

Article

Financing Cooperative Supply Chain Members—The Bank's Perspective

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Abstract: This paper contributes to the supply chain finance literature with an agent-based Monte Carlo simulation model focusing on the bank's point of view. Our theoretical model assesses how a bank should screen a supply chain (SC) member and whether that requires different considerations and monitoring systems compared with traditional corporate loans. In the model, the SC members may cooperate, reducing their bankruptcy risk considerably; thus, the chance for and extent of inter-entity financial aid are critical to consider when assessing bankruptcy risk. A cooperative SC member cannot just be financed from debt taken by other members, but it may also offer protection to other SC members using its operating cash flow. Thus, based on our results, bankruptcy risk is SC-specific, rather than a characteristic of an individual firm. Therefore, to finance an SC member is a quasi-joint decision of its peers, so particular care should be paid to estimating and monitoring the correlations between the operational cash flows of cooperative SC members. One of the key results is that of edge default exposure of the bank; it might be optimal to limit the amount of the loan made available to a given collaborative SC member instead of charging higher rates or financing the most attractive SC member only. Another SC member offering an additional guarantee with its assets will provide the remaining need for financing. As this solution also reduces the total bankruptcy risk of the SC, the SC itself should prefer this financing structure.

Keywords: supply chain finance; bank loan; He methodology; corporate liquidity; Monte Carlo simulation; bankruptcy



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1. Introduction

The literature on supply chain finance (SCF) usually focuses on a simple supply chain (SC) of one supplier, one retailer, and a bank where one of the SC members has scarce funding. Previous research contributed to determining the optimal combination of trade credit or reverse lease and bank loan, and showed how certain factors might change the optimum. However, little is known about how it is different from the bank's perspective to judge lending risks when the borrower is an SC member. Our research focuses on separating the effects of various factors on the lending risk of an SC member. Another novelty of the paper is to set up a three-member supply chain in which the financial position of a single SC member (borrower) may show heterogeneous correlation patterns instead of the deterministic (usually positive perfect correlation) approach of the classic models.

The literature on supply chain finance (SCF) suggests that firms within the same supply chain cooperate. For example, [Gelsomino et al. \(2016\)](#) report how peers provide liquidity to their peers during crises. For further details, see Section 2. As a result, the material flow is coordinated jointly, contributing to further efficiency in inventory management, and the cash flows across the chain are subject to cooperative strategies.

An optimization of cash management (default risk) at the supply chain (SC) level is a frequent practice, resulting in a transfer of (short-term) financing sources from peers providing external sources to SC members with stricter financial constraints at the same time. From the side of the trade credit literature, one can assume that firms can have an

informational advantage in screening the creditworthiness of their partners compared to banks; thus, it is a rational decision for firms within the same supply chain to provide peers with trade credit.

This paper investigates the unique questions that a bank should consider when lending to cooperative SC members. In Tirole's theoretical model (Tirole 2006), where there is informational asymmetry among agents, which creates a place for moral hazard, banks can decide on an individual firm's creditworthiness based on their pledgeable income and the volume of their assets. Assets can serve as collateral, and the pledgeable income incorporates the expected revenue of the project with an adjustment for the private benefit of the borrower's insufficient efforts (called misbehaving in Tirole's model). The loan contract should create such construction, which incites the borrower to perform and repay the loan.

The present paper models how the classic model can evolve when borrowers cooperate at a cash management level to absorb better shocks affecting their pledgeable income. Since the pledgeable income is not a category within the financial statements, one has to find the link between this rather theoretical term and the practical categories of financial statements.

The paper interprets the pledgeable income as cash flow or cash liquidity resulting from cash management. For to highlight that the amount of available income for debt service is a question of liquidity and cash management as well, the article uses the theoretical term 'pledgeable income' instead of free cash flow to the firm (FCF) or gross cash flow that banks usually monitor when calculating debt service coverage ratios.

The paper develops an agent-based model and uses Monte Carlo simulation to quantify the role of cooperation in reallocating and maintaining corporate pledgeable cash flow and decreasing the default risk of participating firms and the expected loss of the lender on credit risk. In this model, the required liquidity of the agent is not a threshold but an interval. Therefore, the agents must manage their liquid assets to reach the targeted minimum liquidity (cash) reserves and maintain business continuity.

The cash balance of agents at the end of each period is the result of their business activity and cooperative cash management. The model considers the changes in cash balance as exogen shocks. The firms' efforts on their projects, their financial and operative performance or the related risk profile are included in the model only through the expected values and the standard deviations of corporate cash flow. The research begins at the point where the periodic cash flow is realized. Then, we focus on how the debt service of agents can be improved by reallocating cash reserves among supply chain members and how the cooperative strategy of agents enhances the bank's position through lower default rates and lower expected loss to the credit portfolio.

The agents of the same SC are necessarily interlinked. To show the connection between the SC members, the model assumes a given level of agent cash flow correlation: a positive connection is typical for members of the same supply chain (supplier-buyer). In contrast, a negative correlation is likely to emerge when describing agents within a conglomerate holding or an SC with various competing entities at some position. Finally, the paper covers how the default frequency or bankruptcy rate of correlated agents with diverse financing opportunities and risk level develops with and without their cooperative cash management.

Cooperation, even in the case of identical agents, pays off. Smoothing of cash balance and liquidity aids among agents with different financing opportunities enhance the chain's financial stability. It can help reduce agents' default rates, hence the contagion within the chain and disruptions of the SC. As cooperative cash management can partially offset the need for external financing, the volume of bank loans can reduce slightly. Thus, the value of cooperation shows off for the lender: when financing one member of a cooperative SC, the borrower's dependence on its peers within the SC should be considered.

