risks

MDPI

Article

Optimal Choice between Defined Contribution and Cash Balance Pension Schemes: Balancing Interests of Employers and Workers

Vanessa Hanna * and Pierre Devolder

Institute of Statistics, Biostatistics and Actuarial Sciences, Université Catholique de Louvain, 1348 Louvain-la-Neuve, Belgium; pierre.devolder@uclouvain.be
* Correspondence: vanessa.hanna@uclouvain.be

Abstract: In the context of pension plans, the employer and the worker have distinct interests and face different risks. The worker seeks higher retirement benefits, while the employer aims to minimize the cost of fulfilling his obligations. To address these diverse needs, the defined contribution plan managed with participating life insurance (DC-PL) and the cash balance plan managed with unit-linked insurance (CB-UL) serve as suitable choices. The multi-criteria analysis is conducted using the cumulative prospect theory model to measure the utility of the parties involved toward a mixed product combining these two pension plans. By assigning weights to risk measures and maximizing utilities, the paper employs both additive utility and Nash equilibrium approaches. The results reveal that the CB-UL plan aligns with employers' interests, offering potential financial gains, while the DC-PL plan attracts workers due to its profit-sharing aspect. Significantly, when equal importance is given to both parties, the CB-UL plan emerges as the prevailing choice. This study contributes to the understanding of pension plan design and decision-making dynamics between employers and workers, providing valuable insights for achieving a balance between retirement benefits and cost management.

Keywords: cash balance; defined contribution; participating contract; unit-linked contract; cumulative prospect theory; multi-criteria approach

check for updates

Citation: Hanna, Vanessa, and Pierre Devolder. 2023. Optimal Choice between Defined Contribution and Cash Balance Pension Schemes: Balancing Interests of Employers and Workers. *Risks* 11: 135. https://doi.org/10.3390/risks11070135

Academic Editor: Mogens Steffensen

Received: 20 June 2023
Revised: 14 July 2023
Accepted: 18 July 2023
Published: 21 July 2023

1. Introduction

Many traditional pension plans were defined in the past as DB plans (defined benefit); in this kind of scheme, the employer bears the investment risk and is responsible for providing a specified level of retirement benefits to workers. As life expectancies and inflation have increased and investment returns have become less predictable, the costs and risks associated with defined benefit plans have become more and more challenging for employers to manage (Munnell et al. 2007; Perlman and Reddick 2022; Turner and Hughes 2008). Even if these plans offer the best guarantee to the workers, this solution has become too risky for many employers. Therefore, not surprisingly, DC (defined contribution) plans have emerged as a popular alternative to DB plans, with the goal of shifting investment risk from the employer to the worker (Munnell and Sundén 2004; Poterba et al. 2007). In a pure DC scheme, the employer contributes a set percentage of the worker's salary into the worker's account without any further liability, and the worker bears the investment risk as the benefit is directly tied to the performance of the plan's investments. However, the reliance on worker investment decisions and potential market volatility can expose workers to substantial financial risks. In particular, if these contributions are invested in risky assets or in unit-linked insurance products, the worker can suffer a big loss at retirement. Consequently, the financial literacy of the worker can have profound implications for their overall retirement well-being, directly impacting their capacity to save and invest effectively, which reinforces their preference for DC plans. Other factors favoring the DC plans are that they facilitate labor market mobility and are more transparent (Broadbent et al. 2007; Brown and Weisbenner 2014; Munnell and Sundén 2004). Another solution could be the target-date funds that were suggested as a safe investment option to provide as the default in their DC plans. This strategy can be an improvement for people with relatively high risk aversion, as shown by Bodie and Treussard (2007). As a result of the aforementioned challenges, we can observe that pure DB and pure DC pension schemes can be seen as extreme solutions where the risks are concentrated on a single party (the employer or the worker).

Hybrid solutions have emerged in order to realize better risk sharing between employer and worker. A first way is to introduce a minimum guarantee in a DC plan (see, e.g., Boulier et al. 2001). For instance, the sponsor can manage the scheme through an insurance company that offers participating life insurance contracts. A participating life insurance contract is a type of life insurance policy that provides the policyholder with both insurance coverage and the opportunity to participate in the profits of the insurance company. Under this type of contract, the insurer pools the premiums paid by policyholders and invests them in a general fund to offer a guaranteed return with an additional bonus in the form of profit sharing depending on the investment's performance. This type of plan will be called DC-PL (defined contribution managed with participating life insurance). Depending on the level of the parameters, this kind of product can offer the worker a good compromise between guarantee and return while giving safety to the employer through a constant contribution rate. On the other hand, the employer will not take advantage of good financial returns, with all the eventual profit sharing being paid to the workers. Another way to implement safety within a pure DC philosophy is to switch from DC to CB (cash balance); this can be seen as a perfect example of a hybrid product between DB and DC Brown et al. (2001). Cash balance plans offer the investment risk management of a pure defined contribution plan with the predictable retirement benefit of a defined benefit plan. The main difference between a cash balance plan and a defined contribution plan is the way employer contributions and returns are defined Coronado and Copeland (2003).

In a cash balance plan, the employer bears the investment risk, and the worker is guaranteed a well-defined rate of return on their account balance, which is not, in general, equal to the real financial return on the assets. The employer attributes a set amount of money to an individual account for each worker. The account earns a predetermined interest rate, and the worker is guaranteed a rate of return on their account balance. In some sense, we could say that the contributions allocated to the account and the return given are virtual. They can be (and are, in general) different from the real contributions paid by the employer and the real investment returns of the assets (see, Hardy et al. 2014). These contributions are allocated to individual accounts for each worker and capitalized at predetermined times, according to the rules of the pension plan. The capitalization process involves calculating the amount that has accumulated in each individual account based on the tariff rules. The tariff rules determine a fixed interest rate or a yield defined by reference to a specified index. This guaranteed interest rate is referred to as the crediting interest rate. This kind of product can offer the worker great safety through pre-determined returns while giving the employer an expected cost close to a DC formula but with the possibility of making profit from additional investment returns. Indeed, in a DC scheme, all the returns are assigned to the workers; in a CB scheme, all the extra return above the crediting interest rate goes back to the employer and can decrease future costs to the pension plan. These returns could be generated, for instance, through a pension fund or through unit-linked insurance products without guarantee. This type of plan will be called CB-UL (cash balance managed with unit-linked insurance).

Cash balance plans have received significant attention in the financial literature as well as in actuarial research. Kopp and Sher (1998) examined the benefits offered by DB and CB plans and suggested that CB plans were more advantageous for younger participants with shorter service, while traditional DB plans provided better benefits for older members. Coronado and Copeland (2003) focused on the implications of transitioning and its practical aspects and concluded that the conversion from DB to DC plans is driven by labor market conditions rather than reducing benefit generosity. Moreover, Gold (2001) approached CB design from the perspective of maximizing shareholder value, focusing on equity yields and optimal asset allocation. He showed that shareholders benefit by defining liabilities in terms of equities. The actuarial valuation of CB plans posed challenges, and different conclusions were drawn by researchers. Applying valuation techniques designed for traditional DB plans to the CB design was found to cause issues. For example, Murphy (2001) shows that traditional actuarial methods sometimes generate a loss on termination where the account value exceeds the actuarial valuation. Broeders et al. (2007) explore a model for valuing hybrid pension liabilities, where the return on these liabilities is a combination of the pension fund's asset return and the risk-free return. They distinguish between two types of pension schemes and find that the valuation of hybrid pension liabilities is influenced by contract nature, investment strategy, and asset return volatility. They conclude that the impact of these factors on valuation differs between the two schemes, which should be taken into account in labor compensation negotiations between employers and workers. Furthermore, Hardy et al. (2014) approach CB benefits as financial liabilities of the employer and apply financial economics and risk management models to analyze and value these liabilities in a market-consistent manner. They argue that market-consistent valuation, based on objective market data rather than subjective judgement, provides a better measure of cost and risk compared to traditional actuarial valuation techniques. They conclude that the investment risk associated with CB plans is significant, especially when crediting rates are based on long bond rates or fixed. Stable costs can be achieved by using shorter rates for crediting, and the claim that CB plans cost less than notional contributions is shown to be untrue based on market value and corporate bond rate valuations. Also, when employing financial theory methods to calculate benefits, their analysis reveals dramatic variations in contributions and discounted benefits compared to an equivalent DC plan. Under the same approach, Brown et al. (2001) examine the funding aspects of CB pension liabilities, emphasizing the use of techniques for valuing and hedging floating-rate bonds to determine the present-value cost and effective duration of these liabilities. They find that the present-value cost of funding a cash balance liability and its effective duration vary significantly depending on the chosen crediting rate.

As mentioned before, DC pension plans managed with participating life insurance have been very popular and more favorable to the worker as it provides a higher benefit in the long-term; however, CB pension plans managed with unit-linked insurance are more beneficial to the employer because, in years of good market performance, they end up with a surplus in their assets. The main contribution of this paper is to compare these two intermediate hybrid solutions, namely DC-PL and CB-UL, and determine an optimal mix that balances the interests of both employers and workers. We recognize that the employer and the worker have different objectives and risk profiles when it comes to retirement benefits. The worker seeks higher benefits upon retirement, while the employer aims to minimize the supplementary costs of fulfilling their obligations. In fact, several papers focus on valuing CB plans and others on determining fair valuations for DC plans with participating contracts. However, the novelty of this study lies in its multi-objective perspective, which considers the pension plan as a contractual agreement between employers and workers, trying to model the equilibrium between these two parties. In order to analyze and evaluate the preferences and behaviors of the individuals involved in our study, we use a descriptive behavior model known as the cumulative prospect theory (CPT), and we formulate the maximization problem based on additive utility and Nash equilibrium approaches, seeking a fair agreement that maximizes the product of the two players' utilities. Indeed, by conducting a multi-criteria analysis, this study contributes to the existing literature on pension plan design by providing insights for employers and workers, particularly in the current landscape where numerous countries are considering the implementation of mandatory second-pillar pension systems. The decision between different pension systems becomes crucial in this context, emphasizing the importance of studies like ours.

