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Article

Research on Formation Control Method of Heterogeneous AUV Group under Event-Triggered Mechanism †

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Abstract: The time-sampling control strategy has communication discontinuities in the control of multiple AUVs (autonomous underwater vehicles). To overcome this problem, a distributed event-triggered communication mechanism is proposed to make each AUV communicate only when its own state is updated, which reduces the frequency of communication and improves the stability. This mechanism has better adaptability for formation control between heterogeneous AUV groups. At the same time, two consistency control algorithms based on event-triggered for homogeneous and heterogeneous AUV groups are studied, respectively. The known consistency algorithms are applied to the control of heterogeneous AUV groups for comparative analysis. The simulation results demonstrate that the number of communication among AUVs under the event-triggered control strategy can be significantly reduced. Therefore, the stability of the system is improved. Compared with the traditional consensus algorithm, the algorithm proposed in this paper has advantages in the control of heterogeneous AUV groups.

Keywords: homogeneous AUVs; heterogeneous AUVs; event-triggered control; consistency control

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1. Introduction

With the exploration of the ocean by human beings, a single AUV is no longer capable of handling the tasks with high operation volume and complexity, such as deep-sea resource exploration, underwater rescue, and deep-sea scientific research [1,2]. Therefore, the cooperative control of multiple AUVs has become a popular control problem [3]. Multi-AUV groups are mainly divided into homogeneous and heterogeneous groups [4]. The homogeneous AUV swarm (HOAUVs) has the same type of AUVs, and the form of the group is simple and easy to study; thus, most of the research on cooperative control methods of multi-AUV groups is focused on HOAUVs [5,6]. However, in the actual mission scenario, due to the different manufacturers of AUVs, their underlying technical architectures, equipment usage management, mission load functions, and other differences, the realistic AUV groups are mostly a heterogeneous swarm (HEAUVs). Consequently, it is of great theoretical value and practical significance to research this type of group [7]. In the case of harsh hydrographic environments and complicated detection tasks, the high intelligence and multi-functionality of HEAUVs can accomplish tasks that cannot be accomplished by single AUVs or HOAUVs or are difficult to accomplish. HEAUVs are the inevitable trend of AUV field development [8].

Multi-AUV systems face many difficulties when operating underwater. In [9], we can find that the quality and reliability of underwater communication are lacking, and the communication topology needs to be improved. In [10], there are phenomena such as packet loss during communication in underwater wireless sensor networks. In [11], The interference of underwater acoustic communication has a greater impact on the multi-AUV system. To reduce the communication frequency, a periodic pulse control strategy is proposed [12]. Neighboring AUVs communicate only at pulse moments, and this strategy also causes unnecessary communication due to the use of a fixed pulse period and does not address the continuous update problem [13]. In practical engineering applications, the behavior of AUVs is mainly controlled by an embedded digital controller [14]. In this controller role, the sampling of data, control law updates, and information transfer occurs at the moment of the fixed sampling period [15]. We call this approach a time-triggered control strategy. This control method has been widely used for some time because of its ease of controller design and implementation [16]. It can be found that when using the time-triggered control strategy, the fixed sampling period is generally chosen to be relatively small to take into account the system stability and performance, which inevitably results in frequent controller updates and data transfer. In the operation process, when the difference between the two sampled state values is small and the system tends to be stable, there is no need for the system to update the controller frequently and pass the information several times [17]. Although the time-triggered control will reduce the number of communications and controller updates to a certain extent, it also causes unnecessary consumption of computational and communication resources. Therefore, a more effective control method, event-triggered control, has been proposed [18].

Event-triggered control strategy, compared with the time trigger, does not need to set a fixed sampling period in the state acquisition [19]. Event-triggered control needs to meet the system stability for the premise by setting a trigger condition [20]. When the state error associated with the system does not meet this preset condition, the system will only perform a corresponding action, that is, controller update, as well as information transmission [21]. Event-triggered control can be divided into centralized event triggering and distributed event triggering [22]. In centralized event-triggered control, there is only one trigger condition in the system, and a control center is needed to obtain the state information of all the agents. When the system does not satisfy this trigger condition, all the agents update the controller at this moment and transmit the information to other neighboring intelligences [23]. For the distributed event-triggered control, each agent has its trigger condition; thus, each agent controller updates and broadcasts its trigger sampling state at different moments [24]. Compared with the centralized trigger control, the distributed trigger control requires only local information and has better flexibility and immunity to interference.

At present, the problem of the coherent control of multi-intelligent systems based on event-triggered mechanisms has received the attention of many scholars at home and abroad, and many research results have been achieved. Deng studied the event-triggered tracking control problem for fully driven AUVs in the vertical plane [25]. Gao studied a fixed-time pilot-following formation control method for a set of AUVs with event-triggered acoustic communication [26]. Kim proposed a distributed event-driven adaptive formation control strategy for the 3D formation tracking of a group of AUVs [27]. Li proposed asynchronous and synchronous communication strategies and proved the consistency of the algorithm with limited information exchange and distributed communication delays [28]. Mu studied the event-triggered consensus problem for multi-intelligent systems with input time delays [29]. Su studied the fixed-time event-triggered formation control problem for multi-AUV systems with external uncertainty [30]. Su proposed an event-triggered integral sliding mode fixed-time control method to solve the trajectory tracking problem for AUVs with disturbances [31]. Xu studied the event-triggered distributed adaptive dichotomous consensus control problem for a multi-AUV system with fixed topology [32].

From these studies, we can find that most researchers focus on the study of HOAUVs, and there are fewer research results involving the control strategy of HEAUVs. In designing the event triggering control strategy, it is necessary to ensure that the interval between any two consecutive event triggering moments is greater than zero, to avoid the occurrence of the zeno phenomenon, which refers to a system with numerous triggers in a finite period of time. How to effectively avoid this phenomenon is key in event triggering control. The periodic event triggering strategy can be a good solution to this problem [33]. Unlike the trigger control in the above literature, the periodic event triggering strategy combines the time triggering strategy and the event triggering strategy. The strategy judges the event triggering conditions at the sampling moment, thus further saving the computational resources. Since the event triggering moments all occur at the sampling moment, the minimum event triggering interval is a fixed period. The difference between event-triggered and time-triggered is illustrated in Figure 1.