The financing of an SC member is a quasi-joint decision from the bank's side to implicitly provide sources for other peers linked to the original borrower. Nevertheless, it is worth financing members of a cooperative SC. As SC members reallocate their liquidity across the chain, a lower expected loss on credit risk can be achieved through the decline in

default rate (probability of default) and the total volume of outstanding loans (Exposure at default). Therefore, supply chain finance (SCF) does not mean a threat for the bank but an opportunity to reduce its expected loss due to the cooperative financing strategies of the SC.

The paper is structured as follows. Section 2 offers a literature overview. Then, Section 3 defines the framework of an agent-based model and distinguishes two versions: in the base case, the agents' financing opportunities and operational risk levels are identical. The participants' financing terms and operational risk vary in the second model version. Section 4 describes the process driving the liquidity shocks using the He methodology, which allows us to pre-determine correlations among the random shocks. After having designed the liquidity management of the agents in Section 5, a Monte Carlo Simulation is run to quantify the success of the different liquidity management practices in base and crisis scenarios for both versions (Section 6). Finally, Section 7 concludes the findings.

2. Literature Overview

Tirole (2006) provided a detailed analysis of firms' external financing. His primary model explains credit rationing, where the pledgeable income of profitable projects (with positive net present value) is not high enough to raise capital from investors or the lender (bank). Tirole's results focus on how entrepreneurs can increase the pledgeable income of their projects and boost the ability to borrow. One of his models assumes that borrowers do not have only one project to be financed. Cross-pledging works where projects are independent, or at least they do not show perfect correlation. Therefore, it is worthwhile for both the lender and borrower to finance the whole firm as a portfolio of projects instead of funding the projects separately in the form of project financing. Tirole's conclusion on not perfectly correlated projects correlates with our result, according to which the credit risk of financing a cooperative SC's member is more moderate than the financing of an individual firm with the same operational risk because cooperative SC members may counterbalance liquidity shocks affecting their partner company.

This idea appears in the large body of supply chain finance (SCF) literature. However, as the present research does not focus on a sole firm as Tirole (2006) did, but instead on cooperating agents, the four-agents model of this paper is appropriate to describe the stylized facts of firms' cash management within the same supply chain or holding. Therefore, a short review of the relevant works from the SCF literature follows.

The large body of literature on financial supply chain management (FSCM) provided essential aspects to our model. According to Mentzer et al. (2001), there is a consensus on the definition of the supply chain. They define a supply chain as 'a set of three or more entities (organizations or individuals) directly involved in the upstream and downstream flows of products, services, finances, and/or information from a source to a customer' (Mentzer et al. 2001, p. 18). The definitions for supply chain management are more heterogenic than that of the SC. Mentzer et al. (2001) use the following version: 'supply chain management is defined as the systemic, strategic coordination of the traditional business functions and the tactics across these business functions within a particular company and across businesses within the supply chain, to improve the long-term performance of the individual companies and the supply chain as a whole' (Mentzer et al. 2001, p. 18).

Wuttke et al. (2013) describe FSCM 'as optimized planning, managing, and controlling of supply chain cash flows to facilitate efficient supply chain material flows' and identify the theory of transaction cost economics as the theoretical framework of their approach (Wuttke et al. 2013, p. 2).

The SCF is an approach that aims to optimize financial flows at an inter-organizational/SC level instead of an organizational/firm level (Hofmann 2005). Financial flows are adjusted to material (product), and information flows across SC members, resulting in improved cash management at the SC level (Wuttke et al. 2013).

In Camerinelli (2009), one finds insight into the solutions implemented by financial institutions to promote these aims of SCF. Once the SCF approach is applied, the following

benefits are listed in the literature: lower debt costs, new opportunities to obtain loans (especially for ‘weak’ supply chain players), and reduced working capital in the SC (Randall and Farris 2009). Gelsomino et al. (2016) refer to the three-dimensional framework of the ‘SCF cube’, where they define the lower cost of debt and a more favorable duration and volume of financing as a value-creating contribution of SCF. In addition, they report that the reduced default risk of SC members is a well-known benefit of factoring and reverse factoring.

As SCF focus on optimal product and cash flows, it shortens the cash-to-cash cycle of its members. However, Gelsomino et al. (2016) add a fascinating insight to the practice of specific SCs. SCF may increase the cash-to-cash cycle of SC members, particularly in economic distress, because some firms may act as liquidity providers toward their peers. In this case, they accept the increase of their cash-to-cash cycle to promote the overall liquidity position of the SC. This finding of Gelsomino et al. (2016) incited our model where firms within the chain can provide their peers with their excess cash to offset the peers’ need for bank finance.

Gelsomino et al. (2016) list two main directions in the SCF literature. Finance-oriented works describe SCF as a set of financial solutions usually offered by financial institutions. Second, SC-oriented works focus on the collaboration of SC members. Within this field, special attention is given to inventory management. Jin et al. (2019) structure their literature review according to the three directions of research: external financing of SC, internal financing of SC (trade credit, early payment), and the comparison of external and internal SC financing. Jin et al. (2019) identify the question of linking inventory (or investment) and funding decisions to each other among SC’s members within the first part of the literature (external financing of SC).

Jin et al.’s (2019) contribution forms part of the literature where authors assess collaborative financing decisions in external financing. They collected several empirical examples where one of the SC members applied for bank financing to reallocate capital within the SC or act as a guarantor toward a bank to help its suppliers’ and distributors’ partners. In their model, Jin et al. (2019) assessed whether there could be more efficient strategies than the described empirical practices. Their conclusion is as follows. First, bank finance may enhance the supplier’s and retailer’s performance when SC members finance themselves separately. Access to bank finance is vital when both firms are capital-constrained. Second, collaborative strategies improve SC efficiency but do not necessarily reduce the bank’s credit risk. Third, the two SC members may have diverse incentives at the individual level. The retailer benefits more from separate financing, but in cooperative financing, the retailer is better off when the supplier acts as a guarantor and not as an intermediary creditor. Finally, incentives push the supplier to provide a guarantee rather than reallocate credit from the bank.

