

$0^- \rightarrow 0^+$ FIRST FORBIDDEN β -DECAY MATRIX ELEMENTS IN SPHERICAL NUCLEI

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Abstract- In this study, the first forbidden beta decay matrix elements have been directly calculated within Saxon-Woods potential. Procedures for calculating the relevant matrix elements and combining them to form the decay rate are described. Calculations have been performed by two different methods. The values of our single particle matrix elements have been compared with the calculated values in the two different tables.

Keywords- Beta decay matrix element.

1. INTRODUCTION

The first forbidden beta decay matrix elements have been studied "for single-particle configurations in the region of ^{208}Pb " by Bohr [1].

The use of intermediate virtual excitations, other than the allowed ones has been advocated by some authors [2-9] in spite of the fact that the leptonic wave functions would include terms which are proportional to the product of the electron or neutrino velocity and the nuclear radius. Many researchers preferred "the use of virtual excitations which include additional terms instead of "allowed ones" [10,11].

In Ref. [11], the matrix element of the relativistic beta moment $M^\pm(\rho_A, \lambda=0)$ has not been calculated analytically, but assumed to be proportional to matrix element of non-relativistic beta moment $iM^\pm(j_A, k=1, \lambda=0)$. Generally, calculations have been made in the base of harmonic oscillator [11,12].

In our study, the first Forbidden β -Decay Matrix Elements $0^- \rightarrow 0^+$ for some nuclei have been studied. Our calculations have been done by two different methods. In the first method, the relativistic beta moment has been calculated directly without any assumption. In the second method, it was assumed that the relativistic beta moment is proportional to the non-relativistic one. Accordingly, our calculations have been done in the Saxon-Woods potential base. $M^\pm(\rho_A, \lambda=0)$ matrix element has been calculated analytically.

The theoretical approach for our calculations is given in Sect.2. Dependency of the radial parts of the matrix element on the Saxon-Woods potential and some conclusions are discussed in sect.3.

2. FORMALISM

In the "ξ approximation" the decay rates for $0^- \rightarrow 0^+$ transition can be written in form [1].

$$f_0 t \cdot B(\lambda\pi = 0^-) = D g_V^2 / 4\pi \quad (1)$$

where

$$B(\lambda\pi = 0^-) = \frac{1}{2I_i + 1} \left\| \left\langle I_f \right\| \pm M^\mp(\rho_A, \lambda = 0) - i \frac{m_e c}{\hbar} \xi \cdot M^\mp(j_A, \kappa = 1, \lambda = 0) \right\| I_i \right\|^2.$$

“First” matrix element is:

$$M^\mp(\rho_A, \lambda = 0) = (4\pi)^{-1/2} \frac{g_A}{c} \sum_k t_\mp(k) (\sigma(k) \cdot v_k). \quad (2)$$

“Second” matrix element is:

$$M^\mp(j_A, \kappa = 1, \lambda = 0) = g_A \sum_k t_\mp(k) r_k (Y_1(\hat{r}_k) \sigma(k))_0, \quad (3)$$

$$\xi \approx 1.2Z.A^{-1/3}, D=6250\text{sec}.$$

In these equations the upper and lower signs refer to β^- and β^+ decays, and g_V , g_A are vector and axial vector coupling constant, respectively.

The following formulas have been used in the calculations of the matrix elements of the operators that are given in Eqs. (2) and (3) [13].

$$\langle J, p \| r_k (Y_1(\hat{r}_k) \sigma(k))_0 \| J, n \rangle = (-1)^{\ell_n + J - 1/2} \cdot \sqrt{\frac{6(2J+1)(2\ell_n+1)}{4\pi}} \cdot \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \ell_n & \ell_p & J \end{Bmatrix} \langle \ell_n 0 1 0 | \ell_p 0 \rangle F_{np}, \quad (4)$$

where F_{np} is the neutron-proton overlap integral:

$$F_{np} = \int_0^\infty R_{nlp}^*(r) \cdot r^3 \cdot R_{nl_n}(r) dr,$$

R_{nl} is the radial part of single particle wave function, and

$$\begin{aligned} \langle J_1(p) \| (\vec{\sigma} \cdot \vec{\nabla}) \| J_2(n) \rangle &= \sqrt{2J_2 + 1} \delta_{J_1 J_2} \delta_{m_1 m_2} (-1)^{J_2 + \ell_1 - 1/2} \cdot \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \ell_2 & \ell_1 & J_2 \end{Bmatrix} \\ &\quad \sqrt{6} (\sqrt{\ell_2 + 1} A_{\ell_1 \ell_2} \delta_{\ell_1, \ell_2 + 1} - \sqrt{\ell_2} B_{\ell_1 \ell_2} \delta_{\ell_1, \ell_2 - 1}) \end{aligned} \quad (5)$$

