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Dynamic Sales Price Control Model for Exclusive Exquisite Products within a Time Interval

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Abstract: When information regarding the effective evaluation of the value of exquisite products is lacking, the market demand function for such products at a given time point is affected by the diffusion of historical transaction price information before the time point. This is because historical transaction prices play an active role in influencing the internal reference price (IRP) of customers, and the continuous diffusion of historical transaction price information leads to the continuous correction, adjustment, and updating of customers' IRPs. Given the varying rates of such information diffusion, the speed at which customers adjust their IRPs also varies across individuals and contexts. By considering the exponential distribution of potential customers' IRPs as an example to establish the dynamic demand function that considers the effect of historical transaction prices, this paper discusses the effect of different information diffusion rates on the demand function at a time point. On the basis of this demand function, a sales price control model that maximizes the discounted profitability for businesses in the patent term of an exquisite product is then constructed to provide businesses with an operation method to cultivate prices and increase profits.

Keywords: dynamic pricing; exclusive brand; historical transaction price; internal reference price; diffusion rate

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1. Introduction

Exquisite products, defined as high-quality products in this study, differ from common consumer products in that customers have increasingly high expectations of the quality of most consumer products, whereas they consider exquisite products to already be of high reference quality when they first use them. Therefore, pricing strategies for exquisite products differ from the periodic high–low pricing strategy commonly adopted for a consumer product. Specifically, a business maintains the price of an exquisite product at a high level to "cultivate" the price. In another scenario, the business prices a new exquisite product higher than the price expected by potential customers upon product launch to attract these customers' attention, leading the customers to discuss the product without actually buying it. Marketing events are subsequently held to prompt the potential customers to purchase the product, leading to a quick sell-out of the new product and thereby helping the business achieve its sales target [1].

Under such a price-cultivation model, businesses do not lower their product prices to promote sales even if they are at losses; such losses are seen as investments in price cultivation. The purpose of price cultivation is to anchor a reference price in the minds of customers and to establish customer evaluations of products. However, before making investments for adopting the price-cultivation practice, businesses must consider the constant updating of reference prices for such exquisite products in customers' minds as well as supplier competition (competition between brands) and retailer competition (competition within brands) in the product market [2–4].

By solving the above questions, a business will be able to emphasize the scarcity of products (limited editions or limited-time offers) or exclusively sell or manufacturing the products of a brand (hereafter referred to as "exclusive brand") for price cultivation. For example, businesses offer large discounts on their products on various occasions, such as during their anniversaries, on key shopping days (e.g., 11 November and 12 December), or in celebration of Taiwanese sports teams winning championships. Such offers can help businesses achieve sales that would otherwise require an entire year to achieve within a short period of time of one week. Although numerous studies have explored product quality and reference prices, few have investigated the effect of the information on historical price (i.e., historical transaction price or historical sales price) continually diffused through advertising on customers' internal reference prices (IRPs) of exquisite products. Studies have also rarely assessed how businesses determine when to cultivate the price of exquisite products labeled as an exclusive brand or exclusive agency and when, once customers' reference prices have been set at a relatively high point, to offer some discounts for a sell-out. The study was conducted to fill this research gap. The purpose of this study is to provide businesses with an operation method for cultivating prices and increase profits.

2. Literature Review

Reference price, a critical factor affecting customers' purchasing decisions, is a diverse concept and has remained an interesting topic in marketing research. The authors in [5] reviewed the research on reference prices and discussed them from three dimensions: (1) the formation of reference prices, (2) the search and use of reference prices, and (3) the effect of reference prices on different types of decision-making and evaluation. Based on these dimensions and the trends and patterns of reference price research over recent years, the present study modified and supplemented the dimensions of reference prices and adjusted three dimensions: (1) the formation of reference prices, (2) the search and evaluation of reference price, and (3) the association between product or brand quality and reference prices. First, relevant studies on the aforementioned three dimensions are presented, and then the association between the reference price and product or brand quality as well as the continuous updating of the reference price of exquisite products are discussed.

2.1. Formation of Reference Prices

Studies on reference price formation can be roughly divided into two categories based on the concepts they discuss, namely expected and fair reference price. A fair reference price is a product price that is identical or similar to the price paid by other society members [6], namely a normative sales price that is deemed fair or just for the seller to charge for a product [7]. Expected reference price is the most widely discussed concept of reference price [5]. With the increasing research on the theoretical aspect of reference price, researchers have paid increasing attention to the formation and applications of reference price in marketing. Theoretical research on expected reference price has its origin in the field of psychology; it was pointed out in [8] that according to the adaptation-level theory, customers respond to product sales price by comparing the sales price and the price level they are adapted to (i.e., the reference price). Building on this theoretical basis, the authors in [9] proposed that reference price serves as the standard for purchase price. Extensive research has consistently stated that product purchase is limited by the change in IRP. Some studies have suggested that potential customer's perceived value of a product is built on the difference between the market price of the product and the IRP [10–13].

2.2. Search and Evaluation of Reference Price

According to [14], reference price is the expected price shaped by the past experience of customers and the immediate shopping environment. In other words, reference price is a combination of the memory-based IRP and the stimulus-based (e.g., stimulation from useful external price information or from the shop environment) external reference price [15–17]. Some research has focused on the association between external reference price and IRP and pointed out that the retail price suggested by a retail store is absorbed by customers as the IRP, which in turn affects their purchase evaluation or behavior [18,19]. Some other studies have suggested that historical transaction price observable to customers must also be considered [20,21]. Another study [22] recommended that the reference price of the field salesperson should be considered as a factor because it also affects the reference price of customers.

According to [23,24], customers are willing to buy a product when their IRP for the product is higher than the product's sales price indicates a gain to customers, whereas a lower IRP indicates a loss to them, and customers are more sensitive to a price that causes a loss than to a price that causes a gain. The authors in [25,26] maintained that retailers must consider the reference price of customers and provided relevant managerial implications. Reference price is a result of customers comparing the prices they can recall or past price experience with the product price displayed on the shelf [27,28]. Past price information forms the expectation of reference price [29,30], and such information from the nearer past has a stronger influence on the expectation [31,32]. Accordingly, promotional events of retailers may imperceptibly reduce customers' expectations of future price and enhance their price sensitivity [33–35]. When the price is lower, customers engage in less research on reference price and exhibit a slower response to the sales price [36,37]. In [38] it is recommended that businesses establish a higher normal price, offer different discounts occasionally, and leave no trace of the discounts when the offers end. In [39], a solution model is proposed for businesses to prevent large drops in the reference price following price promotions.

Customers' searches for reference price differ across individuals and contexts. This is because each piece of historical transaction price information is diffused at a different rate, and thus some potential customers will receive new transaction price information first and some later [40,41]. Advertising is a common approach used by businesses to diffuse product information (including price information) and to induce purchase actions by customers [25]. The authors in [42] revealed that when the product price returns to a normal level after a price competition among businesses, customers consider the current price relatively high and likely to cause a loss to them; advertising can alleviate such a sense of loss in customers.

