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Article

A Novel Early Warning Method for Handling Non-Homogeneous Information

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Abstract: Early warnings are an indispensable part of emergency management, which is a powerful way to eliminate or reduce the negative impacts caused by emergencies in advance. Early warning problems have been discussed from different perspectives and have obtained fruitful results. Information plays a critical role in all kinds of decision problems, with no exception for the early warning problem. There are various information types related to emergencies in real-world situations; however, existing early warning studies only considered a single information type, which might not describe the problem properly and comprehensively. To enrich existing early warning studies, a novel early warning method considering non-homogeneous information together with experts' hesitation is proposed, in which numerical values, interval values, linguistic terms, and hesitant fuzzy linguistic terms are considered. To facilitate the computations with non-homogeneous information, a transformation process needs to be conducted. On such a basis, a fuzzy TOPSIS method based on alpha-level sets is employed to handle the transformed fuzzy information due to its superiority in obtaining information and its capacity to contain as much information as possible during the early warning process. Additionally, two different options are provided to analyze the status and tendency of early warning objects. Finally, an illustrative example about early warnings about landslides and a related comparison are conducted to demonstrate the novelty, superiority, and feasibility and validity of the proposed method.

Keywords: non-homogeneous information; early warning; fuzzy TOPSIS method; alpha-level sets

MSC: 91B06; 91B05

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1. Introduction

Early warnings are regarded as an effective way to prevent or avoid the occurrence of emergencies in advance to eliminate or reduce loss of life and property, and the negative impacts caused by emergencies in advance [1]. Therefore, it plays an indispensable and important role in emergency management. With an indispensable and important role in emergency management, early warning problems have drawn great attention from around the world, particularly with the frequent occurrence of various emergencies in recent years. Topics related to early warning have become an active research field and has obtained fruitful results from various perspectives, such as research on the development or improvement of an early warning system [2–7], models and methodologies of early warning [8–14], early warning data monitoring and acquisition [15–20], and so on.

Regarding early warning problems, information is an indispensable and important element in the early warning process, which determines the quality, reliability, and reasonability of the early warning results. Extant studies have discussed such an issue from different perspectives and has obtained fruitful results [8–19]. However, with respect to such an important issue, extant early warning studies just focus on a single information type, i.e., numerical value [14,21], interval value [22], or linguistic information [23], which is gathered by monitoring or sensors or is provided by experts. In a real-world situation, early warning problems are usually under uncertain and complex situations that are described from both qualitative and quantitative contexts, and the use of just a single type of information might not describe the early warning objects properly and comprehensively. It may lead to unreliable early warning results; miss chances to take actions in advance; and then cause potential loss of life and property, and negative impacts. Therefore, it seems necessary to consider and pay attention to different information types related to early warning objects. However, to date, there are seldom extant early warning studies discussing this issue. Thus, the motivation behind this study is to fill the gap in extant early warning studies by considering different information types from both qualitative and quantitative contexts to describe early warning problems.

On such a motivation basis, this study proposes a novel early warning method for handling non-homogeneous information together with experts' hesitation, in which numerical values, interval values, linguistic information, and hesitant fuzzy linguistic information are considered. The first two types describe the early warning problem from a quantitative context, in which interval values are employed to describe the uncertain quantitative information; the other two types describe qualitative information, in which hesitant fuzzy linguistic information is employed to express information about the experts' hesitations due to its closeness to natural language and its easy understandability. To handle non-homogeneous information, it is transformed into a unified form: trapezoidal fuzzy numbers. On such a basis, to avoid the discounts of considering non-homogeneous information purpose, an alpha-level set-based fuzzy TOPSIS method [24] is used to conduct the related computations. Additionally, two different options are provided to analyze the status and tendency. Finally, an illustrative example and comparisons are provided to highlight the feasibility and validity of the proposed method.

