

THE EFFECTS OF ADDENDUM MODIFICATION COEFFICIENT ON TOOTH STRESSES OF SPUR GEAR

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ABSTRACT

The effects of addendum modification coefficient on the root stresses of spur gear are investigated. By considering positive and negative addendum modified gears, distributions of root stresses are determined by the finite element method. The root stresses of addendum modified gears are compared with those of standard gears. The problem is analyzed as one in the plane stress and, also the plane strain.

Key words : Tooth Stresses. Addendum Modification Coefficient. Spur Gear Analysis.

1. INTRODUCTION

In the study, the effects of addendum modification coefficient on spur gear tooth stresses are investigated by the finite element method. The tooth stresses of positive and negative addendum modified gears are compared with those of standard gear.

There are various methods for increasing of load carrying capacity and reducing noise in mating gears, and for obtaining certain distance between two axes of gears. These methods are *a*) changing the pressure angle, *b*) modifying whole depth, *c*) modifying tooth thickness and *d*) modification using addendum modification coefficient.

In the method of changing the pressure angle, tooth thickness is increased by increasing the pressure angle and resulting in decrease of root stresses of the tooth. On the other hand, the contact ratio and tip thickness is decreasing. Disadvantage of this kind of modification is the need for special cutting tools. In modifying of whole depth of the tooth, depending on the factor related to whole depth, whether thinner and higher tooth or thicker and pump tooth is obtained. Disadvantage of the later method is also the need for special tools. The modification by changing tooth thickness is rarely used in application. The most commonly used modification method is the addendum modification. The most important advantage of this method is that manufacturing of modified gears can be made by the base rack. The first systematic studies on addendum modification are realized by R. Buchanan, C. H. Wiebe, P. Hoppe, M. Fölmer, M. Maag, K. Kutzbach [1].

Principle of the addendum modification depends on changing the position of the base circle center. It is possible to employ involutes belonging to two base circle center as matched in accordance with the fundamental law of gearing. If we assume one of the gears as a rack cutter whose tooth number is infinite, we can employ this gear in different axes positions as matched. We can rematch gears having same reference profile. If rack cutter is shifted (modified) as $+x_m$ from the pitch circle of the gear, positive addendum

modification is resulted. Conversely, if rack cutter is shifted as $-xm$ from the pitch circle of the gear, negative addendum modification is resulted. Here, x represents addendum modification coefficient.

The aim of the addendum modification involves preventing undercutting, rearranging the distance between axes of the gear pairs, improvement of gears from the point of view of load carrying capacity and surface pressure, and changing the contact ratio for reducing noise in gear sets. Generally, if addendum modification is implemented, several aims are considered, simultaneously. The factors must be taken into account in addendum modification are given below :

- a) modification must be realized in some limitations of tolerance of addendum(h_b)
- b) the contact ratio (ϵ) must be greater than 1 ($\epsilon > 1$)
- c) active profile must be possible high and in a suitable region
- d) modification in high power and speedy gears must be realized either from the point of view of equivalent tooth fillet strength or minimum wear.

In the positive addendum modification, if x increases, tooth fillet stresses will be decreased, tip thickness of the tooth and the contact ratio will be decreased. Negative addendum modification is advised to be used in only necessity of certain axes distance since it decreased the load carrying capacity of tooth. The contact ratio increases in the negative addendum modification. It is limited by undercutting practically. It is only possible for great tooth number.

It can be easily seen that the lower limit is determined by undercutting while the upper limit is determined by tip thickness.

In the existing design formulas of tooth strength, the effect of addendum modification on bending strength is calculated multiplying the bending strength of standard gear by form factor which depends on the addendum modification coefficient [2]. Total addendum modification coefficient given as;

$$X_t = Z_t / (2 \tan \phi) [(\tan \phi_v - \tan \phi) - (\phi_v - \phi)] \quad (1)$$

Total addendum modification coefficient is expressed in Eq. (1) depending on tooth number and pressure angle. Here, the distance between axes can be calculated using ϕ_v after solving addendum modification coefficients.