The authors in [43] employed the mean IRP of potential customers as the external variable for a utility function; change in the mean IRP causes the entire utility function curve to shift. Accordingly, the market demand is better reflected by a demand function that incorporates reference price [31]. Various studies have developed reference price evaluation models under the assumption that reference price is an external variable affecting customer utility function. In [26,27], simple linear demand models were developed on the basis of the reference price effect. In [44], a demand function was proposed based on triangular distribution. The authors in [14] performed a confirmatory comparison of different reference price models and suggested that the optimal reference price model should be established on the basis of the IRP mechanism. Regarding the establishment of reference price models, the authors in [45] stated that an acceptable price range is determined by the ends of reference price distribution. The authors in [46] maintained that the price distribution pattern must also be considered, and that price refers to a range of prices or the latitude of acceptable prices, rather than a single price point. Whenever a buyer observes a reference price, the distribution pattern of the particular reference price changes continuously [21,47].

Many studies have highlighted the fact that customers do not make judgements based only on the absolute price; rather, they evaluate the price according to a reference point, namely the reference price as a price point [48]. Several studies [27,41,49] considered reference price to be a weighted mean of previous prices. The authors in [38] proposed a model integrating the reference price range and price point. The authors in [50] pointed out that although reference price points are applicable to most marketing fields, decision-makers lack the understanding of how reference price points are formed in a price series. Reference price points are time-variable and form an S-shaped weighted function; the first and last prices in the time series are attached the highest weight and the price in the middle and lower weights. In [51] customers' reference prices were considered to be weighted means of the lowest past price and the latest price. The authors in [52] believed that customers, to a certain extent, research the expected price distribution of a product before buying it and verified that a higher expected price, despite not prompting purchase actions, affects the customer's reference price. In was explained in [53] that the reference price is affected through the behavior of customers setting their reference price according to external factors when the product price is unknown to them, and customers' expected maximum utility of the external choice was compared with the product's marginal utility observed internally.

Customers constantly take in new price information, including from advertisements, etc. [54], which causes their reference price to change with time; therefore, the initial reference price of customers changes. A new piece of price information may not result in a complex process of reference price formation; instead, it may prompt a new reference price directly [55,56]. Although the reference price effect is easily observed by businesses, the exact amount of a customer's reference price is not directly available to them. Therefore, businesses should use demand information to estimate customers' reference prices and constantly adjust the estimate [57], because the customer demand function, considering the changes in customers' IRPs, is dynamic. A study on the quality and effect of reference prices pointed out that studies have predominantly focused on one-off pricing and emphasized the importance of including dynamic pricing, because all variables of dynamic pricing involve the functions of time and system parameters [58]. Dynamic pricing is also a topic of interest for future research on reference prices or pricing. Previous studies have predominantly assumed that reference price is an external variable that affects the customer utility function. Therefore, they have rarely addressed the effect of advertising diffusion, particularly continuous diffusion, on price information.

2.3. Association of Reference Price with Product or Brand Quality

In the face of fierce market competition, businesses need to emphasize product quality and sales price as the main market variables, making quality management and product pricing strategies the most critical operational tools for most businesses. In fact, a dynamic relationship exists among the market demand for, quality of, and sales price of a product. By controlling the sales price, which substantially affects product sales, businesses can boost their sales and profitability [59,60]. Various empirical studies [61] have revealed that customers often rely on their reference price and the sales price to judge information related to product quality. This explains why customers do not prefer products with the lowest price. In other words, the sales price and reference price concurrently affect customers' resource allocations and provide information to them; such a complex relationship indicates the inapplicability of a demand function that considers only a single price factor. Understanding the demand function is fundamental to pricing management; therefore, a wrong assumption regarding the demand pattern can lead to an incorrect decision, and consequently, business losses.

Studies on the relationship between product quality and reference price can be divided into two types based on the attributes of products they have discussed. The first type of study discussed fresh, or deteriorating, products whose "freshness" (i.e., quality) declines with time; examples of these products are food, flowers, and medications. This type of study focused on how product freshness affects customer utility and in turn affects customer demand [62–64]. Some retail chains offer discounts on products according to their expiration dates (for example, PX Mart in Taiwan provides a 20% off discount on all fresh products that will expire soon); such a practice is not appropriate, because it neglects the fact that customers' different requirements for product expiration date (i.e., freshness) may reflect customers' different considerations apart from price. The negligence has a significant effect on retailers' price discrimination and expected profitability [65,66]. Various promotional models have been proposed, but these studies have predominantly focused on the optimization of price reduction and rarely discussed the optimization of dynamic pricing [63,67,68]. Some of them also discuss the reference price effect [56,69,70].

Another type of study examined the relationship between the quality and price of products. Conventional studies have believed that customers make rational purchase decisions according to the current price, product quality, and other market factors. The authors in [71] studied the relationship between the expected product quality and reference price. Customers compare their expected product quality (i.e., reference quality) with the actual product quality and accordingly perceive an increase or decrease in the product quality on the basis of the comparison result [57]. This in turn affects the customers' reference prices. The authors in [72] emphasized that the relationship between product demand and quality is dynamic, and that product quality is a result of the tangible quality and the brand's reputation. The authors in [73] consolidated the relationship between the quality and sales price of products and demonstrated that a high price is likely to result in high perceived quality; such a phenomenon is observed for exquisite products marketed as products with a high quality (or high design quality).

A review of the literature showed that despite the large amount of research on reference price, little research has been conducted on the integration between the price effect, product quality, and advertising effect [74]. Therefore, this study established a dynamic demand model that considered the continuous diffusion of historical price information and the association of such information with the constant correction, adjustment, and updating of customers' IRPs. Unlike previous studies that have mostly used simple linear functions or triangular distribution functions to address this topic [27,33,49,50], the present study adopted the exponential distribution of customers' IRPs, which better reflected the characteristics of exquisite products. The study results could help businesses determine the appropriate time to cultivate the price to set the customers' reference price high and the appropriate time to offer discounts for a sell-out. To achieve these objectives of model development, this study (1) expressed the demand function at a time point in terms of historical prices at a given diffusion rate of historical transaction prices/historical sales prices; (2) determined the effect of different information diffusion rates on the demand function at a time point; and (3) based on the dynamic demand established according to points (1) and (2), established a sales price control model that maximized the discounted profit for exquisite products to provide businesses with an operation method for cultivating prices.

3. Dynamic Demand Model Shaped by Historical Prices

3.1. Mathematical Symbols and Assumptions

- c: unit product cost of the business.
- N: number of potential customers, with each potential customer's IRP being greater than the unit cost c.
- z: each potential customer's IRP, which varies across individuals and time.

