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Column-Hemispherical Penetration Grouting Mechanism for Newtonian Fluid Considering the Tortuosity of Porous Media

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Abstract: The intricate morphology of porous media can considerably affect the progression of penetration and the diffusion of grouting slurry. In this study, a Newtonian fluid's refined seepage motion equation was proposed to incorporate the impact of tortuosity on the grouting process into gravel soil, and the column-hemispherical penetration grouting mechanism was analyzed using the method of theoretical analysis. Utilizing secondary-development programming techniques, a numerical simulation program was developed with COMSOL Multiphysics to model the penetration grouting mechanism of a Newtonian fluid in a column-hemispherical pattern, considering a medium's tortuosity. The penetration grouting process of Newtonian cement into gravel soils was then simulated. Finally, the theoretical analysis, experimental values, and numerically simulated values were compared.The findings suggested that incorporating the tortuosity of porous media is more efficacious in depicting the penetration and diffusion behavior for Newtonian fluid grouting in porous media, as compared to omitting the tortuosity. The findings of this study contribute to a better understanding of grouting engineering in porous media strata, guiding practical design and construction.

Keywords: porous media; tortuosity effect; Newtonian fluid; column-hemispherical diffusion; penetration grouting mechanism

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1. Introduction

Porous rock and soil are prone to triggering geological disasters, including landslides, collapses, and debris flows, with severe impacts on regions such as the Niujuangou watershed in Wenchuan, Sichuan, the Jiangjiagou watershed in Dongchuan, and the Dongyue watershed in Nujiang, Yunnan. These disasters result in significant human casualties, environmental degradation, and property losses [1–5]. Grouting technology has emerged as a widely accepted and effective means for tackling various engineering geological problems, with penetration grouting garnering particular attention, as evidenced by empirical studies and theoretical investigations [6–9]. The penetration grouting mechanism was first studied with Newtonian fluids, and rich research results have been achieved. Grouting fluids exhibit diverse rheological characteristics, leading to their categorization into three modes: Bingham, Newtonian, and power-law fluids. Among them, Newtonian fluids, as a grouting material, have been widely applied in geotechnical engineering, and many scholars have conducted extensive research on the penetration grouting of Newtonian fluids. Funehag et al. explored the method of using Newtonian fluids to seal narrow cracks in tunnels by grouting [10,11]. They then further employed Newtonian fluids for grouting to address the issue of tunnel leakage, providing insights into waterproofing solutions for tunnel engineering [12]. Fransson mentioned the effectiveness of Newtonian fluid grouting in achieving rock and soil sealing in their research [13]. Tong et al., elucidated the grouting applications of Newtonian fluids in porous media and fractured media, emphasizing the developmental process from Newtonian to non-Newtonian fluids and from generalized theoretical models to engineering application [14]. These findings indicate that Newtonian fluids have certain research value in theoretical and application domains.

Meanwhile, fluids demonstrate three distinct types of diffusion pattern—spherical, columnar, and column-hemispherical—when grout is injected into a medium [7,8]. Maag [7] theoretically derived the earliest theory of the penetration grouting mechanism—Maag's equation—by assuming that Newtonian fluids permeate and diffuse in a spherical form in an isotropic homogeneous medium as early as 1938. Since then, Karol, Huang, et al. have continued the study in this area [15,16]. On this basis, Zou, Yang et al. proposed the equations of the diffusion radius for a Newtonian fluid's column and column-hemispherical penetration grouting model, respectively [17,18]. As we know, the conductivity of water in soil is influenced by many factors such as soil suction, film flow, pore structure, and soil saturation [19,20]. Similarly, many scholars have conducted research on the influencing factors of the diffusion and penetration of grouting fluids. Zheng, Li, et al. deduced the theoretical spherical diffusion equations and columnar penetration grouting theory (modified the equations of Maag, Karol et al.) for Newtonian fluids, taking into account the changes in viscosity over time, by replacing the initial viscosity with the average viscosity [21,22]. Ding et al. analyzed the diffusion mechanism for Newtonian fluids in spherical penetration grouting, considering the rheological equations and the time-dependent behavior of viscosity in their investigation [23]. Ye et al. conducted an analysis of viscosity degeneration in shield tunnels during the grouting process of Newtonian fluids, taking into consideration the relationship between the diffusion model and viscosity [24]. Kou et al. developed a mechanism for the penetration grouting of Newtonian fluids with time-dependent viscous slurry, considering the impact of void ratios in saturated clay [25]. The tortuosity of rock and soil was then considered by Zhang et al. in their investigation of the diffusion mechanism for the spherical penetration grouting of Newtonian grouts [26,27].

In conclusion, the Newtonian fluid penetration grouting mechanism has become an extensive and profitable study subject. Nevertheless, how the tortuosity of porous media affects the Newtonian fluid's column-hemispherical spatial diffusion pattern during the grouting process has not yet been adequately understood. Furthermore, Newtonian fluids, as a grouting material, hold not only theoretical research significance but also practical application relevance in engineering. Hence, it is necessary to investigate the grouting mechanism of Newtonian fluids. Based on the above-mentioned considerations, the tortuosity of porous media is considered in this article to study the fluid's column-hemispherical penetration grouting mechanism. The Newtonian fluid, which is frequently applied in grouting engineering practice, is used as the study object in this research (for example, pure cement grouts whose water–cement ratio is greater than 1.25 correspond to a typical Newtonian fluid [28]). This research endeavors to furnish theoretical guidance and technical reference for grouting engineering construction in porous media stratum, with practical applicability.