However, in a theoretical model based on a two-member SC, Ding and Wan (2020) showed that it is optimal for the buyer to pre-pay to the suppliers rather than make the supplier use bank loans to finance its operation if possible. Li et al. (2020) contribute to this by incorporating the risk perception of members within the two-stage SC. The agents in their model are maximizing their (expected) profit. According to their results, risk-neutral or risk-seeking retailers can perform better if it is the buyer rather than the bank lending them. Once the retailer’s risk aversion climbs above a given level, he will prefer the bank lending alternative. Even once we assume that the buyer maximizes his profit, he charges the lowest possible interest rate for the offered loan considering his risk attitude and the supplier’s bankruptcy costs. Thus, our model assumes that bank loans are only used once intra-SC lending opportunities are no longer available.

The firm’s positioning in their SC and the SC financing decisions affect the bank’s lending risk, and banks must consider this. Xie et al. (2019) showed in their theoretical model that banks would increase the interest rate on loans provided to an SC member (seller) if banks had no information about the default risk of the other SC members (buyer). They underlined that it is the loan amount and not the interest rate that depends on the

default risk of the buyer (credit rationing), the seller may be interested in hiding information about the buyer to receive a higher loan amount at a more elevated rate. [Yan and Ye \(2020\)](#) investigate an SC where it is the retailer that faces a capital constraint. Their results show that compared with the scenario without financing service, the entire supply chain is better off when combining trade credit and bank loans, which exposes the commercial bank and supplier to the retailer's default risk. [Dong and Pan \(2019\)](#) investigate the difference between SC members taking bank loans separately and collaboratively. They conclude that higher interests lead to lower output but not to a higher bank profit necessary. Additionally, stand-alone lending of SC members generates more bank profit, but collaborative financing would only boost SC profit under certain conditions. They conclude that banks should promote lending to the SC members individually, which is also in line with our results.

No wonder banks pay attention to retailer connections. Using loan-level data, [Hasan et al. \(2020\)](#) showed that banks tend to ask for larger loan spreads, higher intensity of covenants, and request collateral at a higher likelihood when a firm depends more on one principal supplier for inputs. However, a longer relationship between the borrower and the supplier and between the bank and the supplier mitigate lending constraints.

[Zhai et al. \(2020\)](#) investigated the effect of altruism and reciprocity on the efficiency of the SC. They concluded that borrowing a bank loan may significantly increase the performance of the SC when SC members face capital constraints. Furthermore, they underline that a loan limit applied for an SC member will reduce the pressure for the borrower to be too generous with other SC members and reduce the bank's risk. Our results also underpin this conclusion.

The theoretical model of [Raghavan and Mishra \(2011\)](#) is challenging to order to any of the above-cited parts of the literature, but their work is an essential antecedent of our paper because it focuses on the decision-making of the lender. They build a two-stage SC (manufacturer and retailer) model where the lender maximizes its profit. The model compares the lender's profit in the case of individual decision-making, where the lender is aware of the relationship between its two borrowers and makes a joint decision. [Raghavan and Mishra \(2011\)](#) prove that if one SC member has low cash reserves, the collective judgment of the lender–SC financing in practice–outperforms individual funding not only from SC members' point of view but also from SC members that of the lender as well.

The basic idea of the works above is common: SC members do not act as separate agents; they cooperate even in their financing decisions. Therefore, when an investor provides financing to one of them—usually the most prominent—this capital funds the whole chain, and the new financial source will also flow toward other SC members.

This model contributes to the above-cited results in several ways. First, it assesses the value-added of a cooperative strategy among peers. Second, the model consists of three firms instead of a two-stage SC, which is more widespread in the literature. Third, collaboration in financing decisions concerns liquidity management, but the firms' cash balance is not linked to their inventory levels; instead, to a random process with a steady expected value, we solely focus on cash management and the pledgeable cash balance of agents. Therefore, the model is finance-oriented (see [Gelsomino et al. 2016](#)), and it partially belongs to the third direction of literature defined by [Jin et al. \(2019\)](#), namely the comparison of external and internal SC financing. Fourth, the paper investigates how cooperative liquidity management (like a cash pool system) can partially offset the need for external funding, reduce the default rates of SC members, and contribute to a lower risk level of the loan provider bank. Finally, we evaluate the cooperative strategy from the lender's perspective, whether it is worth allowing the collaborative approach among the borrowers (or potential borrowers) within the same supply chain—which finally means a joint decision of financing the whole chain even in the case of individual loan contracts.

At the same time, the paper is not only linked to the actual findings on SCF. The broadest context for this work is provided by [Brealey et al. \(2007\)](#) within the management of cash flows. Corporate liquidity is still a relatively wide area. For example, [Denis \(2011\)](#) and [Havran \(2011\)](#) provide deep insight into liquidity management. From the large volume

of works on corporate liquidity management, we must mention [Subramaniam et al. \(2011\)](#), whose findings are similar to ours. First, they cite that according to the literature, cash reserves serve as a hedge to reduce financing risk, especially during volatile periods. Then they prove that a diversified activity can provide the company with a natural hedge and contribute to lower cash holding. Their results correlate with those of [Berger and Ofek \(1995\)](#), [Lamont \(1997\)](#), [Shin and Stulz \(1998\)](#), and [Khanna and Tice \(2001\)](#). Finally, they show that diversified firms hold less cash than so-called focused firms because diversified firms have access to efficient internal capital markets within the firm, and they have a more significant potential to sell assets. Our model is linked to these works by correlating agents' liquidity shocks. We test the above-cited results in our model by changing correlations among agents' cash flow changes.