where,

$$\begin{aligned} A_{\ell_p \ell_n} &= \int_0^\infty U_{\ell_p}^* \frac{\partial U_{\ell_n}}{\partial r} dr - (\ell_n + 1) \int_0^\infty U_{\ell_p}^* \frac{1}{r} U_{\ell_n} dr \\ B_{\ell_p \ell_n} &= \int_0^\infty U_{\ell_p}^* \frac{\partial U_{\ell_n}}{\partial r} dr + \ell_n \int_0^\infty U_{\ell_p}^* \frac{1}{r} U_{\ell_n} dr \end{aligned}$$

with $U_l = r R_{nl}$.

3. RESULTS AND DISCUSSIONS

In the present calculations, it has been used the eigenfunctions and eigenvalues of Schrödinger equation which is solved to Saxon-Woods Potential [14].

The calculations have been done for transition, namely $3p_{1/2}^n \rightarrow 3s_{1/2}^p$ and $2g_{9/2}^n \rightarrow 1h_{9/2}^p$ for nuclei $^{206-214}\text{Pb}$. The results are given in figures and tables. The wave functions at the radial integral of the first matrix element are shown in Fig. 1. The products with r and the wave functions at the radial integral of the second matrix element are presented in Fig. 2. It has been seen from the figures that the radial integrals

of transitions $3p_{1/2}^n \rightarrow 3s_{1/2}^p$ are very large from radial integrals that have been calculated for transitions $2g_{9/2}^n \rightarrow 1h_{9/2}^p$.