2. Study on Column-Hemispherical Penetration Grouting Mechanism for Newtonian Fluid Considering the Tortuosity of Porous Media

2.1. Tortuosity of Porous Media

In porous media, a fluid flows forward in a tortuous way with a characteristic tortuosity effect, as seen in Figure 1.

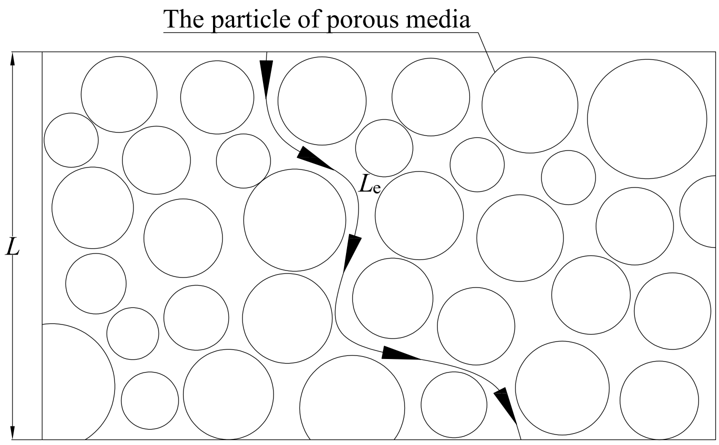


Figure 1. Diagrammatic sketch of fluid's flow paths in porous media.

Tortuosity is a parameter frequently used to describe the extent of meandering when a fluid flows through a porous medium [29]. The following defining equation is generally used:

$$\Gamma = \left(\frac{L_e}{L}\right)^2 \tag{1}$$

where L_e is the length of the fluid's actual flow path in porous media, L is the straight line length of the actual flow path, and Γ is media's tortuosity, which is usually distributed in the range of 2.00–2.50 as studied by Kong [29].

2.2. Seepage Motion Equation for Newtonian Fluid Considering the Tortuosity of Porous Media

The rheological equation for a Newtonian fluid is [7,30]

$$\tau = \eta\gamma \tag{2}$$

$$\gamma = -\frac{du}{dr} \tag{3}$$

where τ is the shear stress, η is the dynamic viscosity, γ is the shear rate, u is the flow velocity of a Newtonian fluid, and r is the flow radius of the fluid in porous media.

A microfluidic unit column is selected from the flow path of the fluid in the porous media's pores, as shown in Figure 1, to analyze the real transport state of the fluid, as shown in Figure 2. We can assume that the radius of the microfluidic columnar unit is r_0 , and p and $p + dp$ denote the pressure on the left and right sides, respectively.

Under the condition of ignoring other external forces, the forces on this infinitesimal column element satisfy

$$\pi r^2 dp + 2\pi r\tau dL_e = 0 \tag{4}$$

Equation (4) can be rewritten as

$$\tau = -\frac{r}{2}\frac{dp}{dL_e} \tag{5}$$

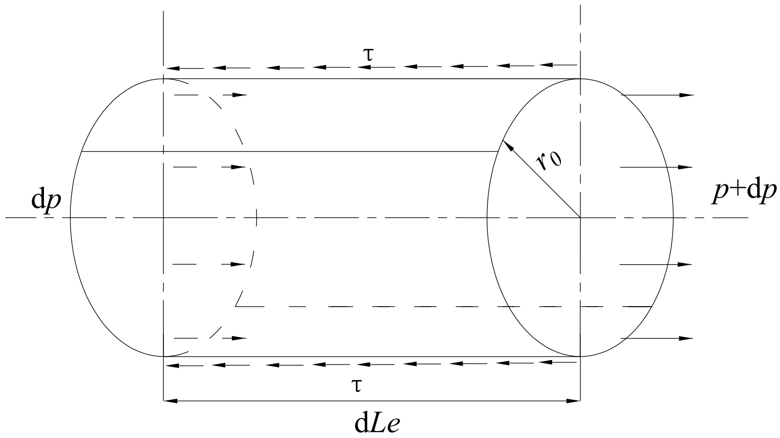


Figure 2. Flow diagrammatic sketch of Newtonian fluid in the pores of porous media.

Considering the boundary condition $r = r_0$, $u = 0$, by combining Equations (2), (3) and (5), separating the variables u and r and then integrating, the velocity equation of Newtonian fluid in porous media is obtained, given by

$$u = -\frac{1}{4\eta}\frac{dp}{dL_e}({r_0}^2 - r^2) \tag{6}$$

The flow rate Q_d of the fluid satisfies the following equation:

$$Q_d = \int_0^{r_o} 2\pi r u dr \tag{7}$$

By substituting Equation (6) into Equation (7) and integrating, we obtain

$$Q_d = -\frac{\pi {r_0}^4}{8\eta}\frac{dp}{dL_e} \tag{8}$$

Therefore, the fluid's average speed traversing the porous media is depicted by

$$\overline{u} = \frac{Q_d}{\pi r_0^2} = -\frac{r_0^2}{8\eta}\frac{dp}{dL_e} \tag{9}$$

Since the seepage velocity of slurry meets the equation $V = \phi\overline{u}$, and r_0 satisfies the equation $k = \phi r_0^2/8$, by combining Equations (1) and (9), the Newtonian fluid seepage motion equation considering the tortuosity of porous media is given by

$$V = -\frac{k}{\sqrt{\Gamma}\eta}\frac{dp}{dL} \tag{10}$$

where ϕ and k are the media's porosity and permeability, respectively. Other symbols are the same as above.