Under this approach, this paper incorporates stochastic interest rates using the Hull & White model to compute the benefits, price the participating contract to generate a fair minimum guaranteed rate, and determine the crediting interest rate. In the first part of the study, we focus on a single premium scenario where a portion of the premium is invested in the DC-PL plan and the remaining portion is invested in the CB-UL plan. In this part, the crediting interest rate is fixed throughout the contract period. Sensitivity analyses are performed to examine the impact of different model parameters on the optimal choices for both employers and workers. We show that the employer is, in all cases, attracted to the CB-UL plan, and the worker is, in most cases, attracted to the DC-PL plan. Interestingly, when considering equal importance between the employer and the worker, it becomes evident that the CB-UL plan emerges as the more favorable choice. This preference is based on the advantages and benefits associated with this plan. Additionally, two extensions are considered: a periodic premium scenario and a stochastic crediting rate based on par yields.

The remainder of the paper is structured as follows: Section 2 develops our model; we describe the financial assumptions of both the DC-PL and CB-UL plans, and we highlight the utility model used in our framework. Moreover, we describe our optimization problem by stating the risk factors and maximizing the utilities of the two players in order to find the optimal mix depending on the weighting coefficient between them. In Section 3, we illustrate a numerical example stated as a basic scenario and carry out sensitivity analyses of various parameters. The study is extended in Section 4 to account for a periodic premium scenario as well as a stochastic crediting interest rate.

2. Model and Assumptions

2.1. Purpose

In a pension scheme, the interests of the worker and the employer are clearly different. Both parties are exposed to different risks at the retirement date: the risk of insufficient benefits for the worker compared to a target, and the risk of insufficient funds for the employer to meet his liability. In other words, the worker aims for a higher level of benefit on retirement, while the employer aims to reduce the supplemental cost of his obligation toward the worker. In this regard, DC-PL and CB-UL plans seem to be suitable intermediate choices to answer the needs of the worker and the employer, so mixing these two plans based on the players' behaviors is a kind of compromise between these two parties and allows us to model the financial risk sharing between the two parties and eventually find the optimal choice depending on the difference of interests between them.

As a result, we have a market where two kinds of people, the employer and the worker, are interested in two different pension plans. The aim is to maximize their utilities based on their preferences by weighting the risk between the two parties, where a part is computed based on a DC-PL contract and the remaining part based on a CB-UL contract. In order to measure the utility, we use a behavior model called the cumulative prospect theory (CPT), a descriptive theory based on experimental evidence that aids in understanding how people make decisions in uncertain situations. This framework is detailed in Section 2.2.3.

2.2. Assumptions

In this paper, we are interested in modeling the financial risks the employer and the worker are exposed to in order to find an optimal choice between their varying interests, so we examine the contracts from a pure financial point of view, ignoring mortality risk and assuming only the risks associated with an uncertain evolution in financial markets.

2.2.1. Defined Contribution

The first system taken into account in our model is the DC plan, whose benefit at maturity is the value of the participant's account balance at the time of retirement. The account balance is made up of contributions made by the worker and employer, as well as credited returns on those contributions. We assume here that the plan is managed with a participating life insurance contract (DC-PL) so the participant receives a minimum guaranteed rate of return in addition to a bonus in years of good investment performance.

In our paper, we use two important concepts in finance as the foundation for our analysis. The first concept is the valuation, as we aim to value the participating contract at zero to fix the control parameters (the parameters chosen by the insurance company), for which we need to define the processes under the risk-neutral measure $\mathbb{Q}$ that is used for the valuation in a complete arbitrage-free market. The second concept is risk management, as we want to compute the utility at maturity, and for this we look at the real situation under the real probability measure $\mathbb{P}$. Therefore, we need to define the different processes in both measures.

In our model, the insurer trades the term structure of interest rates on a complete financial market assuming that it evolves according to the one-factor Hull & White model (Hull and White 1990), which is a stochastic model that describes the evolution of interest rates over time. The model assumes that the short-term interest rate r_t follows, under the risk-neutral measure $\mathbb{Q}$, a mean-reverting process that can be described by the following stochastic differential equation:

$$\mathrm{d}r_t = \theta\,(\nu_t - r_t)\,\mathrm{d}t \,+\, \sigma_r\,\mathrm{d}W_t^r, \quad t \in [0,T] \text{ and } T \in \mathbb{N}, \tag{1}$$

where T is the time of retirement, ν_t is the time-varying long-term mean of the interest rate, θ is the speed of mean reversion, and σ_r is the volatility parameter (θ and σ_r are positive constants). W^r is a Brownian motion under the risk-neutral measure $\mathbb{Q}$ defined on a financial filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_t, \mathbb{Q})$ for all t in the time interval $[0,T]$.

We define under this measure the price of a zero-coupon bond, with no arbitrage, as follows:

$$P(t,T) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T r(s)ds} \mid \mathcal{F}_t\right],$$

where $P(t,T)$ is the price of the zero-coupon bond at time t that matures at time T. It is well known that the explicit form under Hull & White is given by:

$$P(t,T) = \exp\left(- r_t \cdot B_r(t,T,\theta) - \theta \int_t^T \nu_s \cdot B_r(s,T,\theta)ds + \frac{\sigma_r^2}{2}\int_t^T B_r^2(s,T,\theta)ds \right), \tag{2}$$

with $B_r(t,T,\theta) = \frac{1-e^{-\theta(T-t)}}{\theta}$. The price of the zero-coupon bond depends on the current short-term interest rate r_t, as well as the parameters of the model: the time-varying long-term mean of the interest rate ν_t, the speed of mean reversion θ, and the volatility parameter σ_r.

On the other hand, under the real probability measure $\mathbb{P}$, the risk-free rate r_t is a solution of the modified stochastic differential equation:

$$\mathrm{d}r_t = \theta\,(\overline{\nu}_t - r_t)\,\mathrm{d}t \,+\, \sigma_r\,\mathrm{d}\overline{W}_t^r, \quad t \in [0,T], \tag{3}$$

where $\overline{W}^r$ is the Brownian motion defined under $\mathbb{P}$ as $\mathrm{d}\overline{W}_t^r = \mathrm{d}W_t^r - \zeta \cdot \mathrm{d}t$; ζ stands for the market price of interest rate risk, and accordingly, $\overline{\nu}_t = \nu_t + \frac{\sigma_r \cdot \zeta}{\theta}$.

In addition, we assume a fund called G, defined as the insurer's general fund. It generally has a low level of volatility, is not traded directly on the market, and is used as a benchmark for the participating contracts. The fund G is assumed to follow, under the real probability measure, a geometric Brownian motion model, which is a popular model used to describe the dynamics of a fund price:

$$\mathrm{d}G_t = \mu_G\,G_t\,\mathrm{d}t \,+\, \sigma_G\,G_t\,\mathrm{d}\overline{W}_t^G, \quad G_0 = 1,\, t \in [0,T] \tag{4}$$

where G_t is the price of the fund at time t, μ_G is the drift or expected rate of return of the fund, σ_G is the volatility of the fund assumed to be positive and constant, and $\overline{W}^G$ is a standard Brownian motion under the real probability measure $\mathbb{P}$ that models the dynamics of the fund G.

It is trivial that the assets on the financial market are correlated, so we define $\rho_{r,G}$ as the correlation parameter between the Brownian motions $\overline{W}^r$ and $\overline{W}^G$ such that $\mathbb{E}^{\mathbb{P}}[d\overline{W}_t^r d\overline{W}_t^G] = \rho_{r,G}$, which leads to the following stochastic differential equation for the fund G using the Cholesky decomposition for correlated Brownian motions:

$$dG_t = \mu_G\, G_t\, dt \,+\, \sigma_G\, \rho_{r,G}\, G_t\, d\overline{W}_t^r + \sigma_G \sqrt{1-\rho_{r,G}^2}\, G_t\, d\overline{\overline{W}}_t^G, \quad G_0 = 1,\, t \in [0,T] \tag{5}$$

with

$$\overline{W}_t^G = \rho_{r,G} \cdot \overline{W}_t^r + \sqrt{1-\rho_{r,G}^2} \cdot \overline{\overline{W}}_t^G, \tag{6}$$

where $\overline{\overline{W}}^G$ is a new $\mathbb{P}$-Brownian motion generated by Cholesky decomposition. It is the specific risk associated with the general fund and independent from $\overline{W}^r$.

Subsequently, we write down the annual return of G as follows:

$$g_t = \frac{G_t}{G_{t-1}} - 1, \quad t = 1, 2, \ldots, T, \tag{7}$$

where G_t is the solution of Equation (5) under the real probability measure:

$$G_t = G_0 \, \exp\left(\left(\mu_G - \frac{\sigma_G^2}{2}\right) \cdot t \,+\, \sigma_G\, \rho_{r,G} \int_0^t d\overline{W}_s^r + \sigma_G \sqrt{1-\rho_{r,G}^2} \int_0^t d\overline{\overline{W}}_s^G\right).$$

After presenting the financial assumption of the DC-PL insurance, we start by considering a single contribution P paid at the inception of the contract and invested in the participating fund. The contract guarantees an annual minimum guaranteed interest rate i^{PL} in addition to a surplus return δ_t as a participation in the profits of the insurer if a part of the real rate of return, which is a market rate obtained from the investment in the general fund G, exceeds the guaranteed interest rate i^{PL}. The surplus return is computed annually inside the contract and translated into the following formula

$$\delta_t = \max(\beta \cdot g_t - i^{PL}, 0), \quad t \in [0,T]$$

where $\beta \in [0,1]$ is called the participation level that is assumed to be constant for the whole period of the contract. Consequently, the benefit at the time of retirement T, denoted V_T^{PL}, is equal to the contribution capitalized each year in the guaranteed interest rate in addition to the possible bonus as follows:

$$V_T^{PL} = P \cdot \prod_{t=1}^{T} \left(1 + i^{PL} + \delta_t\right). \tag{8}$$

It is worth noting that we assume the couple of parameters (i^{PL}, β) to be fair because the participating contract has to be priced fairly in order not to have a distortion of decision-making based on an arbitrary choice for these parameters that can be too generous as well as too miserly.