This study assumed that the demand function of the potential customer group at each time point, t, during the sales period features an exponential distribution, and the variation in the distribution's expected value u with t, u' (u is related to t), is the number of times the sales price is higher or lower than u at t (the number of times is an indicator of the diffusion rate). The probability density function $f_t(z)$ of z at time point, t, is as follows:

$$f_t(z){:}\ \ f_t(z) = \frac{1}{u_t}\, exp\left(\frac{-z}{u_t}\right),\ \ 0\ \leq z < \infty, \tag{1}$$

where u_t is the mean of z, expressed as follows:

$$u_t = \int_0^{\infty} z f_t(z) dz = \int_0^{\infty} z \frac{1}{u_t}\, exp\left(\frac{-z}{u_t}\right) dz > 0 \tag{2}$$

p_t: transaction price (sales price) of a product at t; p_t is the sales price control variable for the business.
λ: $\lambda \geq 0$; indicator of the diffusion effectiveness of a product's transaction price information. Among them, the greater λ is, the more substantial the diffusion effectiveness is; see Equation (3).
q_t: $q_t = q_t(p_t)$; the amount of demand per unit time for a product at time point t and sales price p_t; $q_t = q_t(p_t)$ is the demand function for the product at t.
r: $r \geq 0$; the discount rate, namely the cost per unit time.

According to previous studies on reference price, the effects of product sales price and the diffusion of transaction price information on reference price have the following two "features":

Feature 1. If a business' product sales price at t is p_t, then, potential customers whose IRP at t is higher (or lower) than p_t will, after they receive the information that the transaction price at t is p_t, correct their IRP downward (or upward). This means that potential customers' IRP distributions at t converge toward p_t with time.

Feature 2. If the potential customers are seen as a whole, then the mean of their IRP distribution at t, u_t, is a representative value of their collective IRP. According to Feature 1, the representative value u_t moves in the opposite direction to $(p_t - u_t)$, and the variation in u_t per unit time is positively correlated with the diffusion effectiveness of transaction price information λ. The relationship can be expressed as follows:

$$\frac{du_t}{d_t} = \lambda\ (p_t - u_t\ ),\ t \in [0, \infty) \tag{3}$$

$\lambda > 0$ is an indicator of the diffusion effectiveness of transaction price information (the greater λ is, the higher the effectiveness becomes). This study established a dynamic demand function that considers the diffusion effectiveness of historical transaction price information by investigating how the difference between the mean u_t of the distribution of potential customers' highest willingness to pay (IRP) at t and the sales price p_t affects du_t/dt (see Equation (3)). The dynamic demand function can be used to estimate the sales and inventory level of a product per unit time at a future time point and to determine the optimal sales price at a future time point. The necessary condition for a customer to purchase a product at price p at time point t is as follows:

$$\text{The highest price a customer is able and willing to pay at } t \text{ is equal to or higher than the sales price } p \text{ at } t \tag{4}$$

In this study, the highest price a potential customer is able and willing to pay at t is defined as the customer's IRP at t. This signifies that a potential customer's IRP is a variable, set as z. The distribution function of z concerns time point t, set as $f_t(z)$.

3.2. Mathematical Models for Developing the Dynamic Demand Function

By replacing t with x in Equation (3) and adding $e^{\lambda x}$ to both sides of the equation, we obtain the following equation:

$$\begin{aligned} \tfrac{d}{dx}\left(e^{\lambda x}u_x\right) &= \lambda e^{\lambda x}u_x + e^{\lambda x}\tfrac{d}{dx}u_x;\ \text{using Equation (3)} \\ &= \lambda e^{\lambda x}p_x.\ \forall\, x \in\ [0.\infty) \end{aligned} \tag{5}$$

By integrating Equation (5), we derive the following equation:

$$\int_0^t \lambda e^{\lambda x} p_x d_x = e^{\lambda t}u_t - u_0 \tag{6}$$

That is,

$$u_t = e^{-\lambda t}\left[\int_0^t \lambda e^{\lambda x} p_x d_x + u_0\right] \forall \mathrm{t} \tag{7}$$

According to Equation (7), the demand function curve for any given business at t must consider the psychological effect of historical transaction prices p_x at various time points x before t that may influence potential customers' willingness to pay (IRP).

Proposition 1. *For an exclusive exquisite product, the following relationships exist between the mean u_t of potential customers' IRP distribution at t and the historical transaction price $p_x, x \in [0, t)$:*

$$\text{(i) } u_t = e^{-\lambda t}\left[\int_0^t \lambda e^{\lambda x} p_x d_x + u_0\right] \tag{8}$$

$$\text{(ii) } \frac{\partial u_t}{\partial u_0} = e^{-\lambda t} > 0 \tag{9}$$

$$\text{(iii) } \frac{\partial u_t}{\partial \lambda} = \frac{1}{\lambda} e^{-\lambda t} \int_0^t e^{\lambda x} u_x' dx \tag{10}$$

Proof. By using Equation (7), we obtain (8) and (9). By taking the partial derivative of λ according to Equation (7), we can derive the following:

$$\begin{aligned}
\frac{\partial u_t}{\partial \lambda} &= -te^{-\lambda t}\left[\int_0^t \lambda e^{\lambda x} p_x d_x + u_0\right] + e^{-\lambda t}\left[\int_0^t e^{\lambda x} p_x d_x + \int_0^t \lambda x e^{\lambda x} p_x d_x\right] \\
&= e^{-\lambda t}\left[\int_0^t (\lambda x + 1) e^{\lambda x} p_x d_x - t\left(\int_0^t \lambda e^{\lambda x} p_x d_x + u_0\right)\right]; \text{ using Equation (7)} \\
&= e^{-\lambda t}\left[\int_0^t (\lambda x + 1) e^{\lambda x} p_x d_x - te^{\lambda t} u_t\right]; \text{ by taking the integral of the parts using Equation (5)} \\
&= e^{-\lambda t}\left[(\lambda x + 1)\left(\tfrac{1}{\lambda} e^{\lambda x} u_x\right)\Big|_0^t - \int_0^t e^{\lambda x} u_x d_x - te^{\lambda t} u_t\right] \\
&= e^{-\lambda t}\left[\tfrac{1}{\lambda} e^{\lambda t} u_t - \tfrac{1}{\lambda} u_0 - \int_0^t e^{\lambda x} u_x d_x\right]; \text{ by taking the integral of the parts using } \int_0^t e^{\lambda x} u_x d_x \\
&= e^{-\lambda t} \tfrac{1}{\lambda} \int_0^t e^{\lambda x} u_x' dx
\end{aligned} \tag{11}$$

Therefore, (10) can be obtained. □

As aforementioned, reference price u_t changes with time t. Therefore, the dynamic demand function for exquisite products can be derived as follows:

Proposition 2. *Dynamic demand function, $q_t(p)$,* with consideration of the diffusion of historical transaction price information.