In DIN 3992, total addendum modification coefficient X_t is determined in accordance with total tooth number Z_t and the properties to be given matching gears. In DIN 3994 and 3995, addendum modification coefficient (x) is advised to be equal $\frac{1}{2}$ for gears. In this method, the load carrying capacity in system improves and gears have properties of a gear set. In evaluating the form factor, the effects of addendum shortened factor K is neglected in DIN 3990. The value of K in the special addendum modification system ($x=0.5$) is greater than in standard gear. So, it is not effective in calculating of the form factor. The finite element method is used for an analysis, after the modeling of the exact tooth geometry as a one to one correspondence with the original tooth shape. So the results of the stress analysis of gear tooth using the finite elements method shows reliable performance.

2. THE FINITE ELEMENT MODEL

For the finite element model, first of all the region of the problem under consideration is determined. In the finite element analysis of the gear, it was shown that the model, consisting of one tooth and one module rim thickness, with simple supported along the boundary is satisfactory to represent the problem region. Thus, it is first obtained the region of one tooth with one module rim thickness. The most important phase of getting the problem region is obtaining involute tooth profile. The involute profile of the tooth obtained by computer program which is prepared by us. The computer program plots the involute profiles of modified gears ($x = -0.5$, $x = -0.3$, $x = 0.3$, $x = 0.5$) and standard gear ($x = 0$). The involute profile program is added to the beginning of Lusas software which makes finite element analysis.

When gear module (m), number of teeth (z), pressure angle (ϕ) and addendum modification coefficient (x) are given, the geometry of the problem is obtained in computer. In addition to these parameters, when face width (F), loading angle (α), the force acting upon the gear (W) are given, calculation can be carried out.

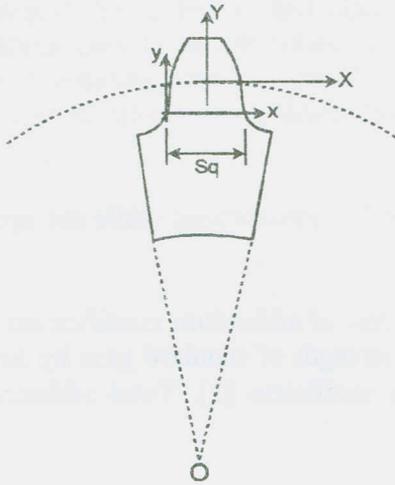


Figure 1. The finite element model and used coordinate system.

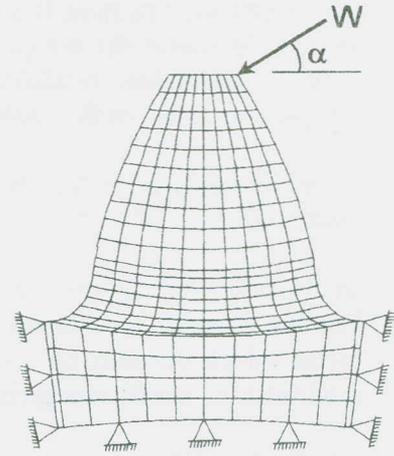


Figure 2. The finite element mesh, boundary conditions and applied load.

Geometry, material properties and loading

The geometry, loading and material properties of the gears are given in Table 1.

Table 1. Geometry, loading and material properties of gears.

Modulus	m	4 mm
Number of teeth	z	26
Addendum	h_b	4 mm
Dedendum	h_t	5 mm
Rim thickness	T_r	4 mm
Load	W/F	1 kN/m
Elasticity modulus	E	210 GPa
Poisson's ratio	ν	0.30

Addendum modification coefficients used in analysis are $x = -0.5$, $x = -0.3$, $x = 0.3$, $x = 0.5$. For each tooth of different addendum modification coefficient a model is established, Fig. (1) through Fig. (4). In our previous study [3] on the modeling for finite element stress analysis of spur gears, it is shown that sufficiently accurate model is that a simple tooth supported along its boundary and having one module rim thickness (Fig. 2). Load is applied at the tip of the tooth as seen in the Fig. (2). Finite element mesh generation is refined at the places where it is expected that stress distribution might change rapidly.

In mesh generation, global coordinates X-Y are used. In the presentation of stress distribution, coordinates x-y are used, Fig. (1). In the finite element analysis, 8-node isoparametric plane finite elements are used.

