf{p}(i-1),2)\right)-\mathbf{G}(\mathbf{p}(i-1)+j,1)\right]^2 \quad (4)$$

in which, $\mathbf{R}(n',2)$ denotes two-dimensional grade line matrix involving x and y coordinates similar to ground line matrix, and $\mathbf{C}(i)$ is the cost value for i^{th} stage. In the second part of the presented algorithm, successive grade line coordinates are determined by dynamic programming approach. In Fig.4, calculation scheme of dynamic programming adoption is presented.

It should be mentioned for Fig.4 that, $\mathbf{F}(i)$ vector stores calculated elevations of the grade line as well as *min* and *opt* are slag variables. Obviously, optimized piecewise grade line is subject to several design constraints determined by project specifications. In accordance with AASHTO (2001) design specifications, following geometric constraints are described in this study:

$$\text{IF } |\mathbf{R}(\mathbf{p}(i+1),1) - \mathbf{R}(\mathbf{p}(i),1)| \le g_{max} \times l \quad (5)$$

where, $g_{\max}$ is maximum allowable gradient. In addition, following constraints are given for crest vertical curves:

$$\begin{aligned}&\frac{|\mathbf{R}(\mathbf{p}(i+1),1)-\mathbf{R}(\mathbf{p}(i),1)|}{l}\le\frac{100\left(\sqrt{2h_d}+\sqrt{2h_0}\right)^2}{L_v-2S} && ,if\ L_v\le S\\ &\frac{|\mathbf{R}(\mathbf{p}(i+1),1)-\mathbf{R}(\mathbf{p}(i),1)|}{l}\le\frac{100Lv\ \left(\sqrt{2h_d}+\sqrt{2h_0}\right)^2}{S^2} && ,if\ L_v> S\end{aligned} \quad (6)$$

and next equations are given for sag vertical curves:

$$\begin{aligned}&\frac{|\mathbf{R}(\mathbf{p}(i+1),1)-\mathbf{R}(\mathbf{p}(i),1)|}{l}\le\frac{400+3.5S}{L_v-2S} && ,if\ L_v\le S\\ &\frac{|\mathbf{R}(\mathbf{p}(i+1),1)-\mathbf{R}(\mathbf{p}(i),1)|}{l}\le\frac{L_v\ \left(400+3.5S\right)}{S^2} && ,if\ L_v> S\end{aligned} \quad (7)$$