2.3. Formulation of Grouting Theoretical Model for Porous Media Incorporating the Tortuosity Effect

In this study, the analysis of the grouting of Newtonian fluid into porous media is grounded on past research works that have adopted certain assumptions concerning the grouts and the structure of the media, as referenced in [18,29–31]:

(1) The porous media is homogeneous and isotropic.
(2) The Newtonian fluid is incompressible. While injecting grouts, it exhibits an invariant flow pattern.

(3) The velocity of the fluid is very low, except for the localized turbulent flow near the grout injection port. In all other areas, the flow pattern of the fluid is laminar.
(4) The tamping method is used for injection. The fluid is injected by segmented top–bottom injection into the medium or through incomplete holes, and it diffuses in the form of a column-hemispherical structure.
(5) The time-varying and gravitational effects on the Newtonian fluid are neglected during grouting.

Based on these assumptions, taking into account the media's tortuosity, Figure 3 illustrates the column-hemispherical grouting model investigated in this study.

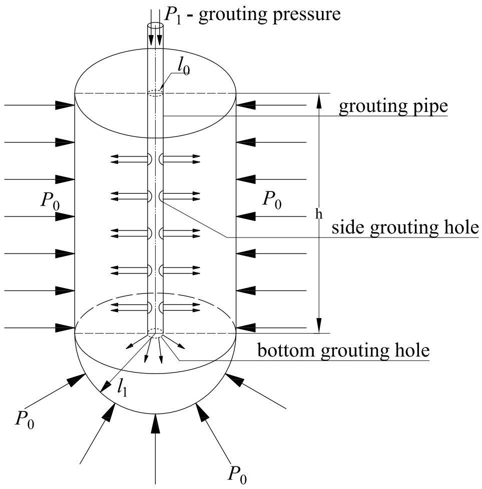


Figure 3. Column-hemispherical penetration diffusion model of Newtonian fluid in porous media.

In Figure 3, P_1 and P_0, respectively, denote the pressure from grouting and groundwater; l_0 is the radius of the grouting pipe; and h and l_1, respectively, represent the fluid's diffusion height of the column part and the diffusion radius of the hemispherical part at time t, respectively.

The grouting volume Q meets the following equation:

$$Q = VA\,t \tag{11}$$

where A is the diffusion surface area of a Newton fluid in porous media and can be characterized by the following equation:

$$A = 2\pi l(l + h) \tag{12}$$

According to the study of Yang et al. [18], in the theoretical column-hemispherical diffusion model shown in Figure 3, the diffusion radius and height satisfy the equation

$$h = \frac{2l}{3}(2g_0 + 1) \tag{13}$$

Equation (12) can be rewritten as follows:

$$A = \frac{2\pi l^2}{3}(5 + 4g_0) \tag{14}$$

where g_0 is the injection holes' quantity located on the upper section of the grouting pipe.

By combining Equations (10)–(14), we have

$$dp = -\sqrt{\Gamma}\frac{3\eta Q}{2\pi l^2(5 + 4g_0)tk}dL \tag{15}$$

Under the boundary conditions $p = p_0$, $l = l_1$; $p = p_1$, $l = l_0$, the following equation can be obtained by using the integral method of separation of variables:

$$\Delta p = p_1 - p_0 = \frac{3\sqrt{T}Q\eta}{2\pi(5+4g_0)tk}(\frac{1}{l_0} - \frac{1}{l_1}) \tag{16}$$

Since the grouting volume at time t meets the equation $Q = \pi l_1^2(h + \frac{2}{3}l_1)\phi = 4\pi l_1^3(g_0+1)\phi/3$, we can obtain

$$\Delta p = p_1 - p_0 = \frac{\sqrt{\Gamma}\,2(g_0+1)\phi l_1^3\eta}{(5+4g_0)kt}(\frac{1}{l_0} - \frac{1}{l_1}) \tag{17}$$

The relationship between the permeability k and permeability coefficient K of porous media can be expressed as the following equation [32]:

$$k = \frac{K\mu}{\rho_w g} \tag{18}$$

where μ is water viscosity, ρ_w is the density of water and takes 1000 kg/m^3 normally, and g is the acceleration of gravity and takes 9.8 m/s^2.

Therefore, Equation (18) can be rewritten as

$$\Delta p = p_1 - p_0 = \frac{\sqrt{\Gamma}\,2(g_0+1)\phi\rho_w g {l_1}^3\eta}{(5+4g_0)Kt\mu}(\frac{1}{l_0} - \frac{1}{l_1}) \tag{19}$$

In practical engineering $l_0 << l_1$, thereby $1/l_0 - 1/l_1 \approx 1/l_0$, Equation (19) can be simplified as

$$\Delta p = p_1 - p_0 = \frac{\sqrt{\Gamma}\,2(g_0+1)\phi\rho_w g l_1^3\eta}{(5+4g_0)Kt\mu l_0} \tag{20}$$

The theoretical diffusion radius l_1 of the hemispherical part can be derived from Equation (20), while the height h of the column part can be obtained by combining it with Equation (14). Namely, the equations of the column-hemispherical penetration grouting mechanism for a Newtonian fluid considering the tortuosity of porous media are Equations (20) and (14).

