

## SOME RENOVATIONS IN TRANSIENT ANALYSIS OF TRANSMISSION LINES BY STATE-SPACE TECHNIQUES

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**Abstract** - A method for transient analysis of single phase transmission lines based on state-space technique is presented. Transmission lines are considered as the interconnection of many lumped parameter sections. By this approach state equations are formulated for the system by choosing the capacitor voltages and inductor currents as the state variables. These equations are solved by state space techniques to compute steady-state and transient responses of transmission lines for various source and load connections. A computer program called LPTLAP (Lumped Parameter Transmission Line Transient Analysis Program) has been prepared for both formulation and solution steps.

### 1. INTRODUCTION

Transmission faults and switching operations in power systems cause sudden changes in voltage and current. Transient overvoltages must be known to indicate system isolation level in the planning stage, to protect system equipment and for the design of protective devices.

Voltage and current at any point on the line can be represented by both space and time dependent partial differential equations. Analytic solutions of these equations are available for a few cases. For this reason, in the past Transient Network Analyzer (TNA) was used for the prediction of overvoltages and many applications are referred [1]. Later, by the use of digital computers numerical methods have got importance and several methods have been developed. These methods may generally be classified as time domain and frequency domain methods.

Dommel combined the method of characteristics for transmission lines and trapezoidal integration for lumped parameters to solve power system transients [2]. The program based on this method is called EMTP (Electromagnetic Transients Program). Discrete time steps are important in the evaluation of numerical integration which may cause errors, and the computations must be carried for all time steps starting from the initial time to calculate the state of the system at any time. These are unwanted conditions for long duration transients.

Transform methods (Laplace or Fourier) can be used to predict transient overvoltages[3-6]; computation is needed for wide range of frequencies to take the inverse transform numerically to obtain time domain responses. Frequency dependent parameters are naturally included in the calculations but it is difficult to implement nonlinear elements and switching operations.

In this study, state-space technique is used for the solution of steady-state and transient voltages or currents at any point on the line. State equations are formulated from the lumped parameter representation of transmission line. Numerical integration solutions for lumped-parameter transmission line model are available in the literature [7-9], but, in this study explicit formulas are used to solve these equations. This way starting from the initial state and initial time the state of the system at any time can be calculated directly without the need of the computations between that time and the initial time. Switching operations, lumped parameters and lossy distributed parameters can easily be cooperated. Nonlinear elements can also be included in the calculations [9]. One limitation of using lumped parameter model of transmission line, as well as the other time domain methods, is that frequency dependent line parameters can not be dealt with directly.

## 2. TRANSMISSION LINE MODEL

In a uniform single phase transmission line four electrical characteristics  $r$ ,  $l$ ,  $g$ , and  $c$  are distributed perfectly along the line. An approximation for this distributed nature is to represent the transmission line as an interconnection of many lumped parameter identical sections. Each section may be in the form of  $\Pi$ , T,  $\overline{\text{T}}$  or  $\Gamma$  and contains a series resistance and inductance, and a shunt conductance and capacitance as seen in Fig. 1.  $R$ ,  $L$ ,  $G$ , and  $C$  are the total resistance, inductance, conductance and capacitance of each section of the transmission line, respectively;  $r$ ,  $l$ ,  $g$ , and  $c$  appearing in the first line represent these parameters for per unit length. The resistance  $R$  for each section is determined by dividing the total resistance of the line by the number of sections  $n$ .  $L$ ,  $C$  and  $G$  can be determined in the same manner [1].

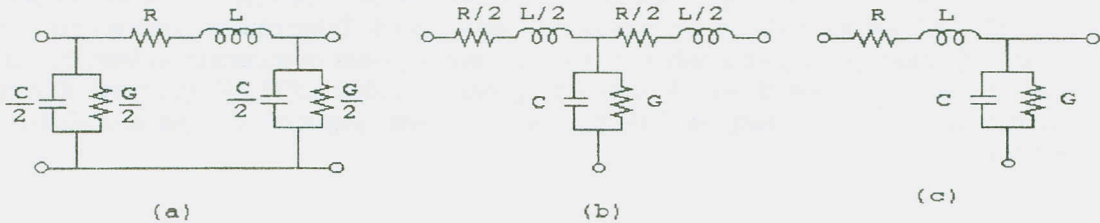


Figure 1. Different lumped parameter network models of each section of a transmission line; a)  $\Pi$ -network, b) T-network, c)  $\overline{\text{T}}$ -network ( $\Gamma$ -network can be constructed by connecting C-G branch of  $\overline{\text{T}}$ -model to the left side of R-L branch).