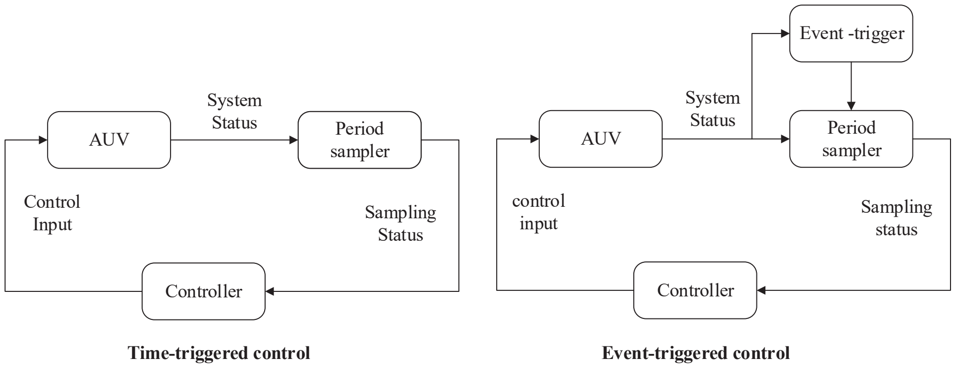


Figure 1. The difference between event-triggered and time-triggered.

In the information exchange process of HEAUVs, the application of the periodic event triggering strategy does not require continuous time communication, but through a pulse time series. However, this strategy also creates unnecessary communication due to the fixed pulse period and does not solve the problem of continuous controller updates.

Motivated by these existing works and observations, this paper proposes a distributed event-triggered control strategy for HOAUVs and HEAUVs. The main contributions can be summarized as follows:

1. A distributed event-triggered communication mechanism is proposed to make each AUV communicate only when its own state is updated.
2. Solve the cooperative control problem of HEAUVs, which can make the state of all AUVs consistent while saving computing resources and communication bandwidth.
3. When analyzing the minimum event trigger interval, a sufficient condition of the trigger condition is obtained. This condition is used to prove that the zeno phenomenon does not occur in the system.

The remainder of this paper is organized as follows: First, in Section 2, the dynamics of AUV and some assumptions are given. Then, the controller is designed for HOAUVs and HEAUVs in Section 3. Section 4 provides a simulation for multi-AUVs to demonstrate the control algorithms based on the event triggering strategy. Finally, Section 5 concludes this paper.

2. Problem Statement

2.1. Dynamics of AUV

For the convenience of problem research, this article assumes that the AUV is fully driven; each AUV can detect its position and angle information, and can receive the status information of the neighbor AUV. To describe the motion state of the AUV, an inertial coordinate system I and a motion coordinate system B are established, respectively. The AUV model is established as shown in Figure 2.

In practical engineering applications, the mathematical model of the AUV has nonlinear strongly coupled characteristics, which adds a great deal of difficulty to the design of the corresponding motion controller. When the equations of motion of a system become complex, it is often possible to develop design models that have less mathematical complexity but still capture the fundamental characteristics of the system. This paper simplifies the design of a model suitable for the design of AUV control algorithms. The AUV equations of motion can be transformed into a nonlinear state space represented by a fixed coordinate system, which is more suitable for linearization. Neglecting the external disturbances, the equations of the AUV model of motion in three degrees of freedom are considered as follows.

$$\begin{cases} \dot{\eta}=J(\eta)\nu \\ \dot{\nu}=M^{-1}(-C(\nu)\nu - D(\nu)\nu - g(\eta) + \tau), \end{cases} \tag{1}$$

where $J(\eta)$ denotes the rotational transformation matrix, M denotes the inertia matrix, $C(\nu)$ denotes the Koch force matrix, and $D(\nu)$ denotes the damping matrix. $\eta = (x, y, z)$ denotes the position state of the AUV, and x, y, z denotes the current northward, eastward, and depth-on coordinates of the AUV in a fixed coordinate system, respectively. $\nu = (u, v, w)$ denotes the linear velocity of the AUV, and u, v, w denotes the longitudinal, lateral, and vertical velocities of the AUV, respectively. $g(\eta)$ denotes the moment vector. τ denotes the thrust force, which is the control quantity to be designed in this paper.

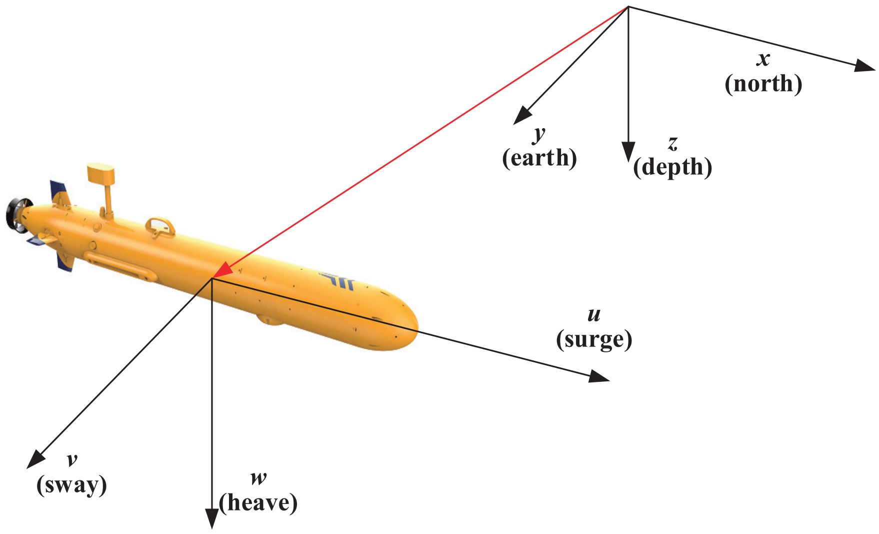


Figure 2. Model diagram of underactuated AUV in inertial and fixed coordinates.

2.2. Assumptions and Lemmas

Assumption 1. *Define the matrix:*

$$Q_i(\eta_i) = R_i^{-T}(\psi_i) M_i R_i^{-1}(\psi_i), \tag{2}$$

where $Q_i(\eta_i) \in R^{3\times3}$ is a positive definite symmetric matrix. There exist positive numbers $K_{Q\min}$ and $K_{Q\max}$, which satisfy $K_{Q\min} x^T x \le x^T Q_i(\eta_i) x \le K_{Q\max} x^T x$ for any x.

Assumption 2. *Define the matrix:*

$$G(\eta_i, \dot{\eta}_i) = R_i^{-T}(\psi_i)\left[C_i\left(R_i^{-1}(\psi_i)\dot{\eta}_i\right) - M_i R_i^{-1}\dot{R}_i(\psi_i) + D_i\right] R_i^{-1}(\psi_i), \tag{3}$$

where $\dot{Q}_i(\eta_i) - 2G(\eta_i, \dot{\eta}_i)$ is a skew-symmetric matrix. There exists a positive number K_G satisfying $x^T G(\eta_i, \dot{\eta}_i) x \le K_G \|\dot{\eta}_i\| x^T x$ for any x.