When the tortuosity effect of porous media is not considered, i.e., Γ is equal to 1, converting the units of pressure and $\beta = \eta/\mu$ to meters, the theoretical diffusion radius equation of the hemisphere part in [18] can be deduced from Equation (20), given by

$$\Delta p = p_1 - p_0 = \frac{2(g_0+1)\phi l_1^3\beta}{(5+4g_0)Ktl_0} \tag{21}$$

Substituting the theoretical diffusion radius obtained from Equation (21) into Equation (14), the theoretical diffusion height of the column part can be calculated.

2.4. Application Scope

The column-hemispherical grouting mechanism for a Newtonian fluid in porous media with the consideration of the tortuosity effect (Equations (20) and (14)) was developed under the premise that the fluid moves in a laminar manner during the penetration and diffusion processes, and therefore it is not applicable to turbulent motion. To determine whether the flow of a Newtonian fluid in porous media is laminar or turbulent, the Reynolds number Re is typically utilized according to [30,33]. The Newtonian fluid is turbulent when $Re > 4000$; The fluid behaves as a laminar flow when $Re < 2000$, and the fluid's flow state is a combination of the two states when $2000 < Re < 4000$. Re is calculated by Equation (22):

$$Re = \frac{\overline{v}d}{\eta} \tag{22}$$

where $\overline{v}$ is the average velocity of a Newtonian fluid in porous media, d is the pore size for the penetration diffusion of a Newtonian fluid in porous media, and other symbols are the same as above.

3. Numerical Simulations

COMSOL Multiphysics is large-scale advanced numerical simulation software developed by COMSOL in Sweden and is widely used in scientific research and engineering calculations in various fields. It is based on the finite element method and solves partial differential equations (single field) or partial differential equation systems (multi-field) to simulate and mimic real physical phenomena. Therefore, using secondary development programming technology, Darcy's law was employed in conjunction with the COMSOL Multiphysics simulation platform. A simulation program has been successfully developed to model the penetration and diffusion process of a Newtonian fluid in porous media (as per Equations (14) and (20)). The program's characteristic is its incorporation of the media's tortuosity, while also ensuring the assumption of the flow being in a laminar motion. This program can more accurately simulate and analyze the fluid's diffusion process in the media.

3.1. Basic Equations and Boundary Conditions

3.1.1. Assumption of Initial Conditions for Numerical Simulations

The following initial condition assumptions are made for the numerical simulation of the penetration and diffusion process based on single-phase flow and Darcy's law:

(1) The seepage movement of grouts in porous media follows Darcy's law.
(2) The pressure at each location along the grouting pipe is assumed to be consistent with the initial grouting pressure, and the pressure loss along the path from the grouting pump outlet to the grouting pipe is ignored.
(3) The rheological model of the grouts remains unchanged, and it is treated as an incompressible fluid during the grouting process. The grouts is assumed to be in a fully permeable mode during the diffusion process, and the influence of filtration effect is ignored. The viscosity of the grouts does not change with time.
(4) The gravity effect is ignored during the diffusion of the grouts.
(5) The permeability coefficient and porosity of the porous medium are assumed to be constant throughout the grouting process. Due to the relatively short time of the grouting process and the limited diffusion range caused by the small radius of the grouting hole (only 0.75 cm), the changes in the porosity and permeability coefficient of the porous medium are insignificant within a small range and short amount of time. Therefore, it can be concluded that the impact of this assumption on the diffusion of the grouts is negligible [30].

3.1.2. Basic Equation

(1) Darcy's law physical equation

$$u = -\frac{k}{\mu}(\nabla p + \rho g \nabla D) \tag{23}$$

The dependent variable in the equation is pressure p. By combining this equation with Equation (18), the following expression can be obtained:

$$u = -\frac{K}{\rho g}(\nabla p + \rho g \nabla D) \tag{24}$$

(2) Control equation

To simulate the penetration diffusion process of cement grouts, Darcy's law can be expressed as

$$\nabla g\left(\rho\left(-\frac{K}{\mu}\nabla P\right)\right) = Qm \tag{25}$$

where ρ is the density of the fluid, Qm represents the source term, and other symbols remain the same as mentioned above. According to Equation (25), the Darcy's law model interface in COMSOL Multiphysics software requires input parameters such as the density of the slurry, permeability of the porous medium, dynamic viscosity of the slurry, pore water pressure, and various boundary conditions and initial conditions.

3.1.3. Boundary Conditions

(1) Inlet boundary conditions

In the internal boundary conditions of Darcy's law, the pressure inlet boundary condition can be chosen, which can also be used to specify a boundary with no restriction or external free flow. The pressure boundary condition is suitable for cases where the fluid velocity is unknown but the grouting pressure is known. The boundary condition of the pressure inlet can be used to simulate cases in which the fluid velocity is unknown in many practical problems.

(2) Outlet boundary conditions

As the physical model experiment is conducted within the range of the porous medium for observation and research, the model does not apply in the situation where the fluid permeates and overflows the porous medium model. Therefore, the outlet boundary condition is set as a no-flow boundary; that is,

$$-\phi \cdot \rho u = 0 \tag{26}$$

From the above equation, it can be seen that the velocity of the grouts is zero; therefore, there is no situation where the grout passes through and overflows by penetration. This boundary condition is consistent with the actual physical model experiment.

