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Online Frequency Estimation on a Building-like Structure Using a Nonlinear Flexible Dynamic Vibration Absorber

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Abstract: The online frequency estimation of forced harmonic vibrations on a building-like structure, using a nonlinear flexible vibration absorber in a cantilever beam configuration, is addressed in this article. Algebraic formulae to compute online the harmonic excitation frequency on the nonlinear vibrating mechanical system using solely available measurement signals of position, velocity, or acceleration are presented. Fast algebraic frequency estimation can, thus, be implemented to tune online a semi-active dynamic vibration absorber to obtain a high attenuation level of undesirable vibrations affecting the main mechanical system. A semi-active vibration absorber can be tuned for application where variations of the excitation frequency can be expected. Adaptive vibration absorption for forced harmonic vibration suppression for operational scenarios with variable excitation frequency can be then performed. Analytical, numerical, and experimental results to demonstrate the effectiveness and efficiency of the operating frequency estimation, as well as the acceptable attenuation level achieved by the tunable flexible vibration absorber, are presented. The algebraic parametric estimation approach can be extended to add capabilities of variable frequency vibration suppression for several configurations of dynamic vibration absorbers.

Keywords: vibration control; dynamic vibration absorbers; building-like structure; frequency estimation; cantilever beam

1. Introduction

Dynamic vibration absorbers are vibration control devices that have been implemented in many engineering systems, such as internal combustion engines, milling machines, compressors, washing machines, Jeffcott rotors, beams, offshore wind turbines, helicopter rotors, and rear wheel drive vehicles [1,2]. Several configurations and practical implementations of dynamic vibration absorbers are described in detail in [3]. An in-wheel powertrain system using dynamic vibration absorbers for electric vehicles is discussed in [4]. Applications of these control devices on offshore platforms are presented in [5,6]. Torsional vibration suppression in the hybrid electric vehicle powertrain system of the single-shaft parallel type, using a dual mass flywheel with centrifugal pendulum dynamic vibration absorbers, is analyzed in [7]. Passive, semi-active, active, and hybrid vibration control schemes for dynamic vibration absorbers have been developed [7,8]. The main advantage of the passive vibration absorbers is that no energy source is required for their operation. However, efficient forced vibration attenuation of the passive vibration absorbers is restricted into a specific small frequency bandwidth around the tuning frequency [9]. On the other hand, variable frequency vibration suppression capabilities of active, semi-active, and hybrid dynamic vibration absorbers could be achieved. A set of actuators and sensors should be suitably connected by a closed-loop control scheme in active vibration control systems [10]. Several robust control design methodologies could then be used for synthesis of active dynamic vibration absorbers [11,12]. A sliding mode control is another alternative to be considered in the design of active vibration control schemes [13,14]. In this regard, a proportional integral derivative (PID) type nonsingular fast terminal sliding mode control method is presented in [13]. A barrier function-based adaptive nonsingular sliding mode control technique is introduced in [14]. A stochastic stable control is another state-feedback loop design approach [15].

Semi-active dynamic vibration absorbers with tunable parameters are semi-passive vibration control devices that could be employed to track variable dominant excitation frequencies in order to reduce undesirable vibrations affecting to the primary mechanical system [16,17]. Semi-active dynamic vibration absorbers can be designed to change their stiffness and damping parameters for efficient suppression of large-amplitude vibrations on container cranes [18]. A tunable and controllable electromagnetic dynamic vibration absorber for vibration suppression on rotational machinery is introduced in [19]. Here, stiffness and damping parameters of this vibration absorption device are tuned to generate the control torques and forces. Centrifugal pendulum dynamic vibration absorbers have been employed for torsional vibration attenuation on rotating shafts by modifying the natural frequencies according to the excitation frequency [20]. A prototype of a rotational-pendulum vibration absorber with variable stiffness is presented in [21]. The stiffness of this pendulum absorber can be adjusted through regulating its rotational speed by means of an electric motor, using information of the excitation frequency. Several semi-active dynamic vibration absorbers with tunable frequencies, and their applications, are described in [22]. Diverse mechanisms for tuning the stiffness of dynamic vibration absorbers to improve their forced vibration attenuation capabilities are described in [23].

Accurate estimations of vibrating excitation frequencies are relevant in the implementation of efficient semi-passive control devices. Efficient frequency identification can be used in numerous applications of semi-active dynamic vibration absorbers, representing the main motivation for synthesis of online frequency estimation techniques. In this fashion, opportune information of the excitation frequency can be employed to tune a semi-active dynamic vibration absorber in real-time. Fast estimation of the excitation frequency could also be used to reconstruct vibrating disturbances affecting the primary system, to implement active vibration control schemes. In [24], an active vibration absorber based on frequency estimation and equivalent dynamic modeling is introduced. Here, the frequency estimation is a critical element of the proposed active vibration absorber algorithm. Several asymptotic frequency estimation techniques were developed in [24,25] and the references therein. A fast convergence rate and an acceptable steady-state frequency estimation error should be simultaneously taken into account [24]. Thus, the tuning of several design parameters should be properly considered in asymptotic estimation methods [25]. Then, asymptotic convergence of the frequency estimation error towards zero should be guaranteed. In this sense, asymptotic frequency estimation in electric voltage signals have been addressed in [26]. Other frequency estimation approaches, in the context of angular velocity controls in direct current electric motors subjected to harmonic vibrations, are reported in [27]. Fast Fourier transform has also been applied for offline frequency estimation [28]. Nevertheless, the synthesis of closed-form formulae to compute, algebraically and explicitly, in real-time and time domain, the vibrating excitation frequencies of applications of nonlinear semi-active dynamic vibration absorbers using measurements of output signals only, without depending on initial conditions of the disturbed nonlinear vibrating system and additional suitable tuning of several design gains, is a challenging issue.

