Mathematical & Computational Applications, Vol. 1, No. 1, pp 97-104, 1996

Computer Aided Constrained Optimisation of Cutting Conditions in Drilling Operations on a CNC Lathe by Using Geometric Programming

A. İhsan SÖNMEZ, Adil BAYKASOĞLU, İ. Hüseyin FİLİZ

Gaziantep Üniversitesi Müh. Fak. Makina Müh. Böl., 27310 GAZİANTEP

Abstract

This paper discusses the use of the geometric programming method to determine the optimum values of cutting speed and feed rate which yield minimum cost in a drilling operation that is performed on a CNC lathe. During the formulation of the problem a number of constraints are considered.

1. Introduction

The determination of economically optimal cutting conditions, i.e. cutting speed and feed rate, is an essential step in computer aided process planning activities.

A survey of the literature on the optimisation of cutting conditions indicates that a few number of researchers have studied the optimisation of cutting conditions in drilling operations In the cases especially where HSS drills are used, drilling operation may have considerable influence on the machining time and therefore optimisation of cutting conditions might be necessary.

Ermer and Shah [1] considered the problem of optimising cutting conditions in drilling. They used both minimum cost and maximum production rate as optimisation criteria.

Arsecularatne [2] and Filiz, Sönmez, Baykasoğlu, Dereli [3] studied the constrained optimisation of cutting conditions in drilling by using minimum cost as optimisation criteria.They used the torque available from machine tool, drill buckling, drill strength, axial-circumferential slips in chuck as the constraints.

In the above mentioned analyses, cost per operation is expressed in terms of cutting speed and feed rate. One of these variables is found by using partial differentiation of the expression with respect to the variable of concern and the other variable is found as the value which satisfy the above mentioned constraints. In these approaches both independent variables could not be treated simultaneously.

In this study, the constrained optimisation of cutting conditions on a CNC drilling operation is successfully and easily treated by the application of a non-linear programming technique, namely, Geometric Programming (GP). In the solution of constrained GP problem Lagrange Multipliers method is used as an additional tool. Minimum cost is used as the objective function and the following restrictions are considered in this work; Maximum machine torque, Limiting torque for the drill, Circumferential slip in the chuck, Axial slip in the chuck, Drill buckling, Maximum and minimum speeds available from machine tool, Maximum and minimum feed rates available from machine tool.

A computer program is written in QBASIC and implemented on an IBM compatible computer for automating the calculations in the optimisation procedure.

2. Elements of Geometric Programming

In Geometric Programming the objective function is written in the following form:

$$g_o(t) = \sum_{j=1}^{q_o} (C_{oj} \prod_{k=1}^{r} t_k^{a_{ojk}}) \qquad \text{for } j = 1,2,3......q_o \qquad 1$$

is a minimum subject to;

$$g_i(t) = \sum_{j=1}^{q_i} (C_{ij} \prod_{k=1}^{r} t_k^{a_{ijk}}) \leq 1 \qquad \text{for } i = 1,2.....p, \quad k = 1,2.....r \text{ and } t_k \geq 0 \qquad 2$$

where q_o: number of terms in the objective function, C_{oj}: coefficients of the objective function, t_k:denotes variables, r: number of variables, p: number of constraint functions, q_i: number of terms in the i'th constraint function.

Duffin, Zener and Peterson [4] showed that the dual of the above stated problem (primal programme) is given by;

$$u(\delta)=(\prod_{j=1}^{q_o}(\frac{C_{oj}}{\delta_{oj}})^{\delta_{oj}})(\prod_{i=1}^{p}(\sum_{j=1}^{q}C_{ij}^{\sum_{j=1}^{q_i}\delta_{ij}})) \qquad 3$$

is a maximum subject to;

$$\sum_{j=1}^{q_o} \delta_{oj} = 1 \qquad \text{(normality condition)} \qquad 4$$

$$\sum_{i=0}^{p} \sum_{j=1}^{q_i} a_{ijk}\, \delta_{ij} = 0 \qquad \text{for } k = 1,2....r \text{ (orthogonality conditions)} \qquad 5$$

where $u(\delta)$ is the dual function and δ_{ij}, denotes dual vectors.

To solve the problem, the optimum value of the dual vectors δ^*_{ij} ,which make the dual objective function maximum, should be found from dual constraint equations.