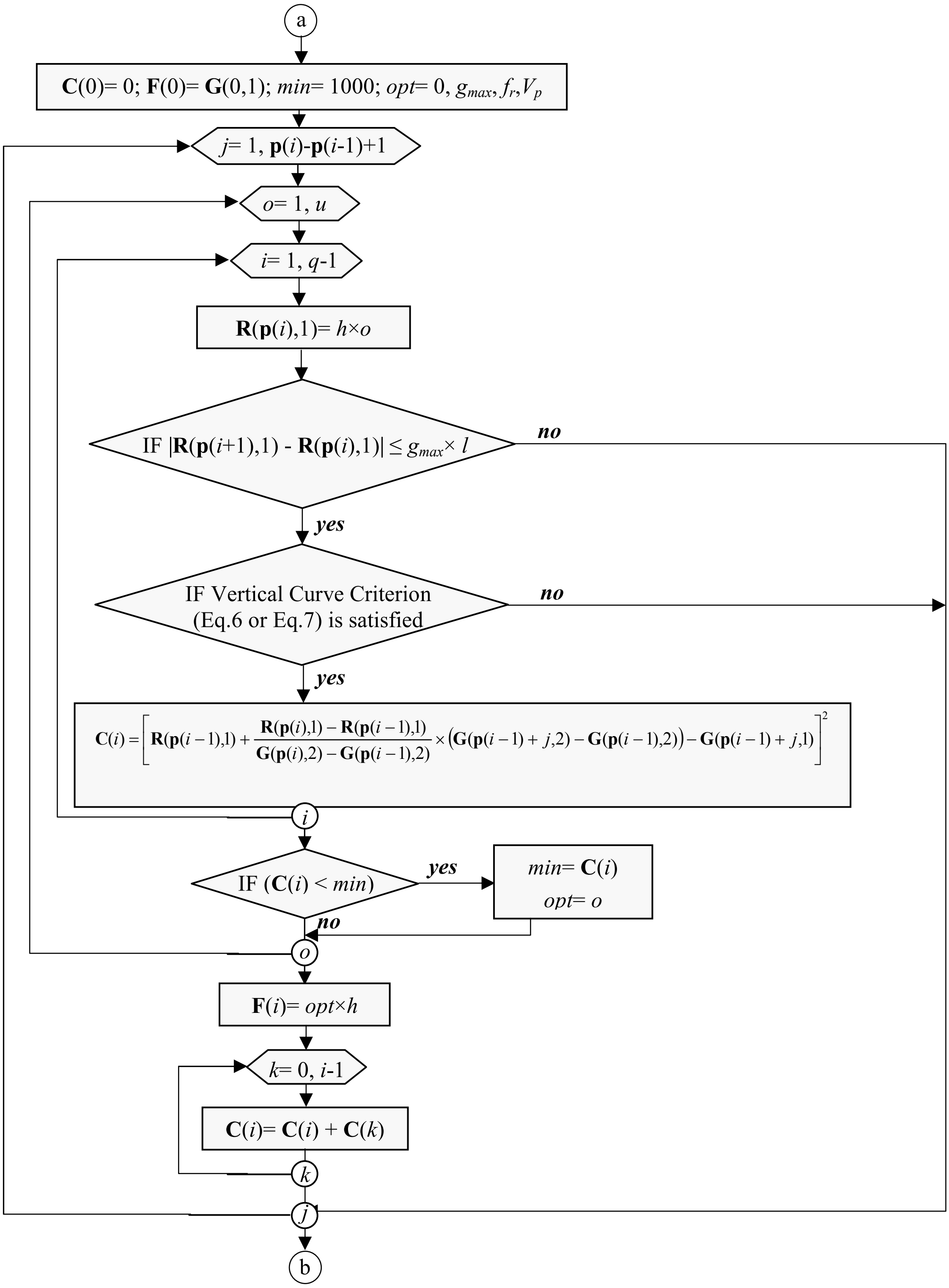

Figure 4. Calculation scheme of the second procedure comprising dynamic programming adoption

in which, L_v is vertical curve length, h_d is height of driver's head above the roadway surface, h_0 is the height of an object above roadway surface, and S is the sight distance. It should be noted that, boundary conditions for implemented algorithm are given by:

$$f_0 = y_0 \text{ and } f_{d+1} = y_{d+1} \tag{8}$$

Due to the backward directional optimization methodology of dynamic programming, the accumulated cost from the end point (x_{d+1}, f_{d+1}) to the processing point is minimized iteratively. Simply, the goal of optimal control is to calculate iteratively the optimum elevation change for each processing stage; thus, cumulative cost (T_i) is minimized. For starting point, optimal grade elevation (f_0) and optimal elevation increment ($\mathbf{R}(\mathbf{p}(i+1),1) - \mathbf{R}(\mathbf{p}(i),1)$) are given by the i^{th} stage by:

$$\mathbf{R}(\mathbf{p}(i+1),1) - \mathbf{R}(\mathbf{p}(i),1) = \chi\left(i, \frac{f_i}{h}\right) \tag{9}$$

where, χ is step size representing the elevational increment performing at the each stage of the dynamic programming progress. Therefore, the optimal grade elevation is obtained by the iterative process controlled by step size.

Consequently, in the last step of proposed algorithm grade line matrix is assigned iteratively. Details of this process are indicated in Fig.5. It should be noted that, final grade line and ground line matrices comprise n' elements.