In this regard, we use the risk-neutral probability measure $\mathbb{Q}$ for the valuation in a complete and arbitrage-free market, and we write the dynamics of the fund G under this measure, as follows:

$$dG_t = r_t\, G_t\, dt \,+\, \sigma_G\, \rho_{r,G}\, G_t\, dW_t^r + \sigma_G \sqrt{1-\rho_{r,G}^2}\, G_t\, dW_t^G, \quad G_0 = 1,\, t \in [0,T] \tag{9}$$

where W^G is a $\mathbb{Q}$-Brownian motion, independent from W_t^r, and defined by

$$dW_t^G \,=\, d\overline{\overline{W}}_t^G \,+\, \frac{\mu_G - r_t - \sigma_G \cdot \rho_{r,G} \cdot \zeta}{\sigma_G \sqrt{1-\rho_{r,G}^2}}\, dt.$$

Consequently, the financial fair valuation price of the benefit, defined in Equation (8), is obtained by computing the risk-neutral expectation of the terminal cash flow as follows:

$$FV_0^{PL} \;=\; \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_s \mathrm{d}s} \cdot V_T^{PL}\right]. \tag{10}$$

In order to solve this expectation, we define the T-forward measure from the risk-neutral measure $\mathbb{Q}$, choosing as a numéraire the zero-coupon bond and using Girsanov's theorem for the change of measure:

$$\frac{\mathrm{d}\mathbb{Q}^T}{\mathrm{d}\mathbb{Q}} \;=\; \exp\left(-\sigma_r \int_0^T B_r(s,T,\theta)\, \mathrm{d}W_s^r - \frac{1}{2}\,\sigma_r^2 \int_0^T B_r^2(s,T,\theta)\, \mathrm{d}s\right).$$

Then, $\mathrm{d}\tilde{W}_t^r \;=\; \mathrm{d}W_t^r + \int_0^t \sigma_r\, B_r(s,T,\theta) ds$ is a $\mathbb{Q}^T$-Brownian motion.

Under this new measure, we write the fair valuation price in Equation (10) as follows:

$$\begin{aligned} FV_0^{PL} \;&=\; P(0,T) \cdot \mathbb{E}^{\mathbb{Q}^T}\left[V_T^{PL}\right] \\ &=\; P(0,T) \cdot \mathbb{E}^{\mathbb{Q}^T}\left[P \cdot \prod_{t=1}^{T}\left(1 + i^{PL} + \delta_t\right)\right] \\ &=\; P \cdot P(0,T) \cdot \mathbb{E}^{\mathbb{Q}^T}\left[\prod_{t=1}^{T}\left(1 + i^{PL} + \max(\beta \cdot g_t - i^{PL},\, 0)\right)\right], \end{aligned}$$

where g_k is the annual return of the fund G defined under the T-forward measure.

Then, we equalize the fair value to the initial contribution P in order to obtain the fairness relation. We note that the result is not explicit due to the dependency between the successive annual returns of the fund G; however, it will be simulated to find the fair couple (i^{PL}, β) to use, thereafter, as a reference for our model.

2.2.2. Cash Balance

The second system accounted for in our model is the CB plan, whose benefit at maturity is expressed based on the notional cash balance account that grows over time with interest credits and is paid out as a lump sum at retirement. At the end of the plan's term or when the worker retires, the benefit at maturity is the accumulated balance in the worker's cash balance account, which includes contributions made by the employer and the guaranteed interest credits earned over the years. We note that the contribution is paid once at the inception of the contract. Let F_0 be the initial account value and i_{t-1}^c be the annual crediting interest rate declared at $t-1$ for the year $t-1$ to t:

$$F_T \;=\; F_0 \cdot \prod_{t=1}^{T}(1 + i_{t-1}^c), \tag{11}$$

where F_T is the benefit of the worker at the time of retirement T and $F_0 = P$ the contribution made at the inception of the contract. The crediting rates can be fixed over the years or stochastic based on Treasury bonds. Both cases will be treated in this paper.

Apart from the worker's guaranteed retirement benefit, we assume that the CB plan is managed with a unit-linked life insurance contract with no maturity guarantee (CB-UL). Hence, the employer receives the return of the investment funds based on the performance of the underlying investments. The value of the unit-linked contract is determined by the fund S, an investment fund chosen by the employer. The fund follows a geometric Brownian motion with the following dynamic:

$$\mathrm{d}S_t = \mu_S\, S_t\, \mathrm{d}t \,+\, \sigma_S\, S_t\, \mathrm{d}\overline{W}_t^S, \quad S_0 = 1,\, t \in [0,T] \tag{12}$$

where S_t is the price of the fund at time t, μ_S is the drift, σ_S is the volatility of the fund assumed to be positive, constant and higher than σ_G, and $\overline{W}^S$ is a standard Brownian motion under the real probability measure $\mathbb{P}$ that models the dynamics of the fund S. We define $\rho_{r,S}$ the correlation parameter between the risk factors $\overline{W}^r$ and $\overline{W}^S$ such that $\mathbb{E}^{\mathbb{P}}[\mathrm{d}\overline{W}_t^r \mathrm{d}\overline{W}_t^S] = \rho_{r,S}$. Analogously to the fund G, using Cholesky decomposition, the differential equation for the fund S is given by:

$$\mathrm{d}S_t = \mu_S\, S_t\, \mathrm{d}t + \sigma_S\, \rho_{r,S}\, S_t\, \mathrm{d}\overline{W}_t^r + \sigma_S \sqrt{1-\rho_{r,S}^2}\, S_t\, \mathrm{d}\overline{\overline{W}}_t^S, \quad S_0 = 1,\ t \in [0,T] \tag{13}$$

with

$$\overline{W}_t^S = \rho_{r,S} \cdot \overline{W}_t^r + \sqrt{1-\rho_{r,S}^2} \cdot \overline{\overline{W}}_t^S, \tag{14}$$

where $\overline{\overline{W}}^S$ is a new $\mathbb{P}$-Brownian motion generated by Cholesky decomposition. It is the risk of the investment fund independent of $\overline{W}^r$.

Note that the annual return of S is

$$s_t = \frac{S_t}{S_{t-1}} - 1, \quad t = 1, 2, \ldots, T, \tag{15}$$

where S_t is the solution of Equation (13) under the real probability measure:

$$S_t = S_0 \exp\left(\left(\mu_S - \frac{\sigma_S^2}{2} \right) \cdot t + \sigma_S\, \rho_{r,S} \int_0^t \mathrm{d}\overline{W}_s^r + \sigma_S \sqrt{1-\rho_{r,S}^2} \int_0^t \mathrm{d}\overline{\overline{W}}_s^S \right).$$

Likewise, we consider a single contribution P paid at the inception of the contract and invested in the unit-linked fund. The returns on this fund are, in general, more volatile than those of the general fund, making the benefit at the time of retirement uncertain, not guaranteed, and expressed as follows:

$$V_T^{UL} = P \cdot \prod_{t=1}^{T} (1 + s_t) = P \cdot \frac{S_T}{S_0},$$

where V_T^{UL} is the value of the unit-linked contract given to the employer at maturity T.

2.2.3. CPT Framework

To solve the optimization problem, we use a psychological theory of decision-making that provides a descriptive account of how people make decisions in situations where the outcomes are uncertain. According to CPT (Cumulative Prospect Theory), people evaluate potential outcomes, denoted by X, based on two main factors: the probability of the outcome occurring and the value or utility of the outcome. The model assumes that people display loss aversion as they tend to value potential losses more than equivalent gains and suggests that gains and losses are relative to some reference point where the utility function is S-shaped and concave to the right of the reference point (the gain part) and convex to the left (the loss part). Based on that, psychologists Tversky and Kahnemann (1992) developed the following subjective utility function:

$$U_J(z) = \begin{cases} (z-\Gamma)^{\gamma_J} & z \geq \Gamma, \qquad J = E \text{ (Employer)} \\ -\lambda_J \cdot (\Gamma - z)^{\gamma_J} & z < \Gamma, \qquad \text{or } W \text{ (Worker)} \end{cases}$$

where γ_J is the risk aversion parameter that measures gains or losses. z refers to the outcome where some values fall above the reference point Γ and the others fall below. λ_J is the loss aversion parameter that represents the sensitivity to losses over gains.

However, CPT assumes that people do not evaluate these factors in a straightforward, rational manner. For example, people tend to overestimate the probability of rare events and underestimate the probability of common events. This is modeled by a probability weighting function proposed by Prelec (1998):

$$w_J(p) = e^{-(-\log p)^{\varphi_J}}, \qquad J = E\,(\text{Employer}) \text{ or } W\,(\text{Worker})$$

where $0 < \varphi_J < 1$ is a free parameter that controls the curvature of the function, w_J is strictly increasing with $w_J(0) = 0$, and $w_J(1) = 1$.

