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Adaptive Control of Advanced G-L Fuzzy Systems with Several Uncertain Terms in Membership-Matrices

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Abstract: In this paper, a set of novel adaptive control strategies based on an advanced G-L (proposed by Ge-Li-Tam, called GLT) fuzzy system is proposed. Three main design points can be summarized as follows: (1) the unknown parameters in a nonlinear dynamic system are regarded as extra nonlinear terms and are further packaged into so-called nonlinear terms groups for each equation through the modeling process, which reduces the complexity of the GLT fuzzy system; (2) the error dynamics are further rearranged into two parts, adjustable membership function and uncertain membership function, to aid the design of the controllers; (3) a set of adaptive controllers change with the estimated parameters and the update laws of parameters are provided following the current form of error dynamics. Two identical nonlinear dynamic systems based on a Quantum-CNN system (Q-CNN system) with two added terms are employed for simulations to demonstrate the feasibility as well as the effectiveness of the proposed fuzzy adaptive control scheme, where the tracking error can be eliminated efficiently.

Keywords: advanced G-L fuzzy systems; nonlinear terms group; adaptive control; membership functions matrices

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1. Introduction

The concept of fuzzy logic was proposed by L. A. Zadeh in 1965 [1] that has received much attention as a powerful tool for interdisciplinary applications, such as clustering [2,3], fault diagnosis [4–6], prediction [7,8] and decision making [9–12], etc. For example, Khoshdast et al. [3] presented an approach to reduce the mesh-induced error for CFD analysis; Krzywanski et al. [9] applied a set of fuzzy logic-based methods to raise the performance of heat transfer in a super-heater for an industrial CFBC; Khoshdast et al. [10] developed a coupled fuzzy logic design for the simulation of a coal classifier in an industrial environment; and in [11], Akbar et al. proposed a fuzzy logic-based model to elucidate the effect of FSW parameters on the ultimate tensile strength and elongation of pure copper joints.

Among the various kinds of applications in the fuzzy area, fuzzy logic control [13–20], especially fuzzy model-based control, is widely used because the design and analysis of the overall fuzzy system can be systematically performed using the well-established classical linear systems theory. The Takagi–Sugeno fuzzy model proposed in 1985 [21] is widely accepted as a powerful tool for modeling, design and analysis of nonlinear dynamic systems [22–36]. This well known mathematical tool aims to build a fuzzy model of dynamic systems where fuzzy implications and reasoning are applied to describe a fuzzy subspace of inputs and its consequence for a linear input-output form.

According to the modeling concept revealed by the T-S fuzzy model, Ge and Li [37] proposed a novel modeling approach—applying the fuzzy concept to model each equation in nonlinear systems. This new modeling approach was proposed in an attempt to raise the efficiency of fuzzy modeling, simplify the modeling procedure, and reduce the numbers of modeling materials and feedback control inputs during fuzzy control. This novel modeling approach has been cited over 50 times in various research fields and applications; for instance, H∞ control design [38–42], design and control of memristive systems [43–46], data encryption [47–50], sampled-data control [51–53] and other applications [54–60]. Moreover, Li et al. [61] extended a new concept of a nonlinear terms group to further improve the effectiveness of the G-L fuzzy model in 2015, called advanced G-L fuzzy modeling strategy (GLT fuzzy system). For all kinds of nonlinear dynamic systems, there will be only two linear subsystems and two membership function matrices in the final output of this advanced fuzzy model. In summary, this proposed modeling approach provides a novel way to model systems with different structures as the same form, and therefore, the fuzzy controllers can be designed in a more efficient way.

Furthermore, in practical engineering, the structure and parameters in a system are usually unobtainable, especially when the parameters are always changing according to the operation of the system. As a result, adaptive control [62,63] has been employed to address this kind of practical issue. Fuzzy logic systems (FLSs) provide a very effective method for coping with uncertainties due to its excellent functional approximation abilities. For example, Liu et al. [64,65] investigated adaptive fuzzy control for MIMO systems with unknown dead-zones, Tong et. al. [66,67] designed a set of state observers for a fuzzy adaptive control system, and Chen et. al. [68] applied a fuzzy neural network to address the adaptive control problem for a class of uncertain nonlinear stochastic systems.

The adaptive control studies mentioned above can be divided into two types: one designs the adaptive control scheme following fuzzy logic theory, and the other derives the stability of adaptive control via transferring the original dynamic systems to a T-S fuzzy system. A novel fuzzy model [37,61] provides an alternative way to model a dynamic system in a more efficient way, and as a consequence, we became interested in how to solve the adaptive control problem by applying a novel fuzzy model. In this paper, a GLT fuzzy system based on an adaptive control strategy is proposed to address the control problem for membership function matrices with mismatched parameters. The concept provided via this set of adaptive control strategies is to consider the unknown parameters in the original dynamic system as extra nonlinear terms and then package them into a nonlinear terms group in each equation. With this approach, the unknown parameters problem is transferred to an uncertain membership function matrices problem. This strategy provides a different perspective for solving the adaptive control problem that is based on a GLT fuzzy model.

The organization of the research can be summarized below. The modeling concept for the GLT fuzzy model theory and the fuzzy adaptive control scheme are introduced in Section 2. The modeling process, simulation results and some further discussion are included in Section 3. Finally, the conclusions are given in Section 4.

2. Materials and Methods

2.1. GLT Fuzzy-Model Theory

According to the proposed fuzzy model [61], to model each nonlinear equation and package the nonlinear terms groups, the consequent part of the proposed fuzzy system is designed to be a linear equation, and the system output can be simply described as weight sums of two linear subsystems as given below:

$$\dot{\boldsymbol{Y}}(\boldsymbol{t}) = \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \boldsymbol{M_i}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) \tag{1}$$

where $\boldsymbol{Y}(\boldsymbol{t}) \in \Re^{n\times 1}$ is the state of the system, revealed to be a vector form, and the coefficient matrices, $\boldsymbol{A_i} \in \Re^{n\times n}$ and $\boldsymbol{B_i} \in \Re^{n\times 1}$, indicate the coefficient of states and constants transformed via modeling, respectively. Moreover, p = 2, the number of linear subsystems and membership function matrices according to the modeling process, where $\boldsymbol{M_i} \in \Re^{n\times n}$ are matrices including the membership functions generated by the proposed fuzzy if-then rules:

$$\boldsymbol{M_i}= \begin{bmatrix} M_{ji} & 0 & 0 & 0 \\ 0 & M_{ji} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & M_{ji} \end{bmatrix}, \tag{2}$$

where j = 1, 2, ... , n; i = 1, 2, $\boldsymbol{M}_1 + \boldsymbol{M}_2$ is the identity matrix and $\sum_{i=1}^{2} M_{ji} = 1$.

2.2. Adaptive Control Scheme

The proposed adaptive control scheme is constructed in this subsection. The unknown parameters are regarded as part of the nonlinear terms, and we package those unknown parameters into nonlinear terms groups, i.e., through this design, the unknown parameters are included in the membership function and no longer appear in the coefficient matrix of linear subsystems of the final output. Equations (3) and (4) are two GLT fuzzy systems with consistent structures, the first is regarded as the drive system, and the second is viewed as a response system with designed fuzzy controllers, where the unknown system parameters are arranged in the membership function matrices in Equation (3):

Master system:

$$\dot{\boldsymbol{Y}}(\boldsymbol{t}) = \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \boldsymbol{M_i}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) \tag{3}$$

Slave system:

$$\dot{\boldsymbol{X}}(\boldsymbol{t}) = \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \hat{\boldsymbol{O}}_{\boldsymbol{i}}(\boldsymbol{A_i}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B_i}) + \boldsymbol{U}(\boldsymbol{t}) \tag{4}$$

where $\boldsymbol{Y}(\boldsymbol{t}) \in \Re^{n\times 1}$, $\boldsymbol{X}(\boldsymbol{t}) \in \Re^{n\times 1}$ are the states of the master and slave systems, respectively; $\boldsymbol{M_i} \in \Re^{n\times n}$ and $\hat{\boldsymbol{O}}_{\boldsymbol{i}} \in \Re^{n\times n}$ are the membership function matrices with unknown and estimated parameters, respectively. Once again, p = 2, the number of linear subsystems and membership function matrices according to the modeling process. It is important to mention that $\boldsymbol{A_i} \in \Re^{n\times n}$ and $\boldsymbol{B_i} \in \Re^{n\times n}$ are the coefficient matrices, which are the same in the master and slave system through the design given above. $\boldsymbol{U(t)} \in \Re^{n\times 1}$ is the adaptive control input, designed to not share the same membership functions as the plant model.