The optimum value of the original objective function $g^*_o(t)$ is obtained from the dual program after finding the optimum values of the dual vector δ^*_{ij}. According to the definition of the geometric programming, δ^*_{ij} are the weight of the terms in the primal objective function, i.e.

$$C_{oj} \prod_{k=1}^{r} t_k^{a_{ojk}} = \delta^*_{oj}.g^*_o(t) \qquad \text{for } j = 1,2,3.....q_o \qquad 6$$

There are q_o equations and r variables. The variables t_k are then found by solving these equations simultaneously.

If (no. of. equations = r +1), then (s-r-1) is termed the degree of difficulty of the problem, where s is the number of terms in the objective and constraint functions. This represents the number by which the independent variables exceed the number of equations in the system of linear simultaneous equations given by normality and orthogonality conditions.

3. The Objective Function

The basic model describing the cost of a drilling operation, as given by many authors, is expressed as follows;

$$C_T = XT_m + (XT_d + Y)(T_m / T) \qquad 7$$

where X is machining cost rate (cost/min), Y: tool cost per cutting edge (in carbide inserts) or drill depreciation cost plus drill resharpening cost (in HSS tools), T_m: machining time (min), T_d: tool change time (min), T: tool life (min).

Machining time for a drilling operation can be written as;

$$T_m = \pi DL/(1000Vf) \tag{8}$$

where D is drill diameter (mm), L: length of cut (mm), f: feed rate (mm/rev), V: cutting speed (m/min).

Taylor's expanded tool life equation for drills has the following form[5];

$$T = \frac{(C_v)^m . D^{(mx_v)}}{V^m . f^{(my_v)}} \tag{9}$$

where C_v, m, x_v, y_v are constants.

The substitution of tool life (T) and machining time (T_m) expressions into equation (7) gives;

$$C_T = \left(\frac{X\pi LD}{1000}\right) V^{-1} f^{-1} + (XT_d + Y)\left(\frac{\pi LD^{(1-mx_v)}}{1000(C_v)^m}\right) V^{(m-1)} f^{(my_v-1)} \tag{10}$$

For convenience, define; $n = m.x_v$, $A = (C_v)^m$, $z = m\, y_v$

Then the cost equation can be written as;

$$C_T(V,f) = \left(\frac{X\pi LD}{1000}\right) V^{-1} f^{-1} + (XT_d + Y)\left(\frac{\pi LD^{(1-n)}}{1000A}\right) V^{(m-1)} f^{(z-1)} \tag{11}$$

This equation is the objective function which will be optimised according to the minimum cost criterion.

4. Derivation of Constraint Functions

There are several constraints which effect the cutting conditions in a drilling operation. The source of these constraints may be machine tool, cutting tool and workpiece specifications. One must keep in mind that the larger the number of constraints, the harder the optimisation problem is to solve.

4.1 Cutting Torque and Thrust in Drilling

Thrust load (Fy) and torque (M) in drilling operations are given by Arshinov et. al. [5] as follows:

$$F_y = 9.81 C_p D^{X_p} f^{Y_p} \quad (N) \tag{12}$$

$$M = 9.81 \cdot 10^{-3} C_m D^{X_m} f^{Y_m} \quad (N.m.) \tag{13}$$

where , C_m , x_m , y_m , C_p , x_p , y_p are constants for a given tool/workpiece pair.

4.2. Constraints

Following constraints are considered in this work;

1) *Maximum machine torque* : Maximum torque which can be provided by a machine is;

$$M_1 = 60P_{max} / (\pi N_{break1}) \tag{14}$$

So; $\quad M_1 \geq M \quad$ (Constraint 1) $\quad$ 15

where, P_{max} is maximum power available from machine (W), N_{break1} is the break speed of the motor after which the power becomes constant (maximum) (rpm).

2) *Limiting Torque of The Drill* : Limiting torque that the drill can withstand is calculated by the formula;

$$M_2 = \pi D_e^3 \tau \, /(16000 f_{s1}) \qquad 16$$

where, D_e is the equivalent diameter for the drill which is equal to 0.7D (mm), f_{S1} is factor of safety , and τ is the shear strength of the drill shank material (MP_a).