From what has been mentioned above, the contributions of this study are as follows: (1) An early warning method considering non-homogeneous information is proposed, which enables the proposed method to comprehensively describe early warning information from qualitative and quantitative contexts. It not only enriches the information consideration, but also provides a new perspective on early warning studies. (2) The proposed early warning method is the first to consider experts' hesitation, which provides new insights into human influence on the early warning process. (3) Two different options for analyzing the status and tendency of early warning objects are provided. It allows the decision maker to figure out the status changes in early warning objects at different alpha levels. The proposed method is the first to provide such options in extant early warning studies.

The structure of the paper is organized as: Section 2 presents related work, the concept of linguistic terms, the hesitant fuzzy linguistic term sets (HFLTS), and the fuzzy TOPSIS method based on alpha-level sets. Section 3 provides the proposed early warning method that handles non-homogeneous information. Section 4 presents an illustrative example and a comparison. The conclusions and future work are provided in Section 5.

2. Preliminary Knowledge

This section briefly presents preliminary knowledge, including the linguistic terms, HFLTS and fuzzy TOPSIS method based on alpha level sets so that the proposed method can be easily understood. Additionally, related work is provided to demonstrate the importance and necessity of this study.

2.1. Linguistic Terms

The concept of linguistic term sets was first proposed by Zadeh in 1975 [25]; it handles information that cannot be described in quantitative forms (i.e., numerical values or interval values) but rather as words or sentences in a natural or artificial language in the real-world. The words or sentences in a linguistic term set are usually scattered over a scale with a defined order [25,26]. For instance, S = {very poor, poor, medium, good, very good} is a linguistic term sets.

To perform a computation with linguistic terms, fuzzy numbers (triangular, trapezoidal, or mixed) are used to express the linguistic terms because of its capacity to contain as much information as possible [27] and have been widely used to solve various problems [23,28–30]. Linguistic terms with related fuzzy numbers are presented in Table 1 and illustrated in Figure 1.

Table 1. Linguistic terms and fuzzy numbers.

S	Linguistic Terms	Fuzzy Numbers
s_0	very poor (VP)	(a, a, b)
s_1	poor (P)	(a, b, c)
s_2	medium (M)	(b, c, d, e)
s_3	good (G)	(d, e, f)
s_5	very good (VG)	(e, f, f)

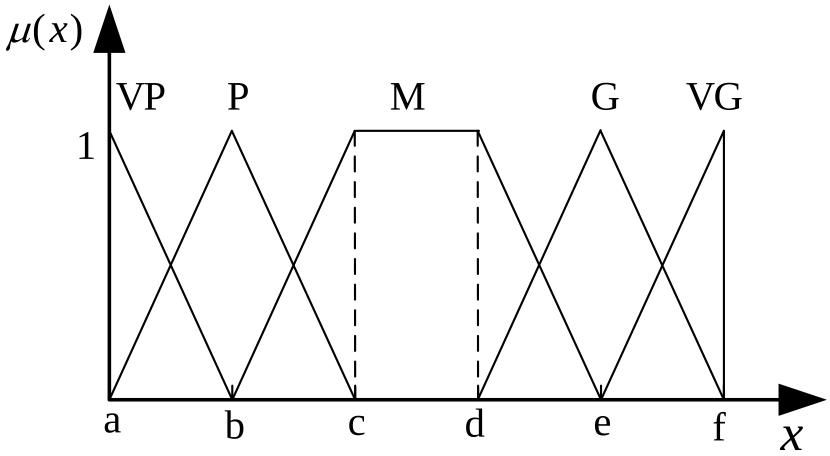

Figure 1. Linguistic terms and related fuzzy numbers.

2.2. Hesitant Fuzzy Linguistic Term Sets

Hesitant fuzzy linguistic term sets (HFLTS) [31], built on the linguistic term sets, was first developed to describe hesitant information in a qualitative context. Because of its closeness to natural languages, it has drawn great attention and been diffusely employed to handle different real-world decision problems that consider hesitant human information in a qualitative context [29,32–34]. It is briefly reviewed as follows.

