

## Obituary

## In Memory of Peter Carr (1958–2022)

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**Abstract:** The editors of this special issue and several of the contributing authors have known Peter for a long time. We thought that the special issue will be enriched by adding a few personal notes and recollections about our interactions with Peter.

## 1. Joe Campolieti—Some Personal Recollections about Peter

I recall the first time I met Peter. It was at an INFORMS Applied Probability conference in NYC in late July of 2001. I was initially introduced to Peter by my colleague, Claudio Albanese who was professor of mathematics at the University of Toronto. At the time, I was teaching at the Masters of Mathematical Finance (MMF) graduate program at the University of Toronto. I taught in that program from 1998 to 2002, before I accepted a tenure-track faculty position at Wilfrid Laurier University as an Associate Professor of Mathematics (and SHARCNET Chair in Financial Mathematics). Prior to meeting Peter, Claudio had mentioned him on a few occasions and always spoke highly of him. Leading up to the 2001 conference, Claudio and I made good progress on the development of some novel so-called solvable models for derivative pricing. We gave talks on this research at the INFORMS 2001 conference. Peter was quite interested in this work as he and Alex Lipton had been working on some ideas related to it. This resulted in a highly referenced paper published in December of 2001 in Risk Magazine, entitled “Black-Scholes goes Hypergeometric” with all four of us as co-authors [Albanese et al. \(2001\)](#). During our stay in NYC, Peter was quick to invite us for dinner at a nice café in the SOHO district. Peter had a great sense of humour and it did not take long to get better acquainted with him. It was very refreshing to see how passionate he was about research and how he was always thinking about new ideas.

I also recall when Peter came to Laurier circa winter 2006. My colleague, Prof. Madhu Kalimipalli, from the Laurier School of Business had invited Peter to give a talk. I made sure to have most of my students in the undergraduate and graduate financial mathematics courses attend his talk. It was an interesting talk that captivated the students. Later that evening, Phelim Boyle (who was also professor at the Laurier School of Business at the time), Madhu and Peter and I had a nice dinner at Sole in Waterloo. Of course, it made for a very interesting conversation. Peter felt at home every time he visited the greater Toronto metropolitan area, thanks to his Canadian roots.

I also recall meeting with Peter at the Bachelier Conference in 2010 in Toronto. It had been a few years since I had seen him. He immediately greeted me with a great smile and enthusiasm. We had a nice chat about what was going on in our lives. He had just rejoined Morgan Stanley at the time. Later in 2011, Laurier began hosting a new series of bi-annual AMMCS conferences. I’ve been the main co-organizer of the financial mathematics special session within the AMMCS since its inception. In 2013, I invited Peter to give a plenary



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talk at the AMMCS and he graciously and enthusiastically accepted. As always, he gave an interesting talk. It was also great to have him participate in the special session. Practically all of the special session speakers joined us for a memorable lunch with Peter. My colleague and collaborator, Prof. Roman Makarov at Laurier, and I had dinner later that evening. We chatted about some new research results. He was impressed with the progress we had made. Later that year Peter generously agreed to write a nice endorsement for a new book I co-authored with Roman (“Financial Mathematics: A Comprehensive Treatment” published in 2014) [Campolieti and Makarov \(2014\)](#).

The last time I met Peter was at the Financial Engineering Conference that took place in early June of 2019. Both of us were given the opportunity to give lengthy back-to-back invited talks in one of the special sessions on mathematical finance. Peter gave a talk on a new model for credit risk. I spoke about spectral expansions for solvable processes with several new results on first passage times, occupation times and other path functionals. This talk allowed me to partly advertise a new book on solvable models that I’m still presently working on completing as it is quite an undertaking. Within that talk I also presented a new structural credit risk model based on a new occupation time hazard model. Peter told me that he really enjoyed my talk and that he was very interested in the general theoretical framework and the applications of the many new analytical formulas I had generated. Who would have thought that this research would soon later be published in Risks within a special issue in memory of Peter in 2022.

He emailed me at the end of December 2020 asking if I had completed my new book and if he could receive a copy of it. Unfortunately, I had not completed it yet. We last corresponded at the very start of January 2021. I mentioned to him that I was taking a six-month sabbatical in the winter of 2021. He said that he would have invited me to NYC if it weren’t for the challenges surrounding COVID at the time. A little over a year later, at the start of March 2022, I got the terrible surprising news of his passing. Peter will always be remembered as a great researcher, a great thinker and such a pleasant person to be around. His spirit will live on as he made such a positive impact on many that knew him.