3.2. Establishment of Numerical Model for the Fluid–Solid Coupling Penetration Grouting

3.2.1. Model Construction

To evaluate the applicability and accuracy of the theoretical mechanism derived in the second section, simulations were performed on the mechanism considering the media's tortuosity (Equations (14) and (20)) and without this consideration (Equations (14) and (21)). These simulations were based on the grouting model experiment outlined in Yang's previous research [18]. The numerical model, a 0.60 m × 0.60 m × 1.00 m three-dimensional cuboid, was defined based on the grouting equipment, experimental results, and numerical simulation scheme. The soil parameters and simulation scheme used in the numerical simulation were selected to be consistent with those of the grouting model experiment, as indicated in Table 1.

Moreover, it should be noted that the Reynolds numbers were calculated instantly when the cement slurries with water–cement ratios of 10, 5, and 2 were prepared. The Reynolds numbers of the three experimental slurries were Re_{G1} = 167.83, Re_{G2} = 505.17, and Re_{G3} = 994.41, respectively. As the three cement grouts were injected into the gravel body, their viscosity gradually increased, causing the pore size and average velocity of the fluid in the gravel soil to decrease progressively. As a result, the instantaneous Reynolds numbers (Res) of the cement slurries also decreased accordingly during grouting. Specifically, the Newtonian cement slurry's instantaneous Reynolds numbers (Res) with the three water-cement ratios all satisfied the relationship Res < 2000 during grouting. These indicate that the cement slurries used in the grouting experiment all meet the theoretical basis of the laminar flow state. Furthermore, the temperature of indoor environment during the grouting model experiment and the temperature of the water used to mix the cement grout were both 20 °C, as documented by Yang et al. in [18]. The cement grout rheology equation from the research results of Ruan [34] is used, and the tortuosity τ is selected as 2.25. For the numerical simulation and modeling of penetration grouting in porous media, we divide the process into two steps considering its complex soil model. The first step is to create a simulated numerical model based on Darcy's law and utilizing Comsol's powerful multi-physics coupling function. The second step is to set the relevant parameters of the model shown in Figure 4. The finite element model for the Newtonian fluid column-hemisphere grouting is a three-dimensional rectangular parallelepiped, with the grouting pipe set in the middle of the x-axis boundary of the model. There are a total of three grouting holes, with one at the bottom of the grouting pipe and two on the side. The radius of the grouting pipe is 0.0075 m.

Table 1. Grouting simulation scheme.

Experiment Number	Soli Properties of the Injected Body			Water-Cement Ratio	Grouting Pressure (Pa)	Grouting Duration (s)
	Particle Size Distribution (mm)	Permeability Coefficient (cm/s)	Porosity (%)			
G1	1∼3	0.65	39.93	10.0	40,000	8
G2	3∼5	2.11	45.05	5.0	60,000	6
G3	5∼10	8.94	50.74	2.0	80,000	4

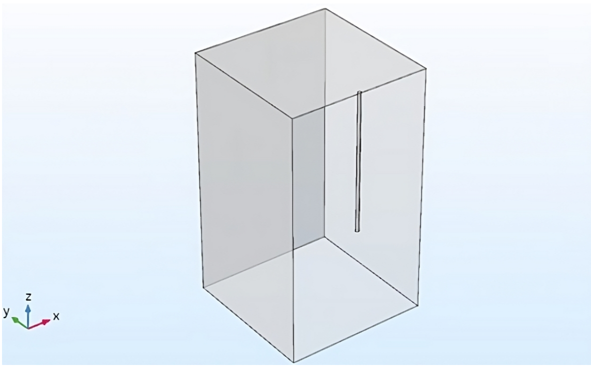


Figure 4. Geometric model of the simulation program.

3.2.2. Meshing of the Finite Element Model

Before solving the numerical simulation model with COMSOL software, the corresponding geometric model needs to be divided into an appropriate mesh. The solve region is divided using a triangular mesh division method. In order to increase the operability of the model and the accuracy of the results, local grid refinement is performed on the pressure setting region of the model, which is the injection holes, to avoid inaccurate solving at these areas, which could affect the accuracy of the simulation results and the convergence of the simulation process. The specific grid division is shown in Figure 5. When calculating the transport and diffusion process of cement grouts in porous media, because the grouts in porous media cannot diffuse without limit, the surrounding boundaries are set as an ideal impermeable boundary—namely, they are set as a no-flow boundary.

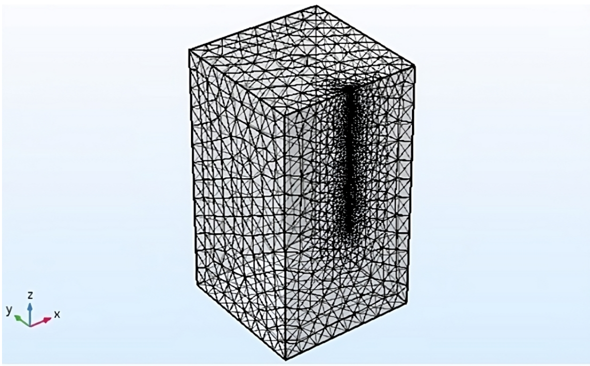
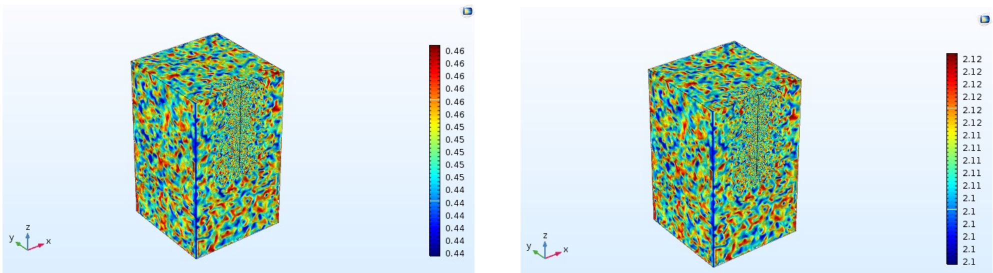


Figure 5. Triangular mesh division of the model.