We often refer to our model's cooperative liquidity management strategy as a cash pool system. Although the research questions of [Berlinger et al. \(2017, 2018\)](#) were slightly different, their conclusions show similarities to ours regarding the reduced default frequency in our model when allowing cooperation in liquidity management. Like ours, their later paper (2018) uses a Monte Carlo approach to evaluate the benefit of centralized cash management, which usually include interest rate savings, economy of scale, and reduced cash flow volatility. Their contribution to interest rate savings demonstrates a new aspect of cash pool systems: this form of centralized cash management reduced the bank's counterparty risk.

3. Methodology

We apply agent-based simulation to assess how different cash management strategies within the SC can affect the chain's liquidity, thus the bank's lending risk. This approach is especially appropriate to describe the processes of complex systems where the interaction of participants affects their environment and where the strategy of one agent is a reaction to the actions of others. This kind of methodology is often used to evaluate the vulnerability or resilience of the system. An emerging body of literature arose in finance using agent-based models for networks (like the interbank market) to assess possible policy measures in the case of crises. (e.g., [Farmer and Geanakoplos 2009](#); [Bookstaber 2012](#); [Bookstaber and Paddrik 2015](#)).

This section follows the general steps of agent-based modeling. First, the agent's environment and the different financing terms appear, which the paper will compare in later sections. Next, a description of the process which drives the liquidity shocks follows. As a final step, we define the heterogeneous agents: three members of a single SC and a bank providing external financing. The text also presents the rules that define the agent's actions. The model applied is based on the one used in [Felföldi-Szűcs et al. \(2021\)](#).

Base and Scenarios for Different Financing Terms

First, the paper models the liquidity management of the agents. Every period a liquidity shock occurs, next, the agents manage their liquidity: they use their credit line at the bank or, in the case of cooperation, have access to internal financing from their peers. (The internal funding may be, for example, a cash pool system, direct borrowing, a reverse lease, or some trade credit.)

Next, we define different terms for bank financing and test the relevance of cooperation in liquidity management under various financing opportunities. The assessment follows the *ceteris paribus* principle; the investigation moves from the base scenario to a stylized SC with different operational risk levels and financing conditions through five steps where we always modify only one parameter in our model. The liquidity changes occur similarly to the base case at the beginning of the period; then, agents must reach their target liquidity by applying their liquidity management strategy. The terms for financing and the different scenarios are as follows.

1. Base case: the operational risk of agents is similar. All three agents can profit from a credit line at a monthly rate of 0.50%, equivalent to a yearly (12-period) rate of 6.16%, and the maximum amount of credit is 100.
2. Heterogenous yield environment: operational risk and maximum level of the credit line is similar for all the agents, but they face different rates for the bank loan.
3. The scenario of various operational risk agents but the same financing terms is a rather theoretical one (compared to the base case), but it is a needed part of our analysis.
4. The various operational risks of agents are paired with diverse terms of financing:
 - a. The difference in the maximal volume of credit lines,
 - b. The difference in the rates for the bank loan,
 - c. The difference in both the rates and the maximal volume of credit lines.

To sum up, we test how the above-defined financing terms affect the three agents' liquidity and how the risk of the loan provider bank evolves. One may consider these agents as stand-alone business units of a larger company, three individual firms held by the same owner, or even three economic sectors of a given country. In the following part of the paper, we will treat the three agents as members of a three-stage SC.

The agents can offer additional liquidity to their fellows representing the only possibility of cooperation in the model. In real life, this peer support may emerge as a cash pool system, renegotiating supplier-buyer payment terms, a jointly owned bank account, the HQ charging management fees and lending to affiliates, or, in another dimension, even rearranging state spending and tax incomes.

We contrast agents with different financing opportunities (the same volume of available financing, but at a different interest rate) to agents with access to identical bank offers (same volume and same rate). This method allows assessing the gains from cooperation already appearing in scenarios with similar agents and financing terms. Based on the literature on trade credit and SCF in the first case, we can assume that cooperation can contribute to an improved liquidity position and a decreased default rate.

4. Modeling of Liquidity Shocks

One could describe shocks or the process of changes in variables in a simulation in general in several ways. If empirical data are available, the model can apply the historical distribution of the variables included in the simulation. For example, in modeling external shocks in Monte Carlo Simulations, authors often assume that the number of events follows a Poisson process where the lambda parameter of the distribution is estimated from historical data. (Bookstaber and Paddrik 2015) A standard Brownian motion drives the stochastic part of the agents' income in Caballero and Panageas (2007), but this choice remains without a detailed explanation. In their agent-based model for financial vulnerability, Bookstaber et al. (2018) cite values for the input parameters from several recent pieces of research or use values according to the authors' choice when developing their agent-based model to assess the vulnerability of financial system in the case of fire sales.

Since no empirical data were available for changes in firms' future pledgeable income or how SC members are linked via trade credit terms, demand trends, or joint shareholder interests, assumptions were needed to generate future changes in agents' income levels. Instead of a continuous process such as the above-cited Brownian motion, we chose the discrete dynamics of He's non-recombining multinomial tree. The process may generate four outcomes for the three agents of the SC for each period. However, it is more illustrative to follow four possible scenarios per period. The convergence of the applied process is shown toward a lognormal process.

The methodology (He 1990) generalizes Cox et al. (1979), where the original CRR-model used binomial processes to describe the price processes of underlying products as the first step of their option pricing model. Several authors extended the original binomial model to a multidimensional solution without a satisfactory economic framework. In He's model, the N-dimensional diffusion process for share prices is modeled by a sequence

of N-variate ($N + 1$) nominal processes, and the model allows the convergence from a discrete-time multivariate multinomial model to a general continuous-time one.