(i) The demand function $q_t(p)$ for a business to determine the sales price p at t according to the historical sales price $p_x, x \in [0, t)$ before t is as follows ($q_t(p)$ is the sales per unit time at t as a result of customers' IRP being higher than the sales price p):

$$q_t(p) = N\int_p^{\infty} \frac{1}{u_t} exp\left(\frac{-z}{u_t}\right) dz = N\, exp\left(\frac{-p}{u_t}\right) \tag{12}$$

Equation (7) shows the relationship between u_t and historical sales price $p_x, x \in [0, t)$ before t.

$$\text{(ii) } \frac{d}{dp} q_t(p) = \frac{-N}{u_t} exp\left(\frac{-p}{u_t}\right) < 0 \tag{13}$$

$$\text{(iii) } \frac{d^2 q_t(p)}{dp^2} = N \frac{1}{{u_t}^2} exp\left(\frac{-p}{u_t}\right) > 0 \tag{14}$$

Proof. Using Equations (2), (3), and (7), we derive (i). Differentiating Equations (7) and (12) and using Equation (2), we obtain (13) and (14). □

Proposition 3. *When a business adopts a linear historical sales price strategy,* $p_x = p_0 + bx,\ x \in [0, t)$ *(if* $b < 0$*, t must satisfy* $t < \frac{p_0}{-b}$*), and the demand function at t* $q_t = q_t(p)$ is as follows:

$$q_t(p) = \mathrm{N}\, exp\left(\frac{-p}{u_t}\right) = \mathrm{N}\, exp\left(\frac{-p}{p_0\left(1 - e^{-\lambda t}\right) + \frac{b}{\lambda}\left(\lambda t - 1 + e^{-\lambda t}\right) + u_0}\right) \tag{15}$$

Proof. According to Equation (7),

$$\begin{aligned} u_x &= e^{-\lambda x}\int_0^x \lambda\, e^{\lambda z}(p_0 + bz)dz + u_0;\ \text{by taking the integral of the parts} \\ &= e^{-\lambda x}\left[(p_0 + bz)e^{\lambda z}\Big|_{z=0}^{z=x} - \frac{b}{\lambda}\ e^{\lambda z}\Big|_{z=0}^{z=x}\right] + u_0 \\ &= (p_0 + bx) - e^{-\lambda x}p_0 - \frac{b}{\lambda} + e^{-\lambda x}\frac{b}{\lambda} + u_0 \\ &= p_0\left(1 - e^{-\lambda x}\right) + \frac{b}{\lambda}\left(\lambda x - 1 + e^{-\lambda x}\right) + u_0\ \ \forall\, x \in\ [0, t) \end{aligned} \tag{16}$$

Using Equations (12) and (16), we derive Equation (15). □

Considering u_x in Equation (16) and differentiating x, we have the following:

$$\frac{d\mu_x}{dx} = p_0\ \lambda\ e^{-\lambda x} + b\left(1 - e^{-\lambda x}\right) \tag{17}$$

From Equation (17), we know how p_0, λ, and b parameters affect $\frac{d\mu_x}{dx}$.

3.3. Sales Price Control Model That Maximizes the Discounted Profit of Exquisite Products in the Patent Term [0,t]

To enable a business to control the sales price p_t at t in a way that maximizes the discounted profitability in $[0, T]$, the following mathematical model, which uses Equations (1) and (12), is used, as follows:

$$\begin{cases} Max_p \int_0^T exp(-rt)(p_t - c)\ N\ exp\left(\frac{-p_t}{u_t}\right)dz \\ s.t.\ \text{ function } \ u_t \text{ differs with feasible solution } \ p_t, t \in [0, T],\ \text{where} \end{cases} \tag{18}$$

$$u_t' = \lambda(p_t - u_t),\ t \in [0, T]; u_0 \text{ and } \lambda > 0 \text{ are both parameters} \tag{19}$$

According to Equation (19), we obtain the following:

$$p_t = u_t + \frac{u_t'}{\lambda},\ \ \forall\, t \in [0, T],\ u_0 \text{ and } \lambda \text{ are both given parameters} \tag{20}$$

After substituting Equation (20) into Equation (18) and using Equation (5), we can rewrite the above model as follows:

$$\begin{cases} Max_u N \int_0^T exp(-rt)\left(u_t + \frac{u_t'}{\lambda} - c\right)\ exp\left(1 - \frac{u_t'}{\lambda u_t}\right)dt \\ s.t.\ \ u_0 \text{ is a fixed value; } \ u_T \text{ is a free value} \\ \quad (\text{i.e., the value of } u_T \text{ differs with feasible solution } u = u_t, t \in [0, T]) \end{cases} \tag{21}$$

Apparently, the model in (21) has the same optimal solution as the following:

$$\begin{cases} Max_u \int_0^T \left[exp\left(-rt - \frac{u_t'}{\lambda u_t}\right)\right]\left(u_t + \frac{u_t'}{\lambda} - c\right)dt \\ s.t.\ \ u_0 \text{ is a fixed value; } \ u_T \text{ is a free value} \end{cases} \tag{22}$$

Equation (22) is a calculus of variations problem. Let the integrated function in Equation (22) be

$$F(t, u_t, u_t') = \left[exp\left(-rt - \frac{u_t'}{\lambda u_t}\right)\right]\left(u_t + \frac{u_t'}{\lambda} - c\right) \tag{23}$$

Then, we can obtain the optimal solution for the calculus of variations problem; the solution, written as $u^* = u_t^*$, has three necessary conditions.

(1) First-order optimality condition (Euler equation): when $u_t = u_t^*$, the following equation is true:

$$\begin{aligned} & exp\left(-rt - \tfrac{u_t'}{\lambda u_t}\right) + \left[exp\left(-rt - \tfrac{u_t'}{\lambda u_t}\right)\right]\tfrac{u_t'}{\lambda}\,\tfrac{1}{u_t{}^2}\left(u_t + \tfrac{u_t'}{\lambda} - c\right) \\ & = F_u = \tfrac{d}{dt}F_{u'} \\ & = \tfrac{d}{dt}\left[\tfrac{1}{\lambda}exp\left(-rt - \tfrac{u_t'}{\lambda u_t}\right) - \tfrac{1}{\lambda u_t}\left(u_t + \tfrac{u_t'}{\lambda} - c\right)exp\left(-rt - \tfrac{u_t'}{\lambda u_t}\right)\right]; \end{aligned} \tag{24}$$

the first and second terms in the square brackets cancel each other out
Thus

$$\begin{aligned} & \tfrac{1}{\lambda}\tfrac{d}{dt}\left[\left(\tfrac{-u_t'}{\lambda u_t} + \tfrac{c}{u_t}\right)exp\left(-rt - \tfrac{u_t'}{\lambda u_t}\right)\right] \\ = & \tfrac{1}{\lambda}\left[exp\left(-rt - \tfrac{c}{u_t} - \tfrac{u_t'}{\lambda u_t}\right)\right]\left[\left(\tfrac{-u_t''}{\lambda u_t} + \tfrac{u_t'^2}{\lambda u_t{}^2} - \tfrac{cu_t'}{u_t{}^2}\right) + \left(\tfrac{-u_t'}{\lambda u_t} + \tfrac{c}{u_t}\right)\left(-r - \tfrac{u_t''}{\lambda u_t} + \tfrac{u_t'^2}{\lambda u_t{}^2}\right)\right] \end{aligned} \tag{25}$$