When  $n$  T-sections are connected in cascade and some series elements are combined, the transmission line model shown in Fig. 2 is obtained. Similar line models can be obtained by using other type of sections. These models will be the fundamental basis for the state-space analysis of the transmission line.

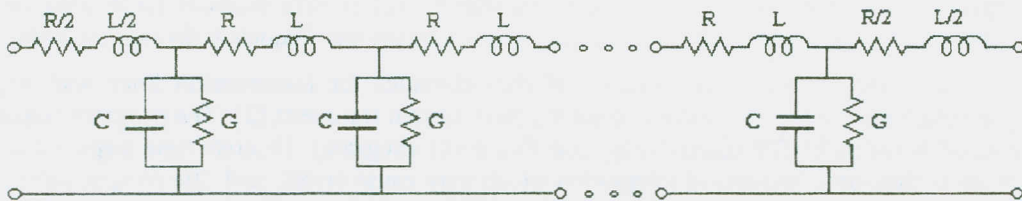


Figure 2. An equivalent circuit of the transmission line model obtained by using T-sections.

## 3. TRANSMISSION LINE TERMINATIONS

It is not always necessary to use a detailed representation of an overall source configuration of a network. The form of source and load representations should be chosen depending on the objectives of the particular study carried out. In view of this, for the studies carried out here simplified lumped parameter equivalent circuits of source and load side networks are used.

For the source models source types are assumed in one of the forms shown in Fig. 3.

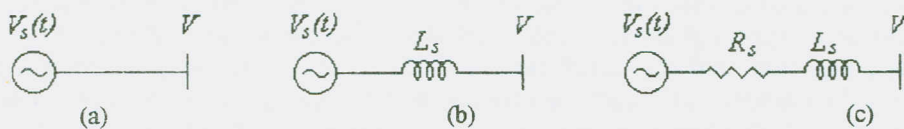


Figure 3. Source Representations; a) Infinite bus-bar, b) Purely inductive finite source, c) Composite Source.

Different types of loads such as residential, commercial or industrial may accumulate to result with load models which can be represented by one of the following equivalent circuits shown in Fig. 4.



The presented method applies for lumped parameter terminations other than shown in Fig. 3 and 4. However without any modification, the prepared program LPTLAP can deal with only the given terminations.

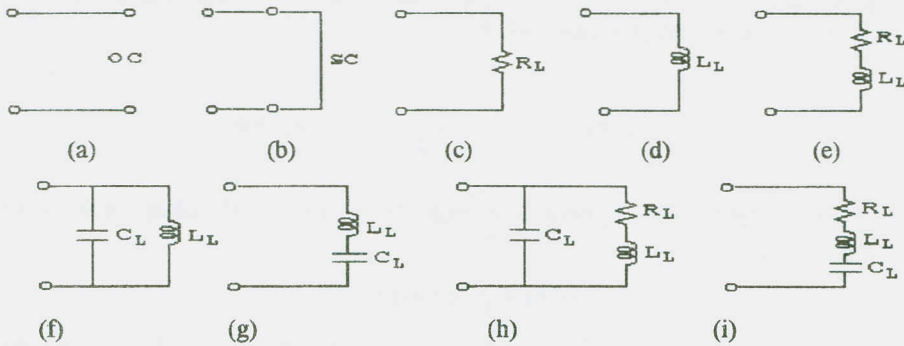


Figure 4. Load types for the termination of the transmission line: a) Open circuit, b) Short circuit, c) Resistive load, d) Inductive load, e) Resistive-inductive load, f) Tank circuit, g) Resonator, h) Lossy tank circuit, i) Lossy resonator.