So; $M_2 \geq M$ (Constraint 2) 17

3) *Drill Buckling* : The maximum load to avoid drill buckling can be calculated by using the formula;

$$F_{a1} = \pi^3 E D^4 / (64 L^2 f_{s2}) \qquad 18$$

where; E is the modulus of elasticity of the drill material (MP_a), f_{s2} is factor of safety.

So; $F_{a1} \geq F_y$ (Constraint 3) 19

4) *Axial Slip in Chuck* : The maximum allowable thrust to avoid axial slip in the chuck can be calculated by using the expression;

$$F_{a2} = \mu \, [\, F_{co} + \sum (m_j r_j)(w_{min})^2 \,] \qquad 20$$

where, μ_a is coefficient of friction of jaw in axial direction, F_{co} is clamping force at zero speed (N), m_j is mass of chuck jaws (kg.), r_j is radial distance of jaws (mm.), w_{min} is minimum spindle speed (rad/sec.).

So; $F_{a2} \geq F_y$ (Constraint 4) 21

5) *Circumferential Slip in The Chuck* : To avoid circumferenctial slip in the chuck, the torque developed in the cutting operation must be less than the frictional torque (M_3) in the chuck which can be calculated by using the formula;

$$M_3 = \mu_c r_g \, [\, F_{co} + \sum (m_j r_j)(w_{min})^2 \,] \qquad 22$$

where, r_g is component gripped radius (mm), μc is coefficient of friction of jaw in the direction of spindle rotation

So; $M_3 \geq M$ (Constraint 5) 23

6) *Maximum-Minimum Rotational Speeds of the Machine Tool* : The rotational speed can be calculated by using the following equation;

$$N_c = 1000V / (\pi D) \qquad 24$$

So; $N_{max} \geq N_c$ (Constraint 6) 25

Where; N_{max} is maximum machine speed.

7) *Maximum-Minimum Feed Rates of the Machine Tool* :

$f \leq f_{max}$ (Constraint 7) 26

Where; f_{max} is maximum feed rate of the machine.

5. The Primal and Dual Programmes

5.1. Primal Programme

The objective function and constraints functions can be written in the geometric programming formats as explained in section 2;

Objective function

$$g_o(t) = C_{01}\, t_1^{a_{011}}\, t_2^{a_{021}} + C_{02}\, t_1^{a_{021}}\, t_2^{a_{022}} \qquad 27$$

where; t_1 denotes cutting speed V and t_2 demotes feed rate f and

$$C_{01} = \frac{X\pi DL}{1000},\ C_{02} = \frac{(XT_d+Y)\pi LD^{1-n}}{1000A},\ a_{011} = -1,\ a_{012} = m-1,\ a_{022} = z-1 \qquad 28$$

Constraint Functions

Constraint 1; $C_{11}\, t_1^{a_{111}}\, t_2^{a_{112}} \le 1$ where; $C_{11} = \dfrac{9.81\text{x}10^{-3}\, C_m\, D^{X_m}}{M_1}$, $a_{111} = 0,\ a_{112} = y_m$ 29

Constraint 2; $C_{21}\, t_1^{a_{211}}\, t_2^{a_{212}} \le 1$ where; $C_{21} = \dfrac{9.81\text{x}10^{-3}\, C_m\, D^{X_m}}{M_2}$, $a_{211} = 0,\ a_{212} = y_m$ 30

Constraint 3; $C_{31}\, t_1^{a_{311}}\, t_2^{a_{312}} \le 1$ where ; $C_{31} = \dfrac{9.81 C_p\, D^{X_p}}{F_{a1}}$, $a_{311} = 0,\ a_{312} = y_p$ 31

Constraint 4; $C_{41}\, t_1^{a_{411}}\, t_2^{a_{412}} \le 1$ where; $C_{41} = \dfrac{9.81 C_p\, D^{X_p}}{F_{a2}}$, $a_{411} = 0,\ a_{412} = y_p$ 32

Constraint 5; $C_{51}\, t_1^{a_{511}}\, t_2^{a_{512}} \le 1$ where; $C_{51} = \dfrac{9.81\text{x}10^{-3}\, C_m\, D^{X_m}}{M_3}$, $a_{511} = 0,\ a_{512} = y_m$ 33

Constraint 6; $C_{61}\, t_1^{a_{611}}\, t_2^{a_{612}} \le 1$ where; $C_{61} = \dfrac{1000}{\pi D N_{max}}$, $a_{611} = 1,\ a_{612} = 0$ 34