The error states $\boldsymbol{e}(\boldsymbol{t}) = \boldsymbol{Y}(\boldsymbol{t}) - \boldsymbol{X}(\boldsymbol{t})$ are defined, $\boldsymbol{e}(\boldsymbol{t}) \in \Re^{n\times 1}$, then we have the following error dynamics description in Equation (5):

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{Y}} - \dot{\boldsymbol{X}} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \boldsymbol{M_i}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) - \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \hat{\boldsymbol{O}}_{\boldsymbol{i}}(\boldsymbol{A_i}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B_i}) - \boldsymbol{U}(\boldsymbol{t}) \tag{5}$$

In observation of the mathematics structure in Equation (5), the structure of the error system is not simple, where the drive system, response system and the designed fuzzy controllers are included. Also, the membership function matrices $\boldsymbol{M_i}$ in the master system comprise unknown parameters, the estimated parameters are designed in $\hat{\boldsymbol{O}}_{\boldsymbol{i}}$ and $\boldsymbol{M_i} \neq \hat{\boldsymbol{O}}_{\boldsymbol{i}}$. Thus, the fuzzy controllers cannot be assigned to remove these uncertain terms directly. According to the observation mentioned above, Equation (5) can be further expanded in the following way:

$$\begin{aligned} \dot{\boldsymbol{e}} = \dot{\boldsymbol{Y}} - \dot{\boldsymbol{X}} = & \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \boldsymbol{M_i}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) - \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \hat{\boldsymbol{O}}_{\boldsymbol{i}}(\boldsymbol{A_i}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B_i}) \\ & - \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \hat{\boldsymbol{O}}_{\boldsymbol{i}}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) + \sum_{\boldsymbol{i}=1}^{\boldsymbol{p}} \hat{\boldsymbol{O}}_{\boldsymbol{i}}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) - \boldsymbol{U}(\boldsymbol{t}) \end{aligned} \tag{6}$$

where $\pm\sum_{i=1}^{p}\hat{\boldsymbol{O}}_i(\boldsymbol{A}_i\boldsymbol{Y}(\boldsymbol{t})+\boldsymbol{B}_i)$ are the extra terms added in Equation (6), then we can obtain the following expression:

$$\begin{aligned}\dot{\boldsymbol{e}}=\dot{\boldsymbol{Y}}-\dot{\boldsymbol{X}}=&\sum_{i=1}^{p}\hat{\boldsymbol{O}}_i(\boldsymbol{A}_i\boldsymbol{Y}(\boldsymbol{t})-\boldsymbol{A}_i\boldsymbol{X}(\boldsymbol{t})+\boldsymbol{B}_i-\boldsymbol{B}_i)\\&+\sum_{i=1}^{p}(\boldsymbol{M}_i-\hat{\boldsymbol{O}}_i)(\boldsymbol{A}_i\boldsymbol{Y}(\boldsymbol{t})+\boldsymbol{B}_i)-\boldsymbol{U}(\boldsymbol{t})\end{aligned}\tag{7}$$

Finally, the error dynamic system can be described in a clearer way in Equation (8):

$$\dot{\boldsymbol{e}}=\dot{\boldsymbol{Y}}-\dot{\boldsymbol{X}}=\sum_{i=1}^{p}\hat{\boldsymbol{O}}_i(\boldsymbol{A}_i\boldsymbol{e}(\boldsymbol{t}))+\sum_{i=1}^{p}(\boldsymbol{M}_i-\hat{\boldsymbol{O}}_i)(\boldsymbol{A}_i\boldsymbol{Y}(\boldsymbol{t})+\boldsymbol{B}_i)-\boldsymbol{U}(\boldsymbol{t})\tag{8}$$

Here, the error dynamic system derived in Equation (8) can be classified into two parts given in Equation (9). The first part is developed via the estimated membership function matrices $\hat{\boldsymbol{O}}_i$ and the coefficient matrices $\boldsymbol{A}_i$ with error states, which can be controlled directly through designing $\boldsymbol{U_1(t)}$. The second part is composed of the difference of the two membership function matrices $\boldsymbol{M}_i$,$\hat{\boldsymbol{O}}_i$, and the linear subsystems of the master system, where the control input in $\boldsymbol{U_1(t)}$ cannot address it directly. Therefore, $\boldsymbol{U_2(t)}$ is needed.

$$\dot{\boldsymbol{e}}=\underbrace{\sum_{i=1}^{p}\hat{\boldsymbol{O}}_i(\boldsymbol{A}_i\boldsymbol{e}(\boldsymbol{t}))}_{\boldsymbol{U}_1(\boldsymbol{t})}+\underbrace{\sum_{i=1}^{p}(\boldsymbol{M}_i-\hat{\boldsymbol{O}}_i)(\boldsymbol{A}_i\boldsymbol{Y}(\boldsymbol{t})+\boldsymbol{B}_i)}_{\boldsymbol{U}_2(\boldsymbol{t})}-\boldsymbol{U}(\boldsymbol{t})\tag{9}$$

Consequently, the fuzzy controllers $\boldsymbol{U}(\boldsymbol{t})=\boldsymbol{U}_1(\boldsymbol{t})+\boldsymbol{U}_2(\boldsymbol{t})$ can be designed as follows:

$$\begin{cases}\boldsymbol{U}_1(\boldsymbol{t})=\sum_{i=1}^{p}\hat{\boldsymbol{O}}_i(-\boldsymbol{C}_i\boldsymbol{F}_i\boldsymbol{e}(\boldsymbol{t}))\\\boldsymbol{U}_2(\boldsymbol{t})=[\boldsymbol{u}_1(\boldsymbol{t}),\boldsymbol{u}_2(\boldsymbol{t}),\boldsymbol{u}_3(\boldsymbol{t})\ldots\boldsymbol{u}_l(\boldsymbol{t})]^T\end{cases}\tag{10}$$

where $\boldsymbol{U}_1(\boldsymbol{t})\in\Re^{n\times1}$ is designed as the fuzzy control input to pre-process the error dynamic systems in Equation (9); $C_i\in\Re^{n\times n}$ is a constant matrix; $F_i\in\Re^{n\times n}$ is a matrix designed to be the control gain satisfying the condition $\boldsymbol{A}_i-C_iF_i=G$, where $\boldsymbol{G}$ is a negative definite matrix. The fuzzy controllers $\boldsymbol{U}_2(\boldsymbol{t})\in\Re^{n\times1}$ should be designed with appropriate parameter update laws, where l is the total number of control inputs in $\boldsymbol{U_2(t)}$.

The Lyapunov function given in Equation (11) is applied,

$$V(\boldsymbol{e}(\boldsymbol{t}),\tilde{\boldsymbol{a}}_j)=\frac{1}{2}\left(\sum_{i=1}^{n}(\boldsymbol{e}_i(\boldsymbol{t})^2)+\sum_{j=1}^{m}(\boldsymbol{r}_j\tilde{\boldsymbol{a}}_j^2)\right)\tag{11}$$

where $\tilde{\boldsymbol{a}}_i$ is the difference of parameters in the membership function; $\boldsymbol{m}$ indicates the number of parameters we have estimated; $\boldsymbol{r}_j$ refer to the adaptation gains, where $\boldsymbol{r}_j > 0$ In this paper, all $\boldsymbol{r}_j$ are set to be $\boldsymbol{r}_j = 1$. Through designing appropriate fuzzy controllers $\boldsymbol{U}(\boldsymbol{t})$ and the adaptation law of system parameters $\tilde{\boldsymbol{a}}_i$, we aim to have the following:

$$\dot{\boldsymbol{V}}=-\sum_{i=1}^{n}(\eta_i\boldsymbol{e}_i(\boldsymbol{t})^2)\le0\tag{12}$$

where η_i is designed to be positive. It is worth to mentioning that the pragmatically asymptotical stability theorem provided by Ge et al. in 1999 [69,70] has proved in a strict way that the error states, as well as the estimated parameters, can approach the uncertain or goal parameters in Equation (12).

3. Simulation Results and Discussion

In this section, a Quantum-CNN system (Q-CNN system) [71] with some extra nonlinear terms is proposed for illustration. This complicated system is presented to show the effectiveness of the proposed modeling and control strategy developed in this article.

$$\begin{cases} \dot{y}_1 = -2b_1\sqrt{1-y_1^2}\sin y_2 \\ \dot{y}_2 = -w_1(y_1-y_3) + 2b_3\frac{y_1}{\sqrt{1-y_1^2}}\cos y_2 + \Delta_1 \\ \dot{y}_3 = -2b_2\sqrt{1-y_3^2}\sin y_4 \\ \dot{y}_4 = -w_2(y_3-y_1) + 2b_4\frac{y_3}{\sqrt{1-y_3^2}}\cos y_4 + \Delta_2 \end{cases} \tag{13}$$

where Δ_1 and Δ_2 are the extra nonlinear terms defined as $\Delta_1 = b_5 y_1 y_4^2$, $\Delta_2 = b_6 y_3 y_2^3$. When the system parameters $b_1 = b_3 = 6.8$, $b_2 = b_4 = 4.3$, $w_1 = 4.7$, $w_2 = 3.9$, $b_5 = 5$ and $b_6 = 2$ and initial states are set as (y_{10}, y_{20}, y_{30}, y_{40}) = (0.1, 0.5, 0.1, 0.5), the system shows chaotic behaviors, as shown in Figure 1.