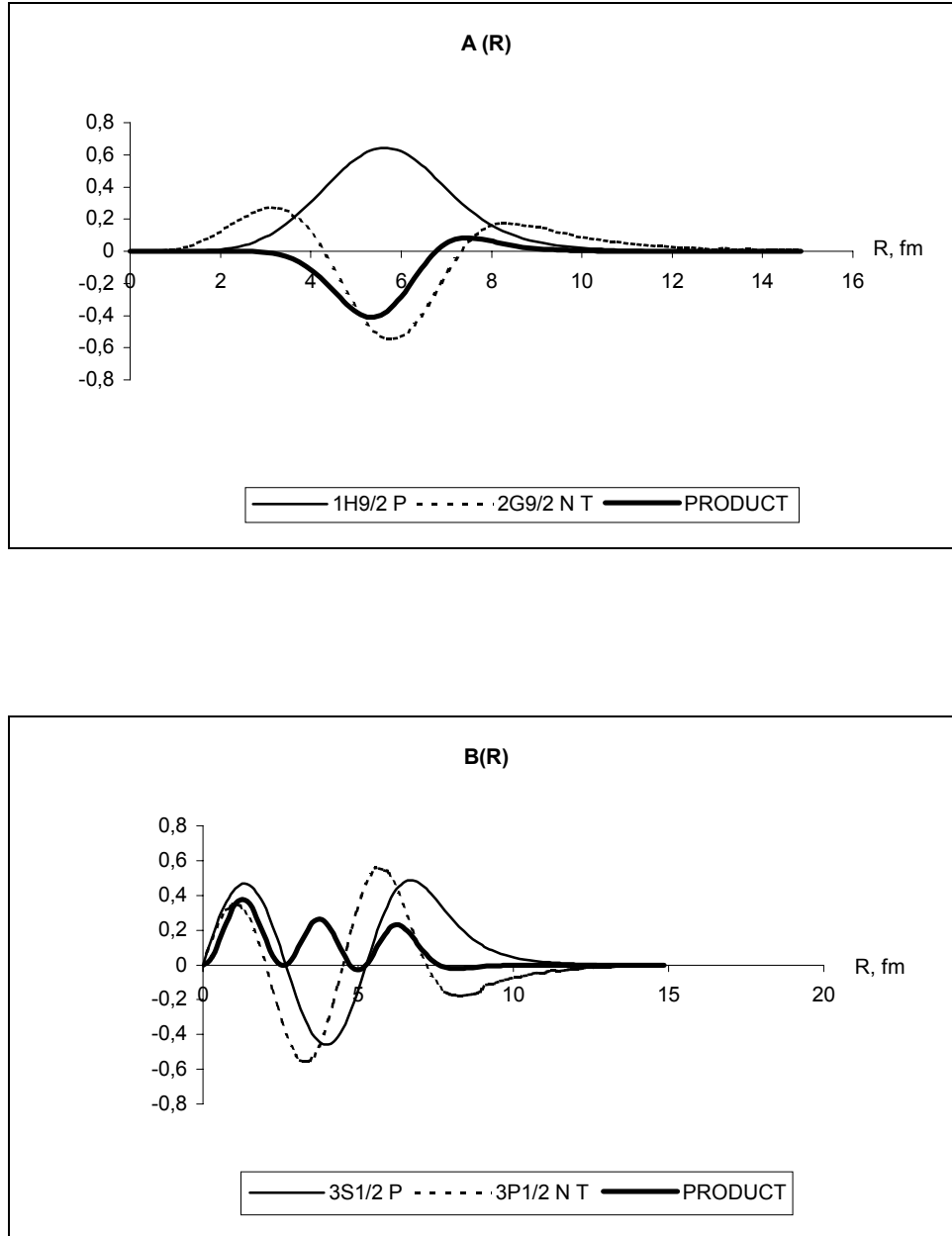


Fig. 1. The wave functions at the radial integral of the first matrix element

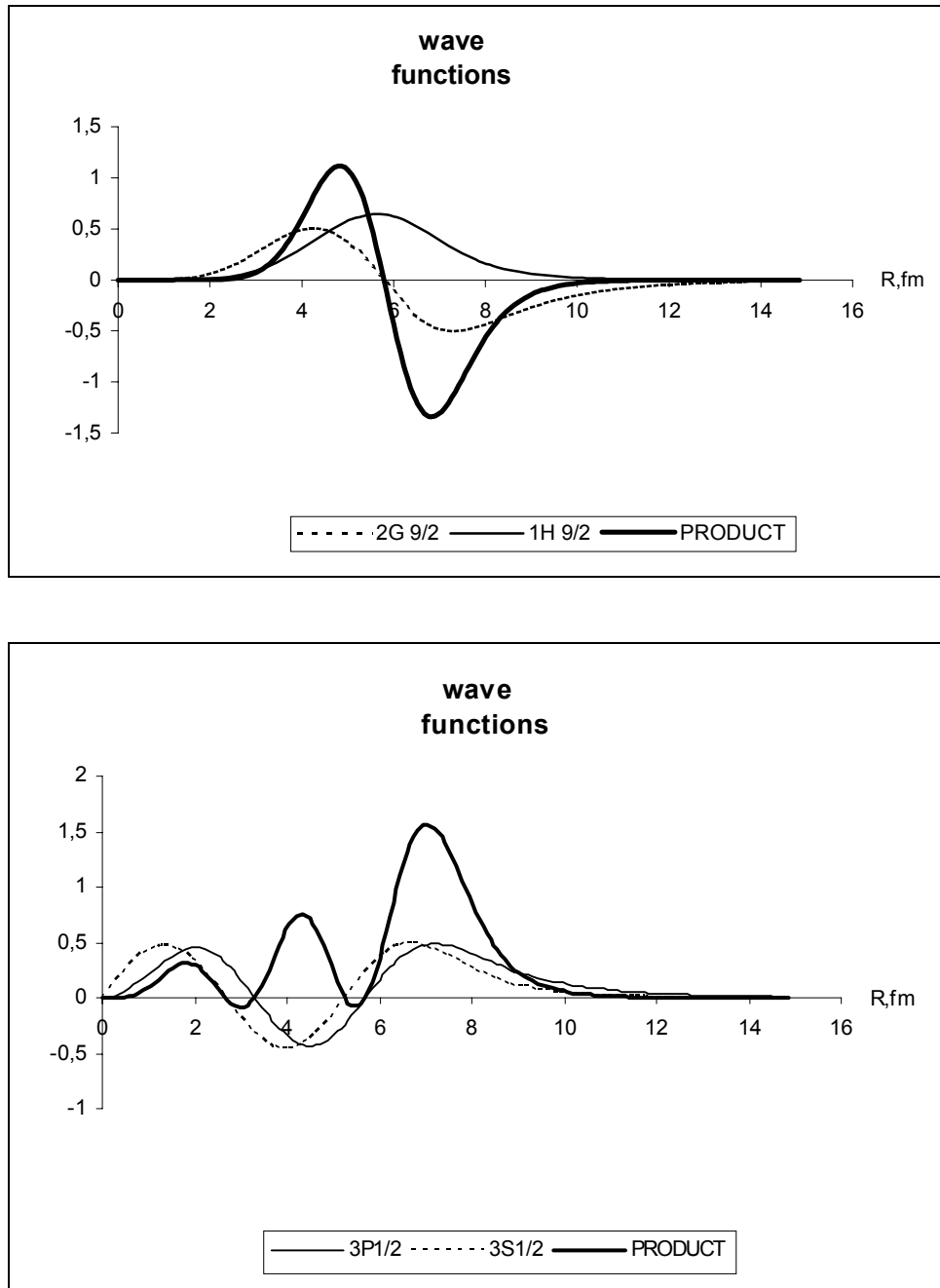


Fig. 2. The wave functions at the radial integral of the second matrix element

In the following tables, values for the single particle of the first and second matrix element calculated to a few nuclei with Saxon Woods potential by using the Chepurinov parameters[14] are given.

Table-1. The values of reduced single particle matrix elements for the

$$\left\langle J_n \| M(\rho_A, \lambda = 0 \| J_P) \right\rangle$$

Nucleus	$3P_{1/2}(n) \rightarrow 3S_{1/2}(p)$		$2G_{9/2}(n) \rightarrow 1H_{9/2}(p)$	
	Our calculations	Ref.(11)	Our calculations	Ref.(11)
^{206}Pb	0.0631	0.1655	0.1252	-
^{208}Pb	0.0636	-	0.1259	-
^{210}Pb	0.0638	-	0.1265	0.0878
^{212}Pb	0.0640	-	0.1272	0.0880
^{214}Pb	0.0643	-	0.1279	0.0879

The corresponding $\left\langle J_n \| M(\rho_A, \lambda = 0 \| J_P) \right\rangle$ matrix element values of Bohr and Mottelson calculation [1] for $^{207}\text{Tl}(3P_{1/2}) \rightarrow ^{207}\text{Pb}(3S_{1/2})$ transition is 0.115, and for $^{209}\text{Pb}(2G_{9/2}) \rightarrow ^{209}\text{Bi}(1H_{9/2})$ transition is 0.181.

Table-2. The values of reduced single particle matrix elements for the

$$\left\langle J_n \| M(j_A, \kappa = 1, \lambda = 0 \| J_P) \right\rangle$$

	$3P_{1/2}(n) \rightarrow 3S_{1/2}(p)$		$2G_{9/2}(n) \rightarrow 1H_{9/2}(p)$	
	Our calculations	Ref.(11)	Our calculations	Ref.(11)