#### 4. STATE-SPACE REPRESENTATION AND SOLUTION OF TRANSMISSION SYSTEMS

Linear, lumped parameter networks containing resistors, capacitors, inductors and voltage and current sources can be represented by the so called state-space equations written in the form [10]

$$x(t) = Ax(t) + Bu(t), \quad x(0) = x_0; \quad (1a)$$

$$y(t) = Cx(t) + Du(t). \quad (1b)$$

In these equations the state vector  $x$  contains some of the capacitor voltages and inductor currents,  $x_0$  is the initial value of this vector, the excitation vector  $u$  represents the input,  $y$  is the vector of output variables which are defined as some of the voltages and/or currents in the whole system;  $A$ ,  $B$ ,  $C$ , and  $D$  are the constant matrices which depend on the values of the lumped parameters of the network. By using  $n$  T-sections in the lumped parameter model of the line and assuming the source termination of Fig. 3a, and load termination of Fig. 4a the state equations can be constructed as

$$\frac{d}{dt} \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_2 \\ \vdots \\ i_n \\ v_n \end{bmatrix} = \begin{bmatrix} -R/L & -1/L & 0 & \dots & 0 \\ 1/C & -G/C & -1/C & 0 & \dots & 0 \\ 0 & 1/L & -R/L & -1/L & 0 & \dots & 0 \\ 0 & 0 & 1/C & -G/C & -1/C & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 1/L & -R/L & -1/L \\ 0 & \dots & \dots & \dots & 0 & 1/C & -G/C \end{bmatrix} \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ v_2 \\ \vdots \\ i_n \\ v_n \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u, \quad (2a)$$

$$V_R = [0 \ 0 \ 0 \ 0 \ \dots \ 0 \ 1] [i_1 \ v_1 \ i_2 \ v_2 \ \dots \ i_n \ v_n]^T \quad (2b)$$

In these equations  $i$ 's ( $v$ 's) indicate the currents (voltages) of inductors (capacitors) in the series (shunt) arms of the lumped parameter model of the line. Similar equations can easily be obtained for different load and source terminations, and can be modified by considering short and open circuit faults [11]. For each possible combination of source and load terminations considered in

Section 3 these equations are obtained in a similar manner. Due to space consideration all of them can not be included here. However the computer program is prepared as to analyze all possible combinations.

The solution of (1a) can be evaluated in terms of the initial state vector  $x_0$  and the excitation vector  $u$ , and is given by the expression [12]:

$$x(t) = e^{A(t-t_0)}x_0 + \int_{t_0}^t e^{A(t-t')}Bu(t')dt'. \quad (3)$$

Consider that a transmission system is excited by a sinusoidal voltage source which can be expressed as

$$u(t) = |u| \cos(\omega t + \phi), \quad (4)$$

where  $|u|$ ,  $\omega$ , and  $\phi$  are real constant numbers representing the amplitude, angular frequency, and the phase of the source, respectively. To simplify the manipulations, the phasor  $U$  is defined as a complex number given by

$$U = |u|e^{j\phi}. \quad (5)$$

Then the input  $u(t)$  can be written as

$$u(t) = \text{Re}\{Ue^{j\omega t}\}. \quad (6)$$

Replacing  $j\omega$  in the above equation by  $p$ , where  $p = \sigma + j\omega$ , we write

$$u(t) = \text{Re}\{Ue^{pt}\}. \quad (7)$$

Inserting (7) into (3), and since all expressions are real in the integration except  $Ue^{pt'}$ , we obtain

$$x(t) = e^{A(t-t_0)}x_0 + \text{Re} \left\{ \int_{t_0}^t e^{A(t-t')}BUe^{pt'}dt' \right\}. \quad (8)$$

Then, by integration the following equation is derived

$$x(t) = \text{Re}\left\{e^{A(t-t_0)}x_0 + (pI - A)^{-1}(e^{pt}BU - e^{A(t-t_0)}BUe^{pt_0})\right\}. \quad (9)$$

This equation, together with Eq. 1b, gives the complete response of transmission line. In (9), the matrix  $(pI - A)$  is assumed to be nonsingular; which is a valid assumption since the eigenvalues of  $A$  are different from the excitation frequency  $p = j\omega$  (and further distinct in all practical applications other than a few exceptional case) [11].

The steady-state solution  $x_{ss}(t)$  is defined by  $\lim_{t \rightarrow \infty} x(t)$  and for a stable system can be written as

$$x_{ss}(t) = \text{Re} \left\{ (pI - A)^{-1}Be^{pt}U \right\}. \quad (10)$$

If the excitation is a step voltage then the step-input solution of the state can be obtained directly from (9) by assuming  $p=0$ , and  $U$  is a real number representing the value of step voltage. Then the complete state response of the system under step excitation can easily be written as

$$x(t) = e^{A(t-t_0)}x_0 + (-A)^{-1}(I - e^{A(t-t_0)})BU. \quad (11)$$

### Computation of $\exp(At)$

$x(t)$  can be obtained from (9) or (11) by matrix operations including inversion and exponentiation. If large number of sections are used, the state dimension will be high and computations of  $\exp(At)$  will require a special attention and care. For all cases, Gauss elimination method with full pivoting gives satisfactory results for the computation of matrix inversion [13].