Definition 1 ([31]). *Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set and an HFLTS, H_S, on S be an ordered finite subset:*

$$H_S = \{s_i, s_{i+1}, \ldots, s_j\}, s_t \in S, t \in \{i, i+1, \ldots, j\} \tag{1}$$

Example 1. *Let $S = \{very\ poor,\ poor,\ medium,\ good,\ very\ good\}$ be a linguistic term set, according to Definition 1, two HFLTSs on S can be expressed as follows:*

$$H_S^1 = \{very\ poor,\ poor\};\quad H_S^2 = \{medium,\ good,\ very\ good\}$$

However, when handling complex real-world decision problems, humans prefer to provide their assessments by using linguistic expressions close to natural languages instead of multiple linguistic terms [25,31]. To meet practical needs, the concept of context-free grammar [35], G_H, is defined, and can produce various linguistic expressions close to those of a natural language.

Definition 2 ([35]). *Let $S = \{s_0, s_1, ..., s_g\}$ be a linguistic term set and G_H be the context-free grammar. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:*

$V_N = \{\langle primary\ term\rangle, \langle composite\ \ term\rangle, \langle unary\ \ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$
$V_T = \{lower\ than,\ greater\ than,\ at\ least,\ at\ most,\ between,\ and, s_0, s_1, ..., s_g\}$
$I \in V_N$
$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$
$\langle composite\ term\rangle ::== \langle unary\ relation\rangle\langle primary\ term\rangle | \langle binary\ relation\rangle$
$\langle primary\ term\rangle\langle conjunction\rangle\langle primary\ term\rangle$
$\langle primary\ term\rangle ::= s_0|s_1|...|s_g$
$\langle unary\ relation\rangle ::= lower\ than|greater\ than|at\ least|at\ most$
$\langle binary\ relation\rangle ::= between$
$\langle conjunction\rangle ::= and\}$

The linguistic expressions, either single linguistic terms $s_t \in S$ or comparative linguistic expressions, S_{ll}, can be produced by G_H.

Example 2. *Let* $S = \{very\ poor,\ poor,\ medium,\ good,\ very\ good\}$ *be a linguistic term set; according to Definition 2, three possible comparative linguistic expressions* S_{ll_1}, S_{ll_2}, *and* S_{ll_3} *could be as follows:*

$$S_{ll_1} = between\ good\ and\ very\ good; \qquad S_{ll_2} = at\ least\ medium; \qquad S_{ll_3} = at\ most\ poor$$

To perform the computation with a comparative linguistic expression, S_{ll}, it should be converted into an HFLTS, H_S, by a transformation function.

Definition 3 ([31]). *Let* E_{G_H} *be the transformation function that can transform* S_{ll} *into* H_S.

$$E_{G_H} : S_{ll} \to H_S \tag{2}$$

Then, S_{ll} *can be converted into* H_S *by*
$E_{G_H}(s_i) = \{s_i|s_i \in S\}$
$E_{G_H}(at\ most\ s_i) = \{s_j|s_j \in S\ and\ s_j \leq s_i\}$
$E_{G_H}(lower\ than\ s_i) = \{s_j|s_j \in S\ and\ s_j < s_i\}$
$E_{G_H}(at\ least\ s_i) = \{s_j|s_j \in S\ and\ s_j \geq s_i\}$
$E_{G_H}(greater\ than\ s_i) = \{s_j|s_j \in S\ and\ s_j > s_i\}$
$E_{G_H}(between\ s_i\ and\ s_j) = \{s_t|s_t \in S\ and\ s_i \leq s_t \leq s_j\}$

According to Definition 3, the S_{ll} in Example 2 can be converted into a related HFLTS as follows:

$$H_{S_1} = \{good,\ very\ good\};\ H_{S_2} = \{medium,\ good,\ very\ good\};\ H_{S_3} = \{very\ poor,\ poor\}$$

When comparative linguistic expressions, S_{ll}, are converted into corresponding multiple linguistic terms, its related fuzzy envelop can be computed by the following:

Definition 4 ([36]). *Let* $env_F(\cdot)$ *be a fuzzy envelop function that can transform* H_S *into its fuzzy membership function,*

$$env_F(H_S) = \Gamma(a, b, c, d) \tag{3}$$

where $\Gamma(a, b, c, d)$ *is a trapezoidal fuzzy membership function, in which an HFLTS,* H_S*, can be expressed by its related trapezoidal fuzzy number* (a, b, c, d) *(see [36] for further details).*

2.3. Fuzzy TOPSIS Method Based on Alpha-Level Sets

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method [37] is a popular multi-attribute decision making method and has been diffusely employed to handle various real-world decision problems [24,37,38].