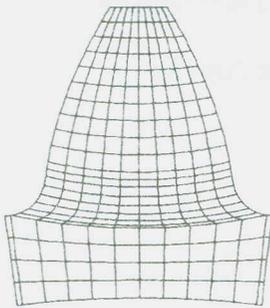


Figure 3. Finite element mesh for $x = +0.3$

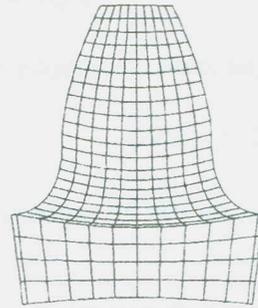


Figure 4. Finite element mesh for $x = -0.3$

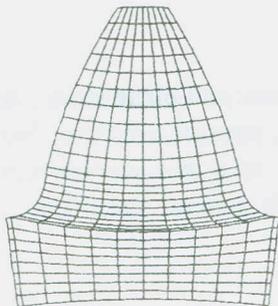


Figure 5. Finite element mesh for $x = +0.5$

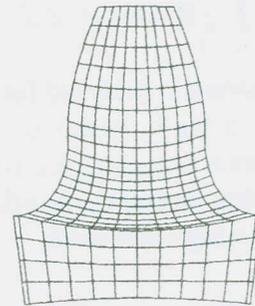


Figure 6. Finite element mesh for $x = -0.5$

3. FINITE ELEMENT FORMULATION AND STRESS ANALYSIS

Tooth stresses of spur gear are analyzed by the finite element method as the plane stress problem and plane strain one as well.

As the geometry of spur gear is always same in every cutting plane perpendicular to face width of the gear, the problem can be treated as two-dimensional one, the plane stress or the plane strain with respect to face width. In the calculation 8-node plane element is used.

Shape functions of the used element, Fig. (7), in the solution are given as [4]

$$\begin{aligned}
 N_1 &= -\frac{1}{4}(1-\xi)(1-\eta)(1+\xi+\eta) & N_5 &= \frac{1}{2}(1-\xi^2)(1-\eta) \\
 N_2 &= -\frac{1}{4}(1+\xi)(1-\eta)(1-\xi+\eta) & N_6 &= \frac{1}{2}(1+\xi)(1-\eta^2) \\
 N_3 &= -\frac{1}{4}(1+\xi)(1+\eta)(1-\xi-\eta) & N_7 &= \frac{1}{2}(1-\xi^2)(1+\eta) \\
 N_4 &= -\frac{1}{4}(1-\xi)(1+\eta)(1+\xi+\eta) & N_8 &= \frac{1}{2}(1-\xi)(1-\eta^2)
 \end{aligned}$$

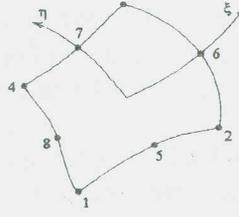


Figure 7. 8-node isoparametric finite element.

The displacement components can be written in matrix form as

$$\mathbf{u} = \sum_{i=1}^8 N_i \mathbf{u}_i = \mathbf{N}\mathbf{U} \quad \mathbf{v} = \sum_{i=1}^8 N_i \mathbf{v}_i = \mathbf{N}\mathbf{V}$$

Strain vector can be expressed as

$$\boldsymbol{\varepsilon} = \mathbf{B}\mathbf{q}$$

Where nodal-displacements vector \mathbf{q} is

$$\mathbf{q} = [\mathbf{u}_1, \mathbf{v}_1, \mathbf{u}_2, \mathbf{v}_2, \dots, \mathbf{u}_8, \mathbf{v}_8]$$

Element stiffness matrix is

$$\mathbf{k}^e = \int_{-1}^1 \int_{-1}^1 \mathbf{B}^T \mathbf{D} \mathbf{B} \det J d\xi d\eta$$

The problem is analyzed for both the case of plane stress and plane strain. As the axis z is normal to the xy -plane, in the plane stress $\sigma_z, \tau_{xz}, \tau_{yz}$ stresses are zero. However, in the plane strain $\varepsilon_z, \gamma_{xz}, \gamma_{yz}$ are zero. In this study, we also take into account von Mises stress (equivalent stress) σ_E in calculations. The stress σ_E , in general stress state, is given with the following expression;

$$\sigma_E = \frac{1}{4} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right]^{\frac{1}{2}}$$

The most critical point of the involute spur gear is at tooth fillet. In the finite element method the critical section of the tooth is obtained by using the coordinate of critical point. At this point, the stress component required to be calculated reaches to its maximum value. Tooth is cut along x -axis by using coordinate of this point, so critical section of tooth has been determined. Along critical section, the magnitude and distribution of the principal stresses ($\sigma_{\max}, \sigma_{\min}$), the maximum shearing stress (τ_{\max}), von Mises stress σ_E , as well as plane stress components ($\sigma_x, \sigma_y, \tau_{xy}$) are obtained.

