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Empirical Testing of Models of Autoregressive Conditional Heteroscedasticity Used for Prediction of the Volatility of Bulgarian Investment Funds

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Abstract: The relevance of the development is determined by the possibility of testing a complex analytical methodology for forecasting the daily volatility of Bulgarian investment funds, which will support the investment community in making adequate investment decisions. The used risk attribution quantification models GARCH (1.1), EGARCH (1.1), GARCH-M (1.1) and TGARCH (1.1) are adapted to predict the volatility of investment funds. The current development focuses on forecasting the risk concentration of investment funds (in Bulgaria) through the testing of complex, analytical and specialized models from the GARCH group. The object of the study includes quantitative analysis, estimation and forecasting of daily volatility through the models GARCH, EGARCH, GARCH-M and TGARCH with specification (1.1). The research covers the net balance sheet value of forty-two investment funds for the period from 13 July 2020 to 13 July 2023, where the results of the research show that according to three of the models GARCH, EGARCH and GARCH-M with the highest risk concentration the investment fund "Golden Lev Index 30" stands out. An exception to the thus formed trend is related to the TGARCH model in which the future conditional volatility is with the "EF Rapid" investment fund. When testing the models, we found that the GARCH model and the EGARCH model successfully optimize the regression parameters of the final equation for all analyzed investment funds, and as a result, valid forecasts are formed. In the case of the remaining two GARCH-M and TGARCH models, the impossibility of applicability of the model for some investment funds was found because of the optimization procedure, in which the parameters of the models have a value of zero. The present study is a unique mechanism for forecasting the daily volatility of Bulgarian investment funds, which further assists investors in risk assessment and is a prerequisite for making adequate and responsible investment decisions. The wide-spectrum toolkit of risk forecasting models allows their testing in investment funds with different risk natures (high-risk, balanced and low-risk). From a research point of view, in future research dedicated to modeling the risk attribution of investment funds, the analytical toolkit can be enriched with the following models: QGARCH, PGARCH, GJR-GARCH, IGARCH, SGARCH, AVGARCH, NGARCH and GAS. From a statistical point of view, we can apply the analyzed models to different probability distributions in order to describe the risky nature of investment funds.

Keywords: mutual funds; risk management; modeling; GARCH; EGARCH; GARCH-M; TGARCH

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1. Introduction

Collective investment schemes (CIS) are an alternative investment instrument. Classic bank savings products (deposits) offered by commercial banks in the country have lost their attractiveness in recent years. This is because of zero and near-zero interest rates on savings

products, which are unable to cover the rate of inflation. The stated circumstances force market entities to redirect their capital into various investment instruments. Investment funds are specialized and licensed financial institutions that professionally invest money in various financial instruments. Twenty-eight management companies, one hundred and forty-six investment funds with different risk profiles (high-risk, balanced and low-risk) leading a specialized investment policy are operating on the territory of Bulgaria as of 4 August 2023. According to the data of the Bulgarian Association of Management Companies as of 4 August 2023, the net value of the assets of the contractual funds amounted to BGN 2483 billion. This undoubtedly proves increased investor interest in collective investing operating on the territory of Bulgaria. Like any investment instrument, CSIs are characterized by a certain yield and risk, strongly determined by the market profile of the specific fund. More and more investors do not pay serious attention to risk. This neglect of risk by the investment community can lead to negative results. The present study aims to empirically establish and forecast the volatility in the net asset value of mutual funds (MF) by testing complex and specialized analytical and econometric models from the group of autoregressive conditional heteroskedasticity (GARCH, EGARCH, TGARCH and GARCH-M). The specified models were formed because of the evolutionary development of risk management worldwide in the 1980s. Management companies of investment funds are facing a serious challenge dictated by strong dynamics of financial markets, high inflation levels in recent months and the deepening military conflict between the Russian Federation and Ukraine, which creates prerequisites for making adequate and timely investment decisions in complex and dynamic market conditions. All activities and operations of investment funds are subject to serious control, supervision and monitoring by the regulatory body in the person of the Financial Supervision Commission (FSC), which "vigilantly" monitors compliance with the rules for evaluation and risk management of the funds, the rules for the evaluation of the portfolio and the determination of the net asset value of the funds and the rules for maintaining and managing the funds' liquidity. The focus of the present study is directly related to risk determination. Investment funds are a strong regulatory constraint in terms of risk management and assessment. A mandatory methodology that every investment fund must perform is related to the assessment of market, credit and liquidity risk. The methods used to evaluate the specified risk objects are based on the classical statistical indicators and the part with the dispersion meters. Each management company has the right to use other methods than the legally regulated methods for risk analysis and assessment for internal company purposes. Part of the management companies implement and test different models for the forecasting and risk assessment of the mutual fund they manage. The current study is a kind of scientific practical experiment related to the implementation of complex econometric models for forecasting the risk of Bulgarian investment funds.

The object of the research is a specialized sample of forty-two Bulgarian investment funds, which is forecasting the daily volatility of Bulgarian investment funds through the GARCH, EGARCH, TGARCH and GARCH-M models.

The purpose of the study is to establish and forecast empirically the variability in the net asset value of the contract funds by testing complex and specialized analytical and econometric models from the group of autoregression conditional heteroskedasticity (GARCH, EGARCH, TGARCH and GARCH-M).

To achieve the intended goal, the authors determine the following research tasks:

- Specialized literature review of current research in the field of forecasting the daily volatility of financial markets;
- Theoretical presentation and complete derivation of the formula apparatus of unconditional dispersion models;
- Empirical application and presentation of the GARCH, EGARCH, TGARCH and GARCH-M models for forecasting the daily volatility of contract funds;
- Analysis and evaluation of the final results of the testing of autoregression conditional heteroskedasticity models.

2. Literature Review

In the specialized scientific literature, in recent years, serious attention has been paid to models for forecasting daily volatility. The evolutionary development of financial markets and, in particular, of investment funds, as an alternative type of investment means for market entities, is a prerequisite for the generation of profitability (Petrova and Radukanov 2021; Nikolaev and Petrova 2021). The yield is an uncertain quantity due to the presence of the risk of a change in its value in a negative aspect for the investment community. In the 1980s, the financial world created, implemented and patented models for quantifying risk attribution (Em et al. 2022). Although autoregressive conditional heteroskedasticity models are more than forty years old, they continue to occupy a leading position in a number of empirical studies in the field of risk management. Jiang (2012) in his autonomous research pays serious attention to risk prediction models. From a theoretical point of view, the author skillfully presents the following quantitative risk assessment models: GARCH, EGARCH and GJR-GARCH. The focus of the research covers several leading capital markets—NASDAQ, S&P 500, FTSE100, HANG SENG and NIKKEI 225. For the empirical application of the models, the author uses two probability distributions of the model—the standard normal cumulative distribution and the t-distribution. For greater representativeness of the results, Wei Jiang also calculates the statistical indicator average standard error of prediction—(RMSE), by which to determine the success rate of the models. Wei Jiang came to the opinion that the best prediction results for the daily volatility were achieved by the GJR-GARCH model, based on the lowest value of the static RMSE criterion. The authors' collective (Ugurlu et al. 2014) tested GARCH family models for the risk forecasting of the following five stock markets: (SOFIX), (PX), (WIG), (BUX) and и (XU100). The results reached by the authors are related to a clearly expressed GARCH effect in the value of the following stock indices: PX, BUX, WIG and и XU100. An exception to this regularity is found in the benchmark of the Bulgarian capital market, the SOFIX stock index. A pronounced GARCH effect is observed in the value of the four stock indices, alerting us to a significant influence of "old news" on the volatility of the daily logarithmic return. In addition to being a powerful tool for quantitative risk assessment, GARCH models can be successfully tested as an adequate methodology for testing the hypothesis of the efficiency of financial markets. Evidence for such a claim overlaps with the study of (Narayan and Liu 2015), in which the authors test for the existence of a unit root in the returns of 156 stocks of US companies listed on the NYSE. The results reached by the authors are related to the fact that 50% of the studied shares follow the random walk hypothesis, that is, the weak form of market efficiency defined by Eugene Fama is found. Cheteni (2017) analyzed the correlation between two capital markets from two different geographic continents, Africa and Asia. The results of the research found a statistically significant correlation between the FTSE/JSE stock indices (Johannesburg Stock Exchange and Shanghai Stock Exchange Composite Index). Based on the application of the GARCH model, the author reaches the opinion that the two stock markets are highly volatile and a constant instability is observed quite tendentially. A serious study in the field of risk management was carried out by Su (2010), in which he presented the impact of the global financial crisis on the value of the Chinese capital market. The author's research thesis states: that in a period of crisis, there is a serious long-term volatility of the financial market compared to the period after the crisis recovery. The methodology used in Su's scientific article includes the application of the following two models: GARCH and EGARCH. Ezzat (2012) analyzed the impact of the political crisis in Egypt in 2011 on the volatility of the Egyptian capital market. The object of the study includes the risk forecasting of the four stock indices EGX 30, EGX70, EGX 100 and EGX 20, and the methodology is represented by the following two models, GARCH and EGARCH. Undoubtedly, the leverage effect of the EGARCH model is measured by serious volatility in the value of stock market indices in the period of political crisis. Chen (2023) demonstrates a parallel analysis of the impact of the global financial crisis of 2008 and the COVID-19 pandemic on the volatility of the leading American stock index S&P 500.

The author formulates the opinion that during the global financial crisis, good forecasting results are observed from the application of the GJR-GARCH model, and during the COVID-19 pandemic a better final performance is the EGARCH model. Natchimuthu et al. (2018) focus on estimating the conditional variance of the Indian stock market for the period from 2006 to 2016. The object of the study covers ten shares of the companies with the largest market capitalization. The authors use the PGARCH volatility forecasting model for the purposes of their research. The results fully confirm the opinion that GARCH models are a suitable mechanism for determining the daily volatility considering the ARCH effect, the leverage effect, in the conditions of the Indian stock market. Zhou (2010) investigated the impact of the global financial crisis on the value of the Swedish Industrial Index for the period from 2000 to 2010. The methodological framework of the study covers the testing of the following models: the TAR model, SETAR model and ESTAR model used to predict the risk attribution of the Swedish Industrial Index. Of the applied modifications of the models with the best results, the ESTAR model is considered. Gerunov (2023) presents the different modes of operation of twenty-four European stock indices for the period from 2006 to 2020. The author implements in his research Multiple Regimes and Markov Switching for forecasting volatility. In the final fragment of the research, the conclusion is reached that in a situation of a Bullish market in the European markets, positive yield and low volatility are found, and in a Bear market, a negative yield and high volatility of the European stock indices are found. Asseiceiro (2019) in his dissertation studied the dependence between three stock markets Euro Stoxx 50, S&P 500 and Nikkei 225 from three geographically separated continents Europe, North America and Asia. The toolkit for establishing dependence between the analyzed stock markets includes the testing of vector autoregression (VAR) and dynamic conditional correlation (DCC). In conclusion, it is concluded that there is a correlation dependence between the leading US index S&P 500 and the authoritative stock market index Euro Stoxx 50. Kallsen and Vesenmayer (2009) for the purposes of risk management constructed their own model for forecasting the daily volatility COGARCH Continuous GARCH Model. As defined by its authors, the operation is driven by an innovation process, unlike the diffusion GARCH model. Jondeau and Rockinger (2006) offer the academic community a new variation of the GARCH model for forecasting the daily volatility of stock markets. In scientific circles, the model is known by its name Copula GARCH, and its idea is to correct the problems related to the lack of a normal probability distribution of the return of financial assets. Caporin and McAleer (2006) further develop the GJR-GARCH model by adding multiple thresholds and an asymmetric effect that is not constant over time. McNeil and Frey (2000) created an EVT-GARCH model for forecasting the volatility of financial instruments, considering extreme values and the presence of price shocks.