## 2. Arash Fahim: Peter Carr’s Take on Dubins-Schwartz Theorem

For a very long time, I assumed that I know Dubins-Schwartz theorem [Dubins and Schwarz \(1965\)](#). The theorem asserts: Every continuous martingale  $M = (M_s)_{s \geq 0}$  can be written as a time-changed Brownian motion  $(B_{[M]_s})_{s \geq 0}$ , where  $[M] = ([M]_s)_{s \geq 0}$  is the (continuous) quadratic variation of  $M$ .

One time, when Peter Carr was visiting University of Michigan in Ann Arbor, where I did my postdoc under Erhan Bayraktar’s mentorship, I mentioned the theorem during a lunch with Peter and Erhan’s group. Peter thought for a minute or two and constructed a bounded function that satisfies Laplace equation as a counter example to the way I describe Dubins-Schwartz theorem. I was feeling embarrassed, and my confidence was shattered. How can a bounded martingale be a time change of Brownian motion which is not bounded. Of course, what Peter did was showing me that the time change can map  $[0, \infty)$  into any interval  $[0, b)$  that you may imagine. In other words,  $[M]$  can be bounded by hitting times of the Brownian motion to some levels, which makes  $(B_{[M]_s})_{s \geq 0}$  a bounded martingale.

Apparently, I should have been more careful in learning the theorem, as it is expected from a good mathematician, such as Peter. Thereafter, I challenged any new results I come upon, particularly, by trying to make counter examples. This may have been a normal practice for many, but for me it was an aha moment and a gift from Peter Carr.

## 3. Dan Pirjol

My first interaction with Peter was sometime in 2011, after giving a talk at Columbia University (on work related to [Pirjol \(2013\)](#)) in the Financial Practitioners seminar ran by Emanuel Derman. I had just moved to JPM and the location in Midtown made it easier to attend seminars, which had started to be held more frequently than during the financial crisis. I did not meet Peter in person at the seminar, but he must have thought that the topic

was sufficiently interesting that he got in touch and told me that he discussed this problem afterwards with the Budapest quant group at Morgan Stanley.

We continued to meet at the NYU seminar organized by Rama Cont and Marco Avellaneda, and at the IAQF and Bloomberg events. I was working on fixed income modeling, and had little overlap with his work which was mostly in equities modeling. I recall attending a talk given by him about the Variance Gamma model, and my ignorance of the topic was such that I believed that the title must refer to one of the Greeks of a variance derivative.

This changed after 2014 when I started working on commodity derivatives. One of the first projects I worked on was a validation study of “Carr randomization”, a method he proposed in Carr (1998) for pricing American options on Black-Scholes assets. The bank was considering using this model for pricing and risk management of American options on commodity futures. My job was to find the weak points of the model, so naturally I shot a message to Peter asking what he thinks are the main weaknesses and if possible to suggest possible enhancements. Model developers are usually very reluctant to admit to any shortcomings of their model, but Peter was open about the limitations of his method and also suggested ways to improve on the published version. He also suggested a topic of research—extend his method to the CEV model. However, as we found out later, this had already been done in Wong and Zhao (2010).

We continued to keep in touch after I moved to an academic position. I invited him to give a talk in the Financial Engineering seminar at Stevens, where he talked about one of his last contributions—“Stoptions” and the logistic model for option pricing proposed in Carr and Torricelli (2021). This he delivered with an interesting spin, emphasizing connections to relativity theory, which reflected his wide ranging curiosity and interests about all branches of science. His online seminar broke all records of attendance, with more than 125 viewers, and lasted for one hour and 30 min. He joked after the seminar that “seminars used to be 1.5 h, and I am old school”. If a passion for communication, curiosity and insistence on seeing the best in others are “old school” skills, then surely he was the best teacher one could hope for. His lesson will continue to live in all those who knew him.

#### 4. Harvey Stein

In 2001, two years after I moved back to NYC from Tel Aviv, I started attending the Mathematical Finance Seminar at NYU’s Courant Institute. It was the premier quantitative finance seminar in NYC, with talks being given by a multitude of famous individuals. Talks were given by Robert Merton, Paul Malliavin, Benoit Mandelbrot, and many other celebrities of the quant world. The research and talks were extremely stimulating and engaging, as were the post-seminar dinners. Of course that was the case, as the seminar was organized by the world renowned quants, Peter Carr and Marco Avellaneda. It was where I met Peter.