Lemma 1. *When trigger function $f_i(t) \le 0$ and $0 < c < \frac{2\omega}{(2|N_i|+\omega)}$, the time interval satisfies $\Delta t_k^i > 0$ [2].*

3. Controller Design

3.1. Event-Triggered Controller for HOAUVs

We designed a distributed event-triggered controller for homogeneous AUV groups.

$$\begin{aligned} \lim_{t\to\infty} \eta_i - \eta_j = 0 \\ \lim_{t\to\infty} v_i - v_j = 0. \end{aligned} \tag{4}$$

Suppose $R_i(\psi_i)v_i = \gamma_i$. Due to $R_i^{-1}(\psi_i) = R_i^T(\psi_i)$, $(R_i^{-1})^T(\psi_i) = R_i^{-T}(\psi_i)$, $v_i = R_i^{-1}(\psi_i)\gamma_i$; Equation (1) can be written as

$$\begin{cases} \dot{\eta}_i = \gamma_i \\ M_i R_i^{-1}(\psi_i)(\dot{\gamma}_i - \dot{R}_i(\psi_i)R_i^{-1}(\psi_i)\gamma_i) \\ = -C_i(R_i^{-1}(\psi_i)\gamma_i)R_i^{-1}(\psi_i)\gamma_i - D_i R_i^{-1}(\psi_i)\gamma_i + \tau_i. \end{cases} \tag{5}$$

Suppose

$$Q_i = R_i^{-T}(\psi_i)M_i R_i^{-1}(\psi_i),$$

$$G_i = R_i^{-T}(\psi_i)[C_i(R_i^{-1}(\psi_i)\dot{\eta}_i) - M_i R_i^{-1}(\psi_i)\dot{R}_i(\psi_i) - D_i]R_i^{-1}(\psi_i),$$

$$p_{ij} = \eta_i - \eta_j,$$

then, Equation (5) can be written as

$$\begin{cases} \dot{p}_{ij} = \gamma_i - \gamma_j \\ \dot{\gamma}_i = -Q_i^{-1}(G\gamma_i - \tau_{qi}), \end{cases} \tag{6}$$

where $\tau_{qi} = R_i^{-T}(\psi_i)\tau_i$ denotes the control input we need to design.

Aiming at the control problems of the above systems, Hu et al. proposed a consistent distributed control protocol under the action of impulses in order to reduce the number of communications [34].

$$\tau(t) = -\sum_{j\in N_i} a_{ij}\big(\eta_i(t) - \eta_j(t)\big)o(t - t_k) - \omega\gamma_i(t), \tag{7}$$

where a_{ij} represents the corresponding element value of the adjacency matrix, ω is a constant, $o(t - t_k)$ represents the impulse function at time t_k and works once. It can be observed that in this control strategy, the control inputs of all AUVs still need to be updated continuously. Based on the shortcomings of this control protocol, the following improvements are made.

The controller is shown below.

$$\tau_{qi}(t) = -\sum_{j\in N_i} a_{ij}(\eta_i(t_k^i) - \eta_j(t_k^j)) - \omega\gamma_i(t_k^i), t \in [t_k, t_{k+1}], \tag{8}$$

Unlike Equation (7), we change the cycle time t to the event trigger (t_k^i), where k denotes the trigger moment, implying that the control volume is updated once only when the state changes. Additionally, if we consider more general motion scenarios, we only need to add depth to the control input in the control protocol. The control input can be changed to $\eta(x, y, z)$, where z is the depth. To describe the variation of the control quantity more precisely, we introduce the error function $e_{\eta i}(t)$ and $e_{\gamma i}(t)$, which denote the errors between the AUV and the leader in position and velocity, respectively. The event-triggered function can be represented as follows.

$$f_i(t) = \|e_i(t)\| - \sqrt{\frac{c\alpha_i\beta_i}{2\theta_i}}\|\gamma_i(t)\| - \delta_i(t), \tag{9}$$

where $\alpha_i, \beta_i, \theta_i$ are all constants greater than 0, and $\delta_i(t)$ denotes compensation function. Next, we will demonstrate the stability of the controller.

Proof. Suppose a Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} p_{ij}^T p_{ij} + \frac{1}{2}\sum_{i=1}^{n} \gamma_i^T Q_i \gamma_i, \tag{10}$$

where Q_i is a symmetric and positive definite matrix. $V > 0$ when p_{ij} and γ_i are not simultaneously 0. The derivative of V is

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} p_{ij}^T(\gamma_i - \gamma_j) + \sum_{i=1}^{n} \gamma_i{}^T Q_i \dot{\gamma}_i + \frac{1}{2}\gamma_i{}^T \dot{Q}_i \gamma_i \\
&\leq -\sum_{i=1}^{n} \omega \gamma_i{}^T \gamma_i - \sum_{i=1}^{n} \gamma_i{}^T \sum_{j=1}^{n} a_{ij} e_{\eta i}(t) \\
&\quad + \sum_{i=1}^{n} \gamma_i{}^T \sum_{j=1}^{n} a_{ij} e_{\eta j}(t) - \sum_{i=1}^{n} \omega \gamma_i{}^T e_{\gamma i}(t).
\end{aligned} \tag{11}$$

Due to

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \|e_{\eta i}(t)\| = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \|e_{\eta j}(t)\|, \tag{12}$$

$$\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{n} \omega \gamma_i{}^T \gamma_i + 2\sum_{i=1}^{n} \left\|\gamma_i{}^T\right\| \sum_{j=1}^{n} a_{ij} \|e_{\eta i}(t)\| - \sum_{i=1}^{n} \omega \gamma_i{}^T e_{\gamma i}(t) \\
&\leq -\sum_{i=1}^{n} \omega \gamma_i{}^T \gamma_i + \sum_{i=1}^{n} |N_i| \left(c\gamma_i{}^T \gamma + \frac{1}{c} e_{\eta i}{}^T(t) e_{\eta i}(t) \right) \\
&\quad + \sum_{i=1}^{n} \frac{|N_i|}{c} e_{\eta i}{}^T(t) e_{\eta i}(t) + \sum_{i=1}^{n} \frac{\omega}{2c} e_{\gamma i}{}^T(t) e_{\gamma i}(t).
\end{aligned} \tag{13}$$