3.2.3. Setting of Porosity and Permeability of Porous Media

Through COMSOL Multiphysics software, the random function is defined to intuitively observe the distribution of porosity and permeability inside the porous media, obtaining three sets of porosity and permeability distribution map of porous media.Taking the particle gradation of 3∼5 mm as an example, the simulated distribution of the porosity and permeability coefficient are shown in Figure 6.



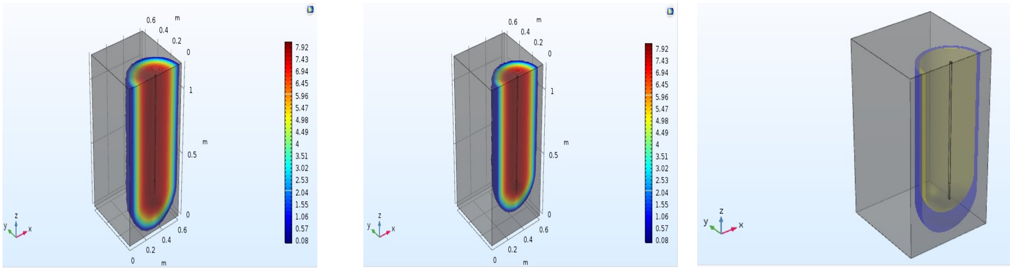
(**a**) (**b**)

Figure 6. The simulation results of porosity and permeability coefficient distribution of gravel soil with particle gradation of 3∼5 mm: (**a**) porosity distribution; (**b**) permeability coefficient distribution.

The results of Figure 6 reveal that the injected gravel soil with a particle gradation of 3∼5 mm exhibits a distribution range of porosity and permeability coefficient from 0.46∼0.44 and 2.12∼2.10 cm/s, respectively, which demonstrates that the gravel soil's porosity and permeability coefficients, as obtained from the numerical simulation conducted in this study, conform to the grouting model experiment requirements described in Yang's work [18]. Subsequently, by utilizing the built-in function definition capability of the COMSOL numerical simulation software, a numerical model for the penetration grouting of the Newtonian fluid in the porous medium was established through secondary development in the programming language. After completing all the aforementioned tasks, three sets of numerical simulation results without considering tortuosity and three sets considering tortuosity were obtained. By comparing the simulation results, the influence of tortuosity on the diffusion effect of Newtonian fluid in porous media during the penetration grouting process was analyzed.

3.3. Simulation Results

The simulation of the grouting mechanism based on the data from the grouting model experiment are shown in Figure 7.



(a) (b) (c)

Figure 7. Numerical simulation results of the column-hemispherical penetration diffusion of Newtonian cement slurry in gravel soil; (**a**) The results without considering the tortuosity effect; (**b**) the results considering the tortuosity effect; (**c**) the comparison of the two results.

By analyzing Figure 7, it is evident that Newtonian cement grout diffuses in a three-dimensional column-hemispherical shape within the gravel soil, both with and without accounting for the soil's tortuosity, as expected from the theoretical penetration and diffusion model illustrated in Figure 3. Additionally, the spatial range of diffusion is reduced when soil tortuosity is taken into consideration.

4. Comparison of Theoretical Analyses, Numerical Simulations and Model Experiments

In this section, we compare the results from theoretical analyses, numerical simulations, and model experiments based on the grouting model experiment data in [18] to examine the correctness of the theory and simulations. The comparison results are shown in Table 2 and Figures 8 and 9.

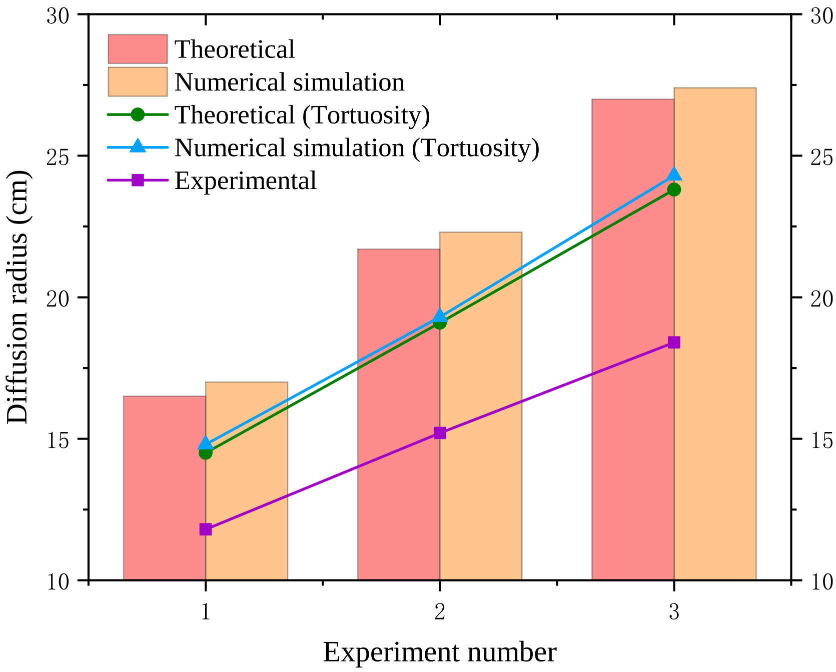


Figure 8. Comparison of theoretically calculated, numerically simulated, and experimental values of the hemispherical diffusion radius.