The paper uses the results of He for the three variables case. Variables X_1 , X_2 , and X_3 variables are independent, their expected value is zero, and their standard deviation is equal to one once the probability of each scenario is 0.25. (Table 1) Once a column is picked, all three variables (X_1 , X_2 , X_3) take their value from the selected row.

Table 1. Variable values for X_1 , X_2 , and X_3 in in the predefined scenarios.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4
X_1	0	0	$\sqrt{2}$	$-\sqrt{2}$
X_2	0	$\sqrt{\frac{8}{3}}$	$-\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{2}{3}}$
X_3	$\sqrt{3}$	$-\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{1}{3}}$

This way, the model creates a non-recombining tree that has four branches in each node with different liquidity shock driver values for each of the agents. After calculating the three independent standardized random variables from the He framework (drivers), they are corrected by a positive drift and the standard deviation of the process (1 and 10, respectively, in the base case) (Table 2). Additionally, the correlation assumed in the given scenario is added to the generated outcome using the Cholesky decomposition (Parker 2017). The model simulates future cash flow every month for 10 years (for 120 periods). The moderate growth rate for cash flows helps us mimic the entire cash flow to the firm. The correlation represents possible relations of agents.

5. Modeling of Agents' Liquidity Management

The model consists of four heterogeneous agents: an SC including three firms, which receive a credit line from the bank, which will be the fourth agent in the model.

Initially, the firms—three agents belonging to the same SC—have liquidity reserves equal to 100 for each of them. After that, to deal with the liquidity shocks of the 120 periods simulated, the firms need to consider the following rules when managing their liquidity.

Case A—Firms may not cooperate

1. The target amount of liquidity is 100.
2. Once a firm has less than 100 cash, it tries to fill reserves by applying for bank credit.
3. A default happens when the liquidity of the agents reaches 0.

Case B—Cooperation in liquidity management

1. The targeted amount of liquidity is 100.
2. Should a firm have less than 100 liquidity after the periodic shock, it first asks its peers for cash support. Only those firms may help that have a cash level exceeding 100. The cost of financial support received from peers costs 0.25% per period.
3. Peer transfers will support the agent further from the required liquidity of 100 first.
4. Once a firm has an after-shock cash balance exceeding 100, it will repay its dues to fellow agents first and only help others later.
5. The extra liquidity available after all those steps will repay bank loans, if any.

In both cases, the commercial bank offers the same credit conditions to all the firms, as described next.

1. The loan has no maturity and is only available to firms below the required cash balance. (Can be viewed as an ever-renewable debt requesting no repayment.)
2. The bank regulation does not allow an exposure toward the individual agents higher than 100.
3. The interest rate charged by the bank amounts to 0.50% per period.
4. The debtor must use all its excess cash above 100 to repay the outstanding principal.

Order of financing:

1. Should there be more than one agent with a liquidity shortage, first, the one with more missing cash will receive help from its peer. Then, the helping peer tries to lift the cash level to 100 and will offer only the remaining excess money to the firm second in need.
2. Should there be more than one firm with excess liquidity, the firm with a more elevated surplus would offer a transfer to the distressed agent first. If the need is higher than the excess of the first lender, resources from the second lender would be used to push the cash balance up to 100.
3. All three agents can borrow from the bank at the same time.

Reimbursement order:

1. First, agents repay (partly) loans from peers.
2. Next, firms must (partly) redeem their bank loans.

6. Results (Simulations of Various Financing Circumstances)

Our simulation tests the effect of the diverse lending environment the SC may face. First, we define a base case to which we will compare various lending scenarios.

We will compare the different scenarios based on the default frequency as the most crucial aspect for the bank and the agents. We use two additional measures to do so. First, a less strict measure for default frequency is reported in Tables 3–8 and 11–18. Here we consider the total number of bankruptcy periods, respecting that we may count 0 to 3 firms in bankruptcy in each simulation period. These numbers can be interpreted as an estimate for individual default rates, a default frequency at the firm level.

Secondly, the model also calculates the proportion of periods that end with at least one firm in bankruptcy (negative cash balance). We could interpret this measure as an SC default frequency. Once one member defaults, disruption can appear in the chain in the product flow, or liquidity shortage can reach other members through contagion. Tables 9 and 19 compare default rates of the SC.

The borrower's riskiness can be estimated by the volatility of corporate cash flows, which can serve to redeem the loan and pay interest. Therefore, our results include the closing cash balances of firms in the different scenarios.

Banks do not only focus on default frequency, but their exposure at default (EAD)—in our model: simply the outstanding volume of the loan—and the loss given default (LGD) are also parameters of expected loss (EL) on credit risk. In our investigation, we can assume that LGD is constant over scenarios. Thus, to measure the EL of the bank, EAD should be compared through different scenarios. We calculate the EAD of the loans in our model; tables report EAD as 'closing bank loan' for all scenarios.

6.1. Base Case

First, we define the base state of the world the same way as in [Felföldi-Szűcs et al. \(2021\)](#). The model assumes that all three agents have similar parameter values. The initial cash balance for all runs and agents remains 100, precisely the same amount the agents aim to maintain during their operation. (Below that level, companies attempt to raise cash from peers or the bank.) Borrowing from other agents (cash pool) costs in all cases 0.25% per period (e.g., month) while the interest rate on the bank credit is set to 0.50%. Those periodic rates are equivalent to 3.04% and 6.16% yearly (for 12 periods).

The model simulates 10 thousand individual runs with 120 periods (10 years) each. Notice that our simulation only assumes a specific correlation in liquidity shocks but defines no path of goods or origin of the shocks.