With the utility and weighting functions, we can define the cumulative prospect theory as follows:

$$CPT_J(X) := \int_{-\infty}^{0} U_J(q)\,\mathrm{d}\big(w_J(F(q))\big) + \int_{0}^{\infty} U_J(q)\,\mathrm{d}\big(-w_J(1-F(q))\big),$$

where $F(q) = \mathbb{P}(X \leq q) = \int_{-\infty}^{q} \mathrm{d}\mu_X$ is the distribution function and μ_X is the probability measure given by the random variable X.

2.3. The Optimization Problem

The employer's role in the pension plan is to make contributions on behalf of the workers, either as a single payment at the beginning of the contract or through periodic payments. The employer's objective is to manage the cost of the pension plan effectively. The cost is primarily determined by the contribution amount, which remains the same for both the DC-PL and CB-UL systems. However, the employer may also be responsible for additional payments at the time of retirement to fulfill the promised liabilities to the worker or to recover any surplus assets, particularly in the CB-UL plan. Therefore, the perception of risk is about this final surplus or final loss.

On the other hand, the worker does not pay contributions. In the DC-PL plan, he receives the value of the participating account, which includes a minimum guaranteed interest rate, whereas in the CB-UL pension plan, the worker receives the guaranteed benefit as specified in the cash balance contract, irrespective of the value of the unit-linked account, which could be higher due to favorable market conditions. At the same time, this provides the worker with protection in case of a market downturn. Essentially, the worker's cash flow is determined by the quality of their final liability, and the choice between the two systems influences the perceived risk associated with that cash flow.

As a result, We recall that the employer wants to lower the supplemental cost of his commitment to the worker, who, in turn, wants a higher level of benefit upon retirement. The criteria used to carry out this analysis are:

- The funding status of the employer, which is the difference between the assets according to the type of management operated by the insurance company and the liabilities, which are the obligations toward the worker. This amount can be positive (i.e., a surplus) or negative (i.e., a deficit), depending on whether the assets in the plan exceed or fall short of the promised benefits; it depends on the plan system along with the type of management as follows:
  - DC-PL contract: 0 as there is no obligation on the employer in the DC plan that does not offer any guarantee to the worker at retirement[1].
  - CB-UL contract: $P \cdot \left(\frac{S_T}{S_0} - \prod_{t=1}^{T}(1 + i^c_{t-1})\right)$ as the employer receives on his asset's portfolio the value of the UL account and has to pay the guaranteed benefit, of the CB side, to the worker.
- The difference between the benefit at maturity and the premium capitalized at a worker target interest rate, called l, is set by the worker in a way that reflects his appreciation for what he would like to receive at maturity:

- DC-PL contract: $P \cdot \left(\prod_{t=1}^{T} \left(1 + i^{PL} + \delta_t\right) - (1+l)^T \right)$ as the worker receives the value of the PL account at the time of retirement.
- CB-UL contract: $P \cdot \left(\prod_{t=1}^{T} (1 + i^c_{t-1}) - (1+l)^T \right)$ as the worker receives the cash balance guaranteed benefit.

We note that the worker can aim, in particular, for capital protection, for which the target l is set to zero.

Before measuring the utility of each party, we combine the two pension plans, where a fixed proportion $x \in [0,1]$ of the initial contribution is invested in the DC-PL plan and the remaining proportion $1-x$ is invested in the CB-UL plan. The investment of the contribution is based on a non-rebalancing case, which means that no additional action is performed after the first setting of the proportion x at the beginning of the contract. So, we see at the time of retirement:

For the funding status of the employer:

$$GE(T) \,=\, P \cdot \left[x \cdot 0 \,+\, (1-x) \cdot \left(\frac{S_T}{S_0} - \prod_{t=1}^{T} (1 + i^c_{t-1}) \right) \right]; \tag{16}$$

For the benefit-target of the worker:

$$GW(T) \,=\, P \cdot \left[x \cdot \prod_{t=1}^{T} \left(1 + i^{PL} + \delta_t\right) \,+\, (1-x) \cdot \prod_{t=1}^{T} (1 + i^c_{t-1}) \right] \,-\, P \cdot (1+l)^T. \tag{17}$$

After setting the mixed risk factor for each player, we measure the utility of the employer and the worker toward those mixed products by computing the expected utility of Equations (16) and (17), respectively, using the CPT framework. We note that the utility function of the employer could be different from the utility function of the worker because the latter, as an individual person seeking a better pension for his retirement, is, in general, supposed to be more risk averse than an employer managing the pension plan, so their risk appetites could be different.

As stated in Section 2.1, the objective is to maximize the utility of both parties under the described framework and indicated assumptions. Therefore, the next step is to weight the risk measures of the worker and the employer, eventually maximizing their utilities in order to find the optimal mix between DC-PL and CB-UL pension plans depending on the weighting coefficient between the two players. In this regard, we approach the problem of bargaining between the two parties based on an additive utility as well as the Nash equilibrium.

Hence, we first model the employer's decision-making process using CPT_E, the cumulative prospect function with the risk aversion parameter γ_E, the loss-aversion parameter λ_E, and the free parameter φ_E that reflects the employer's risk appetite. The worker's decision-making process is modeled using CPT_W, another cumulative prospect function with the risk aversion parameter γ_W, loss-aversion parameter λ_W, and free parameter φ_W adapted to the worker's risk profile.

To capture the multi-criteria nature of the decision problem, we introduce a weighted average of the utilities of the employer and worker, where the weight coefficients $\varrho \in [0,1]$ and $1-\varrho$ reflect the importance or power of each player in the decision-making process. In particular, when $\varrho = 0$, the worker's utility is the only consideration in the decision-making process, and the decision is made based on his preferences alone; however, when $\varrho = 1$, the employer's utility is the only consideration in the decision-making process, and the decision is made based on his preferences alone. For values of ϱ between zero and one, the decision takes into account both the employer's and worker's utilities, with the weight given to each depending on their respective power or importance.

Starting by the additive utility, the maximization problem is written as follows:

$$\max_{x\in[0,1]} \Big[\varrho \cdot CPT_E\big(GE(T)\big) + (1-\varrho)\cdot CPT_W\big(GW(T)\big)\Big]. \tag{18}$$

For the Nash equilibrium, the problem becomes:

$$\max_{x\in[0,1]} \left[\Big(CPT_E\big(GE(T)\big)\Big)^{\varrho} \cdot \Big(CPT_W\big(GW(T)\big)\Big)^{1-\varrho}\right]. \tag{19}$$

In fact, the Nash bargaining problem was introduced by Nash (1950) and is based on the idea that in a bargaining situation, the two players will negotiate over different products. The solution seeks to find a way that is fair to both players and that will lead to an agreement being reached by maximizing the product of the two players' utilities.

3. Numerical Illustrations

To solve numerically our optimization problem described in Equations (18) and (19), we fix a number of parameters to find the optimal mix between the two products, DC-PL and CB-UL, based on the preferences of the employer and the worker and depending on the bargaining power between them. In this regard, we present hereafter the chosen values for the different parameters introduced in Section 2.2 to represent the basic scenario in our paper. Afterwards, we carry out sensitivity analyses for a wide range of parameters in order to study their impact on the optimal result obtained in the basic scenario.

3.1. Basic Scenario

We consider a pension contract that is funded by a single contribution P paid at the inception of the contract with $T = 15$ years to retirement. We assume the following parameters to illustrate our model:

- For the interest rates parameters, we fit the restricted exponential model (used to calibrate the instantaneous forward rate and developed by Cairns (1998)) to the curve of Belgian zero-coupon rates on 28 November 2022, using a virtual adjustment through non-linear regression. The exponential model is given in the following form:

$$f(0,t) = \beta_0 + \sum_{j=1}^{n} \beta_j \cdot e^{-c_j t},$$

  with $n = 3$, a constant that determines the number of parameters in the model. The parameters β_j and c_j are estimated using non-linear least squares analysis, which generates the following values: $\beta_0 = 0.0049$, $\beta_1 = 0.005$, $\beta_2 = -0.0035$, $\beta_3 = 0.02$, $c_1 = 0.1385$, $c_2 = 0.0385$, and $c_3 = -0.02$.
  By defining the instantaneous forward rate, we can easily compute the mean reversion level. On the other hand, we assume that the interest rate volatility $\sigma_r = 0.8\%$, the initial rate $r_0 = 0.5\%$, and the speed of mean reversion $\theta = 15\%$.
  This model generates an initial curve of spot rates displayed in Table 1. These rates are obtained from the following formula: $P(0,t) = \dfrac{1}{(1+R(0,t))^t}$, where $P(0,t)$ is defined from Equation (2).

Table 1. Spot rates for different durations observed at zero.

R(0,1)	R(0,5)	R(0,10)	R(0,15)	R(0,20)	R(0,25)
0.6486%	1.1367%	1.5883%	1.9347%	2.2207%	2.4728%

- For the PL and UL insurance parameters, we recall the values from Hanna et al. (2022), where low and high volatility levels are chosen for the general fund G and the investment fund S, respectively, $\sigma_G = 3\%$ and $\sigma_S = 15\%$, in addition to the following values for the drifts of the two funds, $\mu_G = 3\%$ and $\mu_S = 7\%$. Moreover, we choose as an example a participation level of $\beta = 70\%$ in the profits of the insurer. Based on that and on the risk-free rate, we compute the fair guaranteed interest rate using Equation (10) and we obtain $i^{PL} = 0.7357\%$.