When $f_i(t) \leq 0$,

$$\|e_i(t)\|^2 \leq 2\zeta_i{}^2 \|\gamma_i(t)\|^2 + 2\delta_i{}^2(t). \tag{14}$$

$$\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{n} \beta_i \|\gamma_i\|^2 + \sum_{i=1}^{n} \frac{|N_i|}{c} \|e_{\eta i}\|^2 + \sum_{i=1}^{n} \frac{\omega}{2c} \|e_{\gamma i}\|^2 \\
&\leq -\sum_{i=1}^{n} \beta_i \|\gamma_i\|^2 + \sum_{i=1}^{n} \frac{\theta_i}{c} \|e\|^2 \\
&\leq -\sum_{i=1}^{n} (1-\alpha_i)\beta_i \|\gamma_i\|^2 + \sum_{i=1}^{n} \frac{2\theta_i}{c} \kappa_i{}^2 e^{-2\varepsilon(t-t_0)}.
\end{aligned} \tag{15}$$

Integrating over Equation (15):

$$\sum_{i=1}^{n} (1-\alpha_i)\beta_i \int_0^t \|\gamma_i(\partial)\|^2 d\partial \leq V(0) + \sum_{i=1}^{n} \frac{\theta_i \kappa_i^2}{c\varepsilon_i}. \tag{16}$$

Due to Q_i and G_i being bounded, $\dot{\gamma}_i$ is bounded. Therefore, $\lim_{t\to\infty} \gamma_i = 0$, $\dot{\gamma}_i \to 0, \eta_i(t_k^i) - \eta_j(t_k^j) \to 0$. □

3.2. Event-Triggered Controller for HEAUVs

Supposing HEAUVs contain N AUVs, the kinetic equation can be expressed as follows [35].

$$\dot{\eta}_i(t) = \mathbf{A}\eta_i(t) + \mathbf{B}\tau_i(t) + \psi_i^*, i = 1, 2, ..., N, \tag{17}$$

where η_i denotes state variables, τ_i denotes control inputs, ψ_i^* denotes bounded vectors of AUVi, and **A** and **B** denote system parameters. Based on the event triggering mechanism [36], we design the controller as shown in Equation (18).

$$\tau_i(t) = s_i(t) + K \sum_{j \in N_i} a_{ij}(\hat{\eta}_j(t) - \hat{\eta}_i(t)), t \in [t_k^i, t_{k+1}^i), \tag{18}$$

where K denotes control gain matrix, $\hat{\eta}_i(t) = e^{A(t-t_k^i)}\eta_i(t_k^i)$ denotes estimated value, and a_{ij} denotes the i,j element of the adjacency matrix **A**. To compensate for the heterogeneity, variable $s_i(t)$ is introduced. $s_i(t)$ updates when the event trigger condition is activated [36].

$$\dot{s}_i(t) = \mathbf{H} \sum_{j \in N_i} a_{ij}(\hat{s}_j(t) - \hat{s}_i(t)), \tag{19}$$

where $\hat{s}_i(t) = \psi_i^* + s_i(t_k^i)$, **H** denotes the corresponding control gain matrix. The event trigger function is shown in Equation (20).

$$f_i(t) = \|e_i(t)\| - c_1 \left\| \sum_{j \in N_i} a_{ij}(\hat{\eta}_j(t) - \hat{\eta}_i(t)) + \sum_{j \in N_i} a_{ij}(\hat{s}_j(t) - \hat{s}_i(t)) \right\| - c_2 e^{-\alpha t}, \tag{20}$$

where $e_i(t) = \left[E_{\eta,i}^T(t), E_{s,i}^T(t)\right]^T$, $E_{\eta,i}(t) = \eta_i(t_k^i) - \eta_i(t)$ and $E_{s,i}(t) = s_i(t_k^i) - s_i(t)$ denote the measurement error. Next, we will demonstrate the stability of the designed controller.

Proof. Suppose there exist positive definite matrices $\bar{P} = diag\{\bar{P}_1, \bar{P}_2\}$ and $\bar{M} = diag\{M_1, M_2\}$, then $K = M_1\bar{P}_1^{-1}$ and $\mathbf{H} = M_2\bar{P}_2^{-1}$. According to the event triggering mechanism, we can know that the stability of the system can be proven when Equation (21) is satisfied [37].

$$\begin{cases} 0 < \alpha < \frac{\lambda}{2} \\ 0 < c_1 < \min\left\{ \sqrt{\frac{1}{\lambda_{\max}(\bar{P}^{-1})}}, \frac{1}{N\|L \otimes I_{2n}\|} \right\} \\ 0 < c_2 < \sqrt{\frac{1}{\lambda_{\max}(\bar{P}^{-1})}}. \end{cases} \tag{21}$$

Due to

$$\begin{aligned} \|e_i(t)\| &\leq c_1 \left\| \sum_{j \in N_i} a_{ij}(\hat{x}_j(t) - \hat{x}_i(t)) + \sum_{j \in N_i} a_{ij}(\hat{z}_j(t) - \hat{z}_i(t)) \right\| + c_2 e^{-\alpha t} \\ &\leq c_1 \|(L \otimes I_{2n})\delta(t) + (L \otimes I_{2n})e(t)\| + c_2 e^{-\alpha t}. \end{aligned} \tag{22}$$

Sum of squares for Equation (22):

$$\begin{aligned} \sum_{i=1}^{N} \|e_i(t)\|^2 \leq\ & 4N{c_1}^2 \|(L \otimes I_{2n})\delta(t)\|^2 \\ & + 4N{c_1}^2 \|(L \otimes I_{2n})e(t)\|^2 \\ & + 2N{c_2}^2 e^{-2\alpha t}. \end{aligned} \tag{23}$$

A Lyapunov function is chosen for a fixed topology [38]:

$$V(\delta(t)) = \delta(t)^T \mathrm{P}\delta(t). \tag{24}$$

Taking the derivative of $V(\delta(t))$:

$$\begin{aligned} \dot{V}(\delta(t)) &= 2\delta(t)^T \mathrm{P}[\hat{A}\delta(t) + \hat{B}\delta(t) + \hat{B}e(t)] \\ &\leq \tilde{\eta}(t)^T \prod_1 \tilde{\eta}(t) - \lambda\delta(t)^T \mathrm{P}\delta(t) + e(t)^T \mathrm{P}e(t) \\ &\leq \tilde{\eta}(t)^T \prod_1 \tilde{\eta}(t) - \lambda\delta(t)^T \mathrm{P}\delta(t) + \lambda_{\max}(P)e(t)^T e(t). \end{aligned} \tag{25}$$