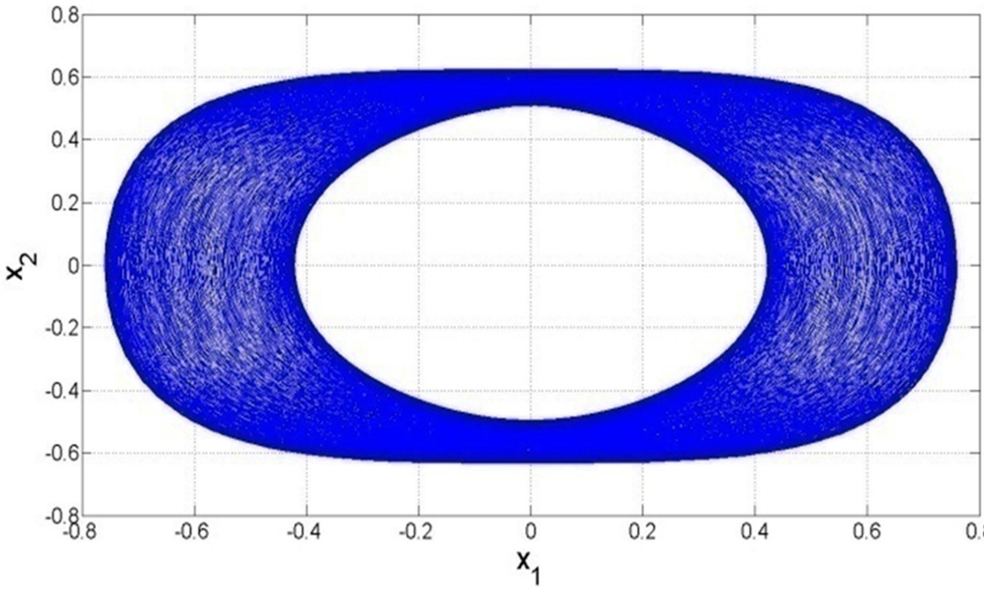

Figure 1. Chaotic behavior of a Quantum Cellular Neural Networks Nano system with extra nonlinear terms.

According to the modeling theory given in Section 2.1, Equation (13) can be expressed in the following form:

$$\dot{\boldsymbol{Y}}(\boldsymbol{t}) = \sum_{\boldsymbol{i}=1}^{2} \boldsymbol{M_i}(\boldsymbol{A_i}\boldsymbol{Y}(\boldsymbol{t}) + \boldsymbol{B_i}) \tag{14}$$

where $\boldsymbol{M_i}$ are membership function matrices, $\boldsymbol{M_1}$ + $\boldsymbol{M_2}$ is the identity matrix and $\boldsymbol{A_i}$, $\boldsymbol{B_i}$ are the coefficient matrices of the GLT fuzzy system:

$$M_1 = \begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{21} & 0 & 0 \\ 0 & 0 & M_{31} & 0 \\ 0 & 0 & 0 & M_{41} \end{bmatrix}, M_2 = \begin{bmatrix} M_{12} & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{32} & 0 \\ 0 & 0 & 0 & M_{42} \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ W_2 - w_1 & 0 & w_1 & 0 \\ 0 & 0 & 0 & 0 \\ w_2 & 0 & W_4 - w_2 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} -2b_1 W_1 \\ 0 \\ -2b_2 W_3 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -W_2 - w_1 & 0 & w_1 & 0 \\ 0 & 0 & 0 & 0 \\ w_2 & 0 & -W_4 - w_2 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 2b_1 W_1 \\ 0 \\ 2b_2 W_3 \\ 0 \end{bmatrix}$$

where $M_{11}, M_{12}, M_{21}, M_{22}, M_{31}, M_{32}, M_{41}, M_{42}$ are fuzzy sets and can be described as:

$$M_{11} = \tfrac{1}{2}(1 + \tfrac{N_1}{W_1}), M_{12} = \tfrac{1}{2}(1 + \tfrac{N_1}{W_1}), M_{21} = \tfrac{1}{2}(1 + \tfrac{N_2}{W_2}), M_{22} = \tfrac{1}{2}(1 - \tfrac{N_2}{W_2}),$$
$$M_{31} = \tfrac{1}{2}(1 + \tfrac{N_3}{W_3}), M_{32} = \tfrac{1}{2}(1 - \tfrac{N_3}{W_3}), M_{41} = \tfrac{1}{2}(1 + \tfrac{N_4}{W_4}), M_{42} = \tfrac{1}{2}(1 - \tfrac{N_4}{W_4})$$

Further, $W_1 = 1$, $W_2 = 30$, $W_3 = 1$, $W_4 = 15$ are the boundary values defined by the ranges for the nonlinear terms groups N_1, N_2, N_3, N_4 separately:

$$\begin{cases} Nonlienar \quad Terms \quad Group \quad 1: N_1 \in [-W_1, W_1] \quad and \quad W_1 > 0 \\ Nonlienar \quad Terms \quad Group \quad 2: N_2 \in [-W_2, W_2] \quad and \quad W_2 > 0 \\ Nonlienar \quad Terms \quad Group \quad 3: N_3 \in [-W_3, W_3] \quad and \quad W_3 > 0 \\ Nonlienar \quad Terms \quad Group \quad 4: N_4 \in [-W_4, W_4] \quad and \quad W_4 > 0 \end{cases}$$

The nonlinear term groups N_1, N_2, N_3 and N_4 are designed as follows, and the time series are given in Figure 2, where parameters b_3 and b_4 have been packaged into nonlinear term groups, because b_3 and b_4 are designed to be the unknown parameters in the following section of fuzzy adaptive control. This step provides the generated GLT fuzzy system two linear subsystems without any uncertainty. Chaotic behavior of a GLT Fuzzy Q-CNN system with extra nonlinear terms is given in Figure 3.

$$\begin{cases} N_1 = \sqrt{1 - y_1^2}\sin y_2 \\ N_2 = 2b_3 \cos y_2 / \sqrt{1 - y_1^2} + b_5 y_4^2 \\ N_3 = \sqrt{1 - y_3^2}\sin y_4 \\ N_4 = 2b_4 \cos y_4 / \sqrt{1 - y_3^2} + b_6 y_2^3 \end{cases}$$

Fuzzy Adaptive Control of GLT Systems

In this subsection, two identical Q-CNN GLT fuzzy systems are illustrated for further discussion. The goal of this section is to control the slave fuzzy system with estimated parameters to achieve a drive system with several unknown parameters via the proposed fuzzy adaptive control strategy devised in Section 2.2.

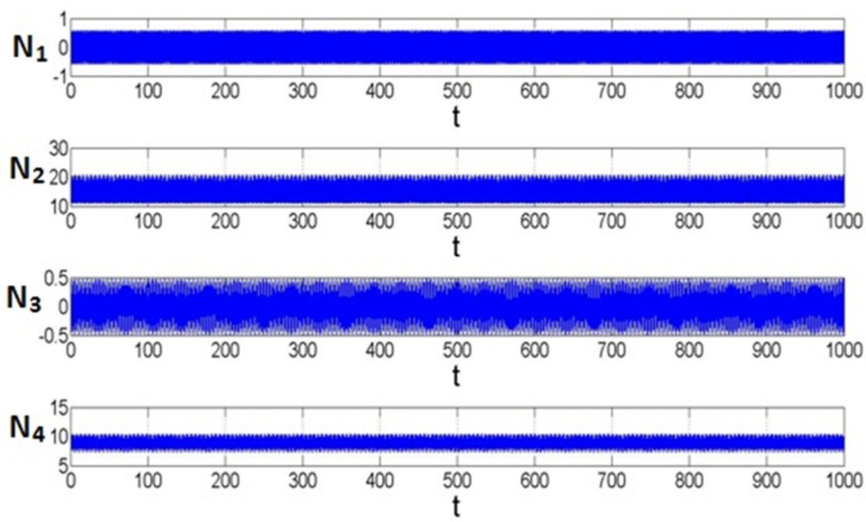

Figure 2. Time series for the nonlinear terms groups N_1, N_2, N_3 and N_4.