The agents expect a periodic cash flow of 1 with a standard deviation of 10 for each period in the base case. The agent cash flows are not correlated, while the maximum credit line available with the bank is 100 for each firm. Table 2 shows four possible outcomes for the periodic liquidity shocks.

Table 2. Periodic liquidity shock outcomes.

Outcomes	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Firm A	1.0000	1.0000	15.1421	−13.1421
Firm B	1.0000	17.3299	−7.1650	−7.1650
Firm C	18.3205	−4.7735	−4.7735	−4.7735

The results of the non-cooperation case indicate that in 2.49% of the cases, at least one period existed at the end of which at least one firm had a negative cash balance (bankruptcy). Even in the worst case, only 0.22% of the total firm periods ended with a default. (One may count 0 to 3 firm bankruptcy periods in each period.) The final closing cash balance averaged 224 for the three agents, ranging from −205 to 636. Here, the model assumes independently operating companies (no cash pool available); changing the correlation of cash flows would not affect the results. (Table 3).

Table 3. Base case results without cooperation.

	Average	Min	Max
Bankruptcy firm periods	0.00%	0.00%	0.22%
Final Cash Balance	224.02	−204.74	635.67
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	9.07	0.00	100.00

Once the model allows agents to cooperate, the bankruptcy rate decreased to 0.03%, and the maximum bankruptcy firm periods ratio lowered to 0.02%. (Table 4) The minimum final cash level climbed, but the cooperation also allowed the agents to accumulate significant debt and loans toward their peers. It is no wonder that the bankruptcy risk decreased because the cash pool offered extra liquidity. Nevertheless, as the expected advantages of the cooperation are moderate when firms consider averages instead of extreme values, there is a limited drive for the agents to help each other, mainly once there are also transaction costs associated with teaming up.

Table 4. Results for the base case with cooperation.

	Min	Mean	Max
Final Cash Balance	2.81	217.03	691.03
Final Pool Debt	−298.76	0.00	289.67
Final Bank Loan	0.00	1.91	100.00
Bankruptcy firm periods	0.00%	0.00%	0.02%

6.2. Heterogeneous Yield Environment

If one of the firms operates in a different country or has a different balance sheet structure (may offer fewer collaterals), it may pay a higher interest rate (0.75% or 1.00%) for the same level of operational risk and the same amount of maximum bank loan.

As a first step, the study investigates how that would modify the risk of the whole SC. While the probability of bankruptcy raised in the non-cooperative cases from 2.49% to 3.66% and 4.31%, respectively, there is hardly any difference in other risk measures (Tables 5 and 6), indicating that level of interest is not a vital driver of risk here.

Based on our results, in line with intuition, higher rates lead to a higher bankruptcy risk for which cooperation could compensate. When correlation was more elevated, closing cash showed a higher standard deviation (see Tables 7 and 8). Collaboration is less efficient when the correlation is positive than when assuming no or negative connection, while in our model, there is a negative correlation of −0.4. was enough to eradicate the risk (see Table 9). Even zero correlation may be sufficient to mitigate most of the risk, as no measurable difference showed across those cases in bankruptcy likelihood.

Based on this, SC decision-makers may prefer to assess loans in firms and countries that receive funding at a lower rate and provide sources via an intra-group system to members with less preferred opportunities. Thus, banks offering service only to one member of a cooperative SC may be better off by considering other financing opportunities of the SC as competitive products instead of only focusing on local market conditions.

Table 5. Results for the base case without any cooperation and with an interest rate difference (0.50%-0.50%-0.75%).

	Min	Mean	Max
Final Cash Balance	−118.86	208.92	658.79
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	0.00	12.10	100.00
Bankruptcy firm periods	0.00%	0.00%	0.27%

Table 6. Results for the base case without any cooperation and with an interest rate difference (0.50%-0.50%-1.00%).

	Min	Mean	Max
Final Cash Balance	−154.37	222.47	635.69
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	0.00	10.83	100.00
Bankruptcy firm periods	0.00%	0.00%	0.28%

Table 7. Results for the base case with cooperation and interest rate difference (0.50%-0.50%-1.00%) (Correlation = 0.00).

	Min	Mean	Max
Final Cash Balance	51.83	221.67	635.82
Final Pool Debt	−271.06	0.00	256.78
Final Bank Loan	0.00	1.23	100.00
Bankruptcy firm periods	0.00%	0.00%	0.01%

Table 8. Results for the base case with cooperation and interest rate difference (0.50%-0.50%-1.00%) (Correlation = 0.40).

	Min	Mean	Max
Final Cash Balance	−132.42	224.98	644.26
Final Pool Debt	−254.09	0.00	233.29
Final Bank Loan	0.00	5.74	100.00
Bankruptcy firm periods	0.00%	0.00%	0.38%

Table 9. Probability of bankruptcy in different cases.

Cases	No Cooperation	Cooperation		
		Corr = 0.0	Corr = 0.4	Corr = −0.4
Base case	2.49%	0.03%	0.16%	0.00%
Interest rates 0.5%-0.5%-0.75%	3.66%	0.04%	0.36%	0.00%
Maximum loan 100-100-100				
Interest rates 0.5%-0.5%-1.00%	4.31%	0.02%	0.75%	0.00%
Maximum loan 100-100-100				

6.3. Diverse Individual Operational Risk

This section assumes the standard deviation of the possible periodic cash flows is uneven across the firms. Instead of the standard deviation of 10 applied to all firms earlier, here, 10, 15, and 20 were set to represent differences in operational risks of the chain members. However, each firm's expected value of the periodic cash flow remained at 1. Table 10 contains the three agents' modified outcomes for liquidity changes (A, B and C).

Table 10. Outcome matrix for the diverse operational risk case.