With respect to the classic TOPSIS version, it has been modified and improved from different perspectives [24,39,40]. Additionally, to handle uncertain real-world complex problems, various fuzzy TOPSIS methods have been presented [41–43]. Some studies converted a fuzzy TOPSIS method into a nonfuzzy multi-criteria decision-making problem by using centroid or distance defuzzification methods [44–46]. Some studies extended the fuzzy TOPSIS method to handle the group decision-making situations by defining a crisp Euclidean distance between any two fuzzy numbers [47]. Both of them [44–47] obtained crisp values of fuzzy relative closeness. However, the conversion of fuzzy information into crisp values might lose information during the decision process.

Regarding such limitations, Wang et al. [24] proposed a fuzzy TOPSIS method based on alpha-level sets, which is a powerful and helpful method among extant fuzzy TOPSIS versions [41,42] due to its superiority and advantages of handling fuzzy information in a reasonable and better manner. Because of this, the fuzzy TOPSIS method based on alpha-level sets has a significant difference from other versions. Therefore, it will be employed in our proposed method to handle fuzzy information for early warnings.

The concept of alpha-level sets is first reviewed before we introduce the fuzzy TOPSIS method based on alpha-level sets.

According to Zadeh's extension principle [27], a fuzzy number/set $\tilde{A}$ can be represented by its related intervals:

$$\tilde{A} = \bigcup_{\alpha} \alpha A_\alpha, 0 \leq \alpha \leq 1 \tag{4}$$

where

$$\begin{aligned} A_\alpha &= \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\} \\ &= [min\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}, max\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}] \end{aligned} \tag{5}$$

where A_α indicates alpha-level sets or alpha-cuts of $\tilde{A}$, and $\mu_{\tilde{A}}(x)$ is the membership function of fuzzy number $\tilde{A}$ [24].

On such a basis, the fuzzy TOPSIS method based on alpha-level sets [24] is briefly introduced as follows:

Step 1: normalize the fuzzy decision matrix $\tilde{X} = (\tilde{x}_{ij})_{n\times m}$. $\tilde{X} = (\tilde{x}_{ij})_{n\times m}$ is an $n \times m$ fuzzy decision matrix, in which $\tilde{x}_{ij}$ is trapezoidal fuzzy number or a triangular fuzzy number, its related membership function is denoted by $\mu_{\tilde{x}_{ij}}(x)$ $(i = 1, \ldots, n,\ j = 1, \ldots, m)$. $\tilde{W} = (\tilde{w}_1, \ldots, \tilde{w}_m)$ indicates fuzzy weights related to the criteria/attributes, and $\{c_1, \ldots, c_m\}$ is characterized by $\mu_{\tilde{w}_j}(x)$ $(j = 1, \ldots, m)$.

If $\tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ $(i = 1, \ldots, n,\ j = 1, \ldots, m)$ are trapezoidal fuzzy numbers, a related normalization can be conducted as follows (the same process for triangular fuzzy numbers):

$$\tilde{r}_{ij} = (\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}), i = 1, \ldots, n; j \in \Omega_b \tag{6}$$

$$\tilde{r}_{ij} = (\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}), i = 1, \ldots, n; j \in \Omega_c \tag{7}$$

where

$$d_j^* = \max_i d_{ij}, j \in \Omega_b, \tag{8}$$

$$a_j^- = \min_i a_{ij}, j \in \Omega_c \tag{9}$$

where Ω_b and Ω_c indicate the set of criteria/attributes of the benefits and costs, respectively.