Due to $c_1^2 c_2^2 \in (0, \lambda_{\max}(P))$, Equation (25) can be written as

$$\begin{aligned} \dot{V}(\delta(t)) &\leq \tilde{\eta}(t)^T \prod_2 \tilde{\eta}(t) - \lambda\delta(t)^T \mathrm{P}\delta(t) + 2Ne^{-2\alpha t} \\ &\leq -\lambda V(\delta(t)) + 2Ne^{-2\alpha t}. \end{aligned} \tag{26}$$

Integrating over Equation (26):

$$\begin{aligned} V(\delta(t)) &\leq V(\delta(t_0))e^{-\lambda(t-t_0)} + 2N\int_{t_0}^{t} e^{-2\alpha t} ds \\ &\leq V(\delta(t_0))e^{-\lambda(t-t_0)} + \frac{2N}{\lambda - 2\alpha}e^{-2\alpha t}. \end{aligned} \tag{27}$$

Suppose $q_1 = \lambda_{\min}(P)$, $q_2 = \lambda_{\max}(P)$

$$q_1\|\delta(t)\|^2 \leq V(\delta(t)) \leq q_2\|\delta(t)\|^2, \tag{28}$$

$$\|\delta(t)\|^2 < \frac{q_2\|\delta(t)\|^2}{q_1}e^{\lambda(t-t_0)} + \frac{2N}{q_1(\lambda - 2\alpha)}e^{-2\alpha t}. \tag{29}$$

In summary, HEAUVs can reach agreement with the control gain of $K = M_1\bar{P}_1^{-1}$ and $H = M_2\bar{P}_2^{-1}$. □

We reflect the anti-interference ability of the controller through the proof of stability. However, this controller still needs further improvement when dealing with the problem of parameter uncertainty.

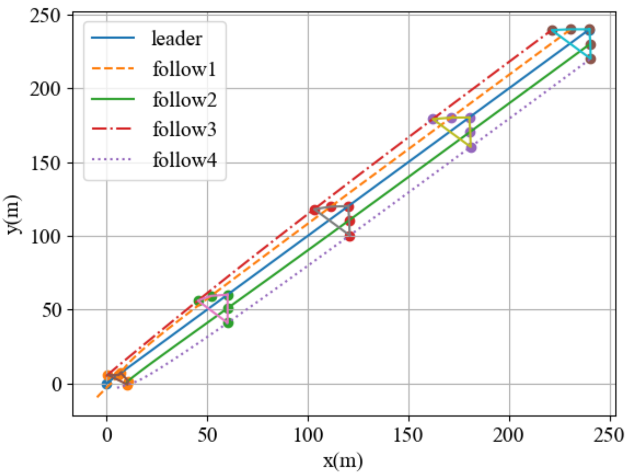
4. Simulation and Analysis

4.1. Simulation of HOAUVs under Event-Triggered Control

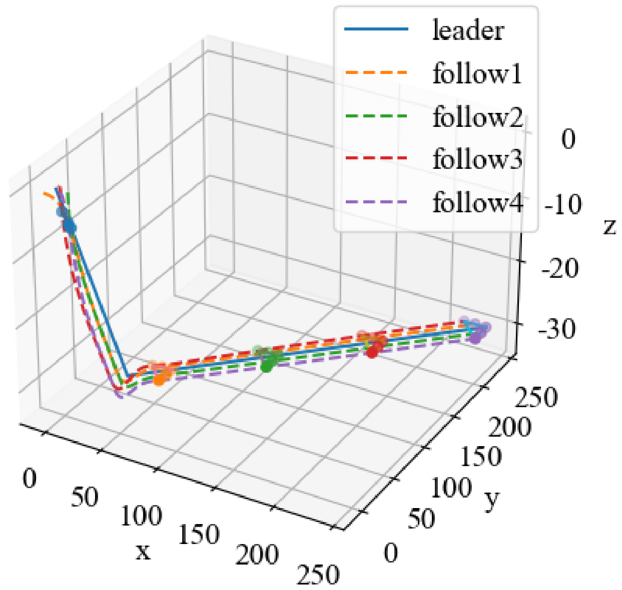
This subsection conducts a simulation analysis for the consistency control algorithm of HOAUVs under event-triggered control. One leader and four followers form an isomorphic multi-AUV swarm [39]. The initial position of the pilot is randomly distributed in the interval $[-5, 5]$, the initial position of each follower is randomly distributed in the interval [−10, 10], the initial combined speed is assumed to be 5 m/s, and the initial values of other state variables are set to 0. The control input of the leader AUV $u_0(t) = t\sin t$ is set, and the trigger interval is set to 0.1 s. According to the controller, select the control gain $\gamma_x = \gamma_y = 1$, $\gamma_z = 1.6$. The system-weighted adjacency matrix is shown as follows [40].

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{30}$$

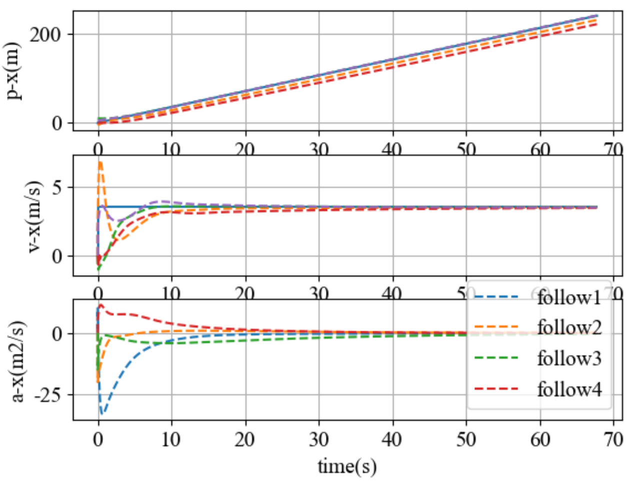
The simulation results are shown in Figure 3. From Figure 3a,b, it can be observed that the formation process of the AUV swarm and the state of the formation keep their navigation. From Figure 3b, it can be observed that the followers in the AUV swarm are able to keep the desired relative distance motion with the navigator. From Figure 3c–e, it can be observed that the velocity of the followers is able to reach the same speed as that of the navigator. The acceleration of the follower eventually converges to zero. The event trigger moments are shown in Figure 3f. It can be observed that the proposed control mechanism can significantly reduce the number of communications. In realistic applications, we can select four or more AUVs of equal size and type as test objects. The control center is deployed on the shore, and control commands are sent to each AUV according to a fixed period. According to the motion data returned by the AUV, the consistency control algorithm is used to adjust the control instructions to realize the consistency control of HOAUVs. Moreover, in the actual underwater communication environment, we can take the time delay into account. In the original control strategy, we introduce a fixed time delay, and its formation change is shown in Figure 4a,b. It can be observed from the figure that when there is a time delay, the original control strategy obviously cannot guarantee that the state of HOAUVs reaches the same state in a limited time. Therefore, we need to improve the control strategy, which is also the direction for further research in this paper.