Peter had recently left Bank of America and was a visiting professor at NYU. He was an iconic figure at the seminar; always asking questions and engaging the speaker. You didn’t need to be familiar with his extensive bibliography to see that he was a leader in the field of quantitative finance; extremely sharp and tremendously knowledgeable. So, in 2003, when I was asked to head and build Bloomberg’s global Quantitative Finance R&D group, recruiting Peter to head up my Quantitative Finance Research group was an obvious move. Luckily, Peter was happy to return to industry.

Peter had a talent for viewing everything as an option, and the ability to reason about options economically and mathematically. But, he wasn’t just a researcher—he lived quantitative finance. He was constantly in contact with other researchers, discussing their work, inviting them to meetings, holding weekly breakfast seminars, and always very excited about it. He was a whirlwind of activity.

Peter loved applying new mathematical techniques to financial problems. During the Covid pandemic, we corresponded about his work with Doug Costa on viewing optionality as a binary operation. Having an algebraic background, I helped him get a deeper

understanding of the Grothendieck group of a monoid and a canonical representation of that group for the monoid he was working with. In response to a proof of Peter's that used the distributive property of plus over max instead of directly appealing to the definition of max, I told Peter that with proofs like that, he was becoming a real algebraist. That was in December of 2020, to which he responded "That is the nicest thing anyone has said to me this year".

With Peter as the head of the research team, the Quantitative Finance R&D group became a force in quantitative finance. We hired a number of well-known industry leaders, including Pat Hagan, Bruno Dupire, and Bjorn Flesaker. We upgraded all of the Bloomberg option models, making them the industry standard. It was a lot of work, but it was very exciting to be on its forefront, and the group became well-known in the industry. Perhaps it became a little too well-known, as it became common for the banks to poach from the team.

Peter was very excited about building such a strong quantitative research group, and hiring so many high-profile renowned quants. He was especially excited about hiring Bruno. He used to say how great it would be to get Bruno out of retirement. Peter also had great respect for Bruno. In retrospect, given Peter's comments before the interview, I think he might have also been a little intimidated. When interviewing people, to gauge their mathematical depth, I would often ask them to define a Martingale. When they mentioned that the conditional expectation of future values is the current value, I would ask what exactly that meant, and start drilling into details and proofs. Before interviewing Bruno, Peter specifically asked me to ask Bruno these questions, so I did. It didn't go over so well with Bruno. None the less, Bruno and Peter subsequently became close friends.

But all good things ultimately have to end. Senior management couldn't decide whether quant groups should be part of the business unit or within the R&D engineering teams. The group was broken up, and, in 2010, Peter went back to Morgan Stanley. But I will always look back on those times as the halcyon days of quantitative finance at Bloomberg, and reminisce about Peter's (and everyone else's) role in it.

## 5. Tai-Ho Wang

My first encounter with Peter was a pure coincidence. I came to New York, among one of probably the worst days in history, on 1 September 2001. I was doing my postdoc at NYU's Courant Institute of Mathematical Sciences (CIMS). CIMS has been well-known for years of its tightness in office spaces. I was finally assigned to an office where Peter Laurence (PL) used to work in, who later became one of my main and long term collaborators. PL and I had several joint works on various topics in quantitative finance.

One day in the afternoon while I was working in the office, Peter came to the office, apparently not finding PL but me, and said, "Hi, I am Peter Carr, I am looking for Peter Laurence". Back then not really knowing who he was but vaguely remembered seeing his name somewhere in the building and on some webpage of CIMS, I tried chatting with him a little. I started, "Hi, my name is Tai-Ho Wang. I am from Taiwan". He immediately replied, "Tai-Ho from Taiwan, easy to memorize". My impression on what he said was like "That's right, how come I never thought of that"? Our first conversation basically ended right there since a moment later PL showed up and they went on to other office for discussions. That first impression on him turned out to be the main theme every time I attended Peter's talks or met with him for discussions.

On a fellowship leave, I returned to CIMS in 2006 as a visiting scholar. Up to this point, jointly with PL I gained extensive experiences in applying Lie symmetry to analyze differential equations originated from problems in quantitative finance. Since Peter and PL had been in close contact, knowing from PL that I would be visiting CIMS for a semester, Peter invited me over for a talk at Bloomberg. Peter was very supportive during the talk and pretty interested in the part of results that I presented. In an after-talk conversation, he pointed out to me a relationship between the Girsanov transformation in stochastic analysis and one of the generators of the six dimensional symmetry group for heat equation. When

I finally got down to the bottom of the argument, I was like “That’s right, how come I never thought of that”?