This paper deals with the variable frequency vibration attenuation problem on a harmonically excited building-like structure using semi-active dynamic vibration absorbers and non-asymptotic online estimations of the excitation frequency, from an algebraic parametrical identification approach. A cantilever beam configuration for a flexible vibration absorber was adopted to attenuate variable frequency vibrations. Nevertheless, the presented adaptive vibration absorption approach could be extended for diverse configurations of dynamic vibration absorbers. Algebraic formulae to calculate the unknown harmonic excitation frequency are proposed. In contrast to other estimation techniques, the frequency estimation was designed to be carried out online and in the time domain, into a small interval of time, employing preferred measurement signals of the position, velocity, or acceleration. Information of initial conditions of the overall nonlinear dynamic mechanical system are unnecessary. Thereby, the introduced online frequency estimation can be reset and updated by monitoring the attenuation level of forced vibrations specified for operative conditions of the main mechanical structure. The harmonic excitation frequency estimation can then be used to tune online a dynamic vibration absorber. In this fashion, the vibration absorption device can be adjusted to converge to the stable system response specified in the mechanical design process. The design of control devices based on dynamic vibration absorbers can consider adjustable parameters according to opportune information of the vibratory disturbance frequency.

The presented non-asymptotic online harmonic vibration frequency estimation approach for a building-like structure using a nonlinear flexible vibration absorber in a cantilever beam configuration is based on the algebraic framework for linear parametrical identification for continuous-time constant linear systems introduced in [29]. This identification scheme is based on the powerful mathematical tools of module theory, Mikusiński operational calculus, and differential algebra. The algebraic identification approach exhibits robustness against structured perturbations and measurement noise, without any additional statistical and probabilistic analysis. The capability of algebraically computing the system parameters almost instantaneously, in the time domain, is another important difference, with respect to other offline or asymptotic parametric identification techniques reported on in the literature, as those described above. Therefore, these outstanding features of the algebraic identification scheme have motivated the synthesis of closed-form formulae to compute the vibrating excitation frequency, which could be used in several applications of semi-active dynamic vibration absorption devices. In this work, an experimental study is introduced to show the effectiveness and efficiency of the algebraic identification scheme to determine the excitation frequency, into a small time interval, for a nonlinear flexible mechanical structure subjected to undesirable harmonic vibrations.

The present manuscript is organized as follows. The building-like mechanical structure with a nonlinear dynamic vibration absorber is described in Section 2. Furthermore, a linear approximation of the overall nonlinear system dynamics is used to algebraically compute the length of the flexible vibration absorber in which it should be adjusted according to the estimated forcing frequency. An adaptive dynamic vibration absorption approach based on algebraic frequency estimations can, thus, be executed. Algebraic formulae for online frequency estimations are presented in Section 3. In Section 4, the effectiveness and efficiency of the frequency estimation and vibration absorption are also proven by simulation and experimental results. An acceptable reduction of the forced vibration magnitude is confirmed. Therefore, the results reveal that an adaptive dynamic vibration absorption scheme, based on online algebraic frequency estimation, is feasible for variable frequency applications. Finally, the conclusions and future research works are presented in Section 5.

2. Building-like Structure with a Flexible Vibration Absorber

In Figure 1, a schematic diagram and the experimental setup are shown. Here, the host structure consists of one rigid floor with mass M, viscous damping c_1 and equivalent stiffness k_{eq} (associated with the four columns connected in parallel). The building-like structure is affected by an external ground motion $x_b = X_b \sin(\Omega t + \varphi)$ with amplitude of the displacement X_b, phase φ and excitation frequency Ω close to the natural frequency of the primary system.

In order to attenuate the resonant vibrations, because of the ground motion $x_b(t)$, a nonlinear flexible dynamic vibration absorber is implemented. The secondary system is composed by a thin beam attached over the building-like structure with equivalent mass m at the end where its lateral motion is restricted to a vertical plane (i.e., gravity effects are considered). The essential properties of the dynamic vibration absorber are its flexural stiffness EI and small viscous damping c_2.

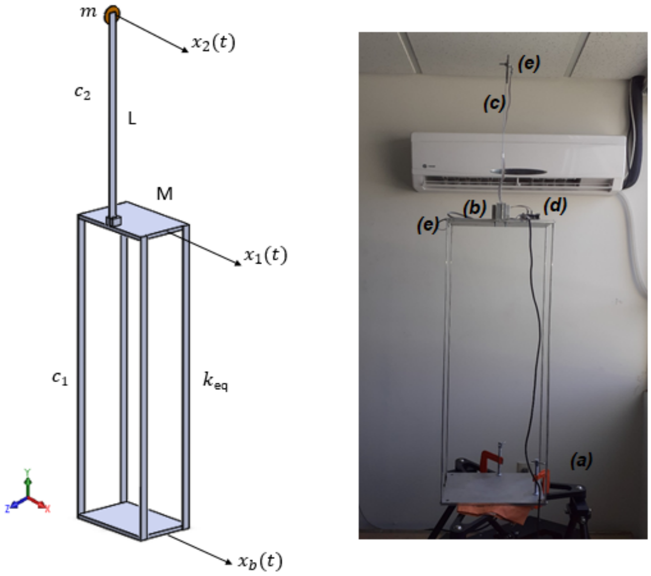


Figure 1. Illustration of the overall system (left side). View of the experimental setup (right side): (**a**) Hexapod Quanser, (**b**) primary system, (**c**) flexible vibration absorber, (**d**) microcontroller, (**e**) accelerometer.