- For the correlations between the general and the investment funds with the interest rate, we fix the following values, respectively: $\rho_{r,G} = -20\%$ and $\rho_{r,S} = -15\%$.
- For the cash balance parameter, we start by assuming that the crediting interest rate is constant for the whole period of the contract ($i^c_{t-1} = i^c \, \forall \, t$) and is chosen to be in line with the risk-free rate such that $i^c = R(0,15) = 1.9347\%$.
- For the target rate parameter, we assume that the worker appreciates a capital protection so we fix $l = 0\%$ as the minimum desired level of return.
- For the CPT parameters, we choose a reference point of $\Gamma = 0$ because we aim to measure the utility of the employer based on his funding status and the utility of the worker based on the difference between the benefit and a target, so the utility is assessed relative to zero. Moreover, we fix the loss aversion parameter $\lambda_E = \lambda_W = 2.25$, as estimated by Tversky and Kahnemann (1992). Several empirical studies estimate, based on experimental data, the choice of the risk aversion value for the utility function and the free parameter for the weighting function as given by Prelec (1998). The range of values for the risk aversion γ_J is between 0.19 and 0.88, and for the free parameter φ_J, it is between 0.533 and 0.94. In our study, we chose an equal value of 0.5 for φ_E and φ_W as well as an equal value of 0.2 for γ_E and γ_W. The choice of a risk aversion of 0.2 could be influenced by the size of the payoffs, as pointed out by Stott (2006). On the other hand, the choice of the same value of risk aversion for the employer and the worker is chosen as a basic scenario; a sensitivity analysis will be conducted later on different values of γ_E and γ_W. In fact, if the two players have the same utility function, it is nevertheless possible for them to have distinct negotiating positions.

We note that the previous parameters' values are fixed for the numerical part unless otherwise noted. In this setting, we present the numerical solution for Equations (18) and (19) as a basic scenario. So, Figure 1 shows graphically the optimal choice between the DC-PL and CB-UL plan schemes for both players, the employer and the worker.

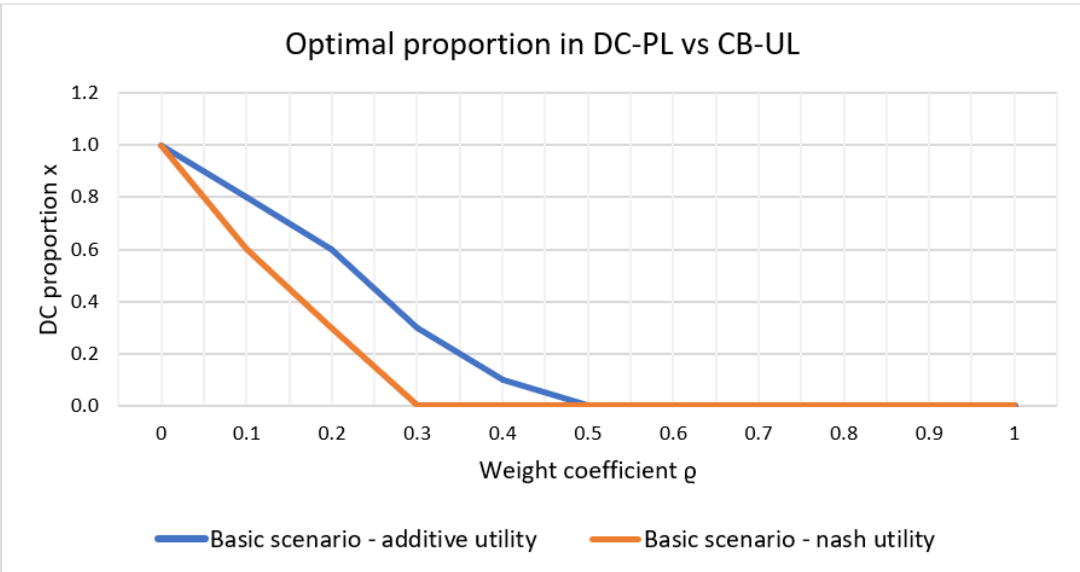

Figure 1. Optimal proportion in the DC (defined contribution) pension plan for the different values of the weight coefficient between employer and worker—basic scenario.

In Figure 1, we see that the worker ($\varrho = 0$) is interested in a DC-PL plan, whereas the interest of the employer ($\varrho = 1$) is in a CB-UL plan. This strong result is rational because when the DC plan is managed with participating life insurance it motivates the worker by giving him the chance to participate in the profits of the insurer on top of the guaranteed return. On the other hand, from the employer's point of view, a CB plan is more attractive because he benefits from good financial returns and makes profits after paying the guaranteed benefit to the worker. Furthermore, when equal importance is given to the employer and the worker, we notice that the CB-UL seems to be the preferable choice, as we see more advantage for this plan. Additionally, expanding the range and looking at ϱ between 0.4 and 0.6, we see the trend towards the CB-UL plan as an optimal choice for both parties. Furthermore, we point out that the results obtained through Nash equilibrium converge faster to the CB-UL plan than the additive utility model.

In the following, we perform sensitivity analyses for the different parameters and compare the results to the basic scenario in order to assess their impact on the optimal results.

3.2. Sensitivity Analysis of the Crediting Interest Rate

To check the sensitivity of the crediting interest rate i^c on the optimal choice of the plan and depending on the bargaining power, we increase and decrease i^c by 0.5%. The results are presented in Figure 2.

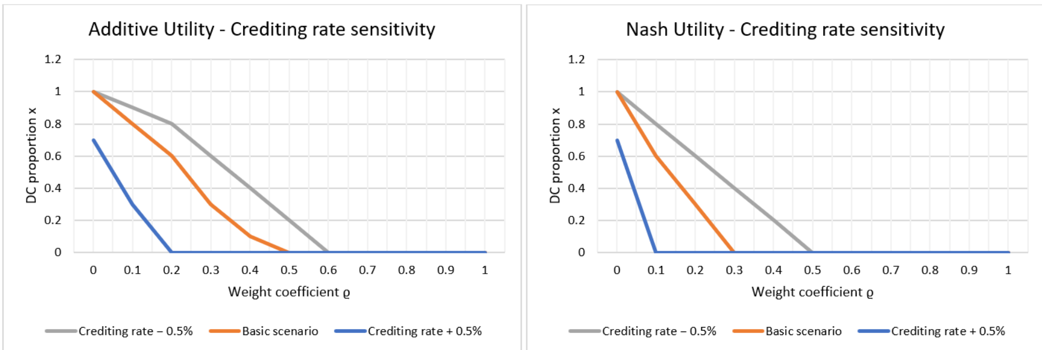

Figure 2. Impact of the crediting interest rate on the optimal results.

Figure 2 shows that by decreasing the crediting interest rate, there is still an advantage for the DC-PL plan. This advantage seems to be more interesting by sharing the power between the worker and the employer compared to the basic scenario. In other terms, if we have equal importance between the employer and the worker ($\varrho = 0.5$) in the additive utility model, the proportion x represents 30% instead of 10% in the DC-PL plan, and it needs a weight coefficient of 0.6 for the employer to reach 100% as an optimal choice in the CB-UL plan. However, by increasing the crediting interest rate by the same percentage, it is clear that the CB-UL plan becomes much more interesting than the DC-PL plan for both parties. Ultimately, we highlight that the impact of the crediting rate is similar for the two graphs in Figure 2, but the Nash equilibrium model tends to move the graph lines faster to a CB-UL plan.

3.3. Sensitivity Analysis of the Worker Target Interest Rate

In this part, we study the impact of the target rate on the optimal results, so we change the value of the target rate by $\pm 1\%$ and illustrate the results in Figure 3.

We point out from Figure 3 that with a higher target, the DC-PL plan meets less the needs of the worker, which leads to a higher preference for the CB-UL plan; however, the worker is attracted to a DC-PL plan with a smaller target. It is worth noting that for a weight coefficient between 0.2 and 0.6, the increasing effect of the target rate keeps the optimal choice between the two parties more in the DC-PL compared to the basic scenario before it converges to a 100% preference for CB-UL, yet it moves very quickly to the CB-UL plan with a decreasing effect.

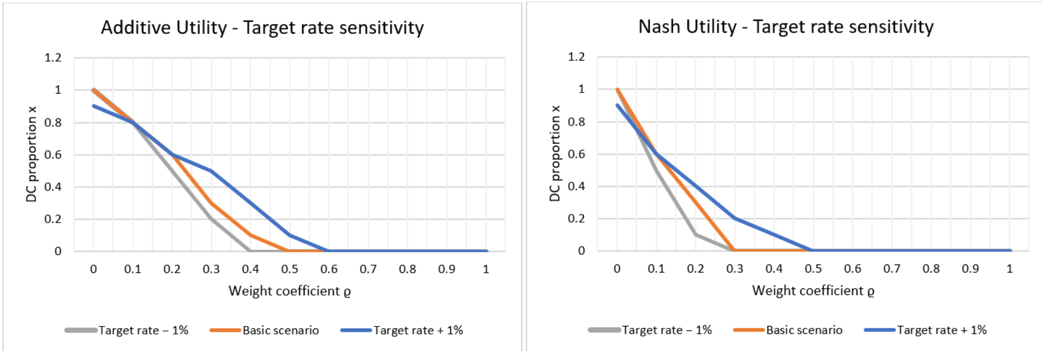

Figure 3. Impact of the target interest rate on the optimal results.

3.4. Sensitivity Analysis of the Risk Aversion

In this section, we focus on the impact of the risk aversion parameters γ_E and γ_W on the optimal choice of pension plans. In this respect, we first increase the risk aversion of the employer, γ_E, from 0.2 to 0.3 to compare its effect to the basic scenario, and then, analogously, we increase the risk aversion of the worker, γ_W, from 0.2 to 0.3, which makes the worker more risk-averse than the employer.