For the matrix exponentiation the fundamental formula for the function of a matrix is directly applicable [12] and it yields

$$e^{At} = \sum_{k=1}^p e^{\lambda_k t} Z_k(A), \quad (12)$$

where  $\lambda_1, \lambda_2, \dots, \lambda_p$  are the distinct eigenvalues of  $A$ , and

$$Z_k(A) = \prod_{\substack{i=1 \\ i \neq k}}^p \frac{(A - \lambda_i I)}{(\lambda_k - \lambda_i)}. \quad (13)$$

For eigenvalue calculation Rutishauser's LR algorithm is applied [13]. For a few exceptional case this algorithm gives satisfactory results. This algorithm incorporates the following features; a) economy of storage, b) special handling of tridiagonal matrices taking the advantage of sparseness of  $A$ , c) double precision real arithmetic.

### Fault Analysis

Short circuit at any point on the line is simulated by removing the capacitance and conductance at node  $n_k$  on the lumped parameter model of the line nearest to the short circuit point. Then, capacitor voltage  $v_k$  becomes zero and hence it will not be a state variable. This results with the reduction of the number of state variables by one; and to represent the faulty system the rows and columns in coefficient matrices corresponding to that state variable must be removed. A similar procedure is followed for the open circuit fault. But, in this case, reduced state variable is the current of inductance connected to the two adjacent nodes of the lumped parameter model, which the open circuit fault occurs between.

Although two sets of uncoupled state equations result for both types of faults, for programming easiness they are treated as a single system of state equations in LPTLAP. Due to the tridiagonal property of  $A$ -matrix this combination does not cause considerable time-loss in computations.

## **5. COMPUTER PROGRAM**

A computer program LPTLAP based on the procedure described has been prepared. The program has three main parts: data input, main part and data output. At first, program reads input data which contains the information about system such as the parameters of overhead line, source type and parameters, load type and parameters, type of study and in case of fault analysis type of fault, the output variables, etc. from a file or directly from the terminal. Then, the coefficient matrices  $A$ ,  $B$ ,  $C$  and  $D$  in the state model are built depending on the identifications given as input data; at the this stage, the steady-state and/or transient analysis of the system is carried out and all the state and output variables of the system are computed. Finally, calculated output variables at each time step are written to an output file. All of or any part of input data in any example may be used (perhaps by small changes) in another example. For this reason, after all calculations, input data are also written as output.

The following types of transmission line analysis problems can be studied by the program:

- a) Steady-state analysis,
- b) Switching transients,



- c) Short and open circuit faults,
- d) Loading and load rejection transients.

The memory requirement and computer run time of the program primarily depends on the system dimension. Considering the dimension of arrays in the program and knowing that one REAL (INTEGER, REAL\*8, COMPLEX\*16) number is stored in 4 (4, 8, 16) bytes [14], the total memory requirement for the state dimension N is  $(16N^3+248N^2+224N)$  bytes.

Computer CPU times of some examples studied in the next section in an IBM AT compatible 386 personal computer are given in Table 1 .

Table 1. Computer CPU times for some examples.

Example no (Type of Analysis)	Number of sections-n	CPU Time (s)
A1 (Steady-state analysis)	10	0.9
B1 (Line energization)	10	128
B2 (Line energization)	8	38
C1 (Short Circuit fault)	8	38
D1 (Loading)	8	50
D2 (Sudden load rejection)	8	38

### 6. APPLICATIONS AND RESULTS

To illustrate the method used, the LPTLAP is applied to several examples. In all case studies, one phase of a 160 km, 400 kV system with parameters  $r=0.032$  W/km,  $l=0.88$  mH/km, and  $g=0.042$  mS/km and  $c=0.013$  mF/km is considered.

#### A. Steady-State Analysis

*Example A1.* The following example is studied to illustrate the steady-state performance of transmission line. A load of 220 MVA with 0.8 power factor (lagging), at a voltage 231 kV is assumed. When the load current and load voltage are selected to be the output variables and the transmission line is approximated by 10 T-sections, the obtained load voltage and current by LPTLAP are shown in Fig. 5a and b, respectively. The phase difference between the voltage and current, i.e.,  $\cos^{-1}0.8=36.87$  is observed in this figure. Maximum load voltage theoretically calculated by using the transmission line terminal equations involving hyperbolic functions is 0.892734 pu. With reference to this value, the relative error involved is in the order of  $10^{-5}$ .