The equations of motion of the building-like structure with a flexible vibration absorber are obtained via the Euler–Lagrange formulation. The total kinetic (T) and potential (V) energies are described as

$$T = \frac{1}{2}(M+m)\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 + \frac{18}{25L^2}m\dot{x}_2^2x_2^2 + m\dot{x}_1\dot{x}_2 \tag{1}$$

$$V = \frac{1}{2}k_{eq}x_1^2 + \frac{3EI}{2L^3}x_2^2 - \frac{3mg}{5L}x_2^2 \tag{2}$$

By computing the Lagrangian $L = T - V$, and developing the Euler–Lagrange equations, considering linear viscous damping and exogenous perturbations due to the ground motion, the equations of motion are given by

$$\begin{aligned} (M+m)\frac{d^2x_1}{dt^2} + m\frac{d^2x_2}{dt^2} + c_1\frac{dx_1}{dt} + k_{eq}x_1 &= -(M+m)\frac{d^2x_b}{dt^2} \\ m\frac{d^2x_1}{dt^2} + m\left(1 + \frac{36}{25L^2}x_2^2\right)\frac{d^2x_2}{dt^2} + \left(c_2 + \frac{36m}{25L^2}x_2\frac{dx_2}{dt}\right)\frac{dx_2}{dt} + \left(\frac{3EI}{L^3} - \frac{6mg}{5L}\right)x_2 &= 0 \end{aligned} \tag{3}$$

where x_1 and x_2 represent the longitudinal motions of the main system and the flexible vibration absorber, respectively. Additionally, the parameters associated to the flexible vibration absorber are the area moment of inertia I, the total length L, and the elastic modulus E.

3. Online Algebraic Estimation of Harmonic Excitation Frequency

Synthesis of an algebraic frequency estimator, to be accomplished using only position measurements, is first developed. Then, a general formulae to algebraically compute the excitation frequency using the preferred measurement signals of the position, velocity, or acceleration is presented.

3.1. Frequency Estimation Using Position Measurements

Consider the first equation from the dynamic model (3)

$$(M+m)\frac{d^2}{dt^2}x_1 + m\frac{d^2}{dt^2}x_2 + c_1\frac{d}{dt}x_1 + k_{eq}x_1 = (M+m)X_b\Omega^2\sin(\Omega t+\varphi) \tag{4}$$

Next, Equation (4) is differentiated twice with respect to time. The resulting equation is then added to Equation (4) multiplied by Ω^2 to obtain

$$\begin{aligned}(M+m)\frac{d^4}{dt^4}x_1 + m\frac{d^4}{dt^4}x_2 + c_1\frac{d^3}{dt^3}x_1 + k_{eq}\frac{d^2}{dt^2}x_1&\\ + \left[(M+m)\frac{d^2}{dt^2}x_1 + m\frac{d^2}{dt^2}x_2 + c_1\frac{d}{dt}x_1 + k_{eq}x_1\right]\Omega^2 = 0&\end{aligned} \tag{5}$$

Equation (5) is multiplied by $\Delta_{t_0} = t - t_0$ to eliminate dependence on initial conditions of the nonlinear dynamic system (3) at $t = t_0$, where $t_0 > 0$ stands for the start time when the frequency estimation is executed. The resulting expression is then integrated by parts, four times, with respect to time, to avoid additional measurements of acceleration and velocity signals. In this fashion, a frequency estimator can be obtained using only position measurements from the resulting equation

$$\xi_4\Omega^2 = \eta_4 + \Delta_{t_0}^4[(M+m)x_1 + mx_2] \tag{6}$$

where ξ_4 and η_4 are output signals provided by the dynamic systems

$$\begin{aligned}\frac{d}{dt}\xi_1 =& -12\Delta_{t_0}^2[(M+m)x_1 + mx_2] + 4c_1\Delta_{t_0}^3x_1 - k_{eq}\Delta_{t_0}^4x_1\\ \frac{d}{dt}\xi_2 =& \xi_1 + 8\Delta_{t_0}^3[(M+m)x_1 + mx_2] - c_1\Delta_{t_0}^4x_1\\ \frac{d}{dt}\xi_3 =& \xi_2 - \Delta_{t_0}^4[(M+m)x_1 + mx_2]\\ \frac{d}{dt}\xi_4 =& \xi_3\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{d}{dt}\eta_1 =& 24[(M+m)x_1 + mx_2] - 24c_1\Delta_{t_0}x_1 + 12k_{eq}\Delta_{t_0}^2x_1\\ \frac{d}{dt}\eta_2 =& \eta_1 - 96\Delta_{t_0}[(M+m)x_1 + mx_2] + 36c_1\Delta_{t_0}^2x_1 - 8k_{eq}\Delta_{t_0}^3x_1\\ \frac{d}{dt}\eta_3 =& \eta_2 + 72\Delta_{t_0}^2[(M+m)x_1 + mx_2] - 12c_1\Delta_{t_0}^3x_1 + k_{eq}\Delta_{t_0}^4x_1\\ \frac{d}{dt}\eta_4 =& \eta_3 - 16\Delta_{t_0}^3[(M+m)x_1 + mx_2] + c_1\Delta_{t_0}^4x_1\end{aligned} \tag{8}$$

with $\xi_k(t_0) = \eta_k(t_0) = 0, k = 1, \ldots, 4$. Since $\Omega^2 > 0$, we have that

$$|\xi_4|\Omega^2 = \left|\eta_4 + \Delta_{t_0}^4[(M+m)x_1 + mx_2]\right| \tag{9}$$

where the notation $|\cdot|$ is used to denote absolute value. Hence, an algebraic formula to compute the excitation frequency into a small interval of time, say $[t_0, t_0 + \varepsilon]$, using only position measurements x_1 and x_2, is proposed as follows

$$\widehat{\Omega}^2 = \frac{\int_{t_0}^{t} e^{-\gamma \Delta_{t_0}} \left| \eta_4 + \Delta_{t_0}^4 [(M+m)x_1 + m x_2] \right| dt}{\int_{t_0}^{t} e^{-\gamma \Delta_{t_0}} |\xi_4| dt} \tag{10}$$

Here, the notation $\widehat{\cdot}$ stands for estimated value and $\gamma > 0$ is a filtering gain used to smooth the numerical frequency estimation process.