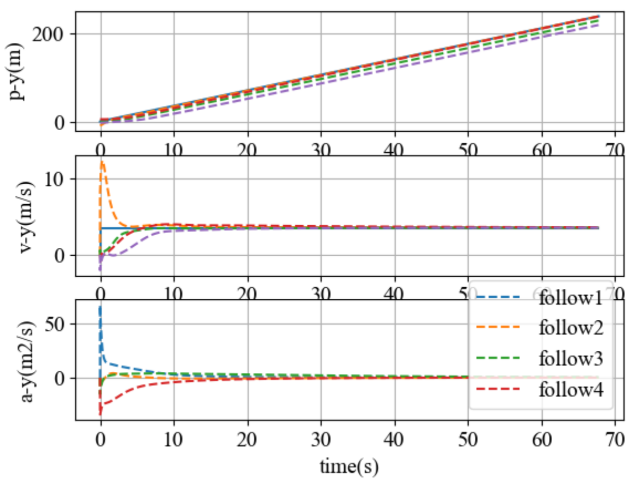
(**a**)



(**b**)

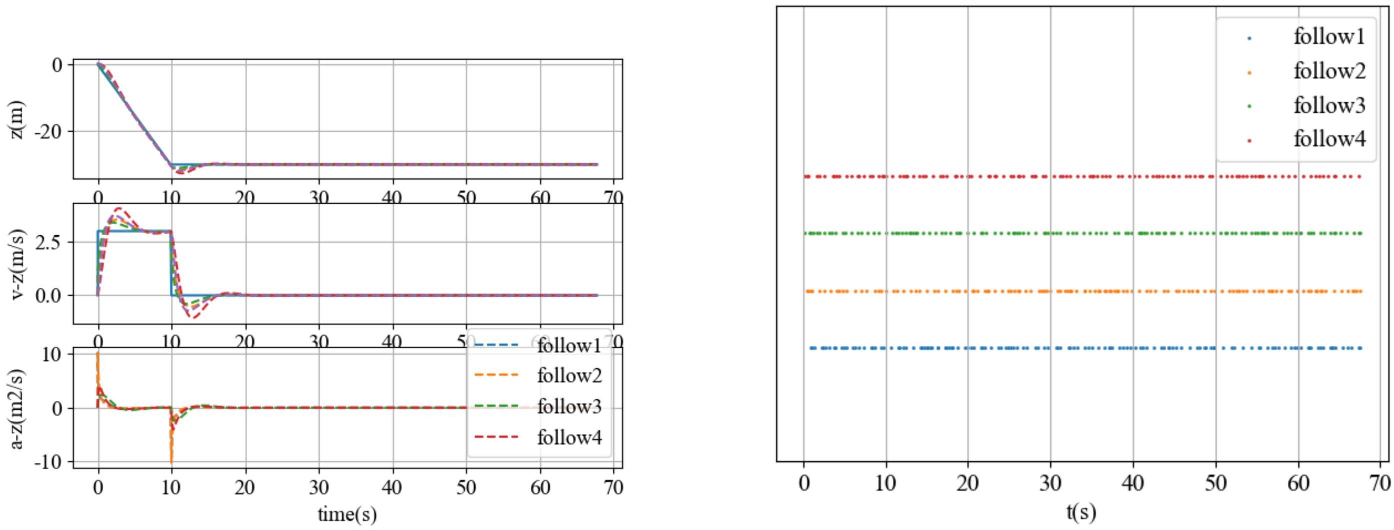


(**c**)



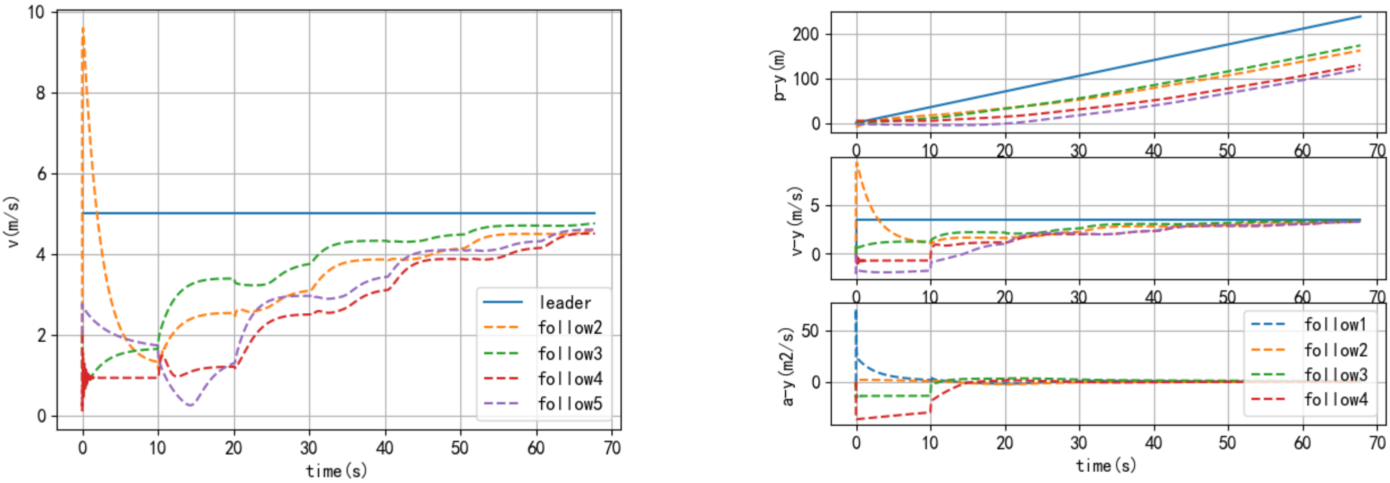
(**d**)

Figure 3. *Cont.*



(**e**) (**f**)

Figure 3. (**a**) HOAUVs' formation (2-D); (**b**) HOAUVs' formation (3-D); (**c**) state in x-direction (HOAUVs); (**d**) state in y-direction (HOAUVs); (**e**) state in z-direction (HOAUVs); (**f**) event trigger time (HOAUVs).



(**a**) (**b**)

Figure 4. (**a**) State in x-direction (HOAUVs with time delay); (**b**) state in y-direction (HOAUVs with time delay).

4.2. Simulation of HEAUVs under Event-Triggered Control

In this section, simulation is performed to verify the event-triggered controller designed for HEAUVs. To contrast with the previous section, the formation and initial condition settings are kept the same as Section 4.1. The parameters of each type of AUV system are as follows.