For 1 pu input, the transmission line voltage, when it is compared by using different number of sections (N) in the lumped parameter representation is recorded and shown in Table 1. As it is seen in the table, when the number of sections are increased the load voltage approaches to a constant value. In fact for  $n \geq 10$ , the load voltage remains constant within 5 significant digits. In general, to satisfy a given precision, it is not needed to increase the number of sections above a certain value, since this increases the computer time and sometimes it may cause the numerical instability.

Table 1. Change in load voltage with respect to N

N	Maximum load voltage (pu)	Computer CPU time (s)
5	0.892744	0.27
10	0.892734	0.66
15	0.892732	1.37
20	0.892731	2.58



## B. Line Energization and Deenergization Transients

*Example B1-Step energization:* The 400 kV transmission line with termination 50 mH inductance at receiving-end is energized by a step voltage. For this example 10 T-sections are used to simulate the transmission line. The receiving end transient voltage obtained by LPTLAP is shown in Fig. 6. Forward traveling wave is reflected after the first arrival to receiving end and there is an exponential decrease up to the next reflection on inductor. High frequency oscillations after the arrival of incident wave are due to the discretization of line by lumped parameters; in fact, these oscillations are not observed in the results obtained by using Fast Inverse Laplace Transform (FILT) directly applied to the distributed line [6].

It is also seen that in the peak variations the results of the lumped parameter approximation deviate from the FILT results, which are validated by the direct evaluation of the inverse Laplace transform integral numerically.

It is important to note that for this example and the following, considering the CPU time of the computer, the LPTLAP takes much longer the time than the other methods; this is due to the computation method of  $\exp(At)$  and research is continuing on its reduction. But LPTLAP computes also the variation of the response at discrete points along the line.

*Example B2-Sinusoidal energization:* In this example, the line has an open circuit at receiving-end is energized by a composite sinusoidal voltage source with parameters  $R_s=0.384 \Omega$ , and  $L_s=48.8$  mH. The sinusoidal source at the instant of energization is at its peak value. Transmission line is modeled by 8 T-sections. Sending-end and receiving-end voltage variations for one cycle of input obtained by LPTLAP and FILT are shown in Fig. 7a and b, respectively. FILT results are obtained by distributed parameter representation of transmission line. Fig. 6b shows that incident wave arrives to line end in approximately  $\tau=\sqrt{lc}=541 \mu s$  and doubled due to positive reflection of open circuit. The peak values of both results are approximately equal but there is a phase difference which can be reduced by using high number of sections for the representation of line.

*Example B3-Line deenergization:* In this example we consider the 400 kV transmission line terminated by a composite source with parameters  $R_s=0.384$ ,  $L_s=48.8$  mH and the load type shown in Fig. 4e with the parameters  $R_L=193 \Omega$ ,  $L_L=0.461$  H. It is dropped by a switch at sending-end. Transmission line is simulated by 8 T-sections. When the line is operating under steady-state conditions, the switch is opened at the time instant  $t=2$  ms. The steady-state voltage for  $0 \leq t \leq 2$  ms and the transients overvoltages occurring in receiving end of line after  $t=2$  ms are shown in Fig. 8. Transients are similar to open circuit fault transients and will be discussed in Example C2.

## C. Fault Transients

*Example C1-Short circuit fault:* The 400 kV transmission line with the termination given in previous example is studied to illustrate the short circuit fault transients. A short circuit is assumed at 100 km from the sending-end at time  $t=2$  ms when the transmission line operates under steady-state conditions. Transmission line is approximated by 8-T sections. Transients occurring in sending-end current and voltage as a result of this fault is shown in Fig. 9a and b for two cycles of sinusoidal input. The peak value of short circuit current is quite agreed with short circuit current  $I=7.57$  kA obtained by dividing the peak source voltage  $400\sqrt{2}/\sqrt{3}=326.6$  kV by the impedance  $(0.384+j15.3+3.2+j27.7) \Omega$ . In this impedance  $(0.384+j15.3)$  represents the source impedance and  $(3.2+j27.7)$  represents the series impedance of the 100 km line section.