3.2. Frequency Estimation Using Position, Velocity, or Acceleration Measurements

The online frequency estimation design process is now extended for applications where velocity or acceleration measurements are available. Then, differentiating Equation (4), with respect to time, twice gives

$$(M+m)\frac{d^2}{dt^2}\dot{x}_1 + m\frac{d^2}{dt^2}\dot{x}_2 + c_1\frac{d}{dt}\dot{x}_1 + k_{eq}\dot{x}_1 = A_1\cos(\Omega t + \varphi) \tag{11}$$

$$(M+m)\frac{d^2}{dt^2}\ddot{x}_1 + m\frac{d^2}{dt^2}\ddot{x}_2 + c_1\frac{d}{dt}\ddot{x}_1 + k_{eq}\ddot{x}_1 = A_2\sin(\Omega t + \varphi) \tag{12}$$

with

$$\begin{aligned} \dot{x}_1 &= \frac{d}{dt}x_1, & A_0 &= -(M+m)X_b\Omega^2 \\ \dot{x}_2 &= \frac{d}{dt}x_2, & A_1 &= A_0\Omega \\ \ddot{x}_1 &= \frac{d^2}{dt^2}x_1, & A_2 &= -A_0\Omega^2 \\ \ddot{x}_2 &= \frac{d^2}{dt^2}x_2 \end{aligned} \tag{13}$$

Notice that Equations (11) and (12) exhibit the same structure of the second-order differential Equation (4), but in terms of either velocity or acceleration signals, i.e., $\dot{x}_j$ or $\ddot{x}_j$, $j = 1, 2$. Then, by applying the same procedure described before to Equations (11) and (12), frequency estimators using either velocity or acceleration measurements can be obtained. Thus, a general formula to compute the excitation frequency is given by

$$\widehat{\Omega}^2 = \frac{\int_{t_0}^{t} e^{-\gamma \Delta_{t_0}} \left| \eta_{4,y} + \Delta_{t_0}^4 [(M+m)y_1 + m y_2] \right| dt}{\int_{t_0}^{t} e^{-\gamma \Delta_{t_0}} |\xi_{4,y}| dt} \tag{14}$$

where

$$\begin{aligned} \frac{d}{dt}\xi_{1,y} =& -12\Delta_{t_0}^2[(M+m)y_1 + my_2] + 4c_1\Delta_{t_0}^3 y_1 - k_{eq}\Delta_{t_0}^4 y_1 \\ \frac{d}{dt}\xi_{2,y} =& \xi_{1,y} + 8\Delta_{t_0}^3[(M+m)y_1 + my_2] - c_1\Delta_{t_0}^4 y_1 \\ \frac{d}{dt}\xi_{3,y} =& \xi_{2,y} - \Delta_{t_0}^4[(M+m)y_1 + my_2] \\ \frac{d}{dt}\xi_{4,y} =& \xi_{3,y} \end{aligned} \tag{15}$$

$$
\begin{aligned}
\frac{d}{dt}\eta_{1,y} =& 24[(M+m)y_1 + my_2] - 24c_1\Delta_{t_0}y_1 + 12k_{eq}\Delta_{t_0}^2 y_1 \\
\frac{d}{dt}\eta_{2,y} =& \eta_{1,y} - 96\Delta_{t_0}[(M+m)y_1 + my_2] + 36c_1\Delta_{t_0}^2 y_1 - 8k_{eq}\Delta_{t_0}^3 y_1 \\
\frac{d}{dt}\eta_{3,y} =& \eta_{2,y} + 72\Delta_{t_0}^2[(M+m)y_1 + my_2] - 12c_1\Delta_{t_0}^3 y_1 + k_{eq}\Delta_{t_0}^4 y_1 \\
\frac{d}{dt}\eta_{4,y} =& \eta_{3,y} - 16\Delta_{t_0}^3[(M+m)y_1 + my_2] + c_1\Delta_{t_0}^4 y_1
\end{aligned}
\quad (16)
$$

with $\xi_{k,y}(t_0) = \eta_{k,y}(t_0) = 0$, $k = 1, \ldots, 4$. Here, y_1 and y_2 stand for the available measurement signals of position, velocity, or acceleration, i.e., $y_j = x_j,\ \dot{x}_j,\ \ddot{x}_j$, $j = 1, 2$.

4. Simulation and Experimental Results

Effectiveness of the online algebraic frequency estimation was also verified through numerical and experimental results on a building-like structure with a coupled flexible vibration absorber. Three simulation and experimental case studies to prove the satisfactory frequency estimation using the proposed formulae are presented. Proposed formulae were performed into a small interval of time of 0.12 s, yielding acceptable frequency estimation results.

4.1. Simulation Case Study

Firstly, the online algebraic estimation of the harmonic excitation frequency was numerically proven for a structure characterized by the set of parameters described in Table 1. For this simulation case study, it was assumed that position measurement signals were only available to compute online and in time domain the harmonic vibration frequency.

Table 1. Parameters of the nonlinear vibrating system.

$M = 2.706$ kg	$E = 69$ GPa
$m = 0.125$ kg	$I = 8.468 \times 10^{-12}$ m^4
$k_{eq} = 224.374$ N/m	$L = 0.5095$ m
$c_1 = 0.832$ Ns/m	$\Omega = 9.1056$ rad/s
$c_2 = 0.048$ Ns/m	$X_b\Omega^2 = 0.58576$ m/s^2

Figure 2 confirms the accurate excitation frequency estimation of vibrating disturbances affecting the primary system dynamics. The capability of vibration attenuation of the nonlinear dynamic vibration absorber on the position response of the primary structure is also shown. The transference of the vibrating energy from the building-like primary structure to the dynamic vibration absorber is then evidenced. For computer simulations, the nonlinear mathematical model (3) and lumped system parameters were considered. Nevertheless, strictly speaking, a flexible beam structure constitutes a distributed or continuous vibrating system. Then, an infinite number of degrees of freedom are present in the distributed system dynamics. Thus, the effectiveness and efficiency of the algebraic frequency estimation is demonstrated by experimental outcomes in the next case study.