AUV1:

$$\mathbf{A}_1 = \begin{bmatrix} 0_{3\times3} & diag\{1,1,1\} \\ 0_{3\times3} & diag\{0.1668, 0.1991, 0.5766\} \end{bmatrix}$$

$$\mathbf{B}_1 = \begin{bmatrix} 0_{3\times3} \\ diag\{-0.01236, -0.0087, -0.0087\} \end{bmatrix}$$

AUV2:

$$\mathbf{A}_2 = \begin{bmatrix} 0_{3\times3} & diag\{1,1,1\} \\ 0_{3\times3} & diag\{0.5253, 0.3690, 0.9395\} \end{bmatrix}$$

$$\mathbf{B}_2 = \begin{bmatrix} 0_{3\times 3} \\ diag\{-0.01162, -0.0068, -0.0070\} \end{bmatrix}$$

AUV3:

$$\mathbf{A}_3 = \begin{bmatrix} 0_{3\times 3} & diag\{1,1,1\} \\ 0_{3\times 3} & diag\{0.0882, 0.1532, 0.1364\} \end{bmatrix}$$

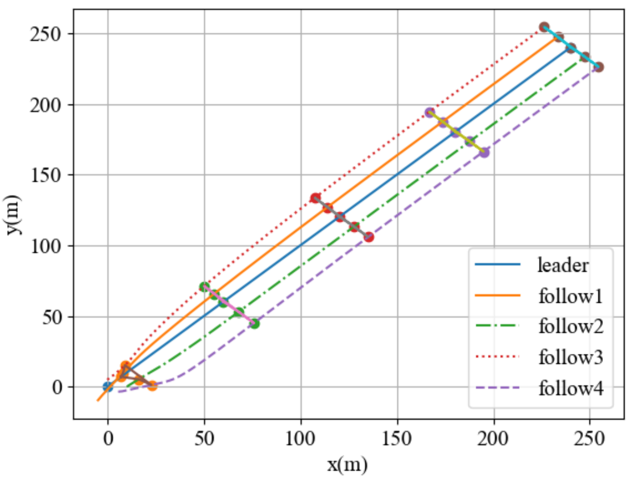
$$\mathbf{B}_3 = \begin{bmatrix} 0_{3\times 3} \\ diag\{-0.0007, -0.0004, -0.0004\} \end{bmatrix}$$

AUV4:

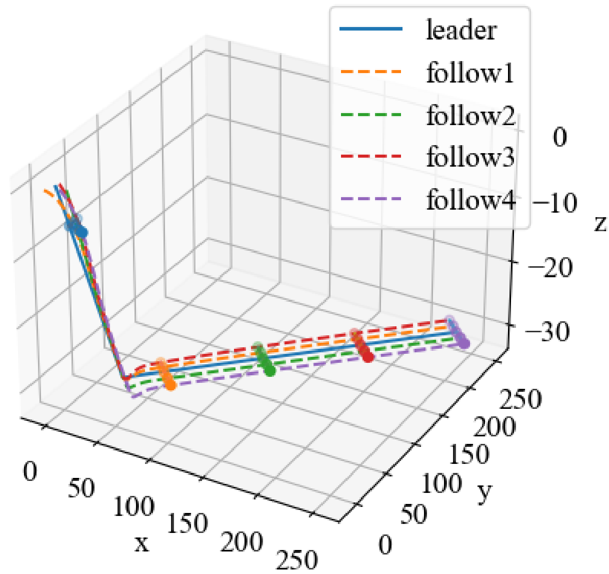
$$\mathbf{A}_4 = \begin{bmatrix} 0_{3\times 3} & diag\{1,1,1\} \\ 0_{3\times 3} & diag\{0.1337, 0.1902, 0.1693\} \end{bmatrix}$$

$$\mathbf{B}_4 = \begin{bmatrix} 0_{3\times 3} \\ diag\{-0.0002, -0.0001, -0.0001\} \end{bmatrix}$$

The formation process of HEAUVs can be observed in Figure 5a,b. The position and velocity of the follower can reach the leader, and the acceleration of the follower eventually converges to zero, as shown in Figure 5c–e. The event trigger moments are shown in Figure 5f. As can be observed from the figure, in the distributed event-triggered control algorithm proposed in this paper, each follower AUV only updates the control signal at the trigger time of its event, without considering the trigger time of its neighbor nodes, which can effectively reduce a large number of redundant AUVs. This can reduce the update frequency of the control signal of the system. Moreover, the estimation-based triggering condition is adopted, which can effectively prolong the release time of event triggering and reduce the number of event triggering controls. The convergence of the final speed and position state is better, and the consistency can be obtained faster.

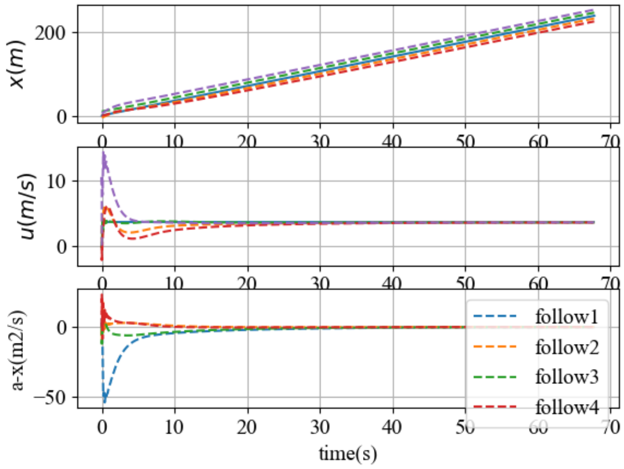


(a)

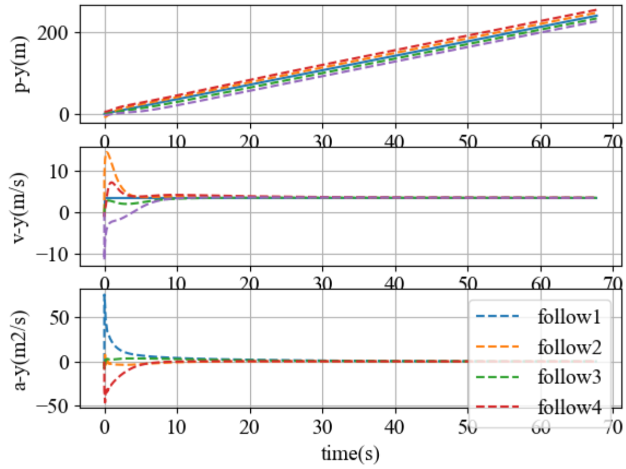


(b)