3. Description of Data and Research Methodology

3.1. Description of Data

The present study is a quantitative assessment of forty-two contract funds operating on the territory of our country. Each of the analyzed investment funds is characterized by a different risk policy. That is, in practice, three groups of investment funds were studied—high-risk, low-risk and balanced. Each of the funds in question has a specific investment policy, which is legally regulated in the rules and prospectus and accordingly approved by the financial supervision committee. Investment fund portfolios include a variety of financial instruments—shares, bonds, deposits, receivables for interest and principal payments, dividends, checking accounts, ETFs, alternative funds, other collective investment schemes, financial derivatives and others. The financial instruments (stocks and bonds) are from different geographically separated markets. We have quite purposefully selected various investment funds to track and empirically establish their volatility. The excerpt covers the period from 13 July 2020 to 13 July 2023, applying a daily basis of the net asset value (NAV) of the funds. The following investment funds are the subject of research: "Select Balance", "Select Regional", "Select Bonds", "Select Dividend", "CKB Active",

"CKB Leader", "CKB Garant", "CKB Private", "DV Balance", "DV Dynamic", "Astra Cash Plus", "Astra Global Equity", "SKY New Shares", "SKY Finance", "Quest Vision", "Prime Assets", "South Market Maximum", "South Market Optimum", "Trend Balanced Fund", "Trend Fund Shares", "Trend Conservative Fund", "Texim Conservative Fund", "Texim Bulgaria", "Texim Balkans", "Texim Commodity Strategies", "Prestige", "Profit", "FIB Avangard", "FIB Classic", "FIB Garant", "First Financial Broker house VOSTOK", "Golden Lev", "Golden Lev Index 30", "EF Rapid", "Arkus Balanced", "Arkus Dynamic", "Concord-1 Stocks and Bonds", "Concord-2 Stocks", "Concord-3 Real Estate", "Concord-4 Energetics", "Concord-5 CEE" and "Concord-6 Bonds". The selection of data for the period from 13 July 2020 to 13 July 2023 of this development aims to present the most up-to-date data on the net asset value of investment funds. The last three calendar years are a completely sufficient period of time for the accurate application of autoregression conditional heteroskedasticity models with the main goal of forecasting the daily volatility of investment funds. When using a larger (for a period of more than three calendar years) sample of data, this will inevitably distort the final estimates from the forecasting of the risk attribution of the contractual funds. This is due to the fact that the market has assimilated (processed) the information in question. This forced us, the authors of the present study, to use the most up-to-date data on the value of investment funds in order to highlight the latest trends and determinants reflecting on them. Of course, the idea of a larger scale of research (for a longer period of three years) is not devoid of logic. The analysis of the risk concentration of the contract funds can be presented in separate sub-periods in which to follow the state and the results formed by the GARCH models. The indicated idea of the segmentation of the individual sub-periods and the inclusion in the analysis of new models for predicting the daily volatility and expanding the scope of the research object with the inclusion of new investment funds from other geographically separated countries is an idea for future scientific research on the current issue. It is quite natural that the COVID-19 pandemic reflects on the risk concentration of investment funds. A significant part of the mutual funds we studied have in their investment portfolios shares of companies from the pharmaceutical sector, which was highly sensitive during the COVID-19 crisis. At the outbreak of the global pandemic, financial markets were initially characterized by uncertainty. This uncertainty and anxiety reflect seriously on the investment behavior of market subjects. The making of hasty and unfounded decisions by the investment community reflects seriously on the volatility of investment funds. In a situation of crisis, which in practice is the COVID-19 pandemic, the risk concentration of financial instruments increases. Almost all spheres of social and cultural life are strongly affected.

3.2. Description of Research Methodology

The present study includes the testing of four models for forecasting the daily volatility of investment funds GARCH, EGARCH, GARCH–M and TGARCH, with specification (1.1). The first empirically tested model was the generalized autoregressive conditional heteroskedasticity (GARCH) model, created by Engle and Bollersev in 1986. The GARCH model is part of the group of symmetric models and includes two components: the autoregression coefficient (p) and the moving average (MA). GARCH models are used in a number of areas of finance in the prediction of risk, the valuation of financial derivatives and their ability to account for the efficiency of financial markets. The formula for calculating GARCH (1.1) is as follows (Bollerslev 1986):

$$v_t^2 = \omega + \alpha * \varepsilon_{t-1}^2 + \beta * v_{t-1}^2; \quad (1)$$

$$\varepsilon_t^2 = v_t^2 + u_t; \quad (2)$$

$$L_{(\mu,\alpha,\beta,\omega)} = \frac{1}{v_t * \sqrt{2 * p}} * \varepsilon^{-\frac{\varepsilon_t^2}{2v_t^2}} \quad (3)$$

where:
v_t^2—conditional return variance;
ω—constant value;
α—regression coefficient considering the ARCH effect;
β—regression coefficient considering the GARCH effect;
ε—parameter accounting for the deviation from the model.

Referring to numerous authoritative scientific studies analyzed in the Section 2 in the field of daily volatility forecasting, the use of autoregressive conditional heteroskedasticity (GARCH) models is observed as a priority. In line with this, a number of researchers in the field of risk management use in their personal research the family of (GARCH) models to model and forecast the risk attribution of various financial assets stocks, bonds and financial derivatives. An essential feature of autoregressive conditional heteroskedasticity models allows them to capture sensitivity of financial instruments related to price shocks in crisis situations of financial instruments. Due to the advantages we analyzed of (GARCH) models for forecasting daily volatility combined with the final estimates of the risk concentration of investment funds formed by us, we implemented them in the present study as a fundamental method for risk forecasting. Of course, in the specialized scientific literature there are many econometric models for forecasting and determining the daily volatility of investment funds. Here, we can highlight the following specialized risk concentration forecasting models: EWMA, ARIMA, ARCH, Power GARCH, IGARCH (Integrated GARCH), APARCH, NGARCH, CGARCH, AVGARCH, SGARCH, QGARCH, MSGARCH, SETAR and others. Naturally, the GARCH models tested in the present study are not without defects. First of all, the researched methodology is difficult to apply in the absence of solid knowledge in the field of statistics, econometrics and finance. Empirical application of autoregressive conditional heteroskedasticity models requires the use of a specialized software product to calibrate the final model equation, especially in multivariate models. Although the software product energetically and precisely calculates the models for predicting daily volatility, this leads to the generation of ready-made conclusions, and in practice, the model is a black box for the risk manager without knowing the sequence of stages for its application. A mandatory condition for the accurate application of the GARCH models is that the investigated values of the investment instruments are stationary. That is, the mean value and the most prominent risk measure of the variance are independent of the time factor. In practice, several methods are known for determining the stationarity of the studied time series Augmented Dickey–Fuller (ADF) test, the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and the Johansen test. The presence of stationarity in the net asset value of investment funds is a confirmation of the fulfillment of the random walk hypothesis. Another significant drawback of the classical version of the GARCH models is the implicit assumption that the prices of financial instruments assume a normal cumulative probability distribution. Due to the strong dynamics of financial markets, it would be difficult to unequivocally accept the opinion that the value of financial assets follows a standard probability distribution. In extreme situations, financial thought has developed various variants of autoregressive conditional heteroskedasticity models with different probability distributions Laplace, Student and GED.

The exponential general autoregressive conditional heteroskedastic (EGARCH) model was created in 1991 by Nelson and is part of the asymmetric models for forecasting daily volatility. A characteristic feature of the model is the inclusion of the leverage effect in its construction. In line with this, a specific feature of the model is related to the different weighting of the positive and negative values of the deviations related to the dynamics of variability (Krasteva 2016). The formula for determining EGARCH (1.1) is as follows (Nelson 1991):

$$Log\, v_t^2 = \omega + \alpha * \left| \frac{\varepsilon_{t-1}}{v_{t-1}} \right| + \theta * \frac{\varepsilon_{t-1}}{v_{t-1}} + \beta * log * v_{t-1}^2; \quad (4)$$

$$\varepsilon_t^2 = v_t^2 + u_t; \quad (5)$$

$$L_{(\mu,\alpha,\beta,\omega)} = \frac{1}{v_t * \sqrt{2 * p}} * \varepsilon^{-\frac{\varepsilon_t^2}{2v_t^2}}; \quad (6)$$

where:

v_t^2—conditional return variance;
ω—constant value;
α—regression coefficient considering the ARCH effect;
β—regression coefficient considering the GARCH effect;
ε—parameter accounting for the deviation from the model;
θ—regression coefficient taking into account the effect of the leverage effect.

GARCH-M (1.1) represents a mean generalized autoregressive conditional heteroskedasticity model. An essential importance of the model is the relationship between the average return and the conditional variance (Nugroho et al. 2019). The application of the GARCH–M (1.1) model is determined according to the following methodology (Bollerslev et al. 1988):

$$v_t^2 = \omega + \alpha * \varepsilon_{t-1}^2 + \beta * v_{t-1}^2; \quad (7)$$

$$\varepsilon_t^2 = v_t^2 + u_t; \quad (8)$$

$$L_{(\mu,\alpha,\beta,\omega)} = \frac{1}{v_t * \sqrt{2 * p}} * \varepsilon^{-\frac{\varepsilon_t^2}{2v_t^2}}; \quad (9)$$

$$r_t = \mu + \delta * v_t^2 + \varepsilon_t; \quad (10)$$

$$r_t = \mu + \delta * v_t + \varepsilon_t; \quad (11)$$

where:

v_t^2—conditional return variance;
ω—constant value;
α—regression coefficient considering the ARCH effect;
β—regression coefficient considering the GARCH effect;
ε—parameter accounting for the deviation from the model.

The TGARCH (1.1) model was created in 1994 by Zakoian (1994). In the specialized literature, the analyzed model resembles the GJR model. The only difference between the two models is related to the fact that it uses the conditional standard deviation. The formula for calculating the TGARCH model (1.1) is as follows (Zakoian 1994):

$$v_t^2 = \omega + \alpha * \varepsilon_{t-1}^2 + \theta * \varepsilon_{t-1}^2 * I * \left(\varepsilon_{t-1}^2 < 0\right) + \beta * v_{t-1}^2; \quad (12)$$

$$\varepsilon_t^2 = v_t^2 + u_t; \quad (13)$$

$$L_{(\mu,\alpha,\beta,\omega)} = \frac{1}{v_t * \sqrt{2 * p}} * \varepsilon^{-\frac{\varepsilon_t^2}{2v_t^2}} \quad (14)$$

where:

v_t^2—conditional return variance;
ω—constant value;
α—regression coefficient considering the ARCH effect;
β—regression coefficient considering the GARCH effect;
ε—parameter accounting for the deviation from the model;

θ—regression coefficient taking into account the effect of the leverage effect.

4. Empirical Results from Model Approbation

To establish the presence of stationarity in the yield of the contract funds, we applied the world-famous Ducky–Fuller test. In order to be as objective as possible, we have presented three varieties of the stationarity test in tabular form as follows: model no drift, model drift and model drift + trend. For the period from 13 July 2020 to 13 July 2023, for all investment funds studied by us, the presence of stationarity in the studied data was observed when testing the three specifications of the Ducky–Fuller test. For the period from 13 July 2016 to 12 July 2020, non-stationarity in the time series of the yield was found only in the case of the investment fund CCB Privit in the specification of the Ducky–Fuller test model drift + trend. The obtained results relevant to the stationarity allow us to apply a methodology for risk prediction including the methods of generalized conditional heteroskedasticity.

Results from the application Ducky–Fuller Test Stationarity (13 July 2016–12 July 2020): model no drift, model drift, model drift + trend and (13 July 2020–13 July 2023) model notrend, model trend, model trend + drift in Appendix A (Tables A1–A6).