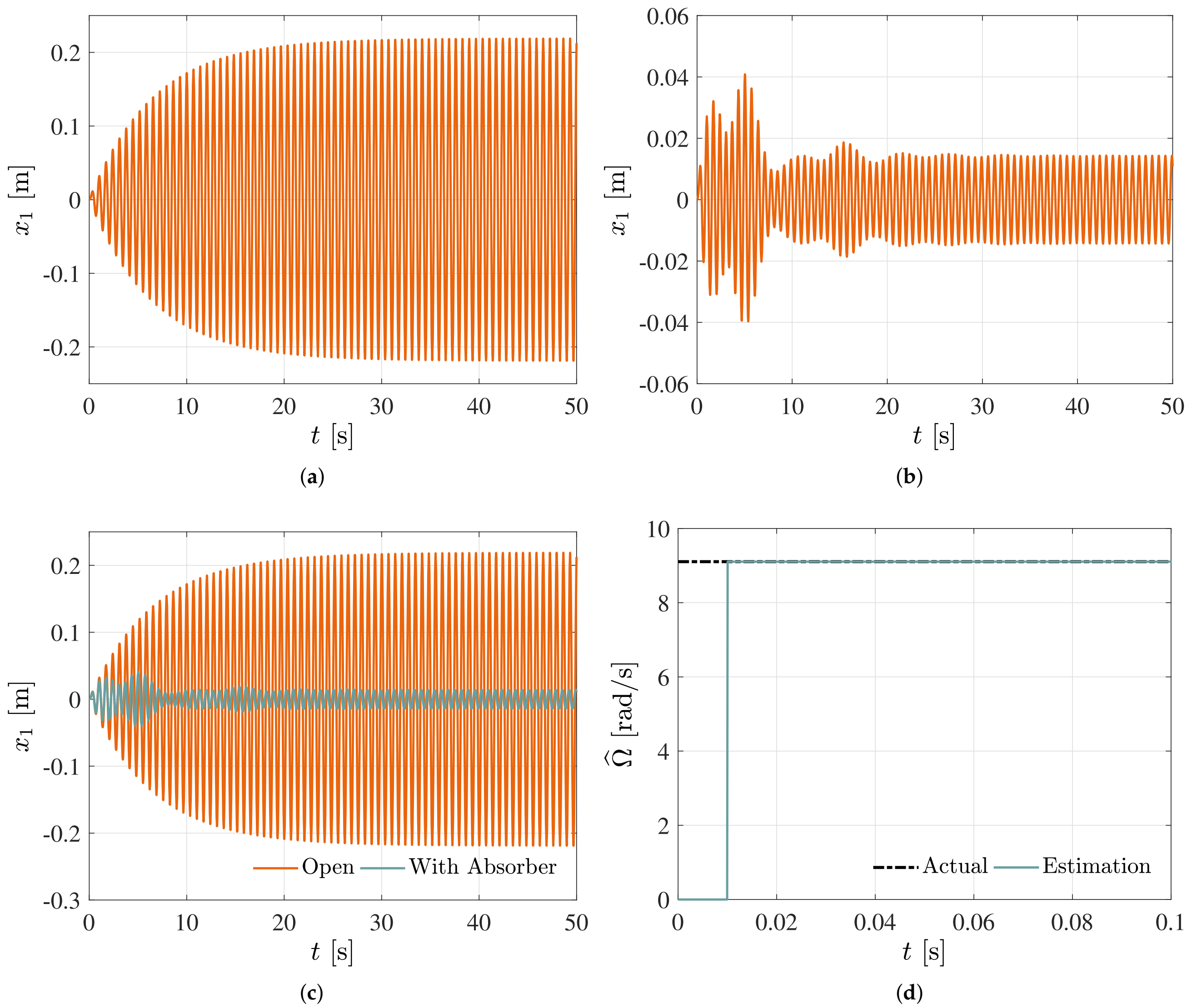


Figure 2. Effective frequency estimations using position measurements for the simulation case study. (**a**) Position the output response of the primary vibrating system without a dynamic vibration absorber; (**b**) position the output response of the primary vibrating system with a dynamic vibration absorber; (**c**) position the output responses of the primary system with and without a dynamic vibration absorber; (**d**) fast accurate estimation of the excitation frequency.

4.2. First Experimental Case Study

In order to experimentally validate the effective and efficient performance of the online algebraic frequency estimation, the experimental setup shown in Figure 1 was built. Moreover, in the present experimental case study, acceleration measurements were used to perform online frequency estimations. In fact, in many applications of vibration control and estimation in practical vibrating systems, it is commonly preferred to use acceleration measurement signals. In this sense, a MEMS three-axes accelerometer ADXL335 was used to measure acceleration signals. The sensor reading was done by a Atmel SAM3X8E ARM Cortex-M3 CPU, which was programmed in Python with a sampling period of 2×10^{-4} s, yielding to satisfactory online estimation results.