(2) Transversality condition for u_T as a free value: when $u_t = u_t^*$, and $t = T$, the following equation is true:
$0 = F_u|_{t=T}$; by using Equation (24), we derive the following:

$$0 = 1 + \frac{u_T^{*\prime}}{\lambda}\frac{1}{u_T^{*2}}\left(u_T^* + \frac{u_T^{*\prime}}{\lambda} - c\right) = 1 + \frac{u_T^{*\prime}}{\lambda}\,\frac{1}{u_T^{*2}}(p_T^* - c) \tag{26}$$

Therefore, $-\lambda u_T^{*2} = u_T^{*\prime}(p_T^* - c)$; $u_T^{*\prime} = -\lambda\frac{u_T^{*2}}{(p_T^* - c)} < 0$; according to Equation (19), we know that

$$u_T^* > p_T^*$$

(3) Legendre's second-order optimality condition: when $u_t = u_t^*$,

$$0 \geq F_{u'u'} = \frac{1}{\lambda}\left[\frac{-1}{\lambda u_t}exp\left(-rt - \frac{u_t'}{\lambda u_t}\right) + \left(\frac{-u_t'}{\lambda u_t} + \frac{c}{u_t}\right)\frac{-1}{\lambda u_t}exp\left(-rt - \frac{u_t'}{\lambda u_t}\right)\right] \tag{27}$$

$$\text{That is, } u_t + c \geq \tfrac{u_t'}{\lambda}, \forall\, t, \text{ and thus } 2u_t^* + c \geq u_t^* + \tfrac{u_t^{*\prime}}{\lambda} = p_t^* \text{ (see Equation (19)).} \tag{28}$$

Removing $exp\left(-rt - \frac{c}{u_t} - \frac{u_t'}{\lambda u_t}\right)$ on both sides of Equation (25), we obtain the following equation, which holds true when $t \in [0, T]$:

$$\begin{aligned} & 1 + \tfrac{u_t^{*\prime}}{\lambda}\,\tfrac{1}{u_t^{*2}}\left(u_t^* + \tfrac{u_t^{*\prime}}{\lambda} - c\right) \\ & = \tfrac{1}{\lambda}\left[\left(\tfrac{-u_t^{*\prime\prime}}{\lambda u_t^*} + \tfrac{\left(u_t^{*\prime}\right)^2}{\lambda\left(u_t^*\right)^2} - \tfrac{cu_t^{*\prime}}{\left(u_t^*\right)^2}\right) + \left(\tfrac{-u_t^{*\prime}}{\lambda u_t^*} + \tfrac{c}{u_t^*}\right)\left(-r - \tfrac{u_t^{*\prime\prime}}{\lambda u_t^*} + \tfrac{\left(u_t^{*\prime}\right)^2}{\lambda\left(u_t^*\right)^2}\right)\right] \end{aligned} \tag{29}$$

By simplifying Equation (29), we have the following:

$$\lambda + \frac{u_t^{*\prime}}{u_t^*} = \left[\frac{-u_t^{*\prime\prime}}{\lambda u_t^*} + \left(\frac{-u_t^{*\prime}}{\lambda u_t^*} + \frac{c}{u_t^*}\right)\left(-r - \frac{u_t^{*\prime\prime}}{\lambda u_t^*} + \frac{\left(u_t^{*\prime}\right)^2}{\lambda(u_t^*)^2}\right)\right] \tag{30}$$

Proposition 4. *Assuming that a business wishes to set the sales price of an innovative product at p_t at time point, t, so that the corresponding u_t (Equation (19)) of p_t maximizes the discounted profitability (Equation (18)) within the patent term $[0, T]$, we must derive the necessary conditions for the optimal solution $p_t^*, t \in [0, T]$ (or u_t^* $t \in [0, T]$, where $u_0^* = u_0$ is a given parameter rather than a decision variable), as shown in Equations (26), (28), and (30).*

Proof. The above discussion through Equation (18) to Equation (30) proves that Proposition 4 is true. □

Proposition 5. *The optimal solution u_t^* of Equation (22) for innovative product in the patent term $[0, T]$ has the following features:*

(*i*) $[0,\ T]$ *has no minimum and only one maximum at most.*
(*ii*) *If a business' sales price p_0^* in the beginning of the term satisfies $p_0^* > u_0$ (u_0 is a parameter in Equation (22) and the starting point for all feasible solutions, u_t), then u_t^* has a maximum $t_1,\ t_1 \in (0, T)$ in $[0, T]$, and $u_{t_1}^* = max_{t \in [0,T]} u_t^*$, where $\frac{\partial u_t^*}{\partial \lambda} > 0, \forall\, t \in [0, t_1]$ (this indicates that when the diffusion rate of historical transaction price information λ increases,u_t^* increases for any given $t \in [0, t_1]$).*
(*iii*) *If a business' sales price p_0^* in the beginning of the term satisfies $p_0^* \leq u_0$, then u_t^* is a strictly decreasing function of t in $[0, T]$, and $\frac{\partial u_t^*}{\partial \lambda} < 0, \forall\, t \in [0, T]$ (this indicates that when the diffusion rate of historical transaction price information λ increases, u_t^* decreases for any given $t \in [0, T]$).*
(*iv*) *$t_2 \in [0, T]$ exists, which makes u_t^* a strictly decreasing function of t in $[t_2, T]$.*

Proof.