*Example C2-Open circuit fault:* Transmission line in the above example with the same terminations is studied for open circuit fault, and calculated transients at sending- and receiving-end voltages are shown in Fig. 10a and b. The open circuit fault is assumed in 100 km from the sending-end. Overvoltages seen in Fig. 10a are due to current chopping. The low frequency transients at receiving-end exponentially decrease due to load resistance and inductance and are in the form of discharges as seen in the figure. Low frequency transients are approximately  $1/2\pi((0.053+0.461)0.78 \times 10^{-6})^{1/2}=251$  Hz where 0.053 and  $0.78 \times 10^{-6}$  represent the inductance and capacitance of the 60 km line section and 0.461 represents the load inductance. High frequency oscillations seen in Fig. 10b are due to the reflections from the two ends of transmission line.

D. Load Switching Transients

*Example D1-Loading:* Sending-end current transients occurring after a load sown in Fig. 4e with  $R_L=193\ \Omega$ ,  $L_L=0.461\ \text{H}$  is connected to the end of transmission line at time  $t=2\ \text{ms}$  is shown in Fig. 11a for two cycles of sinusoidal input. The line is operating under steady-state conditions and it is modeled by 8 T-sections. Sending-end current after the loading operation is similar to short circuit fault transient. Peak value of the current seen from the figure is approximately equal to the current  $1.1\ \text{kA}$  calculated simply by dividing the peak source voltage  $326.6\ \text{kV}$  by  $Z_S+Z_L=214+j204.3$ , where  $Z_S=0.384+j15.3$  is source impedance,  $Z_L=160(0.15+j0.276)$  is the total line impedance and  $Z_L=193+j144.8$  is the load impedance.

*Example D2-Sudden loss of load:* Transient voltage occurring at sending-end, when the load used in the previous example is separated form the transmission line is shown in Fig. 11b. Before switching the line is operating under steady-state. For this study 8 T-sections are used to simulate the transmission line. Transients are similar to open circuit fault transients shown in Fig. 5. Differences are due to distances from the sending-end.

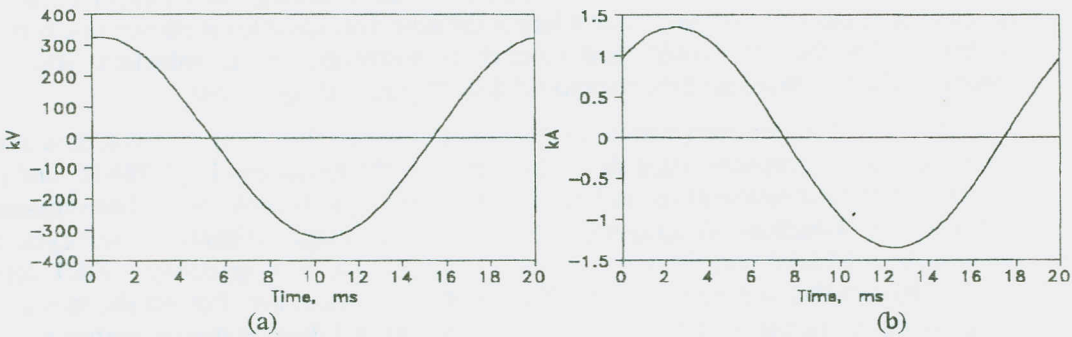


Figure 5. Steady-state response of transmission line; a) Load voltage, b) Load current.

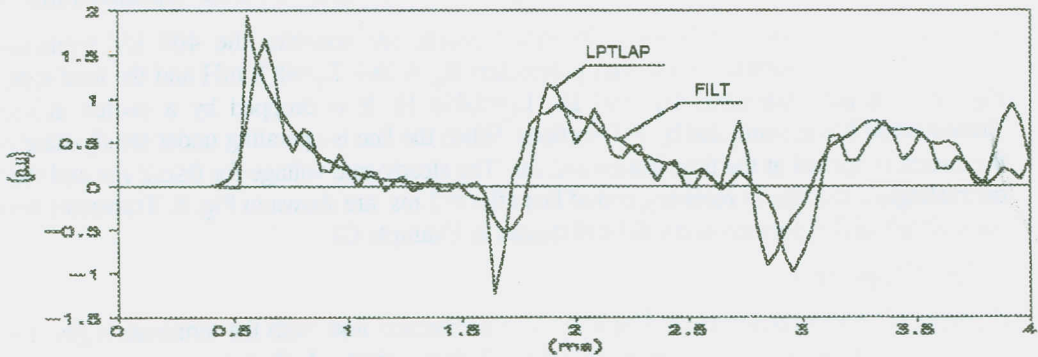


Figure 6. Step response of 50 mH loaded transmission line.