Step 2: determine the ideal and negative ideal solutions. Based on Equations (6) and (7), $\tilde{r}_{ij}$ belong to [0,1]; therefore, the ideal and negative ideal solutions are $P^* = \{1, \ldots, 1\}$ and $P^- = \{0, \ldots, 0\}$, respectively.

Step 3: compute the alpha-level sets of r_{ij}. Different alpha levels are set; then, the alpha-level sets of $\tilde{r}_{ij}$ and $\tilde{w}_j$ based on Equation (5) can be denoted by $(r_{ij})_\alpha = [(r_{ij})^L_\alpha, (r_{ij})^U_\alpha]$ and $(w_j)_\alpha = [(w_j)^L_\alpha, (w_j)^U_\alpha]$, respectively.

Step 4: calculate the fuzzy closeness of each alternative at each alpha level. The fuzzy relative closeness, RC_i, of alternative p_i regarding P^* can be calculated as follows:

$$RC_i = \frac{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2} + \sqrt{\sum_{j=1}^{m}(w_j(r_{ij}-1))^2}} \tag{10}$$

in which

$$(w_j)^L_\alpha \le w_j \le (w_j)^U_\alpha, j = 1, \ldots, m \tag{11}$$

$$(r_{ij})^L_\alpha \le r_{ij} \le (r_{ij})^U_\alpha, j = 1, \ldots, m, i = 1, \ldots, n \tag{12}$$

According to Equation (10), RC_i is an interval value; its related lower bound, $(RC_i)^L_\alpha$, and upper bound, $(RC_i)^U_\alpha$, can be obtained by Equations (13) and (14), respectively, as (see [24] for further details):

$$\begin{aligned} (RC_i)^L_\alpha &= Min \frac{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})^L_\alpha)^2}}{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})^L_\alpha)^2} + \sqrt{\sum_{j=1}^{m}(w_j((r_{ij})^L_\alpha - 1))^2}} \\ s.t. \quad & (w_j)^L_\alpha \le w_j \le (w_j)^U_\alpha, j = 1, \ldots, m \end{aligned} \tag{13}$$

$$\begin{aligned} (RC_i)^U_\alpha &= Max \frac{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})^U_\alpha)^2}}{\sqrt{\sum_{j=1}^{m}(w_j(r_{ij})^U_\alpha)^2} + \sqrt{\sum_{j=1}^{m}(w_j((r_{ij})^U_\alpha - 1))^2}} \\ s.t. \quad & (w_j)^L_\alpha \le w_j \le (w_j)^U_\alpha, j = 1, \ldots, m \end{aligned} \tag{14}$$

Step 5: defuzzify the fuzzy relative closeness.

Based on Equation (4), the fuzzy relative closeness of alternative p_i with respect to related alpha levels, $\tilde{RC}_i$, from 0 to 1 is presented as follows:

$$\begin{aligned} RC_i &= \textstyle\bigcup_\alpha \alpha \cdot (RC_i)_\alpha \\ &= \textstyle\bigcup_\alpha \alpha[(RC_i)^L_\alpha, (RC_i)^U_\alpha], 0 \le \alpha \le 1 \end{aligned} \tag{15}$$

The average level cuts [48] is employed to defuzzify RC_i. Let $\alpha_1, \ldots, \alpha_K$ be different alpha levels; then, the defuzzified values, $m(RC_i)$, can be obtained as follows:

$$m(RC_i) = \frac{1}{K}\textstyle\sum_{k=1}^{K}\left(\frac{(RC_i)^L_{\alpha_k} + (RC_i)^U_{\alpha_k}}{2}\right), i = 1, \ldots, n \tag{16}$$

where K is the number of alpha levels.

Step 6: rank alternatives according to the defuzzied values $m(RC_i)$ of alternatives p_i.