Fast forward to 2019, I had joined Baruch College for ten more years. A friend of mine came down to New York and was invited to speak at NYU Tandon. I attended my friend’s talk, one the one hand for reunion; on the other hand to meet and say hello to Peter since it had been a while. Peter was so kind to invite me to join their dinner with the speaker and some of his friends and colleagues. In the restaurant, at some point among the multi-threading conversations before the meals were served, Peter came up with this question: Assume under Black-Scholes model with zero interest and dividend rate, what is the distribution for at-the-money Black-Scholes delta at the half-time point prior to expiry? The first reaction that came to my mind was, since Black-Scholes’ delta is always between zero and one, a beta distribution. The answer turned out to be yes and no. No, because it is a uniform distribution; yes, because uniform is a special case of beta. This tiny observation was probably not as fancy as his other numerous fascinating works, a good interview question though. However, my impression at that moment was again “Interesting, how come I never thought of that?” It showed Peter had a rare and unique talent in discovering special structures or symmetries hidden behind the models and formulas, which I admire and respect to the highest level.

## 6. Lingjiong Zhu

I have known Peter for over ten years. I first met him when I was still a PhD student at NYU’s Courant Institute of Mathematical Sciences, in the fall of 2012. We met during a reception for the master in financial math program at Courant, where he was an adjunct professor. I got invited to this reception because I was a teaching assistant for Stochastic Calculus, and most of the students of that class were in the financial math program. At the reception, Peter asked me what I was doing for my research and I told him about the Hawkes process papers that I had been writing under the supervision of Prof. S.R.S. Varadhan. He invited me to give a talk to the credit risk group at Morgan Stanley and was so impressed that he asked me to give the same talk to a much larger audience there, which eventually led to my six-month stint at Morgan Stanley in 2013, and gave me a chance to talk to him on an almost daily basis. Peter’s enthusiasm for doing research and learning new things had an immense impression on me. He had tremendous energy. He often got up at 4 o’clock in the morning to do research, which was several hours before he came to work!

While at the company, he would continue to work till the evenings. A few times, I would have dinner with him on the 11th floor at the company when everyone else was gone. Peter continued to talk to me on a regular basis and support me academically even after I left Morgan Stanley and moved back to academia, and we started working together on a paper when I was a postdoc at University of Minnesota. I kept close contact with Peter even after I moved to Florida State University in 2015. In February 2017, I had the pleasure of inviting him to visit FSU. We discussed our working paper and got some significant progress during his visit. Both of us gave talks about this work at many conferences and seminars; but it is sad that the paper has not yet been finished. Hopefully I will be able to complete this paper on my own in the future as a way to honor him.

In the summer of 2018, Peter visited China for the first time. He visited Shanghai, my hometown, before flying to Chengdu to visit Southwestern University of Finance and Economics. I happened to be in Shanghai during that summer.

I picked him up at the famous Peace Hotel, and had the pleasure to treat him to lunch at the equally famous, Lv Bo Lang, known for the Shanghai-style dim sum. After lunch, he headed to give a talk at Shanghai Jiao Tong University. The next day, Peter gave a lecture at Fudan University, invited by Jian Sun, a former colleague of ours at Morgan Stanley. I went there as well.

On 25 July, Peter and I met again. We discussed research at a Starbucks near Peace Hotel, before I took him to lunch, this time at Shunfeng Harbour Restaurant. Peter visited

Shanghai again in the summer of 2019. I was also in Shanghai during that summer; but unfortunately we did not meet up this time because Peter got the timing wrong. After all, there was a 12-h time difference!

We used to Skype on Sunday afternoons at 4 pm every other week or sometimes once a month. *Among all the famous people I know in academia, Peter was one of the very few that I could count as a personal friend.* He was not only incredibly smart, but also was super kind and exceedingly generous, especially in terms of helping young colleagues and junior people. Every time I went to New York, I would stop by his office. The first time, I was in his Morgan Stanley's office, and then at his temporary office at Jane Street, and finally at his Brooklyn office at NYU's Tandon School of Engineering. He even invited me to give a talk there once after I finished the visit to Rutgers University. A couple of months before Peter passed, he was invited to give a talk at a machine learning conference targeting quants, and since Peter was not doing much work on machine learning, he instead suggested to the organizers to invite me to give a talk because he knew I had been working on some theoretical machine learning problems. He was such a nice person and always thinking about helping junior people. The last time I spoke to him on Skype was in early January 2022. We agreed to talk on Zoom on 6 February. He didn't show up, and I thought he was too busy and forgot the appointment. When I finally received the bad news from my friends weeks later, it was a complete shock to me. Peter's untimely passing was such a tragedy and a big loss to the community. He will be dearly missed.

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