(i) Using proof by contradiction to prove that u_t^* has no minimum value: If u_t^* has a maximum value in $[0, T]$, t_0, then $u_{t_0}^{*'} = 0$, and $u_{t_0}^{*''} \geq 0$. When $t = t_0$, the left side of Equation (30) is $\lambda > 0$, and the right side of the equation < 0, which contradict each other. Accordingly, u_t^* has no minimum value. If u_t^* has two maximum values, then, given that u_t^* is a continuous function, a minimum value must exist between the two maximum values, which contradicts the proven fact that u_t^* has no minimum value.
(ii) If $p_0^* > u_0$, then we have $u_0^{*'} > 0$ according to Equation (19) and $u_T^{*'} < 0$ according to Equation (26). Therefore, given Feature (i) in this Proposition, we know that u_t^* has one maximum in $[0, T]$, and $t_1 \in\ (0, T)$. Considering Feature (i) in the Proposition and (iii) in Proposition 1, we obtain $\frac{\partial u_t^*}{\partial \lambda} > 0, \forall\, t \in [0,\ t_1]$.
(iii) If $p_0^* \leq u_0$, we have $u_0^{*'} \leq 0$ according to Equation (19); according to (i), we know that $u_t^* \in [0, T]$ has no minimum value. Therefore, u_t^* in $[0, T]$ is a strictly decreasing function of t. According to Features (i) and (ii) of this Proposition and (iii) in Proposition 1, we obtain $\frac{\partial u_t^*}{\partial \lambda} < 0, \forall\, t \in [0, T]$.
(iv) Combining (ii) and (iii), we prove that Feature (iv) is true.
□

Proposition 6. *A business' optimal sales price function $p_t^*, t \in [0, T]$ for an exquisite product in the patent term $[0, T]$ has the following features:*

(i) *If a business' sales price p_0^* in the beginning of the term satisfies $p_0^* > u_0$, then $t_0\ \in (0, T)$ exists, which makes $p_t^* > u_t, \forall\, t \in [0,\, t_0)$, $p_{t_0}^* = u_{t_0}^*$, $p_t^* < u_t^*$, $\forall\, t \in (t_0, T]$, and p_t^* is a strictly decreasing function of t at any given point near and on the right of t_0.*
(ii) *If a business' sales price p_0^* in the beginning of the term satisfies $p_0^* < u_0$, then $p_t^* < u_t^*$, $\forall\, t \in (t_0,\, T]$.*
(iii) *$t_0\ \in (0, T)$ exists, which makes $p_t^* < u_t^*, \forall\, t \in (t_0,\, T]$.*

Proof.

(i) According to (ii) in Proposition 5, we know that u_t^* has a maximum value t_0; therefore, u_t^* and $u_t^{*'}$, near and on the right of t_0, are both strictly decreasing functions of t. Based on Equation (20), we obtain that $p_t^* = u_t^* + \frac{u_t^*}{\lambda}$ strictly decreases near and on the right of t_0.

(ii) Using (iii) in Proposition 5, we prove that Feature (ii) of Proposition 6 is true.

(iii) Combining Propositions 1 and 2, we prove that Feature (iii) of Proposition 6 is true. □

4. Conclusions

Businesses set their product sales price according to historical prices; this seemingly simple pricing decision actually involves an intricate process. Each piece of historical price information is diffused at a different rate, which makes customers adjust their IRPs at different rates, depending on the individuals or contexts. In practice, businesses selling exquisite products may cultivate product prices by emphasizing product scarcity (limited editions or limited-time offers) and using the exclusive brand strategies of an exclusive agency, exclusive dealing agreement, and, within the patent term, exclusive manufacturing from the manufacturer of the brand. Before cultivating the price of such products, businesses should consider, first, the product differentiation strategy and pricing of the competitors, and, second, how the potential customers' willingness to pay (IRP) may be affected by historical transaction price $p_x, x \in [0, t)$ at a time point x before t. This study focuses on the latter part of the foregoing.

Unlike previous studies that have predominantly assumed that reference price is an external variable that affects the customer utility function, this study considers (1) that product price information will be affected by the effect of advertising diffusion, and (2) the integration between the price effect, product quality, and advertising effect. Therefore, this study used the exponential distribution of potential customers' IRPs as an example to establish the dynamic demand function that considers the effect of historical transaction prices. On the basis of this demand function, a sales price control model that maximized the discounted profitability for businesses in the patent term of an exquisite product was then constructed. Finally, the necessary conditions, namely relevant features, of this optimal sales price control model were discussed.

Therefore, a business will be able to emphasize the scarcity of products (limited editions or limited-time offers) or exclusively sell or manufacture the products of a brand (hereafter referred to as "exclusive brand") for the time-course control of price cultivation. In addition, it is common for businesses to use e-commerce platforms to display their high-price products without actively encouraging purchases in the early stage of product launch; such behavior helps establish price anchors for the products in the minds of potential customers. Such product displays are usually followed by promotional events in other market channels to boost sales. This study provides businesses with an operation method for cultivating prices and contributing to the business's increase in profits. It must be considered that the differences in the environment of each product category or quality, and information diffusion rate at each stage of the marketing, may have varied with changes by the strategy (e.g., peak-season business cycle). These topics and their corresponding numerical analyses should be worthy of further and more in-depth discussion in future studies.