2.4. Related Work

To demonstrate the importance and necessity of this study, this subsection reviews several studies that are related to this study [15–20]. For example, Yan et al. [14] proposed a fuzzy AHP method for the early warning problem regarding coal mining operations, in which the information represented by numerical values was provided by experts. Burchard-Levine et al. [15] proposed a water-quality early warning method by using data-driven models to analyze and fuse the information collected by monitoring stations. Pyayt et al. [21] proposed a flood early warning method by combining data-driven methods and finite element analysis; the information employed in the flood early warning method was collected by sensors. Akwango et al. [16] proposed a drought early warning method, in which the information was collected from 173 households. Li et al. [17] proposed an early warning model for conventional sudden water pollution based on a mainstream algorithm, in which the information was expressed in a quantitative context. Zhang et al. [19] proposed an intelligent COVID-19 early warning model using social media information, in which machine learning methods and natural language processing were used to handle the social media text. Li et al. [20] proposed a novel early warning model for coal and gas outburst, in which the information was different types of sensor data with spatial heterogeneity over time. Zhang et al. [22] proposed a dynamic early warning method for considering the uncertain and fuzzy environment in a quantitative context using interval values. Zhang et al. [23] proposed an early warning method by employing linguistic information, which extends the scope of early warning information to a qualitative fuzzy environment.

Although extant early warning studies have discussed related topics from different aspects and obtained fruitful results, they neglect the practical issue that there are various information types related to early warning objects in a real world situation. Extant early warning studies focus only on a single information type, which might not describe the early warning objects properly and comprehensively. With respect to this practical issue, this study focuses on non-homogeneous information related to early warning objects, including numerical values, interval values, linguistic information, and hesitant fuzzy linguistic information. The novelty of this study is filling the gap in and enriching extant early warning studies.

3. Proposed Method

This section presents a novel early warning method that handles non-homogeneous information, in which numerical values N, interval values I, linguistic terms s_i, and comparative linguistic expressions S_{ll} are considered. Additionally, to cope with the non-homogeneous information properly and reasonably, the fuzzy TOPSIS method based on alpha-level sets is employed. The general framework of the proposed method is illustrated in Figure 2.

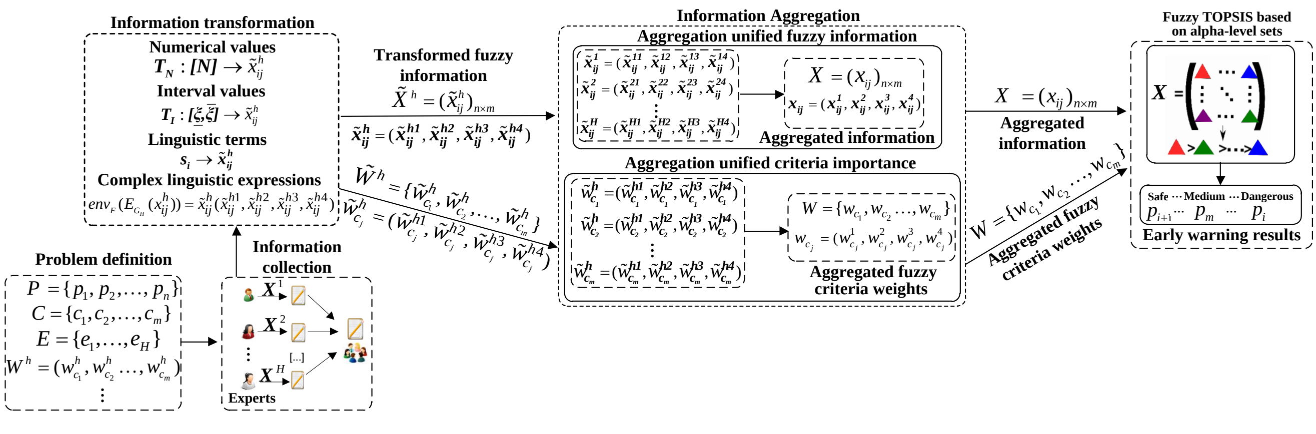

Figure 2. The general framework of the proposed method.

Based on Figure 2, it can be seen that there are five phases in the proposed method. The following subsections will present those phases in detail.

3.1. Problem Definition

The following notations are defined for the early warning problem.