Outcomes	Scenario 1	Scenario 2	Scenario 3	Scenario 4
Firm A	1.0000	1.0000	15.1421	−13.1421
Firm B	1.0000	25.4949	−11.2474	−11.2474
Firm C	35.6410	−10.5470	−10.5470	−10.5470

The new setup produced a bankruptcy probability of 24.55 % when no cooperation was allowed. Using a joint cash pool system kept the likelihood down at 1.21%. Tables 11 and 12 illustrate that all risk measures have climbed compared to the base case, while the average closing bank loan balance increased.

Table 11. Results for diverse operational risk without cooperation.

	Min	Mean	Max
Final Cash Balance	−430.16	207.33	1143.57
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	0.00	20.94	100.00
Bankruptcy firm periods	0.00%	0.02%	0.54%

Table 12. Results for diverse operational risk with cooperation.

	Min	Mean	Max
Final Cash Balance	−225.47	236.68	1103.94
Final Pool Debt	−538.89	0.00	653.19
Final Bank Loan	0.00	6.96	100.00
Bankruptcy firm periods	0.00%	0.00%	0.60%

6.4. Diverse Operational Risk and Bank Loan Limits

As a next step, the model assumed that in line with real-life experience, SC members with higher operation risk receive fewer loans from the market. Therefore, to keep the total debt available for the SC at the same level, the maximum bank debt limit changes to 150, 100, and 50 for the firms with a standard deviation of 10, 15, and 20, respectively.

As the firm with the highest standard deviation is likely to face the most significant liquidity issues and the strictest loan limits, the new rules lead to an even higher chance of bankruptcy (37.77%) (for details, see Tables 6 and 13). However, a bankruptcy risk increase is hard to spot (Tables 7 and 14). While fellow firms are likely to pass over their lines of credit using the cash pool at a lower cost, limited own credit lines reduce the chance of bankruptcy for the SC member with higher operational risk. Thus, the probability of default fell to 1.04%.

Table 13. Results for diverse operational risk and loan limits without cooperation.

	Min	Mean	Max
Final Cash Balance	−497.31	220.45	866.51
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	0.00	13.41	150.00
Bankruptcy firm periods	0.00%	0.03%	0.58%

Table 14. Results for diverse operational risk and loan limits with cooperation.

	Min	Mean	Max
Final Cash Balance	−132.72	244.95	1103.15
Final Pool Debt	−565.37	0.00	666.28
Final Bank Loan	0.00	6.21	150.00
Bankruptcy firm periods	0.00%	0.00%	0.28%

6.5. Diverse Operational Risk and Interest Rates

Differences in operational risk may affect the amount of debt available and the rates at which credit is provided. Thus, the credit lines remained at 100 for each firm, but a rate of 0.50, 0.75%, and 1.00% was applied in line with the risk level. This modification pushed the likelihood of bankruptcy up to 30.9% in the non-cooperative case and to 1.38% when the cash pool was available. Tables 15 and 16 show the same changes in the bankruptcy firm periods (remember: in each period, one may count 0 to 3 firm bankruptcy periods).

Table 15. Results for diverse operational risk and interest rates without cooperation.

	Min	Mean	Max
Final Cash Balance	−526.44	218.54	912.82
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	0.00	17.44	100.00
Bankruptcy firm periods	0.00%	0.03%	0.46%

Table 16. Results for diverse operational risk and interest rates with cooperation.

	Min	Mean	Max
Final Cash Balance	−178.50	234.53	1007.90
Final Pool Debt	−480.67	0.00	651.52
Final Bank Loan	0.00	5.77	100.00
Bankruptcy firm periods	0.00%	0.00%	0.55%

6.6. Diverse Operational risk, Bank Loan Limits, and Interest Rates

Finally, to make our model more realistic, required rates on bank loans should also reflect the increased level of operational risk. Therefore, the rates were modified to 0.50%, 0.75%, and 1.00%, so any firm with a higher risk pays a higher rate than its less risky counterparts. Tables 17 and 18 present the indicators for the two cases. While for the non-cooperative case, only a slight growth showed in bankruptcy likelihood (34.52%), the cooperative case had its probability climbing to 2.00%.

Table 17. Results for diverse operational risk, loan limits, and interest rates without cooperation.

	Min	Mean	Max
Final Cash Balance	−527.57	226.33	1097.57
Final Pool Debt	0.00	0.00	0.00
Final Bank Loan	0.00	19.21	150.00
Bankruptcy firm periods	0.00%	0.03%	0.60%

Table 18. Results for diverse operational risk, loan limits, and interest rates with cooperation.

	Min	Mean	Max
Final Cash Balance	−271.14	231.23	1100.14
Final Pool Debt	−589.64	0.00	677.68
Final Bank Loan	0.00	7.98	150.00
Bankruptcy firm periods	0.00%	0.00%	0.38%

In our model, the amount of debt available is just as significant as a risk driver as the cost of debt when the correlation among SC member cash flows was zero. Nevertheless, once one allows for cooperation, the cash pool reduces the importance of the limited credit lines but cannot compensate for increased interest expenses. Therefore, the rates play a more critical role in determining the financing risk of SCs with cash pool systems.

Thus, a lesson for the banks may be not to judge the risk of bankruptcy of an SC member on an individual basis, as increased individual risk may be waived by the potential help offered by other SC members. Limiting credit lines reduces the risk of bankruptcy for the specific SC member and the whole chain, once another SC member may provide the lost amount (for contrast, please see Tables 12 and 14).

However, offering debt at a higher cost may lead to an increased bankruptcy risk of the entire SC. While Tables 12 and 16 show little difference, the total bankruptcy rate climbed from 1.21% to 1.38%. Therefore, to reduce the expected loss on credit risk, it seems more efficient to lend less to a risky cooperative SC member than to lend at a higher cost.