Figure 4 proves that the increase of γ_W leads to the worker's unwillingness to take risks and therefore increases their preference for the DC-PL plan. Moreover, we observe in the additive utility model that for an equal weight between the two parties, the optimal decision is to invest 40% in the DC-PL plan. Similarly, the increase in the employer's risk aversion speeds up the convergence to the CB-UL plan. Interestingly, the impact is more pronounced in the additive utility than in the Nash utility model.

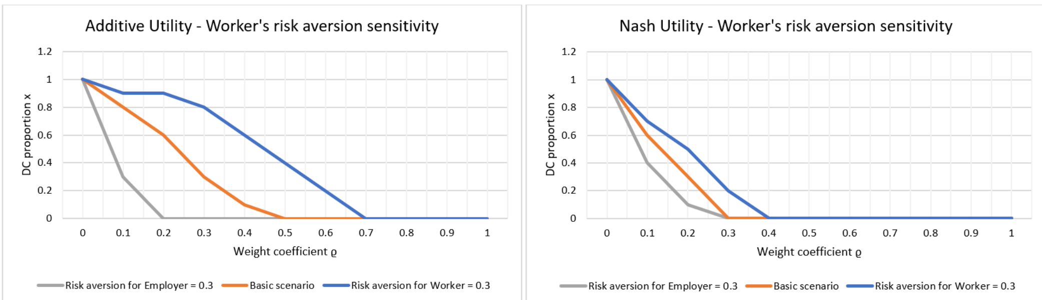

Figure 4. Impact of the risk aversion parameter on the optimal results.

3.5. Sensitivity Analysis of the Participation Level

In this section, we study the impact of the participation level β on the optimal investment decision for a DC pension plan managed with a participating life insurance contract versus a CB pension plan managed with a unit-linked insurance contract. We note that the participating contract must be fairly priced, which means that the change in the parameter β leads to a change in the minimum guaranteed interest rate i^{PL}. To conduct this study, we first decrease β from 70% to 60%, which gives a rate i^{PL} of 1.2285%, and then increase it to 80% and obtain a rate i^{PL} of 0.0240%.

We observe that as the participation level beta increases (decreases), the minimum guaranteed interest rate granted by the insurer decreases (increases). Consequently, when the contract is priced fairly, the return on the participating fund remains relatively consistent. This leads to the worker's continued attraction towards the DC-PL plan, resulting in the same optimal outcome for ϱ values between 0 and 0.2 in the additive utility model, compared to the basic scenario (see Figure 5). However, as the bargaining power increases, there is a corresponding increase in interest towards the DC-PL plan when there is higher participation in the insurer's profits, and vice versa. It is worth noting that the impact remains largely unchanged whether we utilize additive or Nash utility models in our analysis.

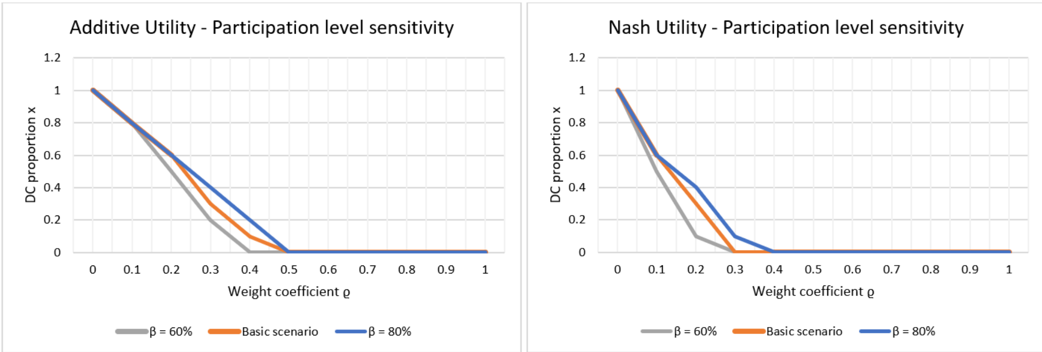

Figure 5. Impact of the participation level parameter on the optimal results.

3.6. Sensitivity Analysis of the Funds' Volatilities

To check the sensitivity of the volatilities of the funds G and S on the optimal utility results under different bargaining power levels, we increase the G-volatility from 3% to 5% and the S-volatility from 15% to 20%. The increase in the G-volatility leads to a decrease in the fair guaranteed interest rate i^{PL}, which is now -0.7861%, and consequently an increase in the surplus return, which intuitively makes the DC-PL plan less attractive but also more preferable to the worker than the CB-UL plan, as proved in Figure 6. On the other hand, the increase in the return of the risky asset and in the surplus return in the participating contract keeps the optimal investment decision in the DC-PL plan longer between $\varrho = 0.2$ and $\varrho = 0.7$ in the additive utility model before it converges to the CB-UL plan. Conversely, in the Nash equilibrium model, the change in the volatilities has no impact on the optimal results for $\varrho \geq 0.4$.

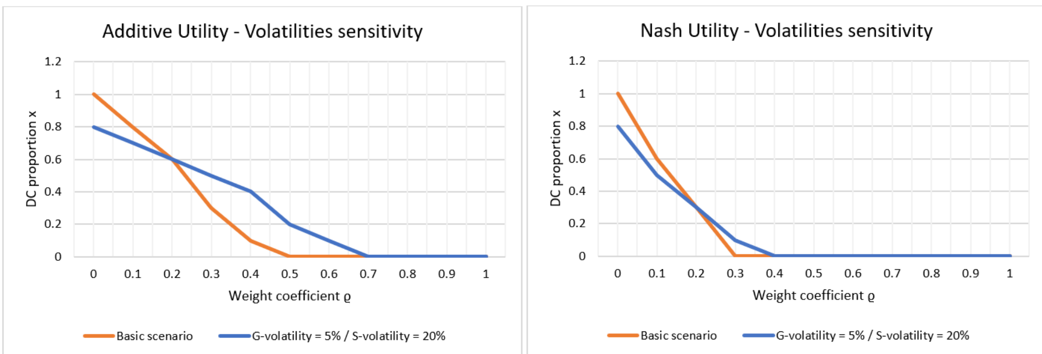

Figure 6. Impact of the funds' volatilities parameters on the optimal results.

3.7. Sensitivity Analysis of the Time to Maturity

To study the sensitivity of the maturity T on the optimal investment decision, we compare a contract with a maturity of 10 years and another with a maturity of 25 years to the basic scenario, whose maturity is 15 years, as illustrated in Figure 7. We note that the change in the time to maturity changes the crediting interest rate (which is chosen to be in line with the risk-free rate in our paper) and the minimum guaranteed interest rate. A contract with maturity $T = 10$ yields a rate i^c of 1.5883% (see Table 1) and a rate i^{PL} of =0.1447%, whereas a contract with maturity $T = 25$ yields a rate i^c of 2.4728% (see Table 1) and a rate i^{PL} of 1.3600%. We note that the crediting interest rate are chosen to be in line with the risk-free rate, as highlighted in Section 3.1.

We point out that when the maturity of the contract decreases, the minimum guaranteed interest rate in the participating life insurance contract decreases. This should make the DC-PL plan less attractive to the worker, as they can earn a higher return with the CB plan. Interestingly, with the participation in the insurer's profits, the DC-PL plan stays more attractive to the worker for all maturity T, as eventually the return on the participating fund is higher than the return on the CB fund. At the same time, with a lower time to maturity, the return on the risky fund S decreases, which makes the CB-UL plan less attractive to the employer. Therefore, we see in Figure 7 that the DC-PL plan is more appreciated for a lower T, and vice versa. On the other hand, with a sharing of power between the worker and the employer, both parties' optimal decisions are more focused on the CB-UL plan. This is because, from one side, the crediting interest rate is higher and more interesting for the worker, and from the other side, the return from the unit-linked contract is higher and potentially more profitable for the employer.

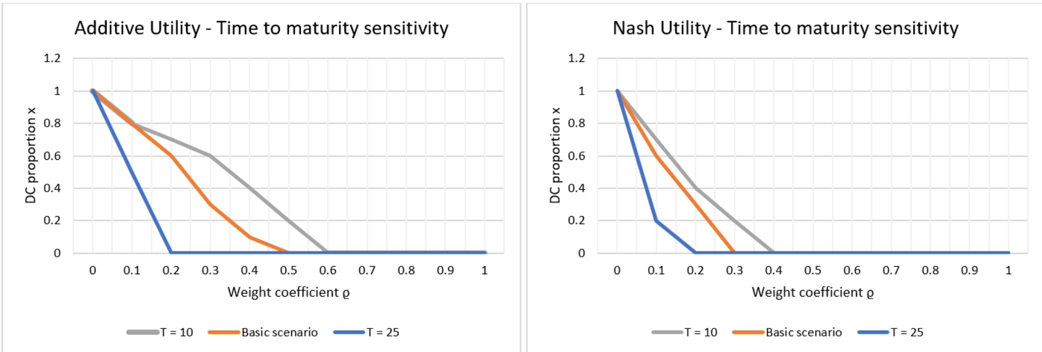

Figure 7. Impact of the time to maturity on the optimal results.

4. Extensions

In this section, we study the impact of premium payment methods and variable crediting interest rates on the optimal choice of the employer and the worker and emphasize the importance of the stochastic crediting rate in the decision-making process.