For comparing the plane stress solution with plane strain solution for a tooth of any facewidth (F), by using a professional FEA software, like *Lusas*, it should be noted that the software, in case of plane strain, takes thickness of elements as one unit in terms of unit used in dimensions of the problem, instead of real thickness of the elements. for this

reason, plane stress solution can be converted to plane strain by replacing the elastic properties in the manner given in the Tab. (2). Because the software uses real thickness in the case of plane stress solution.

Table 2. *Converting the solution of plane stress to plane strain.*

Solution	to convert to	E is replaced by	ν is replaced by
Plane stress	Plane strain	$E / (1-\nu^2)$	$\nu / (1-\nu)$

4. RESULTS AND DISCUSSION

As it is seen in the Fig. (3-6), as the addendum modification coefficient is algebraically reduced, tooth thickness at addendum is being decreased and fillet radius is being increased, and vice versa.

In the curves, the horizontal axes shows the tooth thickness at critical sections. It should be noted that tooth thickness has different value, for each modification coefficient, although all horizontal coordinates change from 0 to 1. In the graphs, horizontal coordinates (χ/S_q) are obtained dividing the coordinate value, in mm, by the related tooth thickness at the critical section, S_q .

The main results obtained from this investigation are summarized as follows :

1. It is possible to improve load carrying capacity of gears or to realize suitable center distance by selecting the proper amount of addendum modification coefficient.
2. The stress concentration factor increases with an increasing addendum modification coefficient x due to a decrease in the radius of curvature at tooth fillet. But the tooth thickness at the critical section becomes higher with an increasing x , therefore tooth fillet stresses are decreasing. In addition to these , with an increasing x , the tip thickness of the tooth is decreasing.
3. A gear having few teeth has undercut. Undercutting can be prevented by using addendum modification method.
4. In the range of negative modifications, the tooth thickness at the critical section becomes smaller with a decreasing x , therefore tooth fillet stresses increase. Negative addendum modification is used in great tooth number and to obtain certain axes distance.
5. The contact ratio of mating gears is decreasing with an increase in addendum modification coefficient.
6. The stress values obtained from the plane stress solution and plane strain solution are very close to each other in this problem but the results of plane strain are a little bit small than those of plane stress.

The variation of the stresses σ_y , τ_{xy} , σ_{max} , σ_{min} , τ_{max} and σ_E are shown in the Fig. 8, 9, ...,13. The stress σ_x not shown in the figures. Because, it has approximately zero values along the tooth thickness, as expected. As seen in the figures, for all stress components, maximum values always occur in the tooth which has, algebraically, minimum addendum modification coefficient. Variations of σ_{max} and contact ratio ϵ , with respect to different addendum modification coefficients, are shown in the Fig. (14).

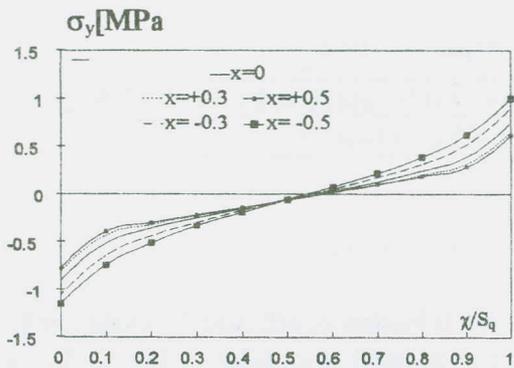


Figure 8. The stress σ_y for different addendum modification coefficients.

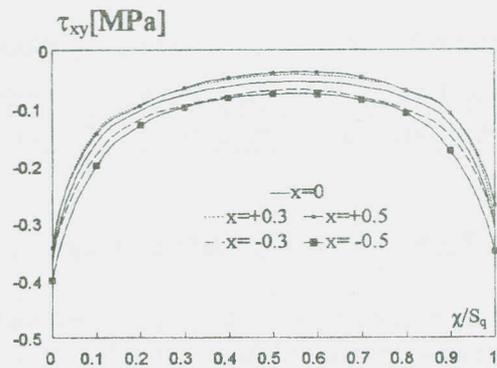


Figure 9. The stress τ_{xy} for different addendum modification coefficients.