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References

1. Chen, M.S.; Li, M.J.; Wang, H.B. Price Cultivation Model for the New Product Unsuitable for Moved to be Displayed Along with its Owner. Ph.D. Thesis, University of Nanhua, Dalin Township, Taiwan, 2017, (unpublished doctoral dissertation).
2. Fumagalli, C.; Motta, M. Exclusive Dealing and Entry, When Buyers Compete. *Am. Econ. Rev.* **2006**, *96*, 785–795. [CrossRef]
3. Fumagalli, C.; Motta, M.; Persson, L. On the Anticompetitive Effect of Exclusive Dealing When Entry by Merger is Possible. *J. Ind. Econ.* **2009**, *57*, 785–811. [CrossRef]
4. Fadairo, M.; Yu, J.; Lanchimba, C. The Choice of Exclusive Dealing: Economic Rationales and Evidence from French Retail Chains. *J. Retail.* **2017**, *93*, 317–335. [CrossRef]
5. Mazumdar, T.; Raj, S.P.; Sinha, I. Reference Price Research: Rreview and Propositions. *J. Mark.* **2005**, *69*, 84–102. [CrossRef]
6. Mezias, S.J.; Chen, Y.R.; Murphy, P.R. Aspiration-Level Adaptation in an American Financial Services Organization: A Field Study. *Manag. Sci.* **2002**, *48*, 1285–1300. [CrossRef]
7. Kopalle, P.; Biswas, D.; Chintagunta, P.K.; Fan, J.; Pauwels, K.; Ratchford, B.T.; Sills, J.A. Retailer Pricing and Competitive Effects. *J. Retail.* **2009**, *85*, 56–70. [CrossRef]
8. Helson, H. *Adaptation-Level Theory*; Harper and Row Publishers: New York, NY, USA, 1964.
9. Monroe, K.B. Buyers' Subjective Perceptions of Price. *J. Mark. Res.* **1973**, *10*, 70–80.
10. Lattin, J.M.; Bucklin, R.E. Reference Effects of Price and Promotion on Brand Choice Behavior. *J. Mark. Res.* **1989**, *26*, 299–310. [CrossRef]
11. Zhang, Q.; Zhang, J.; Tang, W. A Dynamic Advertising Model with Reference Price Effect. *RAIRO-Oper. Res.* **2015**, *49*, 669–688. [CrossRef]
12. Biswas, A.; Wilson, E.J.; Licata, J.W. Reference Pricing Studies in Marketing: A Synthesis of Research Results. *J. Bus. Res.* **1993**, *27*, 239–256. [CrossRef]
13. Krishna, A. A Meta-Analysis of the Impact of Price Presentation on Perceived Savings. *J. Retail.* **2002**, *78*, 101–118. [CrossRef]
14. Briesch, R.A.; Krishnamurthi, L.; Mazumdar, T.; Raj, S.P. A Comparative Analysis of Reference Price Models. *J. Consum. Res.* **1997**, *24*, 202–214. [CrossRef]
15. Erdem, T.; Mayhew, G.; Sun, B. Understanding the Reference Price Sensitive Shopper: A Within and Cross-Category Analysis. *J. Mark. Res.* **2001**, *38*, 445–457. [CrossRef]
16. Chen, C.Y. Who I am and how I Think: The Impact of Self-construal on the Roles of Internal and External Reference Prices in Price Evaluations. *J. Consum. Psychol.* **2009**, *19*, 416–426. [CrossRef]
17. Cheng, L.L.; Monroe, K.B. An Appraisal of Behavioral Price Research (part 1): Price as a Physical Stimulus. *Ams Rev.* **2013**, *3*, 103–129. [CrossRef]
18. Mayhew, G.E.; Winer, R.S. An Empirical Analysis of Internal and External Reference Prices Using Scanner Data. *J. Consum. Res.* **1992**, *19*, 62–70. [CrossRef]
19. Bearden, W.O.; Carlson, J.P.; Hardesty, D.M. Using Invoice Information to Frame Advertised Prices. *J. Bus. Res.* **2003**, *56*, 355–366. [CrossRef]
20. Slonim, R.; Garbarino, E. The Effect of Price History on Demand as Mediated by Perceived Price Expensiveness. *J. Bus. Res.* **1999**, *45*, 1–14. [CrossRef]
21. Lalwani, A.K.; Monroe, K.B. A Re-examination of Frequency-depth Effects in Consumer Price Judgments. *J. Consum. Res.* **2005**, *32*, 480–485. [CrossRef]
22. Bruno, H.A.; Che, H.; Dutta, S. Role of Reference Price on Price and Quantity: Insights from Business-to-Business Markets. *J. Mark. Res.* **2012**, *49*, 640–654. [CrossRef]
23. Kopalle, P.K.; Joan, L.M. The Impact of External Reference Price on Consumer Price Expectations. *J. Retail.* **2003**, *79*, 223–236. [CrossRef]
24. Bell, D.R.; Lattin, J.M. Looking for Loss Aversion in Scanner Panel Data: The Confounding Effect of Price Response Heterogeneity. *Mark. Sci.* **2000**, *19*, 185–200. [CrossRef]
25. Greenleaf, E.A. The Impact of Reference Price Effects on the Profitability of Price Promotions. *Mark. Sci.* **1995**, *14*, 82–104. [CrossRef]
26. Kopalle, P.K.; Rao, A.G.; Assunção, J. Asymmetric Reference Price Effects and Dynamic Pricing Policies. *Mark. Sci.* **1996**, *15*, 60–85. [CrossRef]
27. Fibich, G.; Gavious, A.; Lowengart, O. Explicit Solutions of Optimization Models and Differential Games with Nonsmooth (asymmetric) Reference-price Effects. *Oper. Res.* **2003**, *51*, 721–734. [CrossRef]
28. Chandrashekaran, R.; Grewal, D. Anchoring Effects of Advertised Reference Price and Sale Price: The Moderating Role of Saving Presentation format. *J. Bus. Res.* **2006**, *59*, 1063–1071. [CrossRef]
29. Zeelenberg, M.; Pieters, R. Beyond Valence in Customer Dissatisfaction. *J. Bus. Res.* **2004**, *57*, 445–455. [CrossRef]
30. Ranyard, R.; Missier, F.D.; Bonini, N.; Duxbury, D.; Summers, B. Perceptions and Expectations of Price Changes and Inflation. *J. Econ. Psychol.* **2008**, *29*, 378–400. [CrossRef]
31. Koszegi, B.; Rabin, M. A Model of Reference-dependent Preferences. *Q. J. Econ.* **2006**, *121*, 1133–1165.
32. Koszegi, B.; Rabin, M. Reference-dependent Risk Attitudes. *Am. Econ. Rev.* **2007**, *97*, 1047–1073. [CrossRef]
33. DelVecchio, D.; Henard, D.H.; Freling, T.H. The Effect of Sales Promotion on Post-Promotion Brand Preference: A Meta-Analysis. *J. Retail.* **2006**, *82*, 203–213. [CrossRef]