4.1. Periodic Premiums

In this section, we consider a pension contract that is funded by a periodic premium with a constant contribution paid annually at the beginning of the year. In this context, the single premium is divided by the expected number of annuity payments over a 15-year period to determine the amount of each periodic premium. Similar to the single premium case, a constant proportion x of the periodic premium is invested in the DC-PL plan, while $1 - x$ is allocated to the CB-UL plan. Therefore, the value of the participating contract is determined by the investment performance and contributions made over time, so the benefit at retirement for the worker in a DC-PL plan is the following:

$$\tilde{V}_T^{PL} = \tilde{P} \cdot \prod_{t=1}^{T}\left(1 + \tilde{i}^{PL} + \delta_t\right) + \tilde{P} \cdot \prod_{t=2}^{T}\left(1 + \tilde{i}^{PL} + \delta_t\right) + \ldots + \tilde{P} \cdot \left(1 + \tilde{i}^{PL} + \delta_T\right), \tag{20}$$

where $\tilde{V}_T^{PL}$ is the benefit at the time of retirement, $\tilde{P}$ is the periodic premium, and $\tilde{i}^{PL}$ is the minimum interest rate guaranteed by the insurer for the whole period of the contract. In order to find the fair guaranteed interest rate, we priced the contract fairly by equalizing the fair value of Equation (20) to the fair value of the premiums as follows:

$$\mathbb{E}^{\mathbb{Q}}\left[e^{-\int_0^T r_s \mathrm{d}s} \cdot \tilde{V}_T^{PL}\right] = \mathbb{E}^{\mathbb{Q}}\left[\sum_{t=0}^{T-1} \tilde{P} \cdot e^{-\int_0^t r_s \mathrm{d}s}\right]. \tag{21}$$

After some manipulations, we obtain the following fairness relation:

$$P(0,T) \cdot \left[\mathbb{E}^{\mathbb{Q}^T}\left[\prod_{t=1}^{T}\left(1 + \tilde{i}^{PL} + \delta_t\right)\right] + \ldots + \mathbb{E}^{\mathbb{Q}^T}\left[1 + \tilde{i}^{PL} + \delta_T\right]\right] = 1 + P(0,1) + \ldots + P(0,T-1). \tag{22}$$

On the other hand, the second system is the CB-UL plan, where, under the periodic premium context, the worker is guaranteed a benefit at retirement based on the accumulated cash balance over the years:

$$F_T = \tilde{P} \cdot \left(\prod_{t=1}^{T} (1 + i_{t-1}^c) + \prod_{t=2}^{T} (1 + i_{t-1}^c) + \ldots + (1 + i_{T-1}^c) \right), \tag{23}$$

whereas the employer receives the accumulation of the returns of the investment fund S and the contributions over time:

$$\tilde{V}_T^{UL} = \tilde{P} \cdot \left(\prod_{t=1}^{T} (1 + s_t) + \prod_{t=2}^{T} (1 + s_t) + \ldots + \prod_{t=T-1}^{T} (1 + s_t) + \frac{S_T}{S_{T-1}} \right). \tag{24}$$

Eventually, the benefit of the worker, in both pension plan schemes, is compared to the accumulation of the contribution by the target rate up to the retirement date as follows:

$$\tilde{P} \cdot \left((1 + l)^T + (1 + l)^{T-1} + \ldots + (1 + l) \right).$$

To compare numerically the periodic to the single premium case, we hold the same parameter values assumed in Section 3.1, and we solve Equations (18) and (19) in order to find the optimal mix between the DC-PL and CB-UL plans, where the terms $GE(T)$ and $GW(T)$ are computed according to the risk factors described in this section above.

Comparing the results in Figure 8, we observe that the optimal mix favors the DC-PL plan when paying periodic premiums. This preference can be attributed to the higher minimum guaranteed interest rate offered in the participating contract for policies with periodic premiums. Specifically, the results show that the minimum guaranteed rate is 1.2547% for periodic premiums, while it is lower at 0.7357% for single premium policies. This significant difference in the guaranteed rates encourages workers to choose the DC-PL plan, as it provides the combined benefits of the minimum guaranteed rate and participation in the insurer's profits.

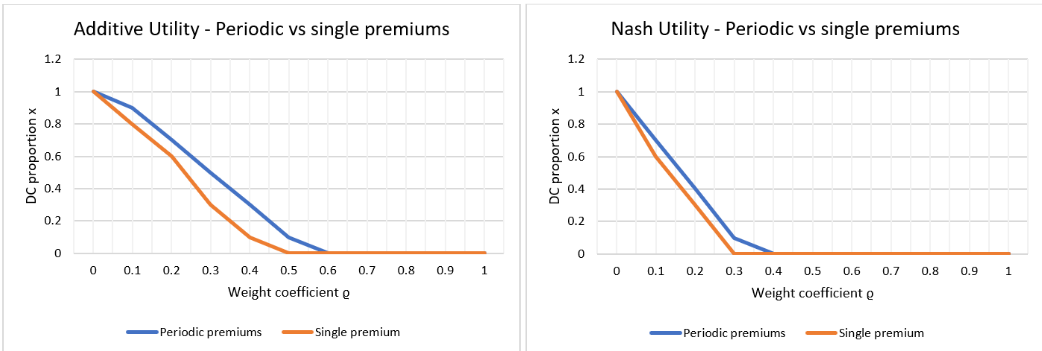

Figure 8. Optimal proportion in the DC pension plan for the different values of the weight coefficient between employer and worker—comparing periodic with single premiums.

The results highlight the importance of considering the premium payment scenario when designing pension plans. Insurers offer a higher minimum guaranteed rate for periodic premium payments to compensate for the steady and predictable cash flow they receive. This allows them to better manage their investments and liabilities, and consequently, workers are motivated to choose the DC-PL plan to benefit from the attractive guaranteed rate and potential participation in insurer profits.

4.2. Stochastic Crediting Interest Rate

Ultimately, we would like to consider the CB pension plan with a stochastic crediting interest rate, which introduces an element of uncertainty in the liabilities. As per the basic scenario, the premium is paid in a single payment at the beginning of the year, and the worker's retirement benefit is determined by the accumulated cash balance over time. Unlike traditional cash balance plans with fixed crediting rates, the stochastic crediting rate introduces variability in the notional investment performance. The crediting rate is subject to market fluctuations and can vary from year to year, influencing the growth of the cash balance. We recall Equation (11), the formula for the accumulated cash balance at retirement, assuming annual crediting interest rates:

$$F_T = F_0 \cdot \prod_{t=1}^{T} (1 + i^c_{t-1}), \tag{25}$$

where i^c_{t-1} represents the stochastic crediting rates in each year of contributions, which is a market-dependent variable. According to Hill et al. (2010), most CB plans offer a crediting rate based on par yields on k-year Treasury bonds with an additional margin m, so we impose

$$i^c_{t-1} = c_k(t-1) + m, \quad t = 1, \dots, T, \tag{26}$$

with $c_k(t-1)$ is the k-year par yield, with yearly coupons defined as follows:

$$c_k(t-1) = \frac{1 - P(t-1, t-1+k)}{\sum_{n=1}^{k} P(t-1, t-1+n)}, \quad t = 1, \dots, T, \tag{27}$$

where $P(t-1, t-1+n)$ is the price of a zero-coupon bond defined in Equation (2) under $\mathbb{Q}$. In order to compute these prices, we simulate the short rates r_{t-1} under the real-world measure $\mathbb{P}$ using Monte Carlo simulation.

To compare numerically the stochastic to the constant crediting rate case, which is the basic scenario, we hold the same parameter values and simulate the short-term interest rates under the real probability measure, choosing a market price of risk $\zeta = -23.04\%$ as suggested in the paper by Barbarin and Devolder (2005), and we choose a maturity reference of one year ($k = 1$) and a margin $m = 1\%$, as it is one of the most popular interest rates used in CB plans (see, Hill et al. 2010).

We observe that the optimal utility solutions for the basic scenario are different when assuming a stochastic crediting interest rate for the CB plan, as illustrated in Figure 9. This result suggests that the choice between the CB and DC plans is influenced by the uncertainty in the credited interest rate. In fact, by incorporating a stochastic crediting rate, the CB plan aims to provide a more realistic representation of the potential gains or losses experienced by the underlying investments, leading to a dynamic retirement benefit estimation for workers. In our scenario, this stochastic framework generates a higher return on the CB plan, which leads to a higher preference for the CB-UL plan since the worker becomes less interested in the DC-PL plan.

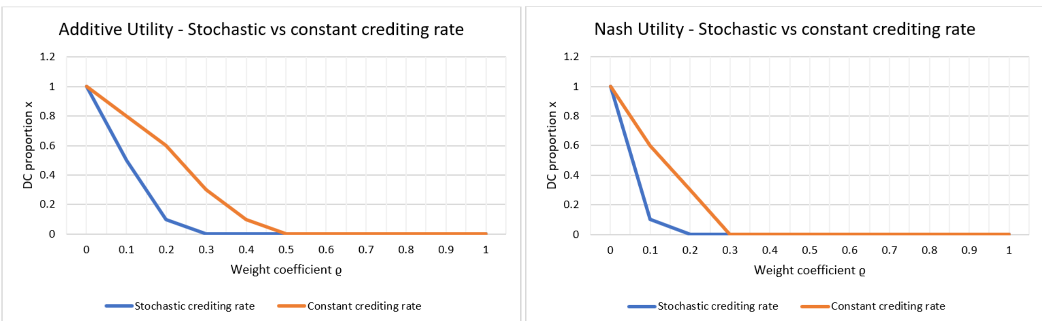

Figure 9. Optimal proportion in the DC pension plan for the different values of the weight coefficient between employer and worker—comparing stochastic with constant crediting rates.

Eventually, we would like to check the effect of the market price of the risk parameter ζ on the optimal allocation between DC-PL and CB-UL pension plans. For this, we choose values of -30%, -23.04% and -15%, and we derive the par yields. The results are illustrated in Figure 10.