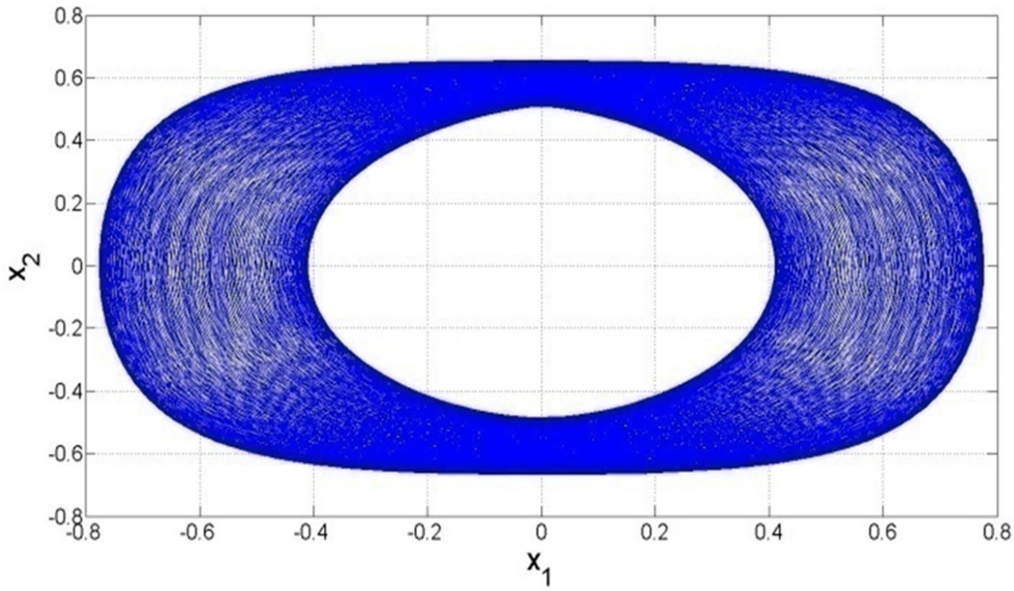

Figure 3. Chaotic behavior of a GLT Fuzzy Q-CNN system with extra nonlinear terms.

CASE I: Parameter b_3 is unknown

Consider the fuzzy system in Equation (14) is the master system, where b_3 is designed as unknown parameters, which leads the nonlinear terms group N_2 to be uncertainties, and the membership functions M_{21}, M_{22} become uncertain membership functions as well. The slave Q-CNN fuzzy system with the designed fuzzy controllers ***U(t)*** and the estimated parameters $\hat{b}_3$ are given below:

$$\dot{\boldsymbol{X}}(\boldsymbol{t}) = \sum_{\boldsymbol{i}=1}^{2} \hat{\boldsymbol{O}}_{\boldsymbol{i}}(\boldsymbol{A}_{\boldsymbol{i}}\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}_{\boldsymbol{i}}) + \boldsymbol{U}(\boldsymbol{t}) \tag{15}$$

where $\dot{\boldsymbol{X}}(\boldsymbol{t}) = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4 \end{bmatrix}^T$. When initial conditions are set as $(x_{10}, x_{20}, x_{30}, x_{40})$ = $(5, -5, 5, -5)$, the coefficient matrices $\boldsymbol{A}_{\boldsymbol{i}}$ and $\boldsymbol{B}_{\boldsymbol{i}}$ are the same to those in the master system in Equation (14), the boundary values (W_1, W_2, W_3, W_4) = (1, 30, 1, 15) and the membership function matrices $\hat{\boldsymbol{O}}_{\boldsymbol{i}}$ with estimated parameters can be organized as follows:

$$\hat{O}_1 = \begin{bmatrix} O_{11} & 0 & 0 & 0 \\ 0 & \hat{O}_{21} & 0 & 0 \\ 0 & 0 & O_{31} & 0 \\ 0 & 0 & 0 & O_{41} \end{bmatrix}, \hat{O}_2 = \begin{bmatrix} O_{12} & 0 & 0 & 0 \\ 0 & \hat{O}_{22} & 0 & 0 \\ 0 & 0 & O_{32} & 0 \\ 0 & 0 & 0 & O_{42} \end{bmatrix}$$

where $\hat{\boldsymbol{O}}_1 + \hat{\boldsymbol{O}}_2$ is the identity matrix and each element is proposed as follows:

$$O_{11} = \tfrac{1}{2}(1 + \tfrac{N_1}{W_1}),\ O_{12} = \tfrac{1}{2}(1 + \tfrac{N_1}{W_1}),\ \hat{O}_{21} = \tfrac{1}{2}(1 + \tfrac{\hat{N}_2}{W_2}),\ \hat{O}_{22} = \tfrac{1}{2}(1 - \tfrac{\hat{N}_2}{W_2}),$$
$$O_{31} = \tfrac{1}{2}(1 + \tfrac{N_3}{W_3}),\ O_{32} = \tfrac{1}{2}(1 - \tfrac{N_3}{W_3}),\ O_{41} = \tfrac{1}{2}(1 + \tfrac{N_4}{W_4}),\ O_{42} = \tfrac{1}{2}(1 - \tfrac{N_4}{W_4})$$

The nonlinear terms groups N_1, N_2, N_3, N_4 in $\hat{\boldsymbol{O}}_{\boldsymbol{i}}$ are as follows:

$$\begin{cases} N_1 = \sqrt{1 - x_1^2} \sin x_2 \\ \hat{N}_2 = 2\hat{b}_3 \cos x_2 / \sqrt{1 - x_1^2} + b_5 x_4^2 \\ N_3 = \sqrt{1 - x_3^2} \sin x_4 \\ N_4 = 2b_4 \cos x_4 / \sqrt{1 - x_3^2} + b_6 x_2^3 \end{cases}$$

It can be observed that the membership functions $\hat{O}_{21}$ and $\hat{O}_{22}$ are composed of the nonlinear terms groups $\hat{N}_2$ comprising the estimated parameter $\hat{b}_3$, i.e., $\hat{N}_2$ is the estimated group and $\hat{O}_{21}$ and $\hat{O}_{22}$ are the estimated membership functions. The initial value of the estimated parameter is set as $\hat{b}_{30} = 6$.

The error and error dynamic systems of the master and slave GLT fuzzy systems provided in Equations (14) and (15) are defined following the form revealed in Section 2.2, where the system dimension $n = 4$. In order to analyze the stability of the error dynamic system, an appropriate candidate Lyapunov function should be defined. We chose the candidate Lyapunov function with square form as follows:

$$V(\boldsymbol{e}(\boldsymbol{t}), \widetilde{a}_i) = \frac{1}{2}\left(\boldsymbol{e}(\boldsymbol{t})^2 + \sum_{i=1}^{n}(F_i\widetilde{a}_i^2)\right) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \widetilde{b}_3^2) \tag{16}$$

where $\widetilde{b}_3 = b_3 - \hat{b}_3$ is the error of unknown parameter b_3 and estimated parameter $\hat{b}_3$. The derivatives of the Lyapunov function in Equation (16) can be described as follows:

$$\dot{V}(\boldsymbol{t}) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \widetilde{b}_3\dot{\widetilde{b}}_3 \tag{17}$$

According to the adaptive control scheme in Section 2.2, we can design the fuzzy controllers $\boldsymbol{U}(\boldsymbol{t}) = \boldsymbol{U}_1(\boldsymbol{t}) + \boldsymbol{U}_2(\boldsymbol{t})$ shown in Equation (18) and the update laws of parameters shown in Equation (19):

$$\begin{cases} \boldsymbol{U}_1(\boldsymbol{t}) = \sum\limits_{i=1}^{2} \hat{\boldsymbol{O}}_i(-\boldsymbol{B}_i F_i \boldsymbol{e}(\boldsymbol{t})) \\ \boldsymbol{U}_2(\boldsymbol{t}) = [\boldsymbol{u}_1(\boldsymbol{t}), \boldsymbol{u}_2(\boldsymbol{t}), \boldsymbol{u}_3(\boldsymbol{t}), \boldsymbol{u}_4(\boldsymbol{t})]^T \end{cases} \tag{18}$$

where $\boldsymbol{B}_1, \boldsymbol{B}_2$ are set as the identity matrix,

$$F_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ W_2 - w_1 & 1 & w_1 & 0 \\ 0 & 0 & 1 & 0 \\ w_2 & 0 & W_4 - w_2 & 1 \end{bmatrix}, \ F_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -W_2 - w_1 & 1 & w_1 & 0 \\ 0 & 0 & 1 & 0 \\ w_2 & 0 & -W_4 - w_2 & 1 \end{bmatrix}$$

$$u_1(t) = (M_{12} - O_{12}) \times 4b_1W_1, \ u_2(t) = b_5y_1(y_4^2 - x_4^2) - 2\hat{b}_3y_1\left(\frac{\cos x_2}{\sqrt{1-x_1^2}} - \frac{\cos y_2}{\sqrt{1-y_1^2}}\right)$$

$$u_3(t) = (M_{32} - O_{32}) \times 4b_2W_3, \ u_4(t) = (M_{41} - O_{41}) \times 2y_3W_4$$

The Update Laws of Parameters

$$\dot{\hat{b}}_3 = -\dot{\widetilde{b}}_3 = \frac{2y_1\cos y_2}{\sqrt{1-y_1^2}}\boldsymbol{e}_2 \tag{19}$$

Some interesting designs can be further discussed in this case. In Equation (18), $\boldsymbol{u}_1(\boldsymbol{t})$, $\boldsymbol{u}_3(\boldsymbol{t})$ are controllers designed in the form of parallel distributed compensation (PDC). Since there are no uncertain parameters in the first and third equations, the membership functions in the master and slave systems can be used to design the corresponding controllers. In addition, $\boldsymbol{u}_2(\boldsymbol{t})$ is a controller designed to let the estimated parameter in the slave system approach the original parameter in the master system. Since the membership functions M_{21}, M_{22} comprise unknown parameters b_3, controller $\boldsymbol{u}_2(\boldsymbol{t})$ should be designed without uncertain membership functions.