A number of researchers in their research use specialized software to determine the specification of the final equations of the GARCH family models. We, the authors of the present study, referring to a number of authoritative scientific studies in the field of risk modeling of financial instruments, point out that the final equations of the models with specification (1.1) stand out with the best results. We, the authors of the development, have empirically applied all the models and tests in the MS Excel environment in relation to the GARCH models we have used to determine the regression parameters through optimization with the additional Solver function. The constructed final equations of the models with specification (1.1) form results that give us reason to claim that they are suitable for forecasting the volatility of investment funds operating in the territory of Bulgaria.

Table 1 presents the formed results of the GARCH model testing.

Table 1. Results from the application of the GARCH model (1.1) (13 July 2016–12 July 2020).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Select Balance	−0.00051%	0.00001	0.3528	0.3037	0.6565	0.438%
Select Regional	0.0364%	0.00001	0.1980	0.7021	0.9001	0.975%
Select Bonds	0.0102%	0.00000	0.2679	0.1150	0.3830	0.127%
Select Dividend	0.0284%	0.00000	0.2160	0.6712	0.8872	0.501%
CKB Active	−0.0027%	0.00001	0.3190	0.2660	0.5849	0.490%
CKB Leader	0.0014%	0.49793	0.2046	0.7025	0.0000	-
CKB Garant	0.0575%	0.00000	0.9974	0.0005	0.9979	2.227%
CKB Private	0.0057%	0.00000	0.0000	0.0000	0.0000	0.100%
DV Balance	0.0069%	0.00000	0.3694	0.4637	0.8331	0.423%
DV Dynamic	0.0508%	0.00000	0.2438	0.6483	0.8920	0.584%
Astra Cash Plus	0.0028%	0.00001	0.12598	0.71267	0.83865	0.782%
Astra Global Equity	-	-	-	-	-	-
SKY New Shares	0.0247%	0.00001	0.21227	0.55516	0.76743	0.559%
SKY Finance	0.0630%	0.00001	0.27814	0.57405	0.85219	0.883%
Quest Vision	0.0106%	0.00000	0.16203	0.47655	0.63858	0.314%
Prime Assets	0.0117%	0.00000	0.21462	0.44049	0.65511	0.341%
South Market Maximum	0.0042%	0.00001	0.13001	0.35011	0.48012	0.437%
South Market Optimum	0.0107%	0.00011	0.00011	0.00011	0.00011	0.011%

Table 1. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Trend Balanced Fund	−0.0113%	0.00000	0.67883	0.15851	0.83734	0.248%
Trend Fund Shares	−0.0140%	0.00000	0.52610	0.19781	0.72391	0.190%
Trend Conservative Fund	−0.0061%	0.00000	0.75959	0.23411	0.99370	1.260%
Texim Conservative Fund	−0.0088%	0.00000	0.00000	0.00000	0.00000	0.032%
Texim Balkans	−0.0429%	0.00001	0.16872	0.83126	0.99998	48.057%
Texim Commodity Strategies	−0.0087%	0.00001	0.15118	0.78136	0.93254	0.980%
Prestige	0.0592%	0.00004	0.15432	0.09701	0.25133	0.732%
Profit	0.0155%	0.00003	0.16678	0.25760	0.42438	0.672%
FIB Avangard	0.0237%	0.00001	0.33795	0.48074	0.81869	0.558%
FIB Classic	0.0091%	0.00000	0.24473	0.54454	0.78928	0.300%
FIB Garant	−0.0007%	0.00000	0.29949	0.17265	0.47214	0.138%
First Financial Broker house VOSTOK	0.0602%	0.00004	0.25179	0.49463	0.74642	1.305%
Golden Lev	0.0175%	0.00001	0.47818	0.29229	0.77047	0.723%
Golden Lev Index 30	0.0935%	0.00021	0.80295	0.14843	0.95138	6.579%
EF Rapid	−0.0416%	0.00001	0.95065	0.00045	0.95110	1.591%
Arkus Balanced	-	-	-	-	-	-
Arkus Dynamic	-	-	-	-	-	-
Concord-1 Stocks and Bonds	0.0195%	0.00000	0.17794	0.44565	0.62360	0.281%
Concord-2 Stocks	0.0160%	0.00001	0.17991	0.48371	0.66362	0.395%
Concord-3 Real Estate	0.0365%	0.00001	0.16069	0.62969	0.79038	0.779%
Concord-4 Energetics	0.1000%	0.00002	0.26404	0.65247	0.91651	1.507%
Concord-5 CEE	−0.0017%	0.00000	0.22032	0.73513	0.95545	0.533%
Concord-6 Bonds	0.0095%	0.00000	0.25482	0.00555	0.26037	0.116%
Texim Bulgaria	0.0327%	0.00002	0.25741	0.08504	0.34245	0.559%

Table 2 presents the formed results of the EGARCH model testing.

Table 2. Results from the application of the EGARCH model (1.1) (13 July 2016–12 July 2020).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Asymmetric Term (Theta)	GARCH (Beta)	Long-Run Volatility
Select Balance	0.0147%	−0.133778	0.037581	−0.141446	0.990120	0.115%
Select Regional	0.0211%	−0.502641	0.208523	0.024147	0.961954	0.135%
Select Bonds	0.0095%	−1.348710	0.217762	−0.023247	0.913321	0.042%
Select Dividend	0.0017%	−0.848907	0.221359	−0.132245	0.936220	0.129%
CKB Active	−0.0409%	−4.784409	0.298394	−0.304587	0.576997	0.350%
CKB Leader	−0.0536%	−5.363801	0.474007	−0.229984	0.519183	0.378%
CKB Garant	−0.0223%	−13.187516	3.160132	−2.179679	0.127200	0.052%
CKB Private	−0.0332%	−0.559091	0.135314	−1.578143	0.957776	0.133%
DV Balance	−0.0062%	−1.164634	0.252830	−0.073216	0.911789	0.136%
DV Dynamic	0.0078%	−0.986865	0.266082	−0.113865	0.924667	0.143%
Astra Cash Plus	−0.0059%	−0.801030	0.018132	−0.229065	0.919221	0.703%
Astra Global Equity	-	-	-	-	-	-

Table 2. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Asymmetric Term (Theta)	GARCH (Beta)	Long-Run Volatility
SKY New Shares	0.0003%	−0.884002	0.089975	−0.132900	0.921351	0.362%
SKY Finance	−0.0036%	−0.991541	0.184349	−0.124715	0.912765	0.340%
Quest Vision	−0.0161%	−0.794969	0.098282	−0.340524	0.935571	0.209%
Prime Assets	−0.0246%	−0.735022	0.092661	−0.313361	0.939164	0.238%
South Market Maximum	0.0100%	−5.692878	0.245647	0.006083	0.492656	0.366%
South Market Optimum	0.0061%	−2.670243	0.253499	0.062935	0.784018	0.207%
Trend Balanced Fund	−0.0056%	−2.524988	0.398429	−0.126233	0.826602	0.069%
Trend Fund Shares	−0.0182%	−0.000182	−0.000182	−0.000182	−0.000182	−0.018%
Trend Conservative Fund	−0.0030%	−6.891154	2.481519	−0.940855	0.576435	0.029%
Texim Conservative Fund	−0.0074%	−11.783716	−0.870808	−0.635879	0.328748	0.015%
Texim Bulgaria	0.0202%	−8.018118	0.397719	−0.030617	0.260571	0.442%
Texim Balkans	−0.2299%	−7.213861	2.684661	0.850072	0.386393	0.280%
Texim Commodity Strategies	−0.0393%	−0.521541	0.108980	−0.104688	0.953087	0.385%
Prestige	0.0791%	−4.898435	0.075700	0.351251	0.506615	0.698%
Profit	0.0238%	−0.416202	0.136427	−0.131487	0.965526	0.239%
FIB Avangard	0.0043%	−1.225117	0.281004	−0.140665	0.905047	0.158%
FIB Classic	−0.0015%	−0.962760	0.197497	−0.065607	0.930171	0.101%
FIB Garant	−0.0039%	−1.408472	0.314174	−0.069676	0.912900	0.031%
First Financial Broker house VOSTOK	0.0118%	−1.407109	0.152858	−0.174206	0.854204	0.802%
Golden Lev	0.0246%	−7.960049	0.654893	0.096034	0.275564	0.411%
Golden Lev Index 30	0.1060%	−2.277610	0.516973	−1.000974	0.768268	0.734%
EF Rapid	0.0000%	−13.204648	0.846037	0.318874	−0.169311	0.353%
Arkus Balanced	-	-	-	-	-	-
Arkus Dynamic	-	-	-	-	-	-
Concord-1 Stocks and Bonds	0.0138%	−3.969594	0.299435	−0.094805	0.682060	0.194%
Concord-2 Stocks	0.0221%	−2.513983	0.287219	−0.042612	0.792382	0.235%
Concord-3 Real Estate	0.0256%	−1.814898	0.244145	−0.158520	0.831085	0.464%
Concord-4 Energetics	−0.0250%	−0.475678	0.154012	−0.105092	0.957326	0.380%
Concord-5 CEE	−0.0068%	−0.452320	0.101433	−0.107245	0.965385	0.145%
Concord-6 Bonds	0.0030%	−0.006785	0.029188	−0.071259	0.999710	0.001%

Table 3 presents the formed results of the GARCH–M model testing.

Table 3. Results from the application of the GARCH–M model (1.1) (13 July 2016–12 July 2020).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Risk Premium (Delta)	Alpha + Beta	Long-Run Volatility
Select Balance	−0.2080%	0.000008	0.373297	0.213271	0.622851	0.586568	0.4284%
Select Regional	0.0897%	0.000001	0.052666	0.934612	−0.099348	0.987278	0.8414%
Select Bonds	0.0051%	0.000001	0.000000	0.000000	0.000000	0.000000	0.1143%
Select Dividend	0.0017%	0.000002	0.141180	0.793347	0.053039	0.934527	0.4814%
CKB Active	−0.3724%	0.000011	0.342677	0.208929	0.929629	0.551606	0.4858%
CKB Leader	−0.4161%	0.000017	0.666357	−0.023683	0.870400	0.642674	0.6952%