- $P = \{p_1, p_2, \ldots, p_n\}$: the set of early warning objects, in which p_i indicates the *i-th* early warning object, $i = 1, 2, \ldots, n$.
- $C = \{c_1, c_2, \ldots, c_m\}$: the set of criteria/attributes, in which c_j indicates the *j-th* criterion/attribute, $j = 1, 2, \ldots, m$.

- $E = \{e_1, e_2, \ldots, e_H\}$: the set of involved experts, in which e_h indicates the *h-th* expert, $h = 1, 2, \ldots, H$.
- $X^h = [x_{ij}^h]_{n \times m}$: the information matrix provided by the expert, e_h, in which x_{ij}^h indicates the assessments provided by the expert e_h on p_i regarding c_j, $h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.
- $\tilde{X}^h = [\tilde{x}_{ij}^h]_{n \times m}$: the transformed information matrix regarding X^h, in which $\tilde{x}_{ij}^h$ indicates the transformed trapezoidal fuzzy numbers related to x_{ij}^h, $h = 1, 2, \ldots, H$; $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.
- $X = [x_{ij}]_{n \times m}$: the aggregated information matrix with respect to $\tilde{X}^h$, in which x_{ij} indicates the aggregated trapezoidal fuzzy numbers, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.
- $W^h = (w_{c_1}^h, w_{c_2}^h, \ldots, w_{c_m}^h)$: the vector with respect to criteria importance assessments provided by expert e_h, in which $w_{c_j}^h$ indicates the assessments regarding criterion c_j, $h = 1, 2, \ldots, H$, $j = 1, 2, \ldots, m$.
- $\tilde{W}^h = (\tilde{w}_{c_1}^h, \tilde{w}_{c_2}^h, \ldots, \tilde{w}_{c_m}^h)$: the transformed information vector regarding W^h, in which $\tilde{w}_{c_j}^h$ indicates the transformed trapezoidal fuzzy numbers related to $w_{c_j}^h$, $h = 1, 2, \ldots, H$, $j = 1, 2, \ldots, m$.
- $W = (w_{c_1}, w_{c_2}, \ldots, w_{c_m})$: the aggregated information vector with respect to $\tilde{W}^h$, in which w_{c_j} indicates the aggregated trapezoidal fuzzy numbers, $j = 1, 2, \ldots, m$.

3.2. Information Collection

The assessments x_{ij}^h and $w_{c_j}^h$ regarding p_i over c_j and criterion importance assessments provided by expert e_h are collected, respectively, as follows:

$$X^h = \begin{array}{c} \\ p_1 \\ p_2 \\ \vdots \\ p_n \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_m \end{array} \\ \begin{bmatrix} x_{11}^1 & x_{12}^1 & \cdots & x_{1m}^1 \\ x_{21}^1 & x_{22}^1 & \cdots & x_{2m}^1 \\ \vdots & \vdots & \cdots & \vdots \\ x_{n1}^1 & x_{n2}^1 & \cdots & x_{nm}^1 \end{bmatrix} \end{array}$$

where x_{ij}^h includes numerical values N, interval values I, linguistic terms s_i, and comparative linguistic expressions S_{ll}.

$$W^h = \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_m \end{array} \\ \begin{bmatrix} w_{c_1}^h & w_{c_2}^h & \cdots & w_{c_m}^h \end{bmatrix} \end{array}$$

where $w_{c_j}^h$ includes linguistic terms s_i and comparative linguistic expressions S_{ll}.

3.3. Information Transformation

In this study, the information employed to describe the early warning problem from quantitative and qualitative contexts includes numerical values N, interval values I, linguistic terms s_i, and comparative linguistic expressions S_{ll}. Due to the non-homogeneous information, information transformation is necessary to convert them into a unified form to conduct the computations.