According to the design in Equations (18) and (19), we have the following derivative of the Lyapunov function, which is a negative semi-definite function of error states:

$$\dot{V}(\boldsymbol{t}) = -e_1^2 - e_2^2 - e_3^2 - e_4^2 \leq 0 \tag{20}$$

The derivative of the Lyapunov function in Equation (20) is a negative definite function of $\boldsymbol{e}$. In the sense of Lyapunov stability theory, the Lyapunov asymptotic stability theorem is not satisfied. We cannot obtain the common origin of error dynamics and the parameter dynamics are asymptotically stable. Through the pragmatic asymptotic stability theorem [60,61] proposed by Prof. Ge, Yu and Chen, D is a 5-manifold, $n = 5$ and the number of error state variables $p = 4$. When $e1 = e2 = e3 = e4 = 0$, $\hat{b}_3$ takes arbitrary values, $\dot{V} = 0$, so X is of 4 dimensions and $m = n - p = 5 - 4 = 1$, $m + 1 < n$ is satisfied. According to the pragmatic asymptotic stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. The simulation results are shown in Figure 4.

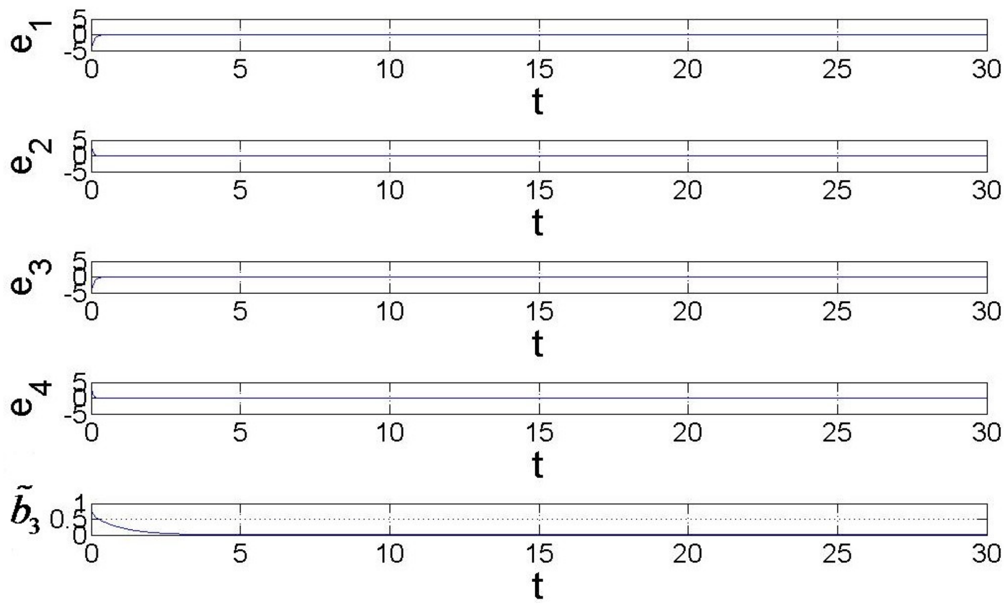

Figure 4. Time series for error states and error of parameters.

CASE II: Parameters b_3 and b_4 are unknown

When the fuzzy system in Equation (14) is the master system, where b_3 and b_4 are designed as unknown parameters, this leads the nonlinear terms groups N_2 and N_4 to be uncertainties, and the membership functions M_{21}, M_{22}, M_{41}, M_{42} become uncertain membership functions as well. The slave Q-CNN fuzzy system with the designed fuzzy controllers ***U(t)*** and the estimated parameters are given below:

$$\dot{\boldsymbol{X}}(\boldsymbol{t}) = \sum_{i=1}^{2} \hat{\boldsymbol{O}}_i(\boldsymbol{A}_i\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}_i) + \boldsymbol{U}(\boldsymbol{t}) \tag{21}$$

where all the system design in Equation (21) is the same as the design in Equation (15), except the initial conditions are set as $(x_{10}, x_{20}, x_{30}, x_{40}) = (0, -10, 10, -10)$, the membership function matrices $\hat{\boldsymbol{O}}_i$, which can be organized as follow:

$$\hat{O}_1 = \begin{bmatrix} O_{11} & 0 & 0 & 0 \\ 0 & \hat{O}_{21} & 0 & 0 \\ 0 & 0 & O_{31} & 0 \\ 0 & 0 & 0 & \hat{O}_{41} \end{bmatrix}, \hat{O}_2 = \begin{bmatrix} O_{12} & 0 & 0 & 0 \\ 0 & \hat{O}_{22} & 0 & 0 \\ 0 & 0 & O_{32} & 0 \\ 0 & 0 & 0 & \hat{O}_{42} \end{bmatrix}$$

where $\hat{\boldsymbol{O}}_1 + \hat{\boldsymbol{O}}_2$ is the identity matrix, and the main differences for $\hat{\boldsymbol{O}}_i$ from those in Equation (15) are $\hat{O}_{41} = (1 + \hat{N}_4/W_4)/2$ and $\hat{O}_{42} = (1 - \hat{N}_4/W_4)/2$, which includes the estimated parameter $\hat{b}_4$, where $\hat{N}_4 = 2\hat{b}_4\cos x_4/\sqrt{1 - x_3^2} + b_6x_2^3$. In this case, $\hat{O}_{21}$, $\hat{O}_{22}$, $\hat{O}_{41}$, $\hat{O}_{42}$ are estimated membership functions. The initial values of the estimated parameters are set as $\hat{b}_{30} = 1$ and $\hat{b}_{40} = 1$.

The error and error dynamic systems of the master and slave GLT fuzzy systems provided in Equations (14) and (21) are defined following the form revealed in Section 2.2, where the system dimension $n = 4$. In order to analyze the stability of the error dynamic system, an appropriate candidate Lyapunov function should be defined. We chose the candidate Lyapunov function with square form as follows:

$$\boldsymbol{V}(\boldsymbol{e}(\boldsymbol{t}), \widetilde{a}_i) = \frac{1}{2}\left(\boldsymbol{e}(\boldsymbol{t})^2 + \sum_{i=1}^{n}(\boldsymbol{F}_i\widetilde{\boldsymbol{a}}_i^2)\right) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + \widetilde{b}_3^2 + \widetilde{b}_4^2) \quad (22)$$

where $\widetilde{b}_3 = b_3 - \hat{b}_3$ and $\widetilde{b}_4 = b_4 - \hat{b}_4$ are the errors for the unknown parameters and estimated parameters, respectively. Then the derivatives of the Lyapunov function in Equation (22) can be described as:

$$\dot{\boldsymbol{V}}(\boldsymbol{t}) = e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 + \widetilde{b}_3\dot{\widetilde{b}}_3 + \widetilde{b}_4\dot{\widetilde{b}}_4 \quad (23)$$

According to the adaptive control scheme in Section 2.2, the fuzzy controllers and the update laws of parameters can be designed in the following form, where $\boldsymbol{B}_1$, $\boldsymbol{B}_2$, are set as the identity matrix:

$$F_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ W_2 - w_1 & 1 & w_1 & 0 \\ 0 & 0 & 1 & 0 \\ w_2 & 0 & W_4 - w_2 & 1 \end{bmatrix}, F_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -W_2 - w_1 & 1 & w_1 & 0 \\ 0 & 0 & 1 & 0 \\ w_2 & 0 & -W_4 - w_2 & 1 \end{bmatrix} \quad (24)$$

$$u_1(t) = (M_{12} - O_{12}) \times 4b_1W_1,\ u_2(t) = b_5y_1(y_4^2 - x_4^2) - 2\hat{b}_3y_1\left(\frac{\cos x_2}{\sqrt{1-x_1^2}} - \frac{\cos y_2}{\sqrt{1-y_1^2}}\right)$$
$$u_3(t) = (M_{32} - O_{32}) \times 4b_2W_3,\ u_4(t) = b_6y_3(y_2^3 - x_2^3) - 2\hat{b}_4y_3\left(\frac{\cos x_4}{\sqrt{1-x_3^2}} - \frac{\cos y_4}{\sqrt{1-y_3^2}}\right)$$