Table 3. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Risk Premium (Delta)	Alpha + Beta	Long-Run Volatility
CKB Garant	0.0097%	0.000007	0.000000	0.000000	0.000000	0.000000	0.2668%
CKB Private	0.0057%	0.000001	0.000954	0.001656	0.000000	0.002610	0.0813%
DV Balance	−0.0670%	0.000003	0.371647	0.428955	0.269456	0.800602	0.4032%
DV Dynamic	0.0778%	0.000003	0.172185	0.744820	−0.084234	0.917005	0.5501%
Astra Cash Plus	−0.0435%	0.000073	0.000000	0.000000	0.000000	0.000000	0.8566%
Astra Global Equity	-	-	-	-	-	-	-
SKY New Shares	−0.0477%	0.000004	0.144890	0.741542	0.148289	0.886432	0.5630%
SKY Finance	−0.0036%	0.000003	0.110155	0.834381	0.058134	0.944536	0.7720%
Quest Vision	0.0119%	0.000007	0.085741	0.076037	0.002797	0.161778	0.2859%
Prime Assets	−0.0429%	0.000004	0.172555	0.481268	0.182577	0.653823	0.3325%
South Market Maximum	−0.2608%	0.000006	0.096085	0.575166	0.642642	0.671251	0.4323%
South Market Optimum	0.0112%	0.000010	0.000000	0.000000	0.000000	0.000000	0.3083%
Trend Balanced Fund	−0.0138%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1752%
Trend Fund Shares	−0.0192%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1768%
Trend Conservative Fund	−0.0464%	0.000005	0.000000	0.000000	0.000000	0.000000	0.2297%
Texim Conservative Fund	−0.0088%	0.000000	0.000000	0.000000	0.000000	0.000000	0.0112%
Texim Bulgaria	−0.2344%	0.000021	0.248803	0.083991	0.520892	0.332794	0.5565%
Texim Balkans	−0.0124%	0.000301	0.159510	−0.018827	0.007116	0.140682	1.8721%
Texim Commodity Strategies	0.1125%	0.000003	0.113383	0.850792	−0.156923	0.964175	0.9673%
Prestige	0.1644%	0.000047	0.176299	−0.026372	−0.147820	0.149927	0.7445%
Profit	−0.2657%	0.000041	0.212225	−0.058938	0.386171	0.153287	0.6968%
FIB Avangard	−0.0865%	0.000002	0.154272	0.780774	0.236418	0.935047	0.5145%
FIB Classic	−0.0736%	0.000001	0.158555	0.704972	0.318816	0.863528	0.2905%
FIB Garant	−0.0039%	0.000002	0.000000	0.000000	0.000000	0.000000	0.1326%
First Financial Broker house VOSTOK	−0.1446%	0.000045	0.243309	0.490369	0.191116	0.733678	1.3039%
Golden Lev	−0.0455%	0.000021	0.548210	0.012409	0.098973	0.560619	0.6916%
Golden Lev Index 30	0.0162%	0.000602	−0.001492	−0.480556	0.010248	−0.482048	2.0149%
EF Rapid	−0.0025%	0.000026	0.285619	−0.021891	−0.003574	0.263729	0.5936%
Arkus Balanced	-	-	-	-	-	-	-
Arkus Dynamic	-	-	-	-	-	-	-
Concord-1 Stocks and Bonds	0.0203%	0.000008	0.000000	0.000000	0.000000	0.000000	0.2747%
Concord-2 Stocks	−0.1010%	0.000004	0.152533	0.565565	0.327284	0.718099	0.3922%
Concord-3 Real Estate	−0.1132%	0.000013	0.153456	0.629427	0.225164	0.782883	0.7689%
Concord-4 Energetics	−0.0259%	0.000006	0.114721	0.837279	0.060528	0.952000	1.1282%
Concord-5 CEE	−0.0519%	0.000000	0.079718	0.889537	0.189971	0.969255	0.3562%
Concord-6 Bonds	0.0073%	0.000001	0.000000	0.000000	0.000000	0.000000	0.0945%

Table 4 presents the formed results of the TGARCH model testing.

Table 4. Results from the application of the TGARCH model (1.1) (13 July 2016–12 July 2020).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Negative TGARCH (Theta)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Select Balance	−0.0039%	0.000002	−0.022063	0.248239	0.775952	0.753889	0.2897%
Select Regional	0.0260%	0.000002	0.067493	0.013904	0.908190	0.975684	0.7933%
Select Bonds	0.0051%	0.000001	0.000000	0.000000	0.000000	0.000000	0.1143%
Select Dividend	0.0101%	0.000001	0.020271	0.161077	0.836001	0.856272	0.3000%
CKB Active	−0.0276%	0.000007	−0.033829	0.560385	0.458438	0.424609	0.3554%
CKB Leader	−0.0425%	0.000015	−0.010572	0.718000	0.247579	0.237006	0.4388%
CKB Garant	0.0097%	0.000007	0.000000	0.000000	0.000000	0.000000	0.2669%
CKB Private	0.0057%	0.000001	0.000902	0.000642	0.001566	0.002467	0.0812%
DV Balance	−0.0063%	0.000004	0.113449	0.477627	0.383255	0.496703	0.2733%
DV Dynamic	0.0322%	0.000002	0.056059	0.173744	0.787324	0.843383	0.3540%
Astra Cash Plus	−0.0204%	0.000138	0.024843	−0.026399	−1.005045	−0.980202	0.8354%
Astra Global Equity	−0.0064%	0.000000	0.065238	0.204743	0.882739	0.947977	0.0507%
SKY New Shares	0.0145%	0.000003	0.021419	0.141342	0.789674	0.811092	0.4172%
SKY Finance	0.0295%	0.000004	0.048009	0.114913	0.817738	0.865746	0.5593%
Quest Vision	0.0129%	0.000006	−0.017085	0.264211	0.212210	0.195126	0.2634%
Prime Assets	−0.0027%	0.000000	−0.032015	0.379581	0.866810	0.834795	0.1694%
South Market Maximum	0.0052%	0.000006	0.084736	0.005157	0.580749	0.665485	0.4302%
South Market Optimum	0.0112%	0.000010	0.000000	0.000000	0.000000	0.000000	0.3083%
Trend Balanced Fund	−0.0138%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1752%
Trend Fund Shares	−0.0192%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1768%
Trend Conservative Fund	−0.0464%	0.000005	0.000000	0.000000	0.000000	0.000000	0.2297%
Texim Conservative Fund	−0.0174%	0.000000	0.000000	0.000000	0.000000	0.000000	0.0141%
Texim Bulgaria	0.0255%	0.000020	0.172733	0.251184	0.086548	0.259281	0.5220%
Texim Balkans	−0.0122%	0.000296	1.000389	−0.888978	−0.062624	0.937765	6.9000%
Texim Commodity Strategies	−0.0398%	0.000003	0.018828	0.133603	0.884378	0.903207	0.5611%
Prestige	0.0639%	0.000047	0.202318	−0.051127	−0.033216	0.169101	0.7561%
Profit	0.0079%	0.000038	0.219634	0.013237	−0.017560	0.202073	0.6944%
FIB Avangard	0.0054%	0.000002	0.074872	0.179624	0.748463	0.823335	0.3556%
FIB Classic	−0.0013%	0.000001	0.079499	0.110227	0.724146	0.803645	0.2420%
FIB Garant	−0.0039%	0.000002	0.000000	0.000000	0.000000	0.000000	0.1326%
First Financial Broker house VOSTOK	0.0139%	0.000040	0.022030	0.294933	0.565766	0.587796	0.9896%
Golden Lev	0.0184%	0.000023	0.733752	−0.498619	−0.018305	0.715447	0.8919%
Golden Lev Index 30	0.0162%	0.000541	−0.002197	0.020168	−0.080124	−0.082321	2.2366%
EF Rapid	−0.0025%	0.000025	0.151297	0.134768	−0.042315	0.108981	0.5349%
Arkus Balanced	0.0019%	0.000018	−0.188803	0.211803	−0.662238	−0.851041	0.3144%

Table 4. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Negative TGARCH (Theta)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Arkus Dynamic	0.0248%	0.000002	−0.047209	0.182892	0.773373	0.726164	0.2580%
Concord-1 Stocks and Bonds	0.0203%	0.000008	0.000000	0.000000	0.000000	0.000000	0.2747%
Concord-2 Stocks	0.0147%	0.000004	0.103760	0.098305	0.614180	0.717940	0.3619%
Concord-3 Real Estate	0.0190%	0.000011	0.040263	0.281957	0.659252	0.699515	0.6058%
Concord-4 Energetics	−0.0005%	0.000005	−0.021698	0.174692	0.881682	0.859984	0.6241%
Concord-5 CEE	−0.0026%	0.000000	0.018418	0.108731	0.906802	0.925220	0.2083%
Concord-6 Bonds	0.0073%	0.000001	0.000000	0.000000	0.000000	0.000000	0.0945%

For development purposes, we have extended the analysis for forecasting the daily volatility of investment funds operating on the Bulgarian capital market. In order to present in maximum objectivity the impact of COVID-19 on the volatility of contract funds, we have expanded the scale by another four calendar years, covering the period from 13 July 2016 to 12 July 2020. This will allow us to assess in detail the impact of COVID-19 on the investment behavior of collective investment schemes and their financial status. Specifically, the formed results allow us to establish that for the reporting period (2016–2020) with the highest risk concentration determined by the GARCH model (1.1), the Texim Balkani investment fund with Long-run volatility in the amount of 48.057% is reported. The main reason for the high estimated values of the volatility of the investment fund is predetermined by the structure of the fund's portfolio. The investment portfolio of Texim Balkani includes shares of companies from countries that are part of the Balkan Peninsula. A characteristic feature of the analyzed financial markets is that they are part of developing markets, for which high individual volatility is clearly expressed. The Balkan capital markets were heavily affected by COVID–19 and generated negative results. Exceptions to this group are the Turkish Capital Market and the Greek Stock Market. Key features of this group of financial markets are low liquidity, insignificant exchange turnover and exchange volume and a minimal number of transactions. The second position in terms of daily volatility is assigned to the investment fund "Golden Lev Index" 30 Long-run volatility in the amount of 6.579%. The investment profile of the fund represents the imitation of the Bulgarian stock index BG TR30. The high-risk concentration of the fund is determined by the serious impact of COVID-19 on the companies that make up the index. The crisis in the health sector influenced by the pandemic has seriously affected all sectors of the Bulgarian economy, which inevitably reflects on the value of investment funds. In the EGARCH (1.1) model with the highest risk attribution, the investment fund PFBK VOSTOK with Long-run volatility in the amount of 0.802% is considered. The reason for the high volatility value of the contract fund is a result of the fund's investment portfolio, which is entirely invested in companies from Russia. As a result of the military conflict between the Russian Federation and Ukraine, the Russian economy suffered numerous sanctions, which reflected negatively on the state of the Russian financial market. The results of testing the GARCH-M (1.1) model confirm the high volatility of the two investment funds "Golden Lev Index" 30 Long-run volatility in the amount of 2.015% and PFBK VOSTOK with Long-run volatility in the amount of 1.304%.

The present study covers quantitative assessment and forecasting of the daily volatility of forty-two investment funds for the period from 13 July 2020 to 13 July 2023[1]. Due to the huge scale of researched and applied risk prediction models in separate tables, results will be presented. With the help of the graphical analysis toolkit, several separate models will be selected related to the forecasting of the daily volatility[2].

Table 5 presents the results of the application of the GARCH model GARCH (1.1).

Table 5. Results from the application of the GARCH model (1.1) (13 July 2020–13 July 2023).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Select Balance	0.0117%	0.000015	0.179990	0.344771	0.524761	0.562%
Select Regional	0.0998%	0.000031	0.259877	0.470541	0.730418	1.074%
Select Bonds	0.0061%	0.000001	0.181995	0.628441	0.810437	0.230%
Select Dividend	0.0260%	0.000006	0.117664	0.677421	0.795086	0.533%
CKB Active	0.0095%	0.000012	0.159047	0.000000	0.159047	0.374%
CKB Leader	0.0096%	0.000012	0.228463	0.000000	0.228463	0.395%
CKB Garant	0.0042%	0.000001	0.431500	0.458081	0.889582	0.301%
CKB Private	−0.0026%	0.000001	0.170830	0.792752	0.963583	0.524%
DV Balance	0.0119%	0.000001	0.198773	0.480975	0.679747	0.177%
DV Dynamic	0.0413%	0.000004	0.153182	0.608414	0.761596	0.434%
Astra Cash Plus	−0.0126%	0.000001	0.246391	0.689840	0.936231	0.396%
Astra Global Equity	0.0050%	0.000001	0.180117	0.803508	0.983625	0.788%
SKY New Shares	0.0523%	0.000010	0.218903	0.478696	0.697600	0.583%
SKY Finance	0.0751%	0.000014	0.237612	0.638733	0.876345	1.061%
Quest Vision	0.0087%	0.000008	0.086143	0.000000	0.086143	0.301%
Prime Assets	0.0256%	0.000011	0.128880	0.140671	0.269552	0.384%
South Market Maximum	−0.0009%	0.000010	0.304325	0.211493	0.515817	0.464%
South Market Optimum	−0.0025%	0.000011	0.270683	0.246504	0.517187	0.479%
Trend Balanced Fund	−0.0044%	0.000009	0.379290	0.207542	0.586832	0.467%
Trend Fund Shares	0.0038%	0.000011	0.268189	0.277552	0.545741	0.486%
Trend Conservative Fund	−0.0154%	0.000003	0.821327	0.018144	0.839471	0.425%
Texim Conservative Fund	−0.0175%	0.000000	0.861974	0.117453	0.979427	0.220%
Texim Balkans	0.0366%	0.000006	0.588560	0.331633	0.920194	0.876%
Texim Commodity Strategies	0.0992%	0.000043	0.240244	0.519203	0.759447	1.332%
Prestige	0.0390%	0.000014	0.263084	0.325033	0.588116	0.574%
Profit	0.0292%	0.000014	0.197122	0.254173	0.451295	0.503%
FIB Avangard	0.0543%	0.000003	0.262425	0.666324	0.928748	0.655%
FIB Classic	0.0333%	0.000003	0.351587	0.479072	0.830659	0.408%
FIB Garant	0.0079%	0.000001	0.532021	0.273502	0.805523	0.227%
First Financial Broker house VOSTOK	−0.0383%	0.000226	0.202744	0.295854	0.498598	2.124%
Golden Lev	0.0168%	0.000007	0.151379	0.194699	0.346078	0.325%
Golden Lev Index 30	0.1000%	0.005432	0.420240	0.000000	0.420240	9.680%
EF Rapid	−0.0336%	0.000199	0.183841	0.061478	0.245319	1.623%
Arkus Balanced	0.0114%	0.000005	0.137214	0.391358	0.528572	0.324%
Arkus Dynamic	0.0184%	0.000001	0.158081	0.714021	0.872101	0.335%
Concord-1 Stocks and Bonds	0.0014%	0.000001	0.124139	0.795209	0.919348	0.352%
Concord-2 Stocks	0.0107%	0.000001	0.124945	0.663540	0.788486	0.265%
Concord-3 Real Estate	0.0329%	0.000003	0.177857	0.698487	0.876344	0.494%
Concord-4 Energetics	0.1000%	0.000100	0.175984	0.352776	0.528760	1.460%