Constraint 7; $C_{71}\, t_1^{a_{711}}\, t_2^{a_{712}} \le 1$ where; $C_{71} = 1/f_{max}$, $a_{711} = 0,\ a_{712} = 1$ 35

5.2 Dual Program

The primal objective function and the constraint functions have been developed in the previous sections. It is seen that the objective function has two terms and there are seven constraint functions. All of the constraint functions have single terms. So the dual objective function turns out to be;

$$U(\delta) = \left(\frac{C_{01}}{\delta_{01}}\right)^{\delta_{01}} \left(\frac{C_{02}}{\delta_{02}}\right)^{\delta_{02}} C_{11}^{\delta_{11}} C_{21}^{\delta_{21}} C_{31}^{\delta_{31}} C_{41}^{\delta_{41}} C_{51}^{\delta_{51}} C_{61}^{\delta_{61}} C_{71}^{\delta_{71}} \quad 36$$

where; δ_{01} and δ_{02} are the dual variables of the objective function.

The dual variables are subjected to the linear constraints. According to the normality condition of the Geometric Programming, first dual constraint function is given by;

$$\delta_{01} + \delta_{02} = 1 \quad 37$$

According to orthogonality condition, the other constraint functions are;

$$a_{011}\,\delta_{01}+a_{021}\,\delta_{02}+a_{111}\,\delta_{11}+a_{211}\,\delta_{21}+a_{311}\,\delta_{31}+a_{411}\,\delta_{41}+a_{511}\,\delta_{51}+a_{611}\,\delta_{61}+a_{711}\,\delta_{71} = 0 \quad 38$$

$$a_{012}\,\delta_{01}+a_{022}\,\delta_{02}+a_{112}\,\delta_{11}+a_{212}\,\delta_{21}+a_{312}\,\delta_{31}+a_{412}\,\delta_{41}+a_{512}\,\delta_{51}+a_{612}\,\delta_{61}+a_{712}\,\delta_{71} = 0 \quad 39$$

and the non-negative constraints are;

$$\delta_{01} \geq 0,\ \delta_{02} \geq 0,\ \delta_{11} \geq 0,\ \delta_{21} \geq 0,\ \delta_{31} \geq 0,\ \delta_{41} \geq 0,\ \delta_{51} \geq 0,\ \delta_{61} \geq 0,\ \delta_{71} \geq 0 \quad 40$$

The dual objective function $U(\delta)$ has to be maximised by using the dual constraint functions. The maximum point obtained from dual objective function is the minimum value of the original objective function. However, the degree of difficulty of the problem is six, therefore an additional method is necessary to solve the problem. Following steps are taken for solving the problem as suggested by Beightler [6] and Nisli [7].

Firstly, the natural logarithm of the dual objective function is taken;

$$F'(\delta) = \ln U(\delta) \quad 41$$

$$F'(\delta) = \delta_{01}(\ln C_{01}-\ln\delta_{01})+ \delta_{02}(\ln C_{02}-\ln\delta_{02})+ \delta_{11}\ln C_{11}+ \delta_{21}\ln C_{21}+ \delta_{31}\ln C_{31}+ \delta_{41}\ln C_{41} + \delta_{51}\ln C_{51}+ \delta_{61}\ln C_{61}+ \delta_{71}\ln C_{71} \quad 42$$

Then, this non-linear optimisation problem can be solved by using the "Generalised Lagrange Multipliers" method. The general formulation of this method is as follows;

$$F(\delta,\lambda) = F'(\delta) - \sum_{j=1}^{N} \lambda_j G_j(\delta) \quad 43$$

where, $G_j(\delta)$ is the constraint function and N is the number of constraints functions.