The Update Laws of Parameters:

$$\begin{cases} \dot{\hat{b}}_3 = -\dot{\widetilde{b}}_3 = \frac{2y_1\cos y_2}{\sqrt{1-y_1^2}}e_2 \\ \dot{\hat{b}}_4 = -\dot{\widetilde{b}}_4 = \frac{2y_3\cos y_4}{\sqrt{1-y_3^2}}e_4 \end{cases} \quad (25)$$

In Equation (24), $\boldsymbol{u}_1(\boldsymbol{t})$, $\boldsymbol{u}_3(\boldsymbol{t})$ are controllers designed in the form of parallel distributed compensation (PDC). Since there are no uncertain parameters in the first and third equations, the membership functions in the master and slave systems can be used to design the corresponding controllers. In addition, $\boldsymbol{u}_2(\boldsymbol{t})$, $\boldsymbol{u}_4(\boldsymbol{t})$ are the controllers designed to let the estimated parameters in the slave system approach the original parameters in the master system. Since those membership functions $M_{21}, M_{22}, M_{41}, M_{42}$ comprise unknown parameters b_3, controllers $\boldsymbol{u}_2(\boldsymbol{t})$, $\boldsymbol{u}_4(\boldsymbol{t})$ should be designed without uncertain membership functions.

With the design of the fuzzy controllers and the adaptation law of parameters mentioned in Equations (24) and (25), we have the following derivative of the Lyapunov function, which is a negative semi-definite function of error states:

$$\dot{\boldsymbol{V}}(\boldsymbol{t}) = -e_1^2 - e_2^2 - e_3^2 - e_4^2 \leq 0 \quad (26)$$

The derivative of the Lyapunov function in Equation (26) is a negative definite function of $\boldsymbol{e}$. In the sense of Lyapunov stability theory, the Lyapunov asymptotic stability theorem is not satisfied. We cannot obtain the common origin of error dynamics and the parameter dynamics are asymptotically stable. Through the pragmatic asymptotical stability theorem [60,61] proposed by Prof. Ge, Yu and Chen, D is a 6-manifold, $n = 6$ and the number of error state variables $p = 4$. When $e1 = e2 = e3 = e4 = 0$, $\hat{b}_3$ and $\hat{b}_4$ take arbitrary values, $\dot{V} = 0$, so X is of 4 dimensions, and $m = n - p = 6 - 4 = 2$, $m + 1 < n$ is satisfied.

According to the pragmatic asymptotic stability theorem, error vector e approaches zero and the estimated parameters also approach the uncertain parameters. The equilibrium point is pragmatically asymptotically stable. Under the assumption of equal probability, it is actually asymptotically stable. The simulation results are shown in Figures 5 and 6.

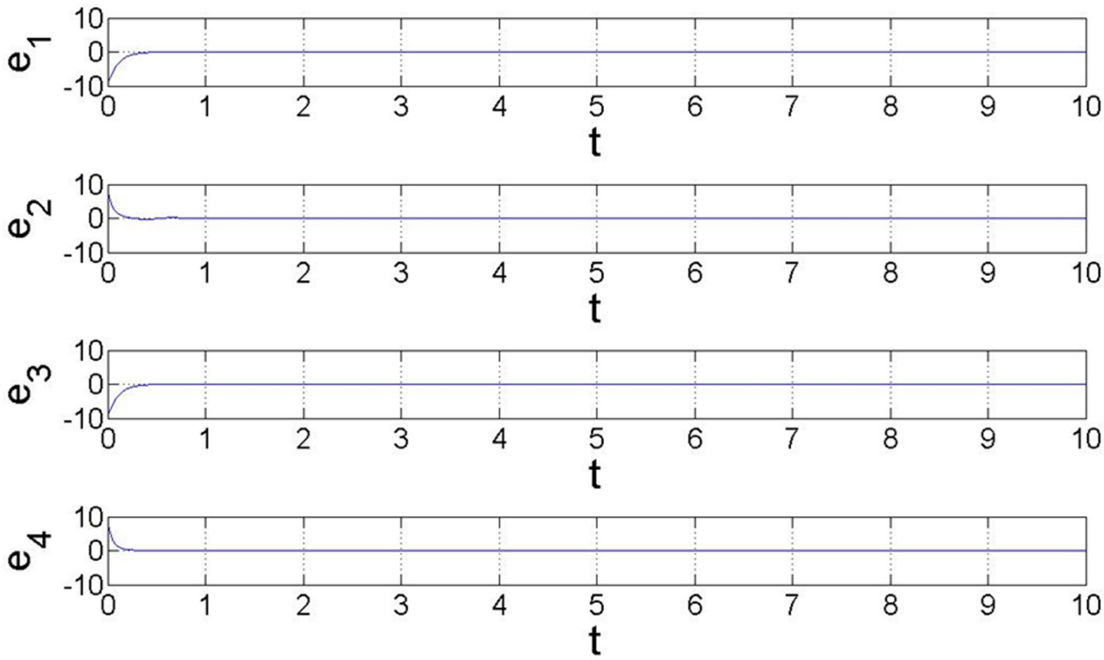

Figure 5. Time series for the error states.

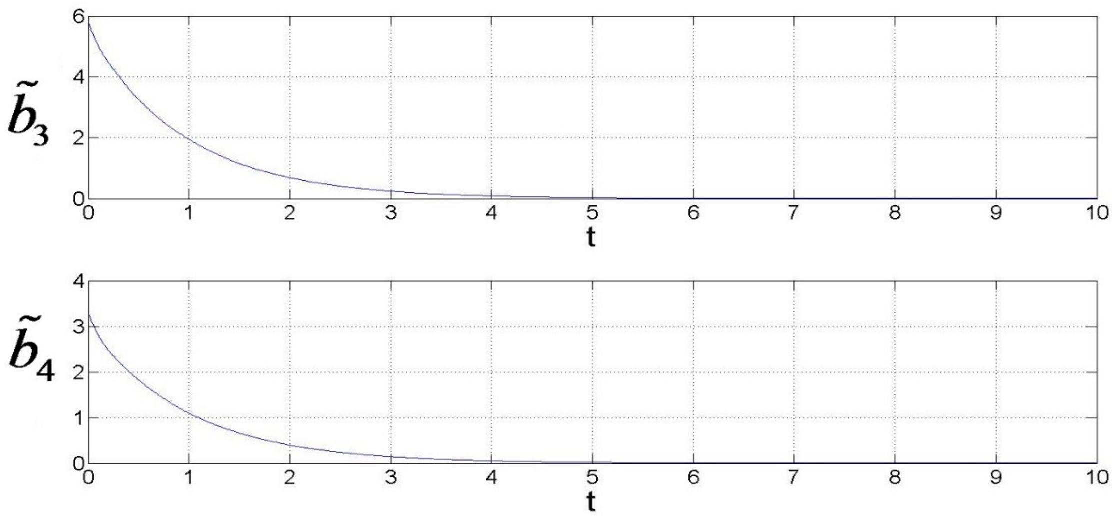

Figure 6. Time series for the error of parameters.

Further discussion: comparing the feedback gains $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ designed in CASES I and II. We can figure out by packaging all unknown parameters into nonlinear terms groups that there are no uncertain terms in the two linear subsystems of the final output fuzzy systems. The uncertainty issue has been technologically transferred into the corresponding membership functions, so that the feedback gains in CASES I and II are developed in the same way. Through this design procedure, adaptive control can be achieved with a set of fixed feedback gains, appropriately designed fuzzy controllers and update laws of parameters.

4. Conclusions

In this paper, an extensive application of the GLT fuzzy system for adaptive control is proposed. Simulation results reveal that the proposed fuzzy adaptive control scheme is feasible, and the control goal as well as parameters identification can be achieved effectively. In fact, by using the fuzzy adaptive control scheme, the GTL fuzzy model played an important role in transforming complicated systems with unknown parameters into two linear subsystems by blending their uncertain membership function matrices. As a consequence, fuzzy controllers and parameter update laws can be constructed in a more convenient way. Furthermore, for those equations without unknown parameters, the first stage controller provides influential control results; and for those equations with parameters that need to be identified, the second stage controller and update laws are prerequisite. Consequently, the proposed adaptive control strategy based on the GLT fuzzy system provides an efficient way for designing controllers as well as achieving tracking goals.