Table 5. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Concord-5 CEE	0.0231%	0.000009	0.277520	0.237865	0.515385	0.423%
Concord-6 Bonds	0.0099%	0.000004	0.305463	0.234500	0.539963	0.308%
Texim Bulgaria	0.0821%	0.000011	0.895789	0.000000	0.895789	1.037%

Let us emphasize that when testing the GARCH (1.1) model, the sum of the two alpha and beta coefficients is below unity. This is proof of the presence of stationarity in the time series of each of the analyzed investment funds. In practice, we can establish the validity of the GARCH model (1.1) as a suitable model for forecasting the daily volatility of Bulgarian mutual funds. The applied optimization of the parameters by the method of maximum likelihood confirms the precise approbation of the model. In 27 of the analyzed investment funds, a higher value of the beta coefficient compared to alpha is observed. This means in practice that "old news" has a serious effect on the volatility of investment funds. In line with this, a stronger sensitivity is observed in the value of the daily yield of investment funds. From the conducted research, we can establish that the investment fund with the highest expected future volatility measured by the GARCH (1.1) model is "Zlaten Lev Index 30."

Table 6 presents the formed results of the EGARCH model testing.

Table 6. Results of the application of the EGARCH model (1.1) (13 July 2020–13 July 2023).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Asymmetric Term (Theta)	GARCH (Beta)	Long-Run Volatility
Select Balance	−0.0145%	−0.0955	0.0453	−0.0940	0.9927	0.148%
Select Regional	0.0981%	−1.6253	0.3606	−0.0325	0.8499	0.446%
Select Bonds	0.0113%	−0.2318	0.0546	−0.0037	0.9841	0.068%
Select Dividend	0.0234%	−1.0355	0.1457	−0.0781	0.9122	0.276%
CKB Active	0.0165%	−18.4154	0.2757	0.1139	−0.6208	0.341%
CKB Leader	0.0059%	−10.7736	0.4465	−0.0256	0.0606	0.323%
CKB Garant	−0.0081%	−0.1825	0.0500	−0.0300	0.9876	0.062%
CKB Private	0.0098%	−0.1903	0.0766	−0.1123	0.9877	0.045%
DV Balance	0.0071%	−0.3305	0.0466	−0.0669	0.9768	0.080%
DV Dynamic	0.0273%	−0.7629	0.1141	−0.1202	0.9386	0.201%
Astra Cash Plus	−0.0052%	−1.4727	0.3539	−0.0253	0.8952	0.089%
Astra Global Equity	−0.0133%	−0.3204	0.2380	−0.0615	0.9823	0.012%
SKY New Shares	0.0324%	−0.7915	0.1784	−0.0859	0.9362	0.203%
SKY Finance	0.0376%	−0.9774	0.1712	−0.1888	0.9108	0.417%
Quest Vision	0.0096%	−22.3111	0.0142	−0.2311	−0.9057	0.287%
Prime Assets	0.0195%	−6.9149	0.3330	0.0153	0.3965	0.325%
South Market Maximum	−0.0321%	−5.0678	0.3425	−0.2277	0.5571	0.327%
South Market Optimum	0.0095%	−0.3334	0.0948	−0.1343	0.9746	0.140%
Trend Balanced Fund	−0.0323%	−1.7067	−0.1136	−0.4585	0.8372	0.529%
Trend Fund Shares	−0.0411%	−0.0004	−0.0004	−0.0004	−0.0004	−0.041%
Trend Conservative Fund	−0.0141%	−10.8532	0.7682	0.0705	0.1543	0.163%
Texim Conservative Fund	−0.0157%	−4.0820	1.7708	0.4022	0.7972	0.004%
Texim Bulgaria	0.0287%	−10.2673	1.1554	−0.4219	0.1291	0.275%
Texim Balkans	−0.0066%	−3.7314	0.0323	−0.4309	0.6635	0.391%

Table 6. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Asymmetric Term (Theta)	GARCH (Beta)	Long-Run Volatility
Texim Commodity Strategies	0.0542%	−2.7020	0.3622	−0.2200	0.7246	0.741%
Prestige	0.0447%	−1.7483	0.4401	−0.0214	0.8616	0.181%
Profit	0.0375%	−3.5292	0.4744	−0.0435	0.6996	0.281%
FIB Avangard	0.0140%	−0.4799	0.0891	−0.1342	0.9611	0.210%
FIB Classic	0.0038%	−1.3712	0.1894	−0.2032	0.8925	0.170%
FIB Garant	−0.0055%	−1.6071	0.3444	−0.0934	0.8941	0.051%
First Financial Broker house VOSTOK	−0.0949%	−0.5371	−0.0746	−0.3174	0.9312	2.023%
Golden Lev	0.0308%	−2.5667	0.3212	0.0291	0.7924	0.206%
Golden Lev Index 30	−0.1255%	−2.1068	6.8298	−5.9521	0.8159	0.328%
EF Rapid	0.0230%	−14.6475	−0.7347	−1.0213	−0.7112	1.385%
Arkus Balanced	0.0024%	−1.0162	0.0571	−0.0477	0.9150	0.253%
Arkus Dynamic	0.0121%	−1.5983	0.1607	−0.0911	0.8712	0.202%
Concord-1 Stocks and Bonds	0.0173%	−0.7787	0.2246	0.0357	0.9448	0.086%
Concord-2 Stocks	0.0138%	−0.9441	0.1561	−0.0115	0.9296	0.122%
Concord-3 Real Estate	0.0488%	−11.0446	0.4003	0.0911	−0.0020	0.404%
Concord-4 Energetics	0.0886%	−4.2033	0.2776	−0.1643	0.5312	1.130%
Concord-5 CEE	0.0374%	−15.7695	0.3642	0.3220	−0.3957	0.352%
Concord-6 Bonds	0.0048%	−6.8613	0.4231	−0.0256	0.4374	0.225%

The results formulated with the asymmetric risk measure EGARCH for all forty-two investment funds show a successful application of the model. This is implied by the generated model parameter values. Here, unlike the GARCH model in the optimization process, some of the coefficients can form negative values. The indicated final forecasts for the risk concentration of the investment funds analyzed by us are strongly influenced by the leverage effect. With the highest expected volatility, the "Golden Lev Index 30" fund stands out again.

Table 7 shows the results of applying the GARCH–M (1.1) quantitative risk assessment model.

Table 7. Results of the application of the GARCH-M model (1.1) (13 July 2020–13 July 2023).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Risk Premium (Delta)	Alpha + Beta	Long-Run Volatility
Select Balance	−0.3180%	0.000054	−0.015316	−0.818872	0.561018	−0.834188	0.5429%
Select Regional	0.2082%	0.000029	0.245575	0.503478	−0.100784	0.749053	1.0671%
Select Bonds	0.0105%	0.000004	0.000000	0.000000	0.000000	0.000000	0.2013%
Select Dividend	−0.2416%	0.000003	0.077059	0.827363	0.526313	0.904423	0.5266%
CKB Active	0.0104%	0.000013	0.146374	−0.109520	−0.004383	0.036855	0.3729%
CKB Leader	−0.0734%	0.000012	0.222467	0.011607	0.225635	0.234075	0.3945%
CKB Garant	0.0000%	0.000005	0.000000	0.000000	0.000000	0.000000	0.2189%
CKB Private	−0.0215%	0.000000	0.084761	0.932215	0.096833	1.016976	0.0543%
DV Balance	0.0071%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1753%
DV Dynamic	−0.0866%	0.000001	0.063527	0.887019	0.306928	0.950546	0.4215%

Table 7. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	GARCH (Beta)	Risk Premium (Delta)	Alpha + Beta	Long-Run Volatility
Astra Cash Plus	−0.0215%	0.000001	0.195043	0.775806	0.044009	0.970850	0.4223%
Astra Global Equity	−0.0164%	0.000000	0.109386	0.880947	0.059509	0.990333	0.1179%
SKY New Shares	−0.0751%	0.000002	0.092792	0.842565	0.247508	0.935356	0.5723%
SKY Finance	−0.2030%	0.000008	0.153735	0.763181	0.346071	0.916915	0.9792%
Quest Vision	−0.0858%	0.000009	0.079459	−0.063860	0.343903	0.015599	0.3002%
Prime Assets	0.0174%	0.000013	0.121975	−0.023988	0.027814	0.097987	0.3840%
South Market Maximum	−0.5798%	0.000014	0.382737	−0.065346	1.540238	0.317390	0.4508%
South Market Optimum	−0.8207%	0.000016	0.208642	−0.037411	2.027426	0.171231	0.4348%
Trend Balanced Fund	−0.6008%	0.000011	0.403150	0.013011	1.647963	0.416161	0.4374%
Trend Fund Shares	−0.7117%	0.000014	0.265404	0.098799	1.738544	0.364202	0.4627%
Trend Conservative Fund	0.0086%	0.000006	0.000000	0.000000	0.000000	0.000000	0.2504%
Texim Conservative Fund	−0.0174%	0.000000	0.000000	0.000000	0.000000	0.000000	0.0599%
Texim Bulgaria	0.0954%	0.000011	0.917246	−0.007563	−0.021089	0.909682	1.1203%
Texim Balkans	0.0035%	0.000034	−0.010302	−0.884337	0.062395	−0.894639	0.4257%
Texim Commodity Strategies	0.0011%	0.000005	0.068636	0.899614	0.067794	0.968250	1.2150%
Prestige	0.1867%	0.000005	0.216711	0.645909	−0.302093	0.862619	0.6031%
Profit	0.0896%	0.000013	0.200603	0.298727	−0.128052	0.499330	0.5042%
FIB Avangard	−0.0246%	0.000001	0.124180	0.843251	0.143501	0.967431	0.5694%
FIB Classic	−0.1050%	0.000002	0.298673	0.560438	0.444921	0.859111	0.3846%
FIB Garant	−0.0056%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1831%
First Financial Broker house VOSTOK	0.3726%	0.000246	0.194242	0.244418	−0.232906	0.438660	2.0940%
Golden Lev	−0.0880%	0.000002	0.207720	0.651029	0.372378	0.858749	0.3857%
Golden Lev Index 30	−0.0880%	0.048913	0.270837	−0.090227	0.002456	0.180610	24.4323%
EF Rapid	0.0053%	0.000214	0.157476	−0.006928	−0.034340	0.150548	1.5854%
Arkus Balanced	−0.1888%	0.000012	−0.061968	−0.118990	0.575710	−0.180958	0.3152%
Arkus Dynamic	10.9458%	0.000000	0.003494	0.980391	−34.580066	0.983885	0.3204%
Concord-1 Stocks and Bonds	0.0093%	0.000002	0.422036	0.507917	−0.023548	0.929953	0.5618%
Concord-2 Stocks	0.0138%	0.000007	0.000000	0.000000	0.000000	0.000000	0.2603%
Concord-3 Real Estate	0.0592%	0.000002	0.128077	0.790698	−0.074630	0.918774	0.4809%
Concord-4 Energetics	−0.8871%	0.000081	0.141761	0.471487	0.700166	0.613249	1.4505%
Concord-5 CEE	0.4256%	0.000011	0.306790	0.025960	−1.091446	0.332749	0.4137%
Concord-6 Bonds	0.0047%	0.000004	0.337246	0.182766	−0.006518	0.520012	0.3008%