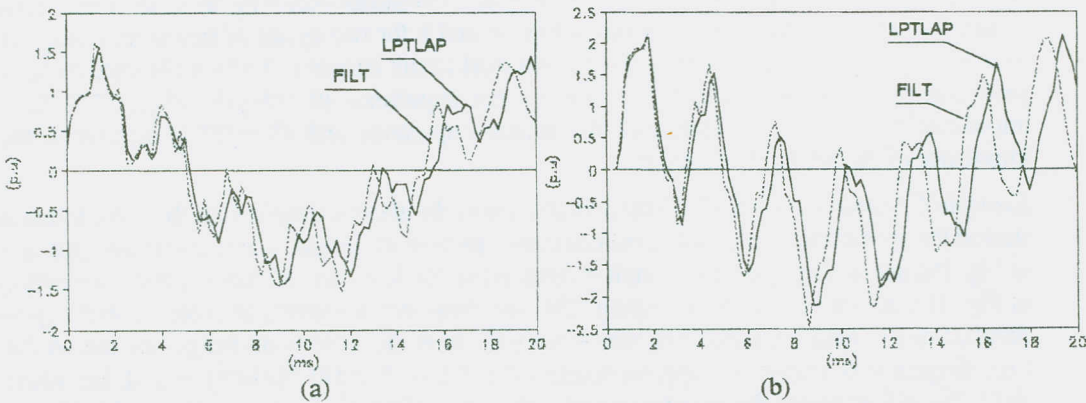


Figure 7. Line energization; a) Sending-end voltage, b) Receiving-end voltage.



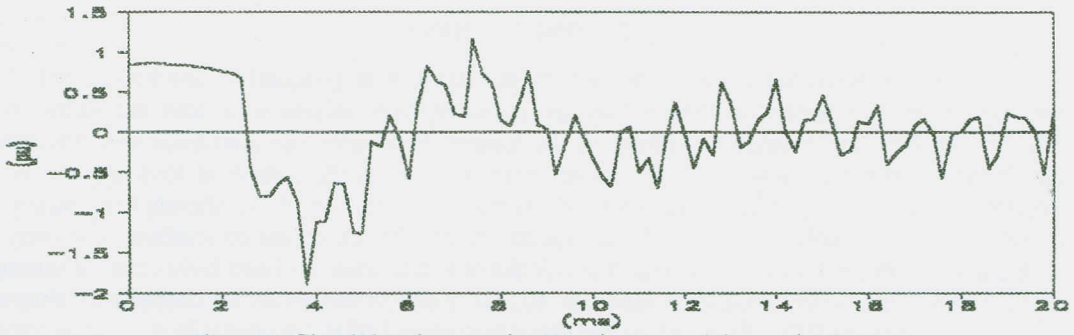


Figure 8. Circuit breaker opening transients, receiving end voltage.

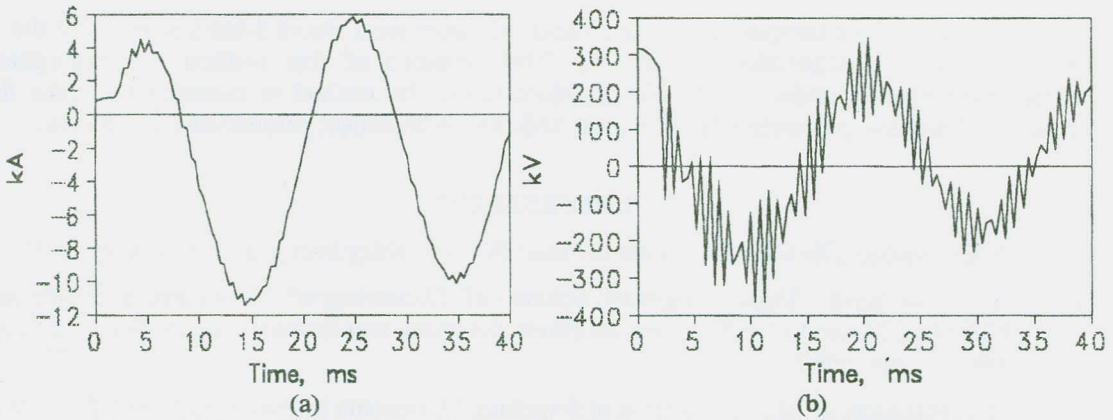


Figure 9. Short circuit fault transients; a) Sending-end current, b) Sending-end voltage.