The online algebraic identification scheme of parameters in the dynamic systems has complete theoretical and practical support [30–32], where the effectiveness of the algebraic scheme, in context of modal analysis, control, and vibration absorption schemes, has been demonstrated. To carry out the technique in practice, software and hardware technology tools are offering increasingly powerful and better-integrated signal acquisitions and processing routines. The limitations of the proposed algebraic technique are technological since its successful implementation relies on a fast and precise acquisition and processing of signals. Alternatives for the application of the technique were proposed in previous works, where the use of direct memory access (DMA) in microcontrollers was proposed in a scheme with buffers of controlled or dynamic lengths, resulting in a quasi-online estimation for embedded systems [33]. In the present work, a microcontroller programmed in a multitasking scheme was used, with a sampling period controlled by interruptions in a precise and deterministic way. A personal computer performs the mathematical operations on the data acquired by the microcontroller, at a fixed sampling rate, to obtain the results of the algebraic estimation.

Table 2 summarizes the obtained online frequency estimation results. An acceptable estimation error can be corroborated in the experimental outcomes. Figure 3 depicts the experimental and simulation frequency estimation processes. The experimental and simulation acceleration responses of the primary structure, $\ddot{x}_1$, and flexible beam vibration absorber, $\ddot{x}_2$, are also portrayed. It can be observed that, despite reasonable differences between experimental and simulation acceleration signals, an acceptable frequency value was computed. These differences are due to the distributed system dynamics of the overall vibrating structure considered in the present study. Thus, the numerical and experimental results reveal that the proposed closed-form formulae is a good choice to compute, explicitly and online, the unknown excitation frequencies for dynamic vibration absorption tasks. Thus, real-time frequency estimation can be advantageously employed to tune semi-active dynamic vibration absorption devices for a wide variety of harmful variable-frequency forced vibration suppression applications, which will be developed in future experimental research works.

Table 2. Performance of the online frequency estimation.

	Ω (rad/s)	$\widehat{\Omega}$ (rad/s)	Error (%)
Simulation	9.1058	9.1147	0.1
Experimental	9.1058	8.878	−2.5

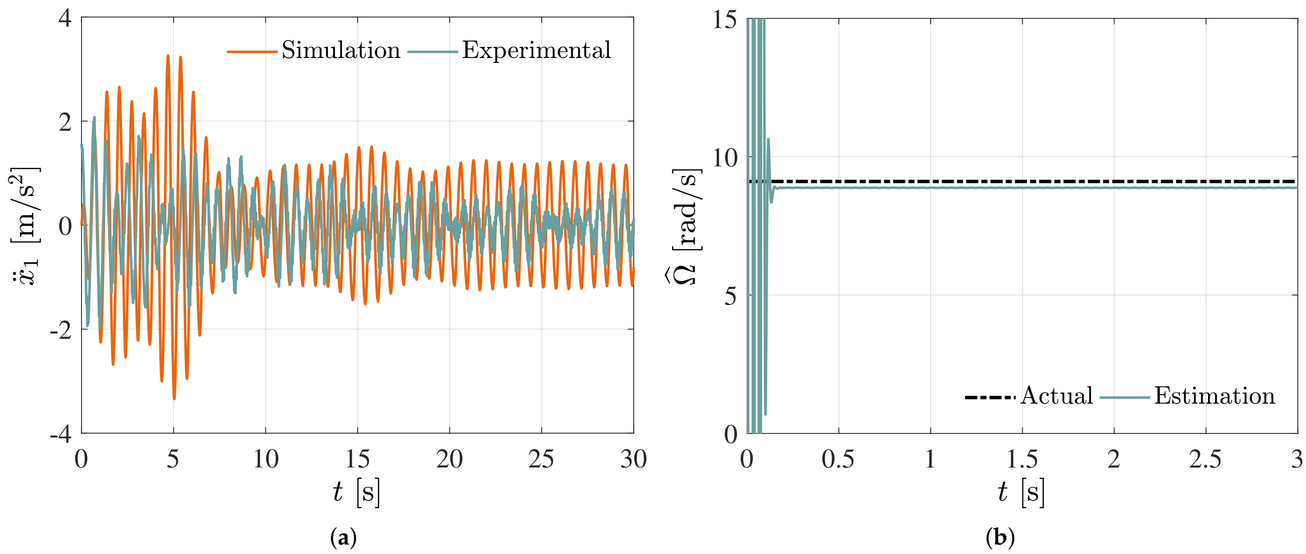
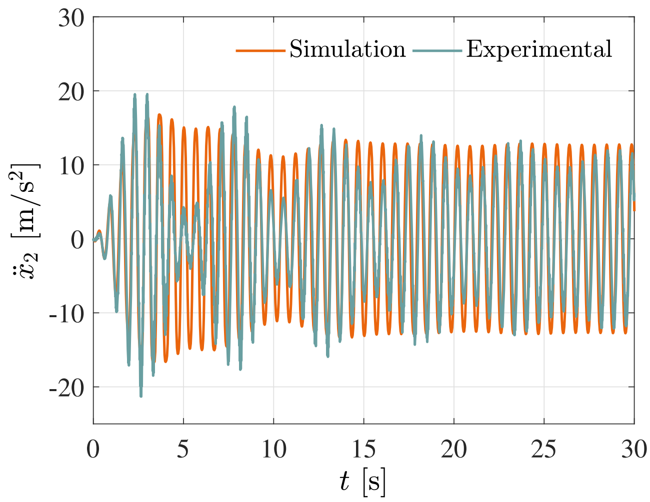
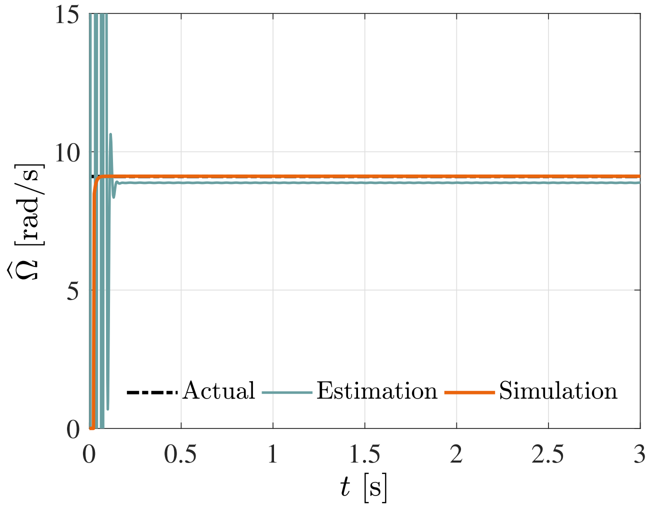