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References

1. Zadeh, L.A. Fuzzy logic. *Computer* **1988**, *21*, 83–93. [CrossRef]
2. Liu, B.; Han, Y.; Jiang, F.; Su, H.; Zou, J. Group controllability of discrete-time multi-agent systems. *J. Franklin. Inst.* **2016**, *353*, 3524–3559. [CrossRef]
3. Khoshdast, H.; Soflaeian, A.; Shojaei, V. Coupled fuzzy logic and experimental design application for simulation of a coal classifier in an industrial environment. *Physicochem. Probl. Miner. Processing* **2019**, *55*, 504–515.
4. Wang, X.-L.; Yang, G.-H.; Li, X.-J. Fault detection filter design with varying gains for multi-delay uncertain systems. *J. Franklin. Inst.* **2016**, *353*, 3699–3721. [CrossRef]
5. Huang, S.-J.; Yang, G.-H.J. Fault estimation for a class of linear parameter varying systems with markovian jumps. *J. Franklin. Inst.* **2016**, *353*, 4680–4700. [CrossRef]
6. He, S.; Liu, F.J. Resilient fault detection observer design of fuzzy markovian jumping systems with mode-dependent time-varying delays. *J. Franklin. Inst.* **2016**, *353*, 2943–2965. [CrossRef]
7. Heydari, G.; Vali, M.; Gharaveisi, A.A. Chaotic time series prediction via artificial neural square fuzzy inference system. *Expert Syst. Appl.* **2016**, *55*, 461–468. [CrossRef]
8. Dash, R.; Dash, P.J. Efficient stock price prediction using a self evolving recurrent neuro-fuzzy inference system optimized through a modified differential harmony search technique. *Expert Syst. Appl.* **2016**, *52*, 75–90. [CrossRef]
9. Sosnowski, M.; Krzywanski, J.; Scurek, R. A Fuzzy Logic Approach for the Reduction of Mesh-Induced Error in CFD Analysis: A Case Study of an Impinging Jet. *Entropy* **2019**, *21*, 1047. [CrossRef]
10. Krzywanski, J. Heat Transfer Performance in a Superheater of an Industrial CFBC Using Fuzzy Logic-Based Methods. *Entropy* **2019**, *21*, 919. [CrossRef]

11. Akbar, H.; Özlem, M.T.; Güldal, G.; Reza, V.B. Development of a fuzzy logic based model to elucidate the effect of FSW parameters on the ultimate tensile strength and elongation of pure copper joints. *J. Manuf. Process.* **2020**, *53*, 250–259.
12. Thillaigovindan, N.; Shanthi, S.A.; Naidu, J.V. A better score function for multiple criteria decision making in fuzzy environment with criteria choice under risk. *Expert Syst. Appl.* **2016**, *59*, 78–85. [CrossRef]
13. Li, S.-Y.; Ge, Z.-M. Generalized synchronization of chaotic systems with different orders by fuzzy logic constant controller. *Expert Syst. Appl.* **2011**, *38*, 2302–2310. [CrossRef]
14. Li, S.-Y. Chaos control of new mathieu–van der pol systems by fuzzy logic constant controllers. *Appl. Soft Comput.* **2011**, *11*, 4474–4487. [CrossRef]
15. Dong, G.; Ren, H.; Yao, D.; Li, H.; Lu, R. Prescribed Performance Consensus Fuzzy Control of Multiagent Systems With Nonaffine Nonlinear Faults. *IEEE Trans. Fuzzy Syst.* **2021**, *29*, 3936–3946. [CrossRef]
16. Zhou, Q.; Wang, W.; Ma, H.; Li, H. Event-Triggered Fuzzy Adaptive Containment Control for Nonlinear Multiagent Systems With Unknown Bouc–Wen Hysteresis Input. *IEEE Trans. Fuzzy Syst.* **2021**, *29*, 731–741. [CrossRef]
17. Li, H.; Wu, Y.; Chen, M. Adaptive Fault-Tolerant Tracking Control for Discrete-Time Multiagent Systems via Reinforcement Learning Algorithm. *IEEE Trans. Cybern.* **2021**, *51*, 1163–1174. [CrossRef]
18. Li, H.; Wang, J.; Lam, H.-K.; Zhou, Q.; Du, H. Adaptive sliding mode control for interval type-2 fuzzy systems. *IEEE Trans. Syst. Man Cybern. Syst.* **2016**, *46*, 1654–1663. [CrossRef]
19. Zhang, J.; Tong, S.-C.; Li, Y.-M. Adaptive Fuzzy Finite-Time Output-Feedback Fault-Tolerant Control of Nonstrict-Feedback Systems Against Actuator Faults. *IEEE Trans. Syst. Man Cybern. Syst.* **2022**, *52*, 1276–1287. [CrossRef]
20. Li, K.; Li, Y.; Zong, G. Adaptive Fuzzy Fixed-Time Decentralized Control for Stochastic Nonlinear Systems. *IEEE Trans. Fuzzy Syst.* **2021**, *29*, 3428–3440. [CrossRef]
21. Takagi, T.; Sugeno, M. Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst. Man Cybern.* **1985**, *SMC-15*, 116–132. [CrossRef]
22. Zhu, B.; Zhang, J.; Zhang, Q.; Zhang, D. Dissipative control for t–s fuzzy descriptor systems with actuator saturation and disturbances. *J. Frankl. Inst.* **2016**, *353*, 4950–4978. [CrossRef]
23. Su, X.; Shi, P.; Wu, L.; Basin, M. Reliable filtering with strict dissipativity for ts fuzzy time-delay systems. *IEEE Trans. Cybern.* **2014**, *44*, 2470–2483. [PubMed]
24. Li, S.-Y.; Hernández, M.A.B. Robust synchronization of chaotic systems with novel fuzzy rule-based controllers. *Inf. Sci.* **2019**, *481*, 604–615. [CrossRef]
25. Derakhshan, S.F.; Fatehi, A.; Sharabiany, M.G. Nonmonotonic observer-based fuzzy controller designs for discrete time ts fuzzy systems via LMI. *IEEE Trans. Cybern.* **2014**, *44*, 2557–2567. [CrossRef]
26. Nagamani, G.; Ramasamy, S.J. Dissipativity and passivity analysis for discrete-time t–s fuzzy stochastic neural networks with leakage time-varying delays based on abel lemma approach. *J. Frankl. Inst.* **2016**, *353*, 3313–3342. [CrossRef]
27. Wang, S.; Fei, J. Robust adaptive sliding mode control of mems gyroscope using t–s fuzzy model. *Nonlinear Dyn.* **2014**, *77*, 361–371. [CrossRef]
28. Ma, Y.; Chen, M.; Zhang, Q. Non-fragile static output feedback control for singular t–s fuzzy delay-dependent systems subject to markovian jump and actuator saturation. *J. Franklin. Inst.* **2016**, *353*, 2373–2397. [CrossRef]
29. Chen, J.; Xu, S.; Li, Y.; Qi, Z.; Chu, Y. Improvement on stability conditions for continuous-time t–s fuzzy systems. *J. Franklin. Inst.* **2016**, *353*, 2218–2236. [CrossRef]
30. Hsueh, Y.-C.; Su, S.-F.; Chen, M.-C. Decomposed fuzzy systems and their application in direct adaptive fuzzy control. *IEEE Trans. Cybern.* **2014**, *44*, 1772–1783. [CrossRef]
31. Xie, X.-P.; Liu, Z.-W.; Zhu, X.-L. Systems. An efficient approach for reducing the conservatism of lmi-based stability conditions for continuous-time t–s fuzzy systems. *Fuzzy Sets Syst.* **2015**, *263*, 71–81. [CrossRef]
32. Zhang, J.H.; Shi, P.; Xia, Y.Q. Fuzzy delay compensation control for t-s fuzzy systems over network. *IEEE Trans. Cybern.* **2013**, *43*, 259–268. [CrossRef] [PubMed]
33. Jee, S.C.; Lee, H. $H-/h\infty$ fault detection and isolation for nonlinear systems with state delay in t–s form. *J. Franklin. Inst.* **2016**, *353*, 2030–2056. [CrossRef]
34. Song, M.K.; Park, J.B.; Joo, Y.H. Systems. Robust stabilization for uncertain markovian jump fuzzy systems based on free weighting matrix method. *Fuzzy Sets Syst.* **2015**, *277*, 81–96. [CrossRef]
35. Wu, Z.-G.; Shi, P.; Su, H.; Chu, J. Sampled-data fuzzy control of chaotic systems based on a t–s fuzzy model. *IEEE Trans. Fuzzy Syst.* **2013**, *22*, 153–163. [CrossRef]
36. Wu, Z.G.; Shi, P.; Su, H.Y.; Chu, J. Reliable h-infinity control for discrete-time fuzzy systems with infinite-distributed delay. *IEEE Trans. Fuzzy Syst.* **2012**, *20*, 22–31.
37. Li, S.-Y.; Ge, Z.-M. Fuzzy modeling and synchronization of two totally different chaotic systems via novel fuzzy model. *IEEE Trans. Syst. Man Cybern.* **2011**, *41*, 1015–1026.
38. Liu, M.; Zhang, S.; Tang, X.; Fan, Z.; Zheng, S. Optimal $H\infty$ filtering for discrete-time-delayed chaotic systems via a unified model. In Proceedings of the 2012 15th International Conference on Information Fusion, Singapore, 9–12 July 2012; IEEE: Piscataway, NJ, USA; pp. 2553–2560.
39. Liu, M.; Zhang, S.; Fan, Z.; Zheng, S.; Sheng, W. Exponential $h\infty$ synchronization and state estimation for chaotic systems via a unified model. *IEEE Trans. Neural Netw Learn Syst.* **2013**, *24*, 1114–1126.