From the point of the whole SC, it appears to be optimal to limit credit lines for members with high operational risk and replace that with increased credit facilities from lower-risk members as the direct borrowing of the earlier may not only be more costly but could also increase the bankruptcy risk of the whole SC. This conclusion also holds for cases where the financing of an SC member is more expensive due to other reasons (e.g., higher taxes, country risk or inflation).

6.7. The Effect of Correlation

As a next step, the study investigates how changes in the correlation among the periodic cash flows of the SC members would influence the earlier results. The models both for a correlation of 0.4 and −0.4 were run. Independent firms simulated in the earlier section may represent companies (business units, industries) selling different, not-interlinked products or services. A positive correlation may reflect interlinked SCs members (buyer-suppliers connection), while a negative link may represent members of a multinational conglomerate including diverse activities located probably in different countries and offering vast diversification opportunities. As in non-cooperative cases, the firms have no links; the re-estimation of the model results was only needed for the cooperative (joint cash pool) cases.

The findings show that a positive correlation among actors increases the risk of bankruptcy while a negative one reduces it. In the model, a medium-level negative connection is already enough to remove the bankruptcy risk.

The most crucial conclusion could be that when addressing the liquidity risk of an SC member that may count on the help of fellow firms, one should also consider how the performance of the given entity correlates with that of the rest of the SC (Table 19).

Table 19. Probability of bankruptcy in different cases.

Cases	No Cooperation		Cooperation					
			Corr = 0.0		Corr = 0.4		Corr = -0.4	
Base case	2.49%		0.03%		0.16%		0.00%	
Diverse operational risk	24.55%		1.21%		8.25%		0.00%	
Diverse operational risk	Diverse interest rates	Diverse loan limits						
	30.90%	33.66%	1.38%	1.04%	8.30%	9.24%	0.00%	0.00%
Diverse op. risk, loan limits, and rates	34.52%		2.00%		12.18%		0.00%	

7. Main Conclusions

Our model's main contribution is to investigate the effect of SC level cash management on bank lending risk not in a two but a three-agent environment while separating the partial impacts of the leading bankruptcy risk drivers. Our most important findings are the following.

1. First, our model reproduced several literature results as follows. Individual firms (no-cooperation cases) are more likely to go bankrupt when (A) their operation risk is higher, (B) their credit lines are limited, and (C) they face higher interest rates to pay. Traditionally, higher operating risk firms obtain foreign financing at higher rates and in more limited amounts than less risky counterparts due to their higher non-leveraged bankruptcy rates. As higher interest rates may be just as dangerous as restricted access to debt, firms with higher operational risk face a higher add-on risk even for the same level of leverage, and they may find it harder to achieve the same level of leverage.
2. When providing a loan to a firm member of a cooperative SC, the classic rules listed earlier may not always be valid. Cooperation among SC members may reduce bankruptcy risk considerably; thus, when assessing bankruptcy risk, the chance and extent of inter-entity financial aid is key to consider. Therefore, contrasting the leverage and rates valid for a cooperative SC member and a traditional firm may be less meaningful. In our experience, analysts rarely consider this limitation when analyzing databases of financial statements.
3. Cash pool systems may offer a defense against both credit limits and higher rates. However, this protection implies that banks serving only one SC member must compete with alternative local capital sources and with all different banks serving other SC members. At the same time, a less than perfect correlation among SC members could lead to a total operating risk even lower than that of the least risky member of the SC. Therefore, offering a higher than elsewhere standard rate makes limited sense even if a member has a higher operational risk. Thus, the market pushes the banks to serve a local SC member at an internationally competitive interest rate. No wonder multinational SCs usually have a strong financing connection with the same banking group in all countries of operation.
4. However, limiting the amount of the loan made available for a cooperative SC member might be optimal to restrict the bank's exposure to default. The remaining need for financing will be taken by another SC member offering an additional guarantee with its assets. This solution reduces the total bankruptcy risk of the SC, so the SC itself would prefer this structure. This result may explain why SCs do not finance themselves in a single country and only through one of their members.
5. When assessing the default risk of cooperative SC members, banks should pay particular care to estimating and monitoring the correlations between the operational cash flows of cooperative SC members. A change (e.g., during a crisis period) may radically affect the risk taken by the lender. Thus, a financier may need extra data

about the operation of the total SC in addition to the standard information gathered in the case of a stand-alone company loan. Additionally, for loans offered to such entities, SC correlation should be an additional bankruptcy risk factor to include in the bank's risk monitoring and mitigation process. As the consequences of the COVID-19 crisis recently showed, spatial diversification should consist of physical, financial (interlinked monetary systems), and social (human movements and personal connections) distances in remote locations.

To sum up, we have several policy recommendations. First, lending to a cooperative SC member requires different considerations and monitoring systems than traditional corporate loans. Therefore, analysts should carefully compare the financial statements of conventional firms and cooperative SC members.

Second, even cooperative SC members should not be treated the same way. For example, the operational cash flow correlation might be positive for firms with a buyer-supplier connection and negative for firms cooperating with entities performing the same tasks ("competitors") within the SC.

Third, analysts must understand that from the point of leverage, a cooperative SC member is not to be judged individually. As they could be financed from debt taken by other members and may offer protection to other SC members using its operating cash flow, operating and funding risk is SC-specific and not a characteristic of an individual firm.

Finally, while in our model, there was only one way the SC members could help each other by explicitly contributing to the SC cash pool, in real life, such aids may take various forms. For example, central management may force an entity to offer subsidies by artificially distorting transfer prices or payment terms, and the SC HQ may charge unjustified management fees that create funds to lend to SC members in need. Therefore, when used with proper monitoring, these techniques may also serve cash management purposes, usually considered a toolset for tax optimization. Thus, all those possible measures should be considered when analyzing statements and estimating operational (default) risk.

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