Figure 3. *Cont.*



(**c**)



(**d**)

Figure 3. Acceptable frequency estimation using acceleration measurements on a damped distributed-parameter flexible structure. (**a**) Experimental and simulated accelerations of the primary structure; (**b**) acceptable experimental frequency estimation; (**c**) experimental and simulated accelerations of the vibration absorber; (**d**) simulation and experimental frequency estimations.

4.3. Second Experimental Case Study

Furthermore, the effectiveness of the proposed frequency estimation approach was satisfactorily verified in another application of the nonlinear flexible dynamic vibration absorber. Thus, a different configuration of the secondary system for the building-like structure was built, as depicted in Figure 4. More details on the experimental setup can also be found in [34].

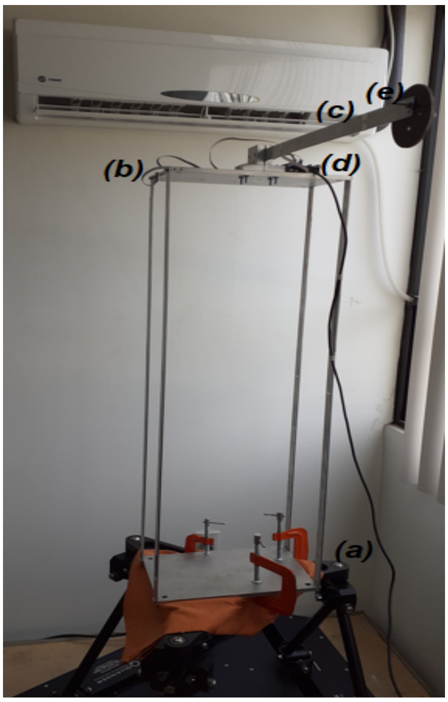


Figure 4. Experimental setup for the second configuration: (**a**) Hexapod Quanser, (**b**) primary system, (**c**) flexible vibration absorber, (**d**) microcontroller, (**e**) accelerometer.

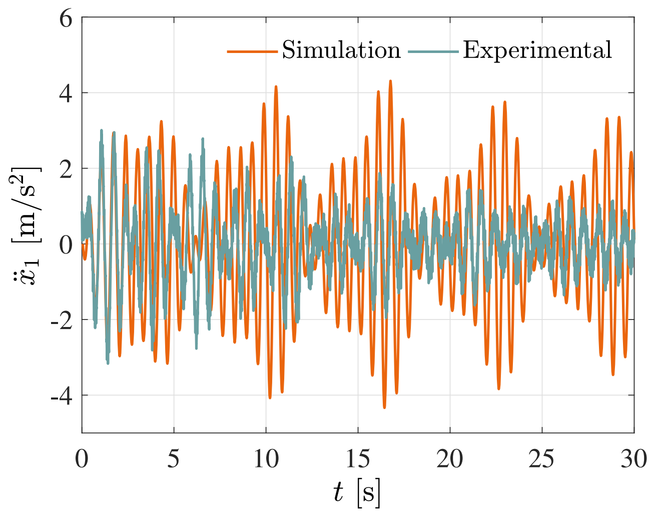
In this second experimental configuration, the vibrating dynamics of the secondary system occurs in a horizontal plane. Hence, the gravitational term in the potential energy (2) is negligible. Then, from the Euler–Lagrange formulation, the nonlinear vibrating system dynamics is given by

$$\begin{aligned}(M+m)\frac{d^2x_1}{dt^2}+m\frac{d^2x_2}{dt^2}+c_1\frac{dx_1}{dt}+k_{eq}x_1 &= -(M+m)\frac{d^2x_b}{dt^2}\\ m\frac{d^2x_1}{dt^2}+m\left(1+\frac{36}{25L^2}x_2^2\right)\frac{d^2x_2}{dt^2}+\left(c_2+\frac{36m}{25L^2}x_2\frac{dx_2}{dt}\right)\frac{dx_2}{dt}+\frac{3EI}{L^3}x_2 &=0\end{aligned} \tag{17}$$

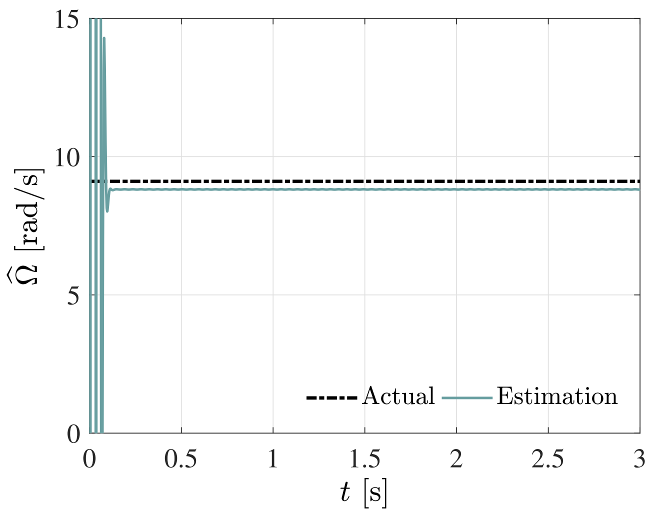
Clearly, the dynamic models represented by the Equations (3) and (17) are very similar. Therefore, the proposed online algebraic harmonic excitation frequency estimation could also be implemented to the nonlinear vibrating system shown in Figure 4. The obtained online frequency estimation results for this second experimental configuration are summarized in Table 3. Here, reasonable small estimation errors could be observed. Finally, the simulation and experimental frequency estimation process, with the dynamic responses, in terms of acceleration signals of the primary and secondary vibrating systems, are portrayed in Figure 5. Thus, the experimental and numerical outcomes confirm the acceptable algebraic excitation frequency estimation.