Unlike the two-pass models for predicting the daily volatility of investment funds in the GARCH–M test, for seven of the studied funds, the deterministic final results are not valid. That is, in the optimization process, the parameters of the models form a value

of zero. In practice, this means no possibility for the real application of the model for modeling the risk attribution of investment funds. Therefore, for the funds in question, GARCH–M cannot form a forecast for the future development of daily volatility. Of course, the model has its merits, as an adequate toolkit for assessing the risk of the remaining thirty-five investment funds. Again, as with the EGARCH model, the parameters of the final equation of the model can take negative values. An interesting regularity is observed in the forecast results of the GARCH-M model, where the "risky nature" of the Zlaten Lev Index 30 fund is once again confirmed. However, here the values of the future daily volatility are several times higher compared to the GARCH and EGARCH models.

Table 8 shows the results of testing the TGARCH model.

Table 8. Results of the application of the TGARCH model (1.1) (13 July 2020–13 July 2023).

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Negative TGARCH (Theta)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
Select Balance	0.0109%	0.000029	−0.021688	0.010346	−0.006121	−0.027810	0.5316%
Select Regional	0.1028%	0.000032	0.179460	0.184087	0.447519	0.626979	0.9220%
Select Bonds	0.0105%	0.000004	0.000000	0.000000	0.000000	0.000000	0.2013%
Select Dividend	0.0172%	0.000003	0.037506	0.081197	0.825570	0.863076	0.4438%
CKB Active	0.0099%	0.000015	0.178402	−0.095663	−0.197261	−0.018859	0.3802%
CKB Leader	0.0082%	0.000012	0.201796	0.049577	−0.006491	0.195306	0.3889%
CKB Garant	−0.0074%	0.000000	−0.022512	0.026374	1.007569	0.985057	0.0730%
CKB Private	0.0047%	0.000000	−0.004683	0.102109	0.956227	0.951545	0.0897%
DV Balance	0.0071%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1753%
DV Dynamic	0.0233%	0.000001	−0.005597	0.133212	0.896868	0.891271	0.2644%
Astra Cash Plus	−0.0124%	0.000001	0.181229	0.035026	0.777557	0.958787	0.3500%
Astra Global Equity	−0.0064%	0.000000	0.065238	0.204743	0.882739	0.947977	0.0507%
SKY New Shares	0.0423%	0.000002	0.008678	0.127356	0.862564	0.871242	0.3950%
SKY Finance	0.0547%	0.000007	0.006564	0.282406	0.771240	0.777804	0.5809%
Quest Vision	0.0129%	0.000013	0.059893	0.053770	−0.506297	−0.446405	0.2960%
Prime Assets	0.0129%	0.000013	0.141844	0.016142	−0.018037	0.123807	0.3816%
South Market Maximum	−0.0281%	0.000011	−0.015546	0.627964	0.240460	0.224914	0.3686%
South Market Optimum	−0.0329%	0.000014	−0.013613	0.693807	0.099705	0.086092	0.3886%
Trend Balanced Fund	−0.0285%	0.000008	−0.028536	0.865890	0.253915	0.225379	0.3311%
Trend Fund Shares	−0.0279%	0.000008	−0.019629	0.357052	0.439461	0.419832	0.3768%
Trend Conservative Fund	−0.0196%	0.000002	0.350596	0.009672	0.293495	0.644092	0.2539%
Texim Conservative Fund	−0.0174%	0.000000	0.000000	0.000000	0.000000	0.000000	0.0599%
Texim Bulgaria	0.0751%	0.000018	0.091742	0.554895	−0.122144	−0.030402	0.4147%
Texim Balkans	−0.0038%	0.000037	−0.002007	−0.066060	−0.936559	−0.938565	0.4341%
Texim Commodity Strategies	0.0490%	0.000050	0.023957	0.386619	0.494451	0.518408	1.0178%
Prestige	0.0424%	0.000004	0.182196	0.054516	0.684140	0.866336	0.5548%
Profit	0.0304%	0.000013	0.213300	−0.034570	0.270084	0.483384	0.5108%
FIB Avangard	0.0185%	0.000001	−0.014412	0.170159	0.895275	0.880863	0.2832%
FIB Classic	0.0089%	0.000002	−0.011679	0.428485	0.621750	0.610072	0.2398%

Table 8. *Cont.*

Investment Fund	Constant (mu)	Unconditional Variance (Omega)	ARCH (Alpha)	Negative TGARCH (Theta)	GARCH (Beta)	Alpha + Beta	Long-Run Volatility
FIB Garant	−0.0056%	0.000003	0.000000	0.000000	0.000000	0.000000	0.1831%
First Financial Broker house VOSTOK	−0.0687%	0.000149	−0.207035	0.604347	0.519742	0.312707	1.4703%
Golden Lev	−0.0501%	0.000018	0.006954	0.022345	−0.536935	−0.529981	0.3457%
Golden Lev Index 30	0.0395%	0.010690	0.400155	−0.002364	−0.013324	0.386831	13.2039%
EF Rapid	0.0089%	0.049984	0.037057	0.046158	−1.006125	−0.969068	15.9325%
Arkus Balanced	0.0019%	0.000018	−0.188803	0.211803	−0.662238	−0.851041	0.3144%
Arkus Dynamic	0.0248%	0.000002	−0.047209	0.182892	0.773373	0.726164	0.2580%
Concord-1 Stocks and Bonds	0.0092%	0.000004	0.638698	−0.510334	0.309354	0.948052	0.8794%
Concord-2 Stocks	0.0138%	0.000007	0.000000	0.000000	0.000000	0.000000	0.2603%
Concord-3 Real Estate	0.0322%	0.000002	0.136676	−0.020833	0.790531	0.927207	0.5098%
Concord-4 Energetics	0.0826%	0.000215	−0.063580	0.307735	−0.129547	−0.193127	1.3426%
Concord-5 CEE	0.0430%	0.000014	0.627237	−0.615696	−0.065036	0.562201	0.5626%
Concord-6 Bonds	0.0047%	0.000004	0.342260	−0.135275	0.344985	0.687245	0.3440%

In the final results of the application of the TGARCH model for the forecasting of the risk concentration of the Bulgarian investment funds, it was found in five of them that it was not possible for its adequate applicability. In practice, the parameters of models after the performed optimization assume a value of zero. As a result, it is not possible to implement a quantitative risk assessment. The coefficients of the final equation of the model under the established calculation algorithm can and, accordingly, take negative values. The "EF Rapid" fund stands out with the highest volatility for the future investment horizon—22.29%, followed by "Zlaten Lev Index 30"—15.30%. For the rest of the funds, values in the interval from 0.05% to 2.00% are observed. The main reason for the high volatility of the investment funds is determined by the different investment policies that they carry out related to the selection of specialized investment instruments with different risk–return characteristics. The investment funds analyzed by us, for which we found high volatility, are the result of the inclusion of high-risk assets in the investment portfolios of the funds. By high-risk assets, we mean public company stocks from various geographical locations. Some of the Bulgarian investment funds invest in shares of companies from the Russian Federation in a situation of military actions and constant tension on a global scale. Due to the sanctions imposed by the international organizations, some investment funds were forced to describe from their portfolios the value of the funds invested in Russian share issues. In practice, the management companies were forced to zero out their positions in Russian public companies. The created unfavorable situation seriously reflected negatively on the results of the investment funds, which overexposed and drastically increased their daily volatility. Specifically, the reason for the highest volatility of the "Golden Lev Index 30" investment fund is dictated by the construction of the fund's investment portfolio, which practically imitates the composition and structure of the Bulgarian stock index BG TR30. The index was constructed by thirty public companies from various spheres of the Bulgarian economy—holding structures, pharmaceuticals, financial institutions, real estate investments and technology companies. The companies in question are the most liquid companies traded on the Bulgarian capital market, and it is typical for them to observe serious daily volatility for research by us over a period of time. Confirmation of

reasoning about the "risky nature" of the Bulgarian capital market is related to the fact that it is part of the so-called emerging markets from Eastern Europe, part of the former Eastern Bloc in the specialized literature. The clustering of the financial markets according to their degree of development classifies the Bulgarian stock market as a market with a smaller scale compared to the markets of Western Europe, due to the smaller realized exchange turnover, exchange volume, successfully executed transactions and the size of the companies (in terms of market capitalization). The low values of determinants of the market liquidity of the companies included in the BG TR30 index are the basis for the higher volatility of the "Golden Lev Index 30" investment fund compared to the other investment funds analyzed by us. Regarding the TGARCH model, the investment fund EF Rapid is considered to have the highest daily volatility. The key importance for the high volatility of the investment fund analyzed by us is rooted in its risk–return characteristic. Regarding the risk concentration, the investment fund is determined as balanced with an average degree of risk. The fund's investment portfolio includes financial instruments, shares, debt instruments, bonds and fixed-income instruments. The financial instruments constituting the contract fund belong to the part of the companies of the Bulgarian capital market. Of course, we, as external observers, have no way of knowing exactly which instruments make up the investment portfolio. But the information that priority is invested in Bulgarian companies allows us to analyze the state of the Bulgarian capital market. Without falling into unnecessary repetition, the main determinants that reflected on the volatility of the contract fund "Golden Lev Index 30" are also confirmed for EF Rapid, low market liquidity, low volume of transactions, low exchange turnover and pine volume and unfortunately low investment culture from the side of local market entities.

Scope (Restrictive Conditions)

The research includes forty-two investment funds operating on the territory of Bulgaria. The data are used to explore models for forecasting daily volatility, covering the period from 13 July 2020 to 13 July 2023, and have a daily frequency. From the many models for forecasting the risk attraction of investment funds, the following analytical models are selected: GARCH, EGARCH, TGARCH and GARCH- M with specification 1.1. The limiting conditions of the present study cover only the assessment of the daily volatility of investment funds from Bulgaria. The main driving motive for this is the fact that there is no similar study in the specialized scientific literature with the object of risk assessment (daily volatility) of funds circulating in the territory of the country. In accordance with this, in Bulgarian practice, according to the rules for assessing the risk of investment funds, the classic risk measures known as part of the descriptive statistic standard deviation, dispersion and coefficient of variation, stress tests and scenario analysis are used. Our empirically tested risk forecasting models GARCH, EGARCH, TGARCH and GARCH–M are alternative tools for identifying daily volatility, allowing investors to define in detail the risky nature of investment funds. Pension funds operating on the Bulgarian capital market fall outside the scope of the study due to their specific management investment policy and the specialized set of assets included in their portfolios. The national contractual funds, the shares of which are publicly traded on the Bulgarian Stock Exchange due to their specific legal status of functioning, fall outside the scope of the present development. Alternative funds, which resemble hedge funds as a business entity, are also outside the focus of the study. The framing of the research period covers the periods of 13 July 2016 to 12 July 2020 and 13 July 2020 to 13 July 2023, and our considerations are related to the use of the most up-to-date data on the net value of the contract funds, which in turn will provide us with precise final results from the application of the models of autoregressive conditional heteroskedasticity, and this will allow us to accurately define the risk attribution of contract funds.