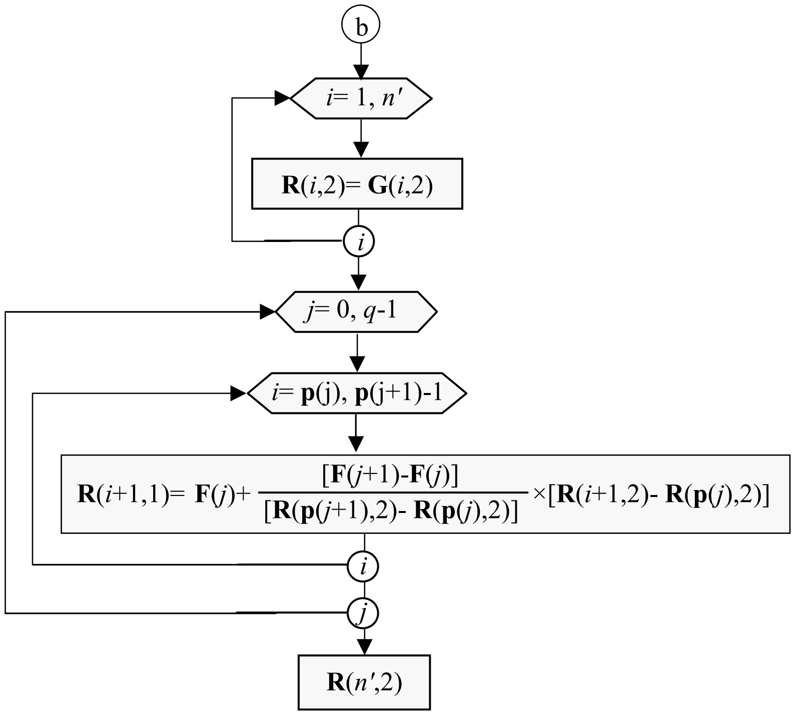

Figure 5. Final procedure for the calculation of final grade line matrix, R(n,2)

4. APPLICATION OF THE IMPLEMENTED ALGORITHM

In order to evaluate the performance of presented dynamic optimization algorithm, a numerical example is explained here. Details of considered roadway section in 2800m length are given in Table 1. Given these data, it is required to determine roadway grades between fixed origin and destination points minimizing the cost fraction. It should be noted that, horizontal alignment is predetermined, and the vertical alignment is performed by the implemented algorithm.

Table 1. Design variables for considered problem

Design variable	Symbol	Value
Maximum allowable gradient	g_{max}	6.00%
Vertical interval	h	0.6m
Horizontal interval	l	60.0m
Driver's height	h_0	1.0m
Object's height	h_d	0.5m
Length of the roadway	L	2800m
Length of vertical curves	L_v	700m
Sight distance	S	300m
Tangents of cut/fill slopes (X:Y)	m_C, / m_F	1:1 / 1:1

The resulting optimized road alignment and related ground line is given in Fig.6. With respect to optimized vertical alignment fill and cut volumes are calculated as 14333m^3 and 14921m^3, respectively. In addition, final R^2 value is obtained by 0.97, which can be considered as successful. Taking into consideration the profile given in Fig.6, calculated cut-fill volumes, and R^2 value obtained, it can be concluded that the presented algorithm is successful for the determination of optimal vertical alignment.