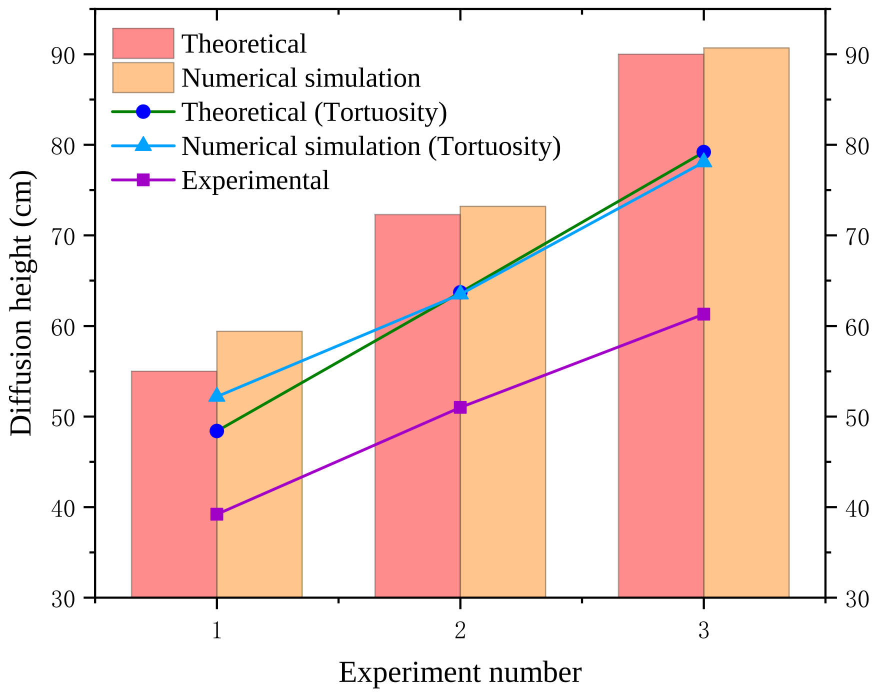


Figure 9. Comparison of theoretically calculated, numerically simulated, and experimental values of the column diffusion height.

Table 2. The comparison between theoretical analyses, numerical simulations, and model experiment results.

<table>
<tr><th rowspan="3">Experiment Number</th><th colspan="5">Diffusion Radius of Hemispherical Part</th><th colspan="5">Diffusion Height of Columnar Part</th></tr>
<tr><th colspan="2">Values Obtained from Equation (20) (cm)</th><th colspan="2">Values Obtained from Equation (21) (cm)</th><th rowspan="2">Experimental Values (cm)</th><th colspan="2">Values Obtained by Combining Equations (14) and (20) (cm)</th><th colspan="2">Values Obtained by Combining Equations (14) and (21) (cm)</th><th rowspan="2">Experimental Values (cm)</th></tr>
<tr><th>Theo-Retical</th><th>Simu-Lated</th><th>Theo-Retical</th><th>Simu-Lated</th><th>Theo-Retical</th><th>Simu-Lated</th><th>Theo-Retical</th><th>Simu-Lated</th></tr>
<tr><td>G1</td><td>14.5</td><td>14.8</td><td>16.5</td><td>17.0</td><td>11.80</td><td>48.4</td><td>52.2</td><td>55.0</td><td>59.4</td><td>39.2</td></tr>
<tr><td>G2</td><td>19.1</td><td>19.3</td><td>21.7</td><td>22.3</td><td>15.20</td><td>63.7</td><td>63.5</td><td>72.3</td><td>73.2</td><td>51.0</td></tr>
<tr><td>G3</td><td>23.8</td><td>24.3</td><td>27.0</td><td>27.4</td><td>18.40</td><td>79.2</td><td>78.1</td><td>90.0</td><td>90.7</td><td>61.3</td></tr>
</table>

The following can be deduced from Table 2 and Figures 8 and 9:

(1) The simulated hemispherical diffusion radius and column diffusion height obtained from the developed Newtonian fluid column-hemispherical penetration grouting mechanism demonstrate a proximity to their corresponding theoretical calculated values, with less than 10% difference between the two, regardless of the consideration of the tortuosity of the gravel soil. This suggests that the numerical simulation program of the mechanism considering the tortuosity effect can reliably simulate the diffusion pattern of the cement grout, which is developed using secondary programming through the COMSOL Multiphysics platform.

(2) The hemispherical diffusion radius and columnar diffusion height, acquired through theoretical calculations and numerical simulations of the Newtonian fluid column-hemispherical penetration grouting mechanism considering the tortuosity effect, are more consistent with the model experimental values than the values obtained without considering the tortuosity.

5. Discussion

Theoretical calculations and numerical simulations considering the tortuosity effect of porous media showed that the theoretical diffusion radius and height are closer to the actual experimental value than the model that does not consider tortuosity. The following factors can explain this.

(1) Within the tortuous porous media, particles' irregular shape, size, and arrangement will cause the Newtonian fluid to encounter obstacles from pores and particles, making the flow path more curved. That is, the tortuosity effect leads to the fluid passing through multiple narrow channels and tiny gaps, increasing the flow resistance and the diffusion path. This convoluted and prolonged path will reduce the grouts' diffusion speed and diffusion range.
(2) The model that considers this effect can more accurately describe the transmission behavior of Newtonian fluids in porous media, thereby better predicting the diffusion radius of the hemisphere part and the diffusion height of the column part. In contrast, the model that does not consider the effect ignores the complex transmission behavior of the fluid in the media, so its predicted results often overestimate the diffusion range.