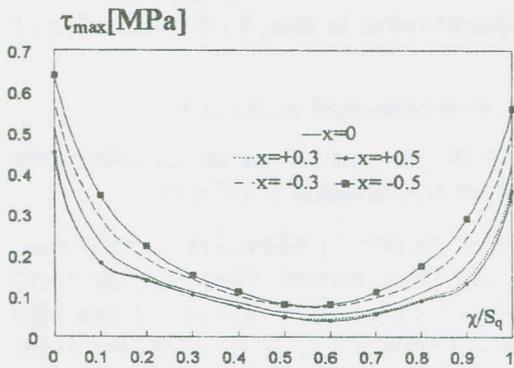


Figure 10. The stress τ_{max} for different addendum modification coefficients.

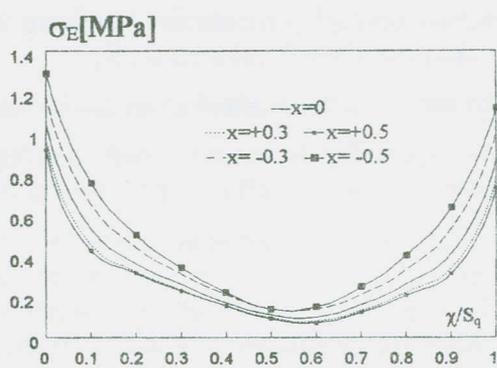


Figure 11. The stress σ_E for different addendum modification coefficients.

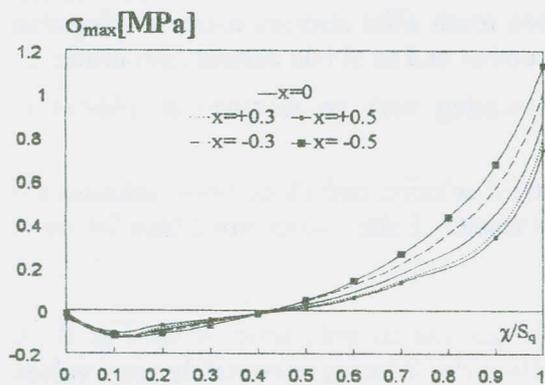


Figure 12. The stress σ_{max} for different addendum modification coefficients.

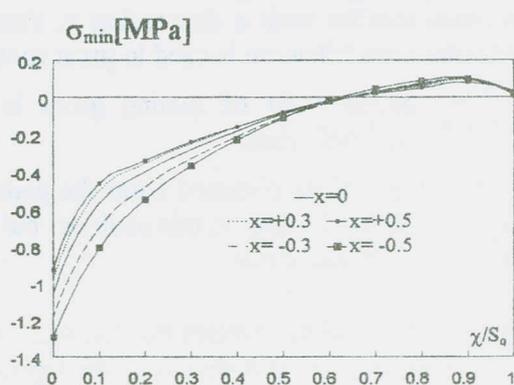


Figure 13. The stress σ_{min} for different addendum modification coefficients.

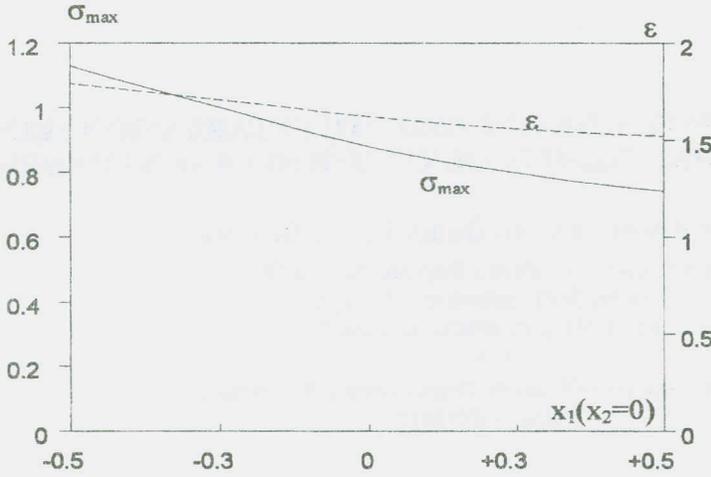


Figure 14. The changing in σ_{max} and in ϵ with addendum modification coefficient.

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