34. Motyka, S.; Suri, R.; Grewal, D.; Kohli, C. Disfluent vs. Fluent Price Offers: Paradoxical Role of Processing Disfluency. *J. Acad. Mark. Sci.* **2016**, *44*, 627–638. [CrossRef]
35. Mead, J.A.; Hardesty, D.M. Price Font Disfluency: Anchoring Effects on Future Price Expectations. *J. Retail.* **2018**, *94*, 102–112. [CrossRef]
36. Ho, H.; Ganesan, S.; Oppewal, H. The Impact of Store-Price Signals on Consumer Search and Store Evaluation. *J. Retail.* **2011**, *87*, 127–141. [CrossRef]
37. Lewis, M.S. Asymmetric Price Adjustment and Consumer Search: An Examination of the Retail Gasoline Market. *J. Econ. Manag. Strategy* **2011**, *20*, 409–449. [CrossRef]
38. DelVecchio, D.; Craig, A.W. Mode Matters: An Exemplar-prototype Hybrid (EPH) Model of Reference Price Formation. *J. Prod. Brand Manag.* **2008**, *17*, 272–279. [CrossRef]
39. Bambauer-Sachsea, S.; Massera, L. Interaction Effects of Different Price Claims and Contextual Factors on Consumers' Reference Price Adaptation after Exposure to a Price Promotion. *J. Retail. Consum. Serv.* **2015**, *27*, 63–73. [CrossRef]
40. Chang, T.Z.; Wildt, A.R. Price, Product Information, and Purchase Intention: An Empirical Study. *J. Acad. Mark. Sci.* **1994**, *22*, 16–27. [CrossRef]
41. Jacobson, R.; Obermiller, C. The Formation of Expected Future Price: A Reference Price for Forward-looking Consumers. *J. Consum. Res.* **1990**, *16*, 420–432. [CrossRef]
42. Zhou, J. Reference Dependence and Market Competition. *J. Econ. Manag. Strategy* **2011**, *20*, 1073–1097. [CrossRef]
43. Putler, D.S. Incorporating Reference Price Effects into a Theory of Consumer Choice. *Mark. Sci.* **1992**, *11*, 287–309. [CrossRef]
44. Chen, M.S.; Chen, C.B. The Optimal Penetration Pricing Strategy Model under the Dynamic Demand Function. *Asia-Pac. J. Oper. Res.* **1999**, *16*, 139–154.
45. Janiszewski, C.; Lichtenstein, D.R. A Range Theory Account of Price Perception. *J. Consum. Res.* **1999**, *25*, 353–368. [CrossRef]
46. Niedrich, R.W.; Sharma, S.; Wedell, D.H. Reference Price and Price Perceptions: A Comparison of Alternative Models. *J. Consum. Res.* **2001**, *28*, 339–354. [CrossRef]
47. Niedrich, R.W.; Weathers, D.; Hill, D.R.C.; Bell, D.R. Specifying Price Judgments with Range-frequency Theory in Models of Brand Choice. *J. Mark. Res.* **2009**, *46*, 693–702. [CrossRef]
48. Rutger, V.O. Why are Consumers Less Loss Averse in Internal than External Reference Prices? *J. Retail.* **2013**, *89*, 62–71.
49. Zhang, J.; Gou, Q.L.; Liang, L.; Huang, Z. Supply Chain Coordination through Cooperative Advertising with Reference Price Effect. *Omega* **2013**, *41*, 345–353. [CrossRef]
50. Baucells, M.; Weber, M.; Welfens, F. Reference-Point Formation and Updating. *Manag. Sci.* **2011**, *57*, 506–519. [CrossRef]
51. Nasiry, J.; Ioana, P. Dynamic Pricing with Loss Averse Consumers and Peak-end Anchoring. *Oper. Res.* **2011**, *59*, 1361–1368. [CrossRef]
52. Wenner, L.M. Expected Prices as Reference Points-Theory and experiments. *Eur. Econ. Rev.* **2015**, *75*, 60–79. [CrossRef]
53. Kim, D.S.; Joo, M.; Allenby, G.M. An Economic Model of Price Comparison with Consumers' Internal Standard. 2020. Available online: https://ssrn.com/abstract=2925491 (accessed on 20 June 2021).
54. Hamelin, J. Le Prix de Référence: Un Concept Polymorphe. *Rech. Et Appl. En Mark.* **2000**, *15*, 75–88. [CrossRef]
55. Chandrashekaran, R.; Grewal, D. Assimilation of Advertised Reference Prices: The Moderating Role of Involvement. *J. Retail.* **2003**, *79*, 53–62. [CrossRef]
56. Ahmetoglu, G.; Furnham, A.; Fagan, P. Pricing Practices: A Critical Review of their Effects on Consumer Perceptions and Behaviour. *J. Retail. Consum. Serv.* **2014**, *21*, 696–707. [CrossRef]
57. Xue, M.; Tang, W.; Zhang, J. Optimal dynamic pricing for deteriorating items with reference-price effects. *Int. J. Syst. Sci.* **2016**, *47*, 2022–2031. [CrossRef]
58. Xue, M.; Zhang, J.; Tang, W.; Dai, R. Quality Improvement and Pricing with Reference Quality Effect. *J. Syst. Sci. Syst. Eng.* **2017**, *26*, 665–682. [CrossRef]
59. Labbé, M.; Violin, A. Bilevel Programming and Price Setting Problems. *4OR-Q. J. Oper. Res.* **2013**, *11*, 1–30. [CrossRef]
60. Hirschmann, D. Optimal Quality Provision when Reputation is Subject to Random Inspections. *Oper. Res. Lett.* **2014**, *42*, 64–69. [CrossRef]
61. Ding, M.; Ross, W.T., Jr.; Rao, V.R. Price as an Indicator of Quality: Implications for Utility and Demand Functions. *J. Retail.* **2010**, *86*, 69–84. [CrossRef]
62. Chen, P.-Y.; Chen, M.S.; Lin, C.Y. The Inventory Model of the Effect of Product Validity on Consumer Purchase Intension. *ICIC Express Lett.* **2013**, *7*, 2115–2120.
63. Chen, P.-Y.; Chen, M.S.; Chen, I.C. The Optimal Price Control Model of Food Sales in a Given Period. *J. Inf. Optim. Sci.* **2013**, *34*, 361–371. [CrossRef]
64. Chen, P.Y. Economic Order Quality Model for Determining the Sales Prices of Fresh Goods at Various Points in Time. *J. Food Qual.* **2017**, *2017*, 6967501. [CrossRef]
65. Herbon, A.; Levner, E.; Cheng, E. Perishable Inventory Management and Dynamic Pricing using TTI Technologies. *Int. J. Innov. Manag. Technol.* **2012**, *3*, 262–266.
66. Herbon, A.; Levner, E.; Cheng, T.C.E. Perishable Inventory Management with Dynamic Pricing using Time-temperature Indicators Linked to Automatic Detecting Devices. *Int. J. Prod. Econ.* **2014**, *147*, 605–613. [CrossRef]

67. Avinadav, T.; Herbon, A.; Spiegel, U. Optimal Ordering and Pricing Policy for Demand Functions that are Separable into Price and Inventory Age. *Eur. J. Oper. Res.* **2014**, *155*, 406–417. [CrossRef]
68. Herbon, A. Optimal Piece Wise-constant Price under Heterogeneous Sensitivity to Product Freshness. *Int. J. Prod. Res.* **2016**, *54*, 365–385. [CrossRef]
69. Hsieh, T.P.; Dye, C.Y. Optimal Dynamic Pricing for Deteriorating Items with Reference Price Effects when Inventories Stimulate Demand. *Eur. J. Oper. Res.* **2017**, *262*, 136–150. [CrossRef]
70. Dye, C.Y.; Yang, C.T.; Wu, C.C. Joint Dynamic Pricing and Preservation Technology Investment for an Integrated Supply Chain with Reference Price Effects. *J. Oper. Res. Soc.* **2018**, *69*, 811–824. [CrossRef]
71. Kopalle, P.K.; Winer, R.S. A Dynamic Model of Reference Price and Expected Quality. *Mark. Lett.* **1996**, *7*, 41–52. [CrossRef]
72. Narasumhan, R.; Méndez, D. Strategic Aspects of Quality: A Theoretical Analysis. *Prod. Oper. Manag.* **2001**, *10*, 514–526. [CrossRef]
73. Mukhopadhyay, S.K.; Setaputra, R. A Dynamic Model for Optimal Design Quality and Return Policies. *Eur. J. Oper. Res.* **2007**, *180*, 1144–1154. [CrossRef]
74. Zhang, J.; Wang, Y.; Lu, L.; Tang, W. Optimal Dynamic Pricing and Replenishment Cycle for Non-instantaneous Deterioration Items with Inventory-level-dependent Demand. *Int. J. Prod. Econ.* **2015**, *170*, 136–145. [CrossRef]