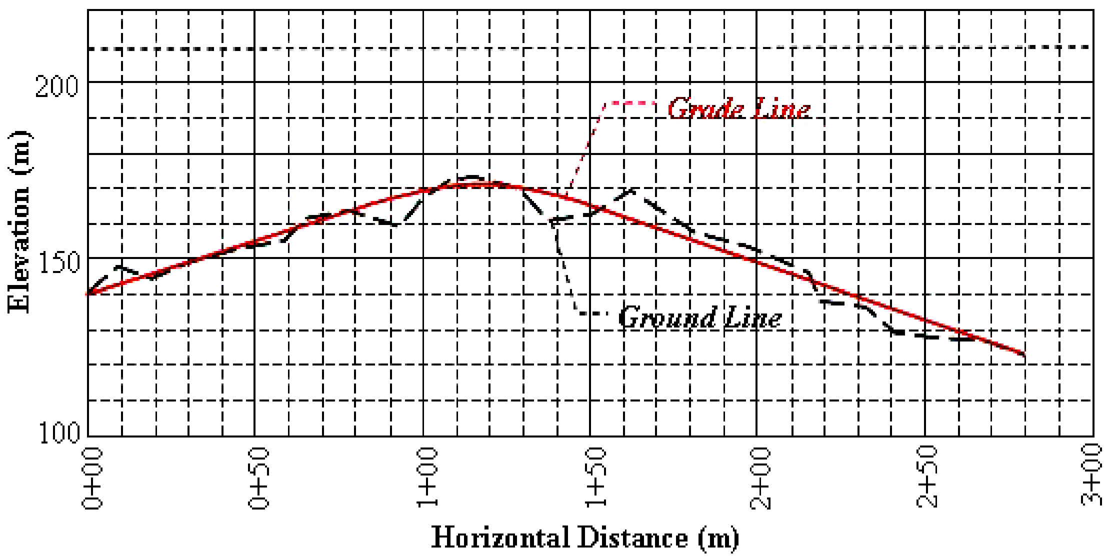

Figure 6. Profile for given numerical example

5. CONCLUSIONS

- In this study, dynamic programming approach was employed for the highway vertical alignment optimization problem. Apart from the previous studies in the literature, least-square based objective function was adopted into the algorithm. Results indicated that, the presented algorithm is quite successful for considered problem.
- It is possible to enhance the introduced algorithm involving additional cost components, such as pavement construction and vehicle operating costs. This can be accomplished by making small modifications in the objective function.
- Implemented dynamic algorithm is not also applicable for highway vertical alignment problem, but also applicable to the solution of a stepwise regression problem involving two-dimensional input-output data.
- The presented earthwork optimization algorithm can also be utilized in the other civil engineering disciplines involving earthwork construction.

REFERENCES

1. J. G. Schoon, *Geometric design projects for highways: an introduction*, ASCE Publication, New York, 2001.
2. A. K. Turner and R.D. Miles, A computer-assisted method of regional route location. Highway Research Record, 348, 1-15, 1971.
3. J. F. B. Shaw and B. E. Howard, Expressway route optimization by OCP, Journal of Transportation Engineering, ASCE, 10, 227-243, 1982.
4. N. R. Trietsch, Comprehensive design of highway networks, Transportation Science, 21, 26-35, 1987.
5. S. M. Easa, Selection of roadway grades that minimize earthwork cost using linear programming Transportation Res. Part A, 22, 121-136, 1988.
6. C. J. Goh and E. P. Chew, Discrete and continuous models for computation of optimal vertical highway alignment, Transportation Research Part B, 22, 399-409, 1988.
7. E. P. Chew, C. J. Goh, T. F. Fwa, Simultaneous optimization of horizontal and vertical alignments for highways. Transportation Research Part B, 23, 315-329, 1989.
8. T. F. Fwa, Highway vertical alignment analysis by dynamic programming, Transportation Research Record No.1239, Transportation Research Board, Washington D.C., 1-9, 1989.
9. A. M. Moreb, Linear programming model for finding optimal roadway grades that minimize earthwork cost, European Journal of Operational Research, 93, 148-154, 1996.
10. C. S. ReVelle, E. E. Whitlatch and J. R. Wright, Civil and Environmental Systems Engineering, Prentice Hall Inc., New Jersey, 1997.

11. T. F. Fwa, W. T. Cha and Y. P. Sim, Optimal vertical alignment for highway design, *Journal of Transportation Engineering*, ASCE, **128**, 395-402, 2002.

12. E. Kim, K. M. Jha and P. S. Schonfeld, Intersection construction cost functions for alignment optimization, *Journal of Transportation Engineering*, ASCE, **130**, 194-203, 2004.

13. A. B. Goktepe and A. H. Lav, Method for balancing cut-fill and minimizing the amount of earthwork in the geometric design of highways, *Journal of Transportation Engineering*, ASCE, **129**, 564-571, 2003.

14. A. B. Goktepe and A. H. Lav, Method for optimizing earthwork considering soil properties in the geometric design of highways, *Journal of Surveying Engineering*, ASCE, **130**, 183-190, 2004.

15. AASHTO, *A policy on geometric design of highways and streets*, American Association of State Highway and Transportation Officials, Washington, D.C, 2001.

16. A. D. Belegundu and T. R. Chandrupatla, *Optimization concepts and applications in engineering,* Prentice Hall Inc, New Jersey, 1999.

17. H. A. Taha, *Operations research: An introduction*, Prentice Hall Inc., New Jersey, 2002.