5. Discussion

Comparing the results of the empirical application of the models for forecasting the daily volatility of Bulgarian contract funds with the authoritative foreign studies dedicated to the studied issues, many points of contact are found. The development of Malmgren and Zhang (2020) considers the application of ARMA-GARCH and GARCH models to different probability distributions. The authors conclude that European and American mutual funds have similar long-term volatility, which is precisely accounted for by autoregressive conditional heteroskedasticity models. By diversifying investment portfolios primarily constructed from shares of public companies, Siaw et al. (2017) found that when modeling the VaR model with DCC GARCH, more efficient investment portfolios are formed. The application of the complex DCC GARCH is indicative of the assessment of the risk concentration of the investigated investment funds. If we have to compare the results of our research with the development of the three authors, equivalent values are observed in terms of the assessment of the risk concentration of investment funds determined by the GARCH model. A study dedicated to the Egyptian stock market and specifically the risk attribution of the stock indices EGX 30, EGX70, EGX 100 and EGX 20, on which investment funds are constructed in a situation of strong sensitivity and price shock Ezzat (2012) applies EGARCH as a methodology to capture the risk concentration in the market. A specific feature of the Egyptian capital market is that it falls into the same category as the Bulgarian capital market in terms of its economic development, which allows us to parallelly compare the results generated by us with the analyzed research. Ezzat (2012), based on the empirical application of the EGARCH model with consideration of the leverage effect, reaches the opinion that during the period of revolution, the Egyptian stock market indices have a clearly expressed risk attribution.

6. Conclusions

Modeling the risk attribution of Bulgarian investment funds is a huge challenge. First, in the conditions of the Bulgarian capital market, there is no established methodology for quantitative assessment, analysis and forecasting of the daily volatility of investment funds. Motivated by this unfavorable trend, we, the authors of the present study, approved a well-established methodology for forecasting the risk of developed capital markets. The methodology we use includes a specialized set of tools and mechanisms for forecasting the risk concentration of investment funds including GARCH, EGARCH, GARCH-M and TGARCH with specification (1.1). When determining the daily volatility of investment funds, highly sensitive and highly risky are the future forecast results of the "Golden Lev Index 30" investment fund measured by the GARCH, EGARCH and GARCH–M models. The risk forecasting results generated by the TGACRH model assign the highest risk concentration to the EF Rapid investment fund. A specific feature is the finding of stationarity in the time series of the investment funds, which is proven by the application of the GARCH model, defined by the sum of the two parameters Alpha and Beta, which have a value lower than unity. Despite the applicability of a complex toolkit of models for predicting the risk of investment funds, there are deviations in the results. In real conditions, the results formed from the testing of the GARCH, EGARCH, TGARCH and GARCH-M models for forecasting the risk of contract funds in Bulgaria would find their logical application as an additional toolkit for assessing volatility. Of course, the implementation of this task requires making additional decisions. First of all, the regulation must be changed in the part with risk management, where all contract funds in the country apply the methodology used by us for forecasting volatility. Without clearly defined rules for risk assessment from a legal point of view, it would be difficult to implement the models analyzed by us in Bulgarian practice. The results of the study are in practice an additional catalyst for assessing the daily volatility of contract funds. In order for the investment industry in the field of financial asset management operating on the territory of Bulgaria to continue its development, more and more emphasis should be placed on the use and implementation of econometric models created in the "scientific research laboratories". A

mandatory condition for the "installation" of the risk assessment models applied by us in the conditions of the Bulgarian capital market is the selection of personnel performing the functions and responsibilities in relation to risk management. That is, every risk manager must possess a specialized set of skills related to knowledge in the field of econometrics, statistics, financial mathematics and information technology in order to be able to apply and analyze the results of testing autoregression conditional heteroskedasticity models. The high-risk concentration determined by the GARCH, EGARCH, TGARCH and GARCH–M models can serve as a guide for investment fund managers in the selection of assets in portfolios and, in critical situations at high-risk levels, portfolio rebalancing can help as well as the entry into the norm of the risk limits. Directions for future research are related to expanding the scale of investment funds studied. Here, we can add investment funds from other countries of Europe, and thus compare the volatility of contractual funds by country. Of course, we will include funds from the same category with a similar asset value. In line with this, we can test the application of the GARCH models in different investment funds, alternative funds (hedge funds), national contract funds and pension funds. From a methodological point of view, we can extend the analysis by applying the following models for forecasting the daily volatility of investment funds: CGARCH, COGARCH, Copula GARCH, CorrARCH) DAGARCH, DTARCH, EVT-GARCH, F-ARCH, FIAPARCH, FIGARCH, GQARCH, GRS-GARCH and HARCH.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Table A1. Results from the application Ducky–Fuller Test Stationarity (13 July 2016–12 July 2020) model no drift.

N	Funds	Model No Drift t-Stat	Model No Drift DF Tau	Presence of Stationarity
1	Select Balance	−37.13150465	−1.941274434	YES
2	Select Regional	−33.61975765	−1.941274434	YES
3	Select Bonds	−31.28403083	−1.941274434	YES
4	Select Dividend	−44.75106395	−1.941274434	YES
5	CKB Active	−39.218544	−1.941277552	YES
6	CKB Leader	−39.02480648	−1.941277552	YES
7	CKB Garant	−51.03769419	−1.941278704	YES
8	CKB Private	−3.706217572	−1.967361129	YES
9	DV Balance	−32.03368791	−1.941288893	YES
10	DV Dynamic	−33.98161344	−1.941275277	YES
11	Astra Cash Plus	−21.92929388	−1.941580096	YES
12	Astra Global Equity	-	-	YES

Table A1. *Cont.*

N	Funds	Model No Drift t-Stat	Model No Drift DF Tau	Presence of Stationarity
13	SKY New Shares	−29.71777222	−1.941288893	YES
14	SKY Finance	−29.60353879	−1.941288893	YES
15	Quest Vision	−20.62633925	−1.941621668	YES
16	Prime Assets	−36.48829483	−1.941621668	YES
17	South Market Maximum	−34.12347221	−1.941275842	YES
18	South Market Optimum	−30.18954771	−1.941277266	YES
19	Trend Balanced Fund	−24.79500236	−1.941283109	YES
20	Trend Fund Shares	−25.31331777	−1.941283408	YES
21	Trend Conservative Fund	−21.64312802	−1.941284007	YES
22	Texim Conservative Fund	−16.34799969	−1.941384898	YES
23	Texim Bulgaria	−22.03523243	−1.941667985	YES
24	Texim Balkans	−33.58838097	−1.941667985	YES
25	Texim Commodity Strategies	−19.0083033	−1.941667985	YES
26	Prestige	−39.49775883	−1.941683255	YES
27	Profit	−39.54871332	−1.941667985	YES
28	FIB Avangard	−29.30077583	−1.941275277	YES
29	FIB Classic	−29.05284419	−1.941275277	YES
30	FIB Garant	−24.61264458	−1.941275277	YES
31	First Financial Broker house VOSTOK	−38.66689223	−1.941275842	YES
32	Golden Lev	−25.68897061	−1.941449234	YES
33	Golden Lev Index 30	−32.3794514	−1.941279283	YES
34	EF Rapid	−43.69172547	−1.941273874	YES
35	Arkus Balanced	-	-	-
36	Arkus Dynamic	-	-	-
37	Concord-1 Stocks and Bonds	−30.46211442	−1.941277552	YES
38	Concord-2 Stocks	−30.63271609	−1.941277552	YES
39	Concord-3 Real Estate	−40.50779356	−1.941274995	YES
40	Concord-4 Energetics	−22.85980491	−1.941368535	YES
41	Concord-5 CEE	−26.33612634	−1.94137314	YES
42	Concord-6 Bonds	−31.29128177	−1.941274995	YES

Table A2. Results from the application Ducky–Fuller Test Stationarity (13 July 2016–12 July 2020) model drift.

N	Funds	Model Drift t-Stat	Model Drift DF Tau	Presence of Stationarity
1	Select Balance	−37.17644643	−2.864460901	YES
2	Select Regional	−33.60286202	−2.864460901	YES
3	Select Bonds	−31.3301487	−2.864460901	YES
4	Select Dividend	−44.72898655	−2.864460901	YES
5	CKB Active	−39.20149234	−2.864493737	YES
6	CKB Leader	−39.00463687	−2.864493737	YES
7	CKB Garant	−51.13651247	−2.864505861	YES
8	CKB Private	−3.562357454	−3.127148876	YES

Table A2. *Cont.*

N	Funds	Model Drift t-Stat	Model Drift DF Tau	Presence of Stationarity
9	DV Balance	−32.02472958	−2.864613078	YES
10	DV Dynamic	−33.96938021	−2.864469784	YES
11	Astra Cash Plus	−21.96357012	−2.867643981	YES
12	Astra Global Equity	-	-	YES
13	SKY New Shares	−29.70260962	−2.864613078	YES
14	SKY Finance	−29.58947774	−2.864613078	YES
15	Quest Vision	−20.63432503	−2.868071620	YES
16	Prime Assets	−36.44718477	−2.868071620	YES
17	South Market Maximum	−34.12780958	−2.864475736	YES
18	South Market Optimum	−30.21531895	−2.864490721	YES
19	Trend Balanced Fund	−24.9070657	−2.864552226	YES
20	Trend Fund Shares	−25.54917607	−2.864555369	YES
21	Trend Conservative Fund	−22.29994247	−2.864561674	YES
22	Texim Conservative Fund	−23.74757295	−2.865619356	YES
23	Texim Bulgaria	−22.19008304	−2.868546668	YES
24	Texim Balkans	−33.5492424	−2.868546668	YES
25	Texim Commodity Strategies	−19.01307433	−2.868546668	YES
26	Prestige	−39.60422352	−2.868702959	YES
27	Profit	−39.74255062	−2.868546668	YES
28	FIB Avangard	−29.29857604	−2.864469784	YES
29	FIB Classic	−29.03868929	−2.864469784	YES
30	FIB Garant	−24.61803521	−2.864469784	YES
31	First Financial Broker house VOSTOK	−38.65056476	−2.864475736	YES
32	Golden Lev	−25.82240162	−2.866289752	YES
33	Golden Lev Index 30	−32.36447839	−2.864511960	YES
34	EF Rapid	−43.67133902	−2.864455010	YES
35	Arkus Balanced	-	-	-
36	Arkus Dynamic	-	-	-
37	Concord-1 Stocks and Bonds	−30.60742226	−2.864493737	YES
38	Concord-2 Stocks	−30.71373025	−2.864493737	YES
39	Concord-3 Real Estate	−40.58275078	−2.864466817	YES
40	Concord-4 Energetics	−22.85039834	−2.865448349	YES
41	Concord-5 CEE	−26.31915222	−2.865496501	YES
42	Concord-6 Bonds	−31.46475307	−2.864466817	YES

Table A3. Results from the application Ducky–Fuller Test Stationarity (13 July 2016–12 July 2020) model drift + trend.