Table 3. Performance of the online frequency estimation for the second configuration.

	Ω (rad/s)	$\widehat{\Omega}$ **(rad/s)**	**Error (%)**
Simulation	9.1058	9.1147	0.1
Experimental	9.1058	8.8190	−3.2



(a)



(b)

Figure 5. *Cont.*

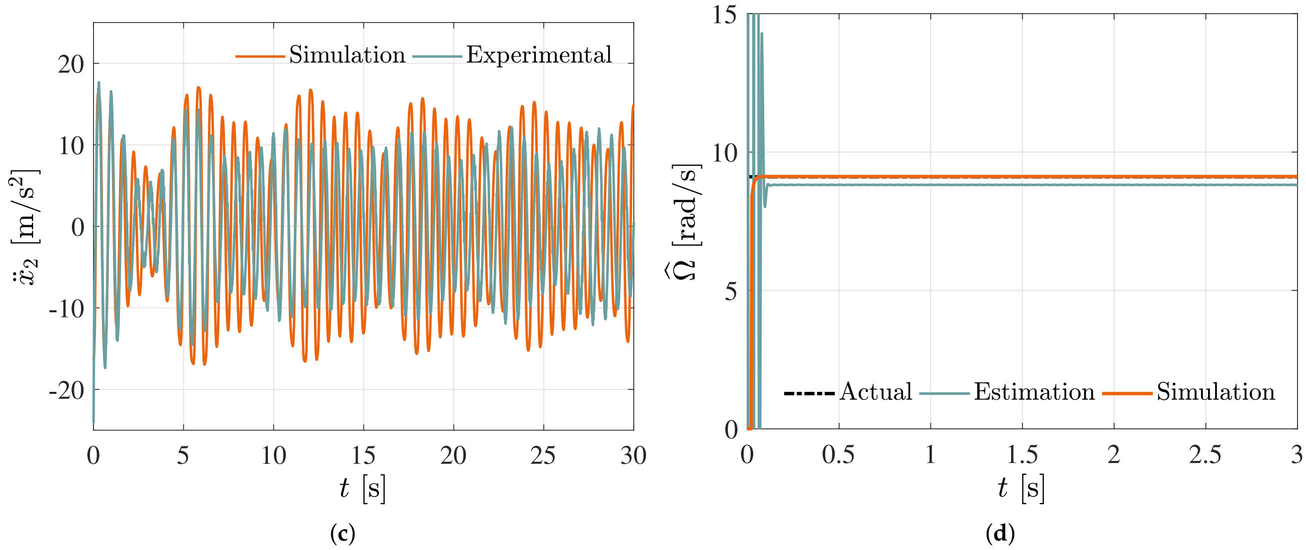


Figure 5. Acceptable frequency estimation using acceleration measurements on a damped distributed-parameter flexible structure for the second configuration. (**a**) Experimental and simulated accelerations of the primary structure; (**b**) acceptable experimental frequency estimation; (**c**) experimental and simulated accelerations of the vibration absorber; (**d**) simulation and experimental frequency estimations.

5. Conclusions

Closed-form formulae to algebraically compute the excitation frequency on a building-like structure, using tunable nonlinear dynamic vibration absorbers in flexible cantilever beam configurations, subjected to unwanted forced harmonic vibrations, were presented. The formulae admitted a wide variety of applications in variable frequency harmonic vibration suppression techniques based on adaptive semi-active dynamic vibration absorbers. In this context, the excitation frequency is the main parameter to tune, online, an important class of adaptive semi-active dynamic vibration absorbers. The fast frequency estimation could then be taken advantage of to tune these dynamic vibration control devices, to hold the optimal tuning conditions to efficiently suppress undesirable variable frequency vibrations. The undesirable harmonic vibration frequency, different from other estimation algorithms, is explicitly identified in the time domain, online, and in a small interval of time. The online frequency estimation could be performed using measurement signals of position, velocity, or acceleration. Thus, the excitation frequency can be directly determined by using the available measurement signals employed to implement semi-active dynamic vibration absorption mechanisms. Moreover, satisfactory experimental outcomes to prove the effective fast frequency estimation on a real prototype of a building-like structure, with a nonlinear flexible thin-beam vibration absorber, were introduced, where gravity effects were also considered. Algebraic estimation robustness was confirmed experimentally, despite flexible beams being elastic bodies with infinite numbers of degrees of freedom. Therefore, from the presented analytical, numerical, and experimental results, we can conclude that the online frequency estimation approach is a very good alternative for tuning, in real-time, a class of semi-active nonlinear dynamic vibration absorbers in flexible cantilever beam configurations for variable frequency harmonic vibration suppression in flexible mechanical structures. Furthermore, the present study provides insight into the extension of the closed-form real-time frequency estimation for other architectures of these adaptive nonlinear mechanical vibration control devices, which will be developed in future experimental research works. Synthesis of new active and semi-active, adaptive, nonlinear vibration absorption schemes, based on real-time frequency estimations for harmful vibration suppression on flexible mechanical structures, will be considered in subsequent research works. Moreover, the design of control laws, to semi-actively and actively tune several architectures of nonlinear dynamic vibration absorbers, will be addressed in future contributions. In this sense, robustness and efficiency comparisons of control laws based on different linear and nonlinear control design methodologies will also be considered.

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