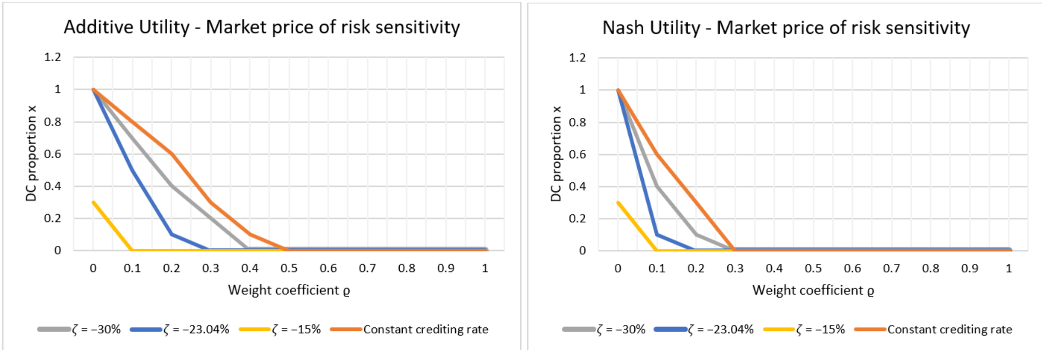

Figure 10. Impact of the market price of risk on the optimal results.

The market price of risk seems to be a significant factor that determines the choice between the two plans. In fact, the increase in absolute value of the market price of risk leads to smaller values of the par yield and consequently decreases the return on the CB fund. Hence, we observe that the worker is attracted to the DC-PL plan for a high value of $|\zeta|$ as the return on the participating fund is higher.

5. Discussion

The results of this study shed light on the preferences and decision-making dynamics between employers and workers in the context of pension plan design. Under this framework, we have presented a basic scenario based on a single premium with a fixed crediting interest rate throughout the contract period.

The findings demonstrate that the interests of the employer and the worker are indeed divergent, with the employer generally leaning towards the CB-UL plan and the worker being more attracted to the DC-PL plan. This aligns with the respective advantages of each plan, as the DC-PL plan offers profit-sharing potential to the worker, while the CB-UL plan provides the employer with the opportunity to benefit from favorable market performance. Particularly when equal importance is assigned to both parties, the CB-UL plan emerges as the more favorable choice, emphasizing its advantages in terms of utility maximization for both employers and workers. This is a strong result, as it holds for all the sensitivity analyses conducted in this research.

In order to examine the impact of various model parameters on the optimal choices for both employers and workers, we carried out sensitivity analyses. Some results indicate that the worker's preference may shift towards the CB-UL plan if the minimum guaranteed rate becomes less attractive, highlighting the influence of guarantee levels on decision-making. Furthermore, it is observed that when the worker desires a level of return at the retirement date higher than the single premium initially paid, there is an inclination towards a higher appreciation of the CB-UL plan, potentially driven by the worker's desire for more certainty to fulfill his aspiration. The convergent results obtained through the Nash equilibrium approach further support the dominance of the CB-UL plan in terms of utility maximization.

These findings provide valuable insights for decision-makers in selecting and designing pension plans that effectively balance the interests of both employers and workers while managing associated risks.

6. Conclusions

In this paper, we looked at two intermediate hybrid pension solutions, DC-PL and CB-UL, with the aim of finding an optimal mix between the interests of both employers and workers, considering the differing interests and risk profiles of employers and workers.

The analysis employs a multi-criteria approach, utilizing CPT as a behavior model and maximizing utilities through both the additive utility and Nash equilibrium approaches. The study examines various scenarios, including sensitivity analyses, periodic premiums, and stochastic crediting interest rates, to assess their impact on the optimal mix between the two pension plans based on the weighting coefficient. We have seen that the optimal choice between the two pension systems depends on market parameters as well as the risk attitudes of both parties.

This paper contributes to the understanding of pension plan design by offering insights into the optimal mix of DC-PL and CB-UL plans, considering the preferences and objectives of both employers and workers. The results provide valuable guidance for decision-making in the design and selection of pension plans, highlighting the distinct preferences and considerations of employers and workers in achieving a balance between retirement benefits and cost management.

The issues addressed in this paper could be very important in the context of occupational pension plans, particularly in systems where there exist mandatory second-pillar pensions. In future research, it could be useful to have a better design for the project of mandatory pension systems. In addition, it would be valuable to expand the scope of our analysis by incorporating additional risk factors, such as mortality risk. Furthermore, considering the impact of inflation on retirement benefits and extending the analysis to include the linkage of periodic premiums with salary would provide a more comprehensive understanding of the impact on optimal choices. By considering these additional factors, the generalizability of the obtained results in this broader setting can be explored.

Author Contributions: The original draft of this article has been written by V.H. under P.D. supervision. The results, conclusions, and main contributions of this article were discussed and agreed by the two authors. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the chair Fondation Louvain—Ethias.

Data Availability Statement: Not applicable.

Acknowledgments: We want to thank the anonymous reviewers for many insightful remarks.

Conflicts of Interest: The authors declare no conflicts of interest.

Note

[1] In some countries, there could be supplemental legal obligations (cf. the Law on Occupational Pensions (LPC) in Belgium).

References

Barbarin, Jérome, and Pierre Devolder. 2005. Risk measure and fair valuation of an investment guarantee in life insurance. *Insurance: Mathematics and Economics* 37: 297–323. [CrossRef]

Bodie, Zvi, and Jonathan Treussard. 2007. Making investment choices as simple as possible, but not simpler. *Financial Analysts Journal* 63: 42–47. [CrossRef]

Boulier, Jean-François, ShaoJuan Huang, and Grégory Taillard. 2001. Optimal management under stochastic interest rates: The case of a protected defined contribution pension fund. *Insurance: Mathematics and Economics* 28: 173–89. [CrossRef]

Broadbent, John, Michael Palumbo, and Elizabeth Woodman. 2006. The Shift from Defined Benefit to Defined Contribution Pension Plans—Implications for Asset Allocation and Risk Management. Paper Prepared for a Working Group on Institutional Investors, Global Savings, and Asset Allocation Established by the Committee on the Global Financial System. Available online: https://www.bis.org/publ/wgpapers/cgfs27broadbent3.pdf (accessed on 20 July 2023).

Broeders, Dirk, An Chen, and David Rijsbergen. 2013. Valuation of liabilities in hybrid pension plans. *Applied Financial Economics* 23: 1215–29. [CrossRef]

Brown, David T., Philip H. Dybvig, and William J. Marshall. 2001. The cost and duration of cash balance plans. *Financial Analysts Journal* 57: 50–62. [CrossRef]

Brown, Jeffrey R., and Scott J. Weisbenner. 2014. Why do individuals choose defined contribution plans? Evidence from participants in a large public plan. *Journal of Public Economics* 116: 35–46. [CrossRef]

Cairns, Andrew. 1998. Descriptive bond-yield and forward-rate models for the British government securities market. *British Actuarial Journal* 4: 265–321. [CrossRef]

Coronado, Julia Lynn, and Phillip C. Copeland. 2003. Cash balance pension plan conversions and the new economy. *Journal of Pension Economics and Finance* 3: 297–314. [CrossRef]

Gold, Jeremy. 2001. *Shareholder-Optimal Design of Cash Balance Pension Plans*. Working Paper 2001-7. Philadelphia: Pension Research Council.

Hanna, Vanessa, Peter Hieber, and Pierre Devolder. 2022. Mixed participating and unit-linked life insurance contracts: Design, pricing and optimal strategy. *Scandinavian Actuarial Journal* 5: 421–46. [CrossRef]

Hardy, Mary, David Saunders, and Xiaobai Zhu. 2014. Market-consistent valuation and funding of cash balance pensions. *North American Actuarial Journal* 18: 294–314. [CrossRef]

Hill, Tomeka, Gaobo Pang, Mark Warshawsky. 2010. *Hybrid Pension Plans: A Comprehensive Look at Their History, Economics and Features*. Arlington: Towers Watson Perspectives Report.

Hull, John, and Alan White. 1990. Pricing interest rate derivative securities. *Review of Financial Studies* 3: 573–92. [CrossRef]

Kopp, Steve J., and Lawrence J. Sher. 1998. A benefit value comparison of a cash balance plan with a traditional final average pay defined-benefit plan. *Pension Forum* 11: 1–16.

Munnell, Alicia H., Kelly Haverstick, and Mauricio Soto. 2007. *Why Have Defined Benefit Plans Survived in the Public Sector?* Chestnut Hill: Center for Retirement Research, Boston College, Brief 2.

Munnell, Alicia H., and Annika E. Sundén. 2004. *Coming Up Short: The Challenge of 401(k) Plans*. Washington, DC: Brookings Institution Press.

Murphy, Raymond J. 2001. The cash balance funding method. In *Cash Balance Symposium Monograph*. Schaumburg: Society of Actuaries.

Nash, John F. 1950. The Bargaining Problem. *Econometrica* 18: 155–62. [CrossRef]

Perlman, Bruce J., and Christopher G. Reddick. 2022. From stand-alone pensions to alternative retirement plans: Examining state and local governments. *Compensation & Benefits Review* 54: 12–30.

Poterba, James, Steven Venti, and David A. Wise. 2007. The shift from defined benefit pensions to 401(k) plans and the pension assets of the baby boom cohort. *Proceedings of the National Academy of Sciences* 104: 13238–43. [CrossRef] [PubMed]

Prelec, Drazen. 1998. The probability weighting function. *Econometrica* 60: 497–528. [CrossRef]

Stott, Henry P. 2006. Cumulative prospect theory's functional menagerie. *Journal of Risk and Uncertainty* 32: 101–30. [CrossRef]

Turner, John A., and Gerard Hughes. 2008. *Large Declines in Defined Benefit Plans Are Not Inevitable: The Experience of Canada, Ireland, the United Kingdom, and the United States.* Bunhill Row, London: Pensions Institute.

Tversky, Amos, and Daniel Kahnemann. 1992. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty* 5: 297–323. [CrossRef]