The dual constraint equations are the constraint equations of the Lagrange Multipliers method;

$$G_1(\delta) = \delta_{01} + \delta_{02} - 1 \quad 44$$

$$G_2(\delta) = a_{011}\,\delta_{01}+a_{021}\,\delta_{02}+a_{111}\,\delta_{11}+a_{211}\,\delta_{21}+a_{311}\,\delta_{31}+a_{411}\,\delta_{41}+a_{511}\,\delta_{51}+a_{611}\,\delta_{61}+a_{711}\,\delta_{71} \quad 45$$

$$G_3(\delta) = a_{012}\,\delta_{01}+a_{022}\,\delta_{02}+a_{112}\,\delta_{11}+a_{212}\,\delta_{21}+a_{312}\,\delta_{31}+a_{412}\,\delta_{41}+a_{512}\,\delta_{51}+a_{612}\,\delta_{61}+a_{712}\,\delta_{71} \quad 46$$

Then the objective function can be written as;

$$F(\delta,\lambda)=\delta_{01}\ln C_{01}-\delta_{01}\ln\delta_{01}+ \delta_{02}\ln C_{02}-\delta_{02}\ln\delta_{02}+\delta_{11}\ln C_{11}+\delta_{21}\ln C_{21}+\delta_{31}\ln C_{31}+\delta_{41}\ln C_{41} +\delta_{51}\ln C_{51}+\delta_{61}\ln C_{61}+\delta_{71}\ln C_{71}-\lambda_1(\delta_{01}+\delta_{02}-1)-\lambda_2(a_{011}\,\delta_{01}+a_{021}\,\delta_{02}+a_{111}\,\delta_{11}+a_{211}\,\delta_{21}+a_{311}\delta_{31}+a_{411}\delta_{41}+a_{511}\delta_{51}+a_{611}\delta_{61}+a_{711}\delta_{71})-\lambda_3(a_{012}\delta_{01}+a_{022}\delta_{02}+a_{112}\delta_{11}+a_{212}\,\delta_{21}+a_{312}\,\delta_{31}+a_{412}\,\delta_{41}+a_{512}\,\delta_{51}+a_{612}\,\delta_{61}+a_{712}\,\delta_{71}) \quad 47$$

Here, λ_j for j=1,2,3 are non negative weighting factors, which are independent of δ's and identifiable as lagrange multipliers.

For optimum solution, the following set of equations must be satisfied,

$$\partial F(\delta,\lambda)\ /\partial\delta = 0, \qquad \partial F(\delta,\lambda)\ /\partial\lambda = 0 \quad 48$$

In this problem;

$$\frac{\partial F}{\partial \delta_{01}} = \ln C_{01} - 1 - \ln\lambda_{01} - \lambda_1 - \lambda_2 a_{011} - \lambda_3 a_{012} = 0$$

$$\frac{\partial F}{\partial \delta_{02}} = \ln C_{02} - 1 - \ln \lambda_{02} - \lambda_1 - \lambda_2 a_{021} - \lambda_3 a_{022} = 0$$

$$\frac{\partial F}{\partial \delta_{11}} = \ln C_{11} - \lambda_2 a_{111} - \lambda_3 a_{112} = 0 , \quad \frac{\partial F}{\partial \delta_{21}} = \ln C_{21} - \lambda_2 a_{211} - \lambda_3 a_{212} = 0$$

$$\frac{\partial F}{\partial \delta_{31}} = \ln C_{31} - \lambda_2 a_{311} - \lambda_3 a_{312} = 0 , \quad \frac{\partial F}{\partial \delta_{41}} = \ln C_{41} - \lambda_2 a_{411} - \lambda_3 a_{412} = 0 \qquad 49$$

$$\frac{\partial F}{\partial \delta_{51}} = \ln C_{51} - \lambda_2 a_{511} - \lambda_3 a_{512} = 0 , \quad \frac{\partial F}{\partial \delta_{61}} = \ln C_{61} - \lambda_2 a_{611} - \lambda_3 a_{612} = 0$$

$$\frac{\partial F}{\partial \delta_{71}} = \ln C_{71} - \lambda_2 a_{711} - \lambda_3 a_{712} = 0 , \quad \frac{\partial F}{\partial \lambda_1} = -\delta_{01} - \delta_{02} + 1 = 0$$

$$\frac{\partial F}{\partial \lambda_2} = -a_{011}\,\delta_{01} - a_{021}\,\delta_{02} - a_{111}\,\delta_{11} - a_{211}\,\delta_{21} - a_{311}\,\delta_{31} - a_{411}\,\delta_{41} - a_{511}\,\delta_{51} - a_{611}\,\delta_{61} - a_{711}\,\delta_{71}$$

$$\frac{\partial F}{\partial \lambda_3} = -a_{012}\,\delta_{01} - a_{022}\,\delta_{02} - a_{112}\,\delta_{11} - a_{212}\,\delta_{21} - a_{312}\,\delta_{31} - a_{412}\,\delta_{41} - a_{512}\,\delta_{51} - a_{612}\,\delta_{61} - a_{712}\,\delta_{71}$$

Now, there are 12 equations and 12 unknowns so, the problem can be solved as explained in previous sections..