N	Funds	Model Drift + Trend t-Stat	Model Drift + Trend DF Tau	Presence of Stationarity
1	Select Balance	−37.16536901	−3.41492952	YES
2	Select Regional	−33.66663972	−3.41492952	YES
3	Select Bonds	−31.5042376	−3.41492952	YES
4	Select Dividend	−44.71567079	−3.41492952	YES
5	CKB Active	−39.18197013	−3.414979457	YES

Table A3. *Cont.*

N	Funds	Model Drift + Trend t-Stat	Model Drift + Trend DF Tau	Presence of Stationarity
6	CKB Leader	−38.9895881	−3.414979457	YES
7	CKB Garant	−51.11858863	−3.414997895	YES
8	CKB Private	−3.394786204	−3.822333241	NO
9	DV Balance	−32.01069539	−3.415160959	YES
10	DV Dynamic	−34.01809379	−3.414943029	YES
11	Astra Cash Plus	−21.94093823	−3.419773419	YES
12	Astra Global Equity	-	-	-
13	SKY New Shares	−29.70215571	−3.415160959	YES
14	SKY Finance	−29.7156486	−3.415160959	YES
15	Quest Vision	−20.61127263	−3.420424643	YES
16	Prime Assets	−36.40603079	−3.420424643	YES
17	South Market Maximum	−34.43584976	−3.414952080	YES
18	South Market Optimum	−30.44660198	−3.414974871	YES
19	Trend Balanced Fund	−24.91019825	−3.415068410	YES
20	Trend Fund Shares	−25.60868541	−3.415073189	YES
21	Trend Conservative Fund	−23.12401147	−3.415082778	YES
22	Texim Conservative Fund	−27.6000697	−3.416691714	YES
23	Texim Bulgaria	−22.19180227	−3.421148190	YES
24	Texim Balkans	−33.5313503	−3.421148190	YES
25	Texim Commodity Strategies	−19.03652166	−3.421148190	YES
26	Prestige	−39.58345268	−3.421386266	YES
27	Profit	−39.7285389	−3.421148190	YES
28	FIB Avangard	−29.61306147	−3.414943029	YES
29	FIB Classic	−29.4925438	−3.414943029	YES
30	FIB Garant	−25.02282015	−3.414943029	YES
31	First Financial Broker house VOSTOK	−38.6549945	−3.414952080	YES
32	Golden Lev	−26.08215108	−3.417711861	YES
33	Golden Lev Index 30	−32.38193329	−3.415007171	YES
34	EF Rapid	−43.64929258	−3.414920560	YES
35	Arkus Balanced	-	-	-
36	Arkus Dynamic	-	-	-
37	Concord-1 Stocks and Bonds	−30.61278613	−3.414979457	YES
38	Concord-2 Stocks	−30.82303627	−3.414979457	YES
39	Concord-3 Real Estate	−40.5621569	−3.414938517	YES
40	Concord-4 Energetics	−22.87466222	−3.416431534	YES
41	Concord-5 CEE	−26.32633677	−3.416504794	YES
42	Concord-6 Bonds	−31.46857374	−3.414938517	YES

Table A4. Results from the application Ducky–Fuller Test Stationarity (13 July 2020–13 July 2023) model notrend.

N	Funds	Model No Drift t-Stat	Model No drift DF Tau	Presence of Stationarity
1	Select Balance	−28.77430347	−1.941366025	YES
2	Select Regional	−30.24940697	−1.941366524	YES
3	Select Bonds	−28.37633276	−1.941366524	YES
4	Select Dividend	−26.09698685	−1.941366524	YES
5	CKB Active	−30.28707953	−1.941368030	YES
6	CKB Leader	−29.81640693	−1.941368030	YES
7	CKB Garant	−22.98364928	−1.941368030	YES
8	CKB Private	−29.11132972	−1.941368030	YES
9	DV Balance	−26.75470332	−1.941366524	YES
10	DV Dynamic	−26.85372723	−1.941366524	YES
11	Astra Cash Plus	−27.51518866	−1.941367527	YES
12	Astra Global Equity	−21.74169838	−1.941536142	YES
13	SKY New Shares	−26.95099707	−1.941372107	YES
14	SKY Finance	−26.50254431	−1.941372107	YES
15	Quest Vision	−36.06264968	−1.941368535	YES
16	Prime Assets	−40.73910333	−1.941368535	YES
17	South Market Maximum	−37.25120345	−1.941368535	YES
18	South Market Optimum	−35.98831376	−1.941366025	YES
19	Trend Balanced Fund	−38.70735494	−1.941368535	YES
20	Trend Fund Shares	−38.11700968	−1.941368535	YES
21	Trend Conservative Fund	−38.66462611	−1.941369041	YES
22	Texim Conservative Fund	−16.01641369	−1.941903440	YES
23	Texim Bulgaria	−22.79620758	−1.941903440	YES
24	Texim Balkans	−19.11692005	−1.941903440	YES
25	Texim Commodity Strategies	−17.69835225	−1.941903440	YES
26	Prestige	−19.37447538	−1.941918853	YES
27	Profit	−19.09119755	−1.941921998	YES
28	FIB Avangard	−26.83177883	−1.941366524	YES
29	FIB Classic	−26.75442102	−1.941367025	YES
30	FIB Garant	−23.04442159	−1.941366524	YES
31	First Financial Broker house VOSTOK	−38.34998169	−1.941368030	YES
32	Golden Lev	−34.41805069	−1.941381077	YES
33	Golden Lev Index 30	−46.29987038	−1.941381077	YES
34	EF Rapid	−46.59001602	−1.941366524	YES
35	Arkus Balanced	−10.05519704	−1.942755838	YES
36	Arkus Dynamic	−9.452665711	−1.942755838	YES
37	Concord-1 Stocks and Bonds	−27.28079721	−1.941367025	YES
38	Concord-2 Stocks	−27.83380675	−1.941367025	YES
39	Concord-3 Real Estate	−30.58542075	−1.941367527	YES
40	Concord-4 Energetics	−17.67584113	−1.94190344	YES
41	Concord-5 CEE	−29.9124813	−1.941468006	YES
42	Concord-6 Bonds	−25.91262975	−1.941367527	YES

Table A5. Results from the application Ducky–Fuller Test Stationarity (13 July 2020–13 July 2023) model trend.

N	Funds	Model Drift t-Stat	Model Drift DF Tau	Presence of Stationarity
1	Select Balance	−28.75501998	−2.865422101	YES
2	Select Regional	−30.23807792	−2.865427323	YES
3	Select Bonds	−28.43682592	−2.865427323	YES
4	Select Dividend	−26.1221695	−2.865427323	YES
5	CKB Active	−30.29478064	−2.865443071	YES
6	CKB Leader	−29.83632854	−2.865443071	YES
7	CKB Garant	−23.00121691	−2.865443071	YES
8	CKB Private	−29.10464133	−2.865443071	YES
9	DV Balance	−26.78138725	−2.865427323	YES
10	DV Dynamic	−26.93592265	−2.865427323	YES
11	Astra Cash Plus	−27.50245481	−2.865437808	YES
12	Astra Global Equity	−21.72931125	−2.867190498	YES
13	SKY New Shares	−27.13406971	−2.865485699	YES
14	SKY Finance	−26.52137717	−2.865485699	YES
15	Quest Vision	−36.13025593	−2.865448349	YES
16	Prime Assets	−40.80293315	−2.865448349	YES
17	South Market Maximum	−37.2633808	−2.865448349	YES
18	South Market Optimum	−35.98547255	−2.865422101	YES
19	Trend Balanced Fund	−38.73641516	−2.865448349	YES
20	Trend Fund Shares	−38.14828867	−2.865448349	YES
21	Trend Conservative Fund	−38.7078285	−2.865453642	YES
22	Texim Conservative Fund	−17.30070381	−2.870939423	YES
23	Texim Bulgaria	−22.96159795	−2.870939423	YES
24	Texim Balkans	−19.20831096	−2.870939423	YES
25	Texim Commodity Strategies	−17.69876967	−2.870939423	YES
26	Prestige	−19.37475576	−2.871094806	YES
27	Profit	−19.11499076	−2.871126501	YES
28	FIB Avangard	−26.83069228	−2.865427323	YES
29	FIB Classic	−26.73648728	−2.865432558	YES
30	FIB Garant	−23.04719217	−2.865427323	YES
31	First Financial Broker house VOSTOK	−38.36720367	−2.865443071	YES
32	Golden Lev	−34.56398444	−2.865579446	YES
33	Golden Lev Index 30	−46.26887112	−2.865579446	YES
34	EF Rapid	−46.55932535	−2.865427323	YES
35	Arkus Balanced	−10.02491324	−2.879330279	YES
36	Arkus Dynamic	−9.442640783	−2.879330279	YES
37	Concord-1 Stocks and Bonds	−27.28483074	−2.865432558	YES
38	Concord-2 Stocks	−27.89222893	−2.865432558	YES
39	Concord-3 Real Estate	−30.78533197	−2.865437808	YES
40	Concord-4 Energetics	−17.72344829	−2.870939423	YES
41	Concord-5 CEE	−29.94001221	−2.866484781	YES
42	Concord-6 Bonds	−25.90094132	−2.865437808	YES

Table A6. Results from the application Ducky–Fuller Test Stationarity (13 July 2020–13 July 2023) model trend + drift.

N	Funds	Model Drift + Trend t-Stat	Model Drift + Trend DF Tau	Presence of Stationarity
1	Select Balance	−28.73712651	−3.416391601	YES
2	Select Regional	−30.22376057	−3.416399544	YES
3	Select Bonds	−28.44177371	−3.416399544	YES
4	Select Dividend	−26.13320599	−3.416399544	YES
5	CKB Active	−30.27754044	−3.416423504	YES
6	CKB Leader	−29.82266603	−3.416423504	YES
7	CKB Garant	−22.98788273	−3.416423504	YES
8	CKB Private	−29.09072987	−3.416423504	YES
9	DV Balance	−26.89974874	−3.416399544	YES
10	DV Dynamic	−27.00064752	−3.416399544	YES
11	Astra Cash Plus	−27.51883075	−3.416415496	YES
12	Astra Global Equity	−21.73317172	−3.419082956	YES
13	SKY New Shares	−27.12050795	−3.416488358	YES
14	SKY Finance	−26.50590063	−3.416488358	YES
15	Quest Vision	−36.10657965	−3.416431534	YES
16	Prime Assets	−40.77581623	−3.416431534	YES
17	South Market Maximum	−37.24282154	−3.416431534	YES
18	South Market Optimum	−35.96168854	−3.416391601	YES
19	Trend Balanced Fund	−38.71235553	−3.416431534	YES
20	Trend Fund Shares	−38.12484842	−3.416431534	YES
21	Trend Conservative Fund	−38.68644981	−3.416439586	YES
22	Texim Conservative Fund	−17.28550569	−3.424794589	YES
23	Texim Bulgaria	−22.92424316	−3.424794589	YES
24	Texim Balkans	−19.19965243	−3.424794589	YES
25	Texim Commodity Strategies	−17.7442352	−3.424794589	YES
26	Prestige	−19.34799485	−3.425031496	YES
27	Profit	−19.09690237	−3.425079822	YES
28	FIB Avangard	−26.82656659	−3.416399544	YES
29	FIB Classic	−26.73221972	−3.416407510	YES
30	FIB Garant	−23.05453061	−3.416399544	YES
31	First Financial Broker house VOSTOK	−38.35241038	−3.416423504	YES
32	Golden Lev	−34.54019229	−3.416630991	YES
33	Golden Lev Index 30	−46.23711083	−3.416630991	YES
34	EF Rapid	−46.53089136	−3.416399544	YES
35	Arkus Balanced	−10.35025222	−3.437606979	YES
36	Arkus Dynamic	−9.841919823	−3.437606979	YES
37	Concord-1 Stocks and Bonds	−27.26689677	−3.41640751	YES
38	Concord-2 Stocks	−27.8736308	−3.41640751	YES
39	Concord-3 Real Estate	−30.78704154	−3.416415496	YES
40	Concord-4 Energetics	−17.71248318	−3.424794589	YES
41	Concord-5 CEE	−29.91755473	−3.418008689	YES
42	Concord-6 Bonds	−25.94189805	−3.416415496	YES

Notes

[1] Data on the net worth of investment funds are available on the following website: https://baud.bg/quotes/, accessed on 13 July 2023.

[2] (Petrova and Todorov 2023) If interested, the authors provide all the calculation files related to the determination of the models in MS Excel environment: Available: https://doi.org/10.7910/DVN/L5RULA (accessed on 26 October 2023).

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