40. Hsiao, F.-H. Delay-Dependent Exponential Optimal Synchronization for Nonidentical Chaotic Systems via Neural-Network-Based Approach. In *Abstract and Applied Analysis*; Hindawi: London, UK, 2013.
41. Liu, M.; Chen, H.; Zhang, S.; Sheng, W. Automation; Systems. H∞ synchronization of two different discrete-time chaotic systems via a unified model. *Int. J. Control. Autom. Syst.* **2015**, *13*, 212–221. [CrossRef]
42. Zhao, Y.; Li, B.; Qin, J.; Gao, H.; Karimi, H.R. H∞ consensus and synchronization of nonlinear systems based on a novel fuzzy model. *IEEE Trans. Cybern.* **2013**, *43*, 2157–2169. [CrossRef]
43. Wen, S.; Bao, G.; Zeng, Z.; Chen, Y.; Huang, T. Global exponential synchronization of memristor-based recurrent neural networks with time-varying delays. *Neural Netw.* **2013**, *48*, 195–203. [CrossRef] [PubMed]
44. Wen, S.; Zeng, Z.; Huang, T.; Chen, Y. Fuzzy modeling and synchronization of different memristor-based chaotic circuits. *Mod. Phys. Lett.* **2013**, *377*, 2016–2021. [CrossRef]
45. Wen, S.; Zeng, Z.; Huang, T.; Simulation, N. Event-based control for memristive systems. *Commun Nonlinear Sci Numer Simul.* **2014**, *19*, 3431–3443. [CrossRef]
46. Wen, S.; Zeng, Z.; Huang, T.; Zhang, Y. Exponential adaptive lag synchronization of memristive neural networks via fuzzy method and applications in pseudorandom number generators. *IEEE Trans. Fuzzy Syst.* **2013**, *22*, 1704–1713. [CrossRef]
47. Hassan, M.F.J.A.M. A new approach for secure communication using constrained hyperchaotic systems. *Appl. Math. Comput.* **2014**, *246*, 711–730. [CrossRef]
48. Chen, C.-K.; Lin, C.-L.; Lin, S.-L.; Chiang, C.-T. Data encryption and transmission based on personal ecg signals. *Sensor Netw Data Commun.* **2015**, *4*, 1–13. [CrossRef]
49. Li, S.-Y.; Hernández, M.A.B.; Tam, L.-M.; Chen, C.S. A Cloud Image Data Protection Algorithm with Multi-Level Encryption Scheme and Automated-Selection Mechanism. *Appl. Sci.* **2019**, *9*, 5146. [CrossRef]
50. Hassan, M.F. Synchronization of uncertain constrained hyperchaotic systems and chaos-based secure communications via a novel decomposed nonlinear stochastic estimator. *Nonlinear Dyn.* **2016**, *83*, 2183–2211. [CrossRef]
51. Wu, Z.-G.; Shi, P.; Su, H.; Chu, J. Local synchronization of chaotic neural networks with sampled-data and saturating actuators. *IEEE Trans. Cybern.* **2014**, *44*, 2635–2645.
52. Wang, Z.-P.; Wu, H.-N. On fuzzy sampled-data control of chaotic systems via a time-dependent lyapunov functional approach. *IEEE Trans. Cybern.* **2014**, *45*, 819–829. [CrossRef]
53. Yan, S.; Qian, C.; Huang, T. Global asymptotic synchronization of a class of non-linear systems via sampled-data feedback. *Trans. Inst. Meas. Control.* **2018**, *40*, 12–21. [CrossRef]
54. Li, S.-Y.; Yang, C.-H.; Lin, C.-T.; Ko, L.-W.; Chiu, T.-T. Chaotic motions in the real fuzzy electronic circuits. In *Abstract and Applied Analysis*; Hindawi: London, UK, 2013.
55. Liu, M.; Chen, H.; Zhang, S.; Fan, Z. Applications. Exponential synchronization of two totally different chaotic systems based on a unified model. *Neural Comput Applic.* **2014**, *25*, 1801–1808. [CrossRef]
56. Li, S.-Y.; Ko, L.-W.; Lin, C.-T.; Tam, L.-M.; Chen, H.-K.; Lao, S.-K. System modeling and synchronization of nonlinear chaotic systems with uncertainty and disturbance by innovative fuzzy modeling strategy. In Proceedings of the 2013 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Hyderabad, India, 7–10 July 2013; pp. 1–5.
57. Manju, A.; Nigam, M.J. Applications of quantum inspired computational intelligence: A survey. *Artif. Intell. Rev.* **2014**, *42*, 79–156. [CrossRef]
58. Li, S.-Y.; Chen, H.-K.; Tam, L.-M.; Huang, S.-C.; Ge, Z.-M. Pragmatical adaptive synchronization–new fuzzy model of two different and complex chaotic systems by new adaptive control. *Inf. Sci.* **2014**, *277*, 458–480. [CrossRef]
59. Shen, Z.-P.; Yan, R.; Wu, Y.-L. Tracking control of Uncertain Chaotic System. *Electro-Opti. Control* **2015**, *11*, 52–55.
60. Tam, L.-M.; Li, S.-Y. Novel-fuzzy-model based modeling and control of nonlinear chaotic systems with uncertainty. In Proceedings of the 2015 International Conference on Advanced Robotics and Intelligent Systems (ARIS), Taipei, Taiwan, 29–31 May 2015; pp. 1–6.
61. Li, S.-Y.; Tam, L.-M.; Tsai, S.-E.; Ge, Z.-M. Novel fuzzy modeling and synchronization of chaotic systems with multinonlinear terms by advanced ge-li fuzzy model. *IEEE Trans. Cybern.* **2015**, *46*, 2228–2237. [CrossRef]
62. Egardt, B. *Stability of Adaptive Controllers*; Springer: Berlin/Heidelberg, Germany, 1979.
63. Landau, Y.D. *Adaptive Control: The Model Reference Approach*; Marcel Dekker, Inc.: New York, NY, USA, 1979.
64. Liu, Y.-J.; Tong, S. Systems. Adaptive fuzzy control for a class of unknown nonlinear dynamical systems. *Fuzzy Sets Syst.* **2015**, *263*, 49–70. [CrossRef]
65. Liu, Y.-J.; Tong, S. Adaptive fuzzy identification and control for a class of nonlinear pure-feedback mimo systems with unknown dead zones. *IEEE Trans. Fuzzy Syst.* **2014**, *23*, 1387–1398. [CrossRef]
66. Liu, Y.-J.; Tong, S.; Li, D.-J.; Gao, Y. Fuzzy adaptive control with state observer for a class of nonlinear discrete-time systems with input constraint. *IEEE Trans. Fuzzy Syst.* **2015**, *24*, 1147–1158. [CrossRef]
67. Tong, S.; Zhang, L.; Li, Y. Observed-based adaptive fuzzy decentralized tracking control for switched uncertain nonlinear large-scale systems with dead zones. *IEEE Trans. Syst. Man Cybern. Syst.* **2015**, *46*, 37–47. [CrossRef]
68. Chen, C.P.; Liu, Y.-J.; Wen, G.-X. Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems. *IEEE Trans Cybern.* **2013**, *44*, 583–593. [CrossRef] [PubMed]
69. Ge, Z.-M.; Yu, J.-K.; Chen, Y.-T. Pragmatical asymptotical stability theorem with application to satellite system. *Jpn. J. Appl. Phys.* **1999**, *38*, 6178. [CrossRef]

70. Ge, Z.-M.; Yu, J.-K. Pragmatical asymptotical stability theorems on partial region and for partial variables with applications to gyroscopic systems. *J. Mech.* **2000**, *16*, 179–187. [CrossRef]
71. Fortuna, L.; Porto, D. Quantum-cnn to generate nanoscale chaotic oscillators. *Int. J. Bifurc. Chaos* **2004**, *14*, 1085–1089. [CrossRef]