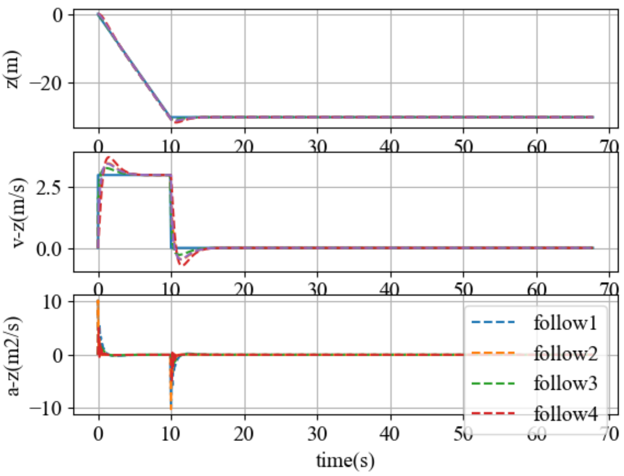
Figure 5. *Cont.*



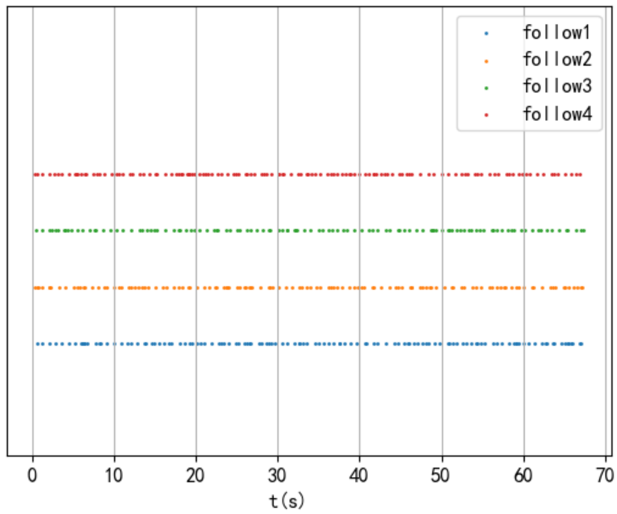
(**c**)



(**d**)



(**e**)

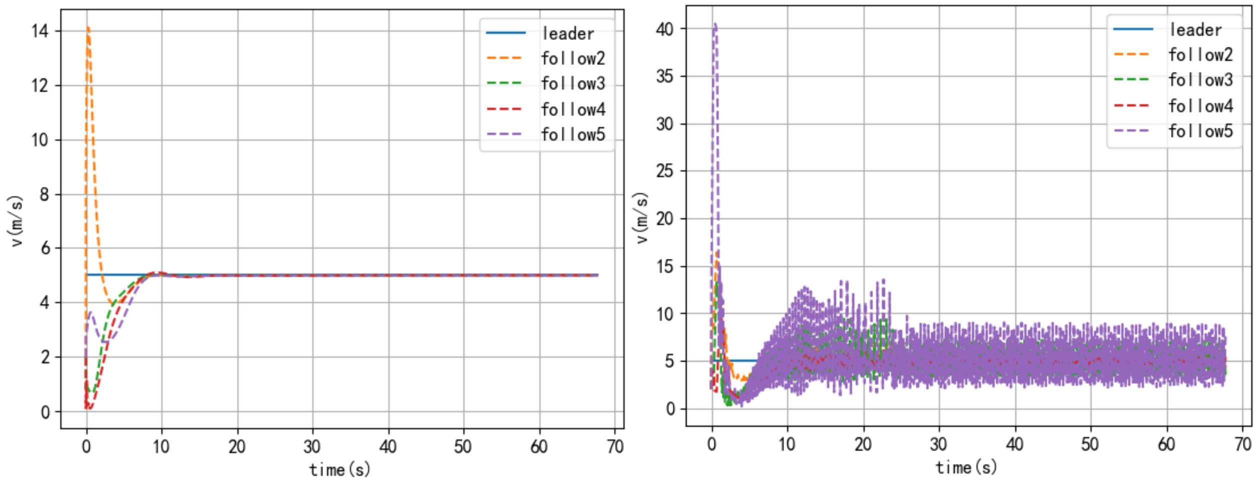


(**f**)

Figure 5. (**a**) HEAUVs' formation (2-D); (**b**) HEAUVs' formation (3-D); (**c**) state in x-direction (HEAUVs); (**d**) state in y-direction (HEAUVs); (**e**) state in z-direction (HEAUVs); (**f**) event trigger time (HEAUVs).

For comparison, we compare with the time-triggered control strategy [4]. The rest of the conditions refer to the settings in Section 4.2 and remain unchanged. We modify the control strategy to the time-triggered control. We select the combined velocity in the x and y directions as a comparison, and the simulation results are shown in Figure 6a,b.

It can be observed from the figure that due to the problem of the trigger mechanism, the state of each AUV cannot complete the convergence, and the overall situation presents a divergent trend. The distributed consistency control strategy proposed in this paper considers the heterogeneity of AUV groups, and has stronger applicability to HEAUVs. In summary, the simulation results illustrate that the proposed event-triggered cooperative formation control scheme is effective.



(**a**) (**b**)

Figure 6. (**a**) Velocity in x-y direction (HEAUVs with event-triggered); (**b**) velocity in x-y direction (HEAUVs with time-triggered).

5. Conclusions

In this paper, the consistency problem of the AUV swarm via event-triggered control is studied. The distributed consistency control algorithms based on an event-triggered mechanism for homogeneous and heterogeneous AUVs are proposed, respectively. Firstly, an AUV dynamics model with three degrees of freedom is given. Then, a simple transformation of the model is performed. An event-triggered control protocol and an event-triggered function are designed for the transformed model. After that, the stability analysis of the algorithm is combined with the Lyapunov stability theory, matrix theory knowledge, and Barbalat's Lemma. It is proven that the algorithm can achieve the consistent state of all AUVs while saving computational resources and communication bandwidth. In analyzing the minimum event trigger interval, a sufficient condition for the trigger condition is obtained.

The current research on the coordinated control of AUVs mainly relies on linear models. However, the singular matrix generated by the linearization process also brings some problems to the controller design. Therefore, the coordinated control method directly using the complex model will become the focus of subsequent research.

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Abbreviations

The following abbreviations are used in this manuscript:

AUV	Autonomous Underwater Vehicle
HOAUVs	Homogeneous AUV swarm
HEAUVs	Heterogeneous AUV swarm

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