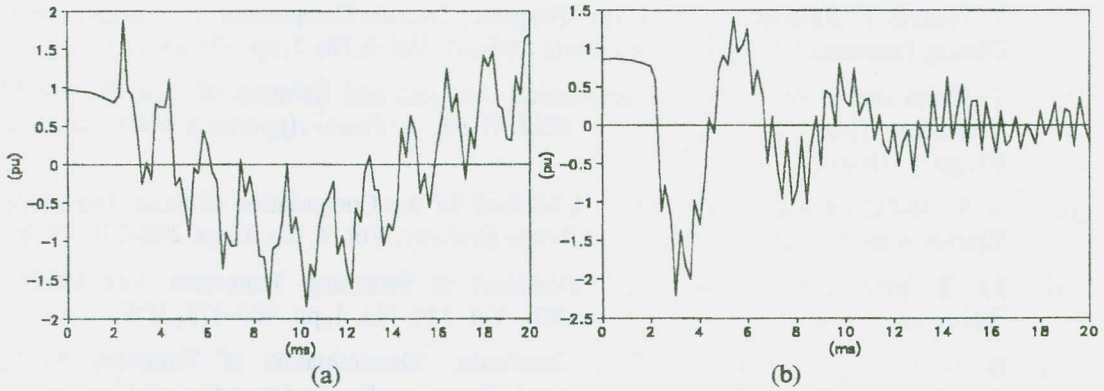


Figure 10. Open circuit fault transients; a) Sending-end voltage, b) Receiving-end voltage.

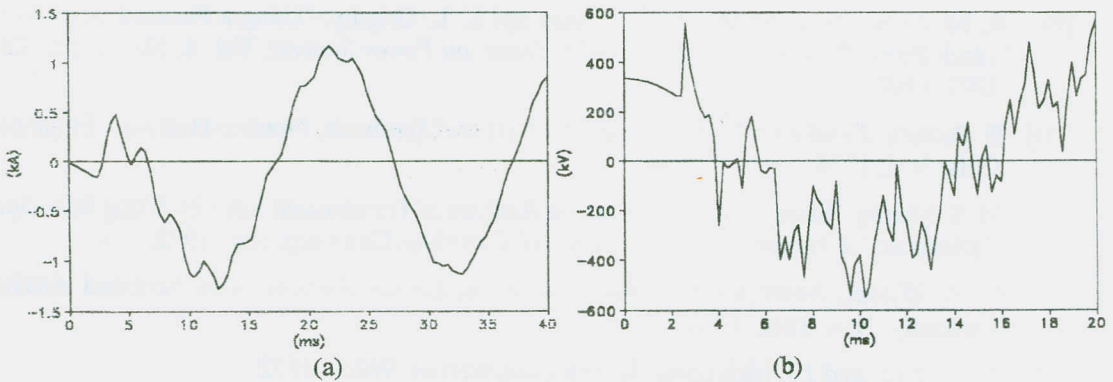


Figure 11. Load switching transients; a) Loading transients, sending-end current.  
a) Sudden load rejection transients, sending-end voltage.

## 7. CONCLUSIONS

By using lumped parameter model of transmission line, state equations are formulated. Then the response of the system is obtained by using closed form solutions of state equations. Both inductor currents and capacitor voltages in the lumped parameter representation of transmission line are obtained simultaneously. Since, closed form solution for the state equations are used in the calculations, possibility of accumulation of errors is eliminated. It is shown that when the sparseness of the coefficient matrix  $A$  and special eigenvalue computation methods are used the state-space technique for the computation of transients in transmission lines have many advantages over the numerical and/or transform methods. Resistive effects can easily be cooperated. Prepared computer program (LPTLAP) based on the procedure described is also capable of solving various types of transients occurring in the power systems due to faults and switching operations.

To decrease the computing time and reduce the numerical errors 3-band property of the  $A$  matrix is used in eigenvalue calculations. The extension of this method for multi-phase transmission lines is under study. For a future work, the method is considered to take the frequency dependent parameters and nonlinear and/or time dependent parameters into account.

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