6. Computer Program and Example

A computer program has been developed for the solution of the optimisation problem which is formulated in the previous sections. QBASIC is used as the programming language in the application.

Inputs:

Maximum spindle motor power : 12 kW, Maximum and minimum spindle speeds: 2500 rpm -10 rpm, Break speed of spindle motor: 500 rpm, Maximum and minimum feed rates: 3 mm/ rev-0.001 mm/rev, Clamping force: 12000 N, Mass of chuck jaws: 2 kg, Coefficients of friction of chuck jaws: 0.35, Tool material: HSS, Shear strength of tool material: 512 MPa, Modulus of elasticity of tool material: 220 GPa, Workpiece material: Free machining carbon steel, Drill diameter: 25 mm, Drill length: 65 mm, Machining length: 50 mm, Component gripped radius: 30 mm, Machining cost rate: 1000 TL/min, Tool depreciation and resharpening cost: 5000 TL, Tool change time: 0.1 min.

Outputs:

Optimum cutting speed	: 60.2 m/min
Optimum feed rate	: 0.05 mm/rev
Cost	: 1380.3 TL
Cutting time	: 1.305 min

A comparison between results obtained from this program and from Ref.[3] (By considering the same objective function, constraints and constants), and from Machining Data Handbook [8] are given in Table 6.1:

Table 6.1 Comparison of Results

	Geometric Programming	Ref.[3]	Ref.[8]
Cutting speed (m/min)	60.2	55	50
Feed rate (mm/rev)	0.05	0.041	0.021
Cost (TL)	1380.3	1772.93	3744.1
Cutting time (min)	1.305	1.741	3.74

As seen in this table, GP algorithm gives better results than the results given in Ref. [3] and [8].

7. Conclusions

In this study a mathematical model has been developed for the constrained optimisation of cutting conditions in drilling operations by using geometric programming technique.

Geometric programming is relatively straight forward and easy to apply in solving algebraic non-linear programming problems subject to non-linear constraints. However, in cases where degree of difficulty is greater than one, geometric programming requires additional effort to optimise the objective function.

Cutting conditions in single pass machining operations with less number of constraints can be easily optimised by using geometric programming technique. In the case of multi pass machining operations and higher number of constraints additional methods are needed for solving the optimisation problem in geometric programming.

In this study drilling operation considered as a single pass machining operation. Seven constraints are used in the optimisation. For solving six degree of difficulty problem Lagrange Multipliers method was used in addition to geometric programming technique.

8. References

1. D.S. Ermer, B.V. Shah, Analytical Sensitivity Studies of the Optimum Machining Conditions for Milling, Drilling, Reaming, and Taping, Transactions of the ASME, (1973), 312-316.
2. Arsecularatne, J.A., Hinduja, S., Barrow, G, Optimum Cutting Conditions for Turned Components, Proc. Ins. Mach. Engrs. 206 (1992), 15-30.
3. Filiz, İ.H, Sönmez, A.İ., Baykasoğlu, A., Dereli, T., Bilgisayar Yardımı ile CNC Tornalama İşlemlerinde Kesme Şartlarının Eniyilenmesi, MATIM (Makina Tasarım ve İmalat Dergisi, ODTÜ) yayınlanaçak.
4. R.J. Duffin, E.L. Peterson, C. Zener, Geometric Programming-Theory and Application, Wiley, N.Y., 1967.
5. Arshinov, V., Aleksiev, G., Metal Cutting Theory and Cutting Tool Design, Mir Publishers, Moscow, 1976.
6. Sertaç, M.N., 1984, Optimisation of Single-Pass Turning Operations by Geometric Programming Method, M.Sc. Thesis, METU, Ankara.
7. C. Beightler, D.T. Philips, Applied Geometric Programming, John Wiley & Sons, Inc., 1976
8. Machining Data Handbook, Met-Cut Research Association Inc., Machinability Data Center, USA, 1980