The values of diffusion radius and height, obtained through theoretical calculations and numerical simulations of the mechanism proposed in this note, exhibit greater sizes than the corresponding model experimental values for three primary reasons.

(1) This study did not consider the time-dependent behavior, filtration effects, and gravitational impact of a Newtonian fluid on grouting diffusion. The grouting slurry's actual penetration and diffusion process during the model experiment were influenced by the continuous changes in concentration, particle distribution, and hydration reaction due to the ongoing water separation.
(2) The particle gradation of the gravel soil used in the grouting model experiment was relatively uniform, and the soil was washed three times in clean water and dried. However, despite these efforts, the soil still needs to meet the assumption of homogeneity and isotropy in order to accurately analyze the grouting mechanism.
(3) The model experiment is subject to certain boundary and size effects, and the penetration diffusion of a Newtonian fluid in porous media is a highly complex process. Moreover, there are several additional variables that can affect the results, such as the conditions of the experimental setup and the behavior of operators. It should be noted that these factors may have a significant impact on the outcomes.

In summary, the discrepancies, as previously stated, suggest that this study may have overlooked certain aspects, such as the time-varying properties of the fluid, the filtering effect, and the influence of gravity in theoretical derivations of the mechanism. Despite these limitations, this research offers new theoretical insights and methodologies for understanding the penetration diffusion of Newtonian fluids in porous media. Future studies could incorporate the time-varying properties of the fluid, the filtering effect, and gravity, aiming to optimize the design of the model experiment to enhance the alignment of theoretical derivations and numerical simulations with actual experimental results.

6. Conclusions

By employing theoretical analysis, computer programming, and comparative analysis, this study developed a column-hemispherical penetration grouting mechanism for a Newtonian fluid in porous media that accounted for the media's tortuosity. The study's primary conclusions are as follows:

(1) According to the tortuosity effect of porous media and the rheological equation of Newtonian fluids, the Newtonian fluid seepage motion equation considering the tortuosity was established, and the column-hemispherical penetration grouting mechanism for a Newtonian fluid incorporating the media's tortuosity was theoretically deduced.

(2) Considering the tortuosity effect, a Newtonian fluid's three-dimensional numerical simulation program was obtained relying on the coupling software COMSOL Multiphysics by secondary development. Numerical simulations were performed to study the grouting process in gravel soil using Newtonian cement with three different water–cement ratios. The simulation outcomes showed that, in the gravel soil, the penetration diffusion morphology of Newtonian cement grout is in accordance with the theoretical model of column-hemispherical penetration grouting, regardless of whether the tortuosity effect is taken into account or not. In comparison, the consideration of the tortuosity effect results in a reduction of the spatial diffusion in the gravel soil.

(3) The validity of the mechanism was confirmed by comparing theoretical analysis, model experiments, and numerical simulations. The validation outcome indicates that, while considering the tortuosity effect in porous media, the program can accurately depict the penetration diffusion behavior of Newtonian fluid. The theoretical values of hemispherical diffusion radius and the column diffusion height, as well as their numerical simulation values obtained by the mechanism proposed in this paper, achieve a closer agreement between the model predictions and experimental measurements.

In summation, the proposed column-hemispherical penetration grouting mechanism for the Newtonian fluid, which considers the tortuosity effect, can better reflects the fluid's diffusion mechanism in porous media compared to the mechanism that ignores the tortuosity effect. This research advances the Newtonian fluid grouting theory in porous media, and its implications for practical grouting engineering design and construction in porous strata are substantial.

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Nomenclature

A	Diffusion surface area, cm^2
d	Pore size through which a fluid passes in an injected medium, cm
g	Gravitational acceleration, 9.8 m/s^2
g_0	Number of grouting holes on the side of the grouting pipe
h	Diffusion height of the column part, cm
k	Permeability of porous media, cm^2
K	Permeability coefficient, cm/s
l	Diffusion radius of the hemispherical part, cm
l_0	Radius of the grouting pipe, cm
l_1	Diffusion radius of the hemispherical part at the time of t, cm
L	Straight line length of the actual flow path, cm
L_e	Length of fluid's actual flow path in porous media, cm
p	Pressure, Pa
P_0	Groundwater pressure, Pa
P_1	Grouting pressure, Pa
Q	Grouting volume, m^3

Q_d	Volumetric flow rate, m^3/s
Qm	Mass source term
r	Flow radius of the fluid in porous media, cm
r_0	Radius of the microfluidic columnar unit, cm
Re	Reynolds number
t	Grouting time, s
u	Flow velocity, cm/s
$\overline{u}$	Average velocity of the fluid flowing through porous media, cm/s
V	Seepage velocity of the fluid in porous loose media, cm/s
$\overline{v}$	Average velocity at which a fluid flows in an injected medium, cm/s
β	Viscosity ratio of grouting cement to water
γ	Shear rate, s^{-1}
$\mathit{\Gamma}$	Tortuosity
η	Dynamic viscosity, Pa·s
μ	Water viscosity, Pa·s
ρ	Density of the fluid, kg/m^3
ρ_w	Water density, kg/m^3
τ	Shear stress, Pa
ϕ	Porosity of porous media

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