^{206}Pb	1.7657	1.5994	0.7273	-
^{208}Pb	1.7578	-	0.6939	-
^{210}Pb	1.7522	-	0.6611	0.8577
^{212}Pb	1.7466	-	0.6293	0.8590
^{214}Pb	1.7416	-	0.5993	0.8604

The corresponding $\left\langle J_n \| M(j_A, \kappa = 1, \lambda = 0 \| J_P) \right\rangle$ matrix element values of Bohr and Mottelson calculation [1] for $^{207}\text{Tl}(3P_{1/2}) \rightarrow ^{207}\text{Pb}(3S_{1/2})$ transition is 1.9218, and for $^{209}\text{Pb}(2G_{9/2}) \rightarrow ^{209}\text{Bi}(1H_{9/2})$ transition is 0.5090.

As seen from the tables, the values we have calculated are different from calculations of Ref. [11], i.e. approximately 1.2-1.4 times larger. The present calculations are very close to the calculations of Bohr and Mottelson. Although our calculations are 2.5 times lower than the ones given in the Ref. [11] for the transition of $3P_{1/2}(n) \rightarrow 3S_{1/2}(p)$, they are 1.5 times larger for transition $2G_{9/2}(n) \rightarrow 1H_{9/2}(p)$. However, the present calculations are also very close to the calculations of Bohr and Mottelson model for the first matrix element.

According the present calculations, the following conclusions can be stated.

The values of first matrix element, which we have calculated directly, are different from the values calculated in the Ref.[11].

The approximate calculations of Bohr and Mottelson for neighboring nucleus to the ^{208}Pb nucleus are very close to our calculations.

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