To retain as much of the fuzzy and uncertain information as possible, the following transformation processes were conducted:

(1) *Numerical values N*

For numerical values $x_{ij}^h \in N$, the assessments provided by expert e_h regarding p_i over c_j belong to a specific scale R, i.e., $x_{ij}^h \in R$. It is first normalized into [0,1] using Equation (17), i.e.,

$$\theta = \frac{x_{ij}^h}{x^*} \tag{17}$$

where $x^* = \max_h\{x_{ij}^h\}, h = 1,2,\ldots,H, i = 1,2,\ldots,n, j = 1,2,\ldots,m$

On such a basis, the transformation function T_N is employed to transform numerical values into trapezoidal fuzzy numbers, i.e.,

$$T_N : [0,1] \to \tilde{x}_{ij}^h \tag{18}$$

$$T_N(\theta) = \tilde{x}_{ij}^h = (\theta,\theta,\theta,\theta) \tag{19}$$

(2) *Interval values I*

For interval values $x_{ij}^h = [d^L, d^U]$, the assessments provided by expert e_h regarding p_i over c_j belong to a specific domain $[\eta^L, \eta^U]$, i.e., $x_{ij}^h = [d^L, d^U] \in [\eta^L, \eta^U]$.

Similarly, normalization should be first conducted as follows:

$$\underline{\xi} = \frac{d^L - \eta^L}{\eta^U - \eta^L} \ and \ \overline{\xi} = \frac{d^U - \eta^L}{\eta^U - \eta^L} \tag{20}$$

where $\underline{\xi}$ and $\overline{\xi}$ are the lower and upper bounds of the normalized interval values, respectively, $\underline{\xi} \leq \overline{\xi}$.

On such a basis, the transformation function T_I is employed to transform interval values into related trapezoidal fuzzy numbers, i.e.,

$$\begin{aligned} &T_I : [\underline{\xi}, \overline{\xi}] \to \tilde{x}_{ij}^h \\ &T_I(\underline{\xi}, \overline{\xi}) = \tilde{x}_{ij}^h(\underline{\xi}, \underline{\xi}, \overline{\xi}, \overline{\xi}) \end{aligned} \tag{21}$$

where $i = 1,2,\ldots,n, j = 1,2,\ldots,m, h = 1,2,\ldots,H$.

(3) *Linguistic terms s_i*

For linguistic terms $x_{ij}^h, w_{c_j}^h \in S = \{s_0, s_1, \ldots, s_g\}$, the experts' assessments x_{ij}^h and $w_{c_j}^h$ can be presented by trapezoidal fuzzy numbers directly, i.e.,

$$\begin{aligned} &x_{ij}^h \to \tilde{x}_{ij}^h(\tilde{x}_{ij}^{h1}, \tilde{x}_{ij}^{h2}, \tilde{x}_{ij}^{h3}, \tilde{x}_{ij}^{h4}) \\ &w_{c_j}^h \to \tilde{w}_{c_j}^h(\tilde{w}_{c_j}^{h1}, \tilde{w}_{c_j}^{h2}, \tilde{w}_{c_j}^{h3}, \tilde{w}_{c_j}^{h4}) \end{aligned} \tag{22}$$

(4) *Comparative linguistic expressions S_{ll}*

For comparative linguistic expressions, $x_{ij}^h, w_{c_j}^h \in S_{ll}$, the experts' assessments are transformed into corresponding trapezoidal fuzzy numbers by Definitions 3 and 4, i.e.,

$$\begin{aligned} &env_F(E_{G_H}(x_{ij}^h)) = \tilde{x}_{ij}^h(\tilde{x}_{ij}^{h1}, \tilde{x}_{ij}^{h2}, \tilde{x}_{ij}^{h3}, \tilde{x}_{ij}^{h4}) \\ &env_F(E_{G_H}(w_{c_j}^h)) = \tilde{w}_{c_j}^h(\tilde{w}_{c_j}^{h1}, \tilde{w}_{c_j}^{h2}, \tilde{w}_{c_j}^{h3}, \tilde{w}_{c_j}^{h4}) \end{aligned} \tag{23}$$

Based on Equations (17)–(23), the collected information $X^h = [x_{ij}^h]_{n\times m}$ and $W^h = (w_{c_1}^h, \ldots, w_{c_m}^h)$ can be converted into related trapezoidal fuzzy numbers $\tilde{X}^h = [\tilde{x}_{ij}^h]_{n\times m}$ and $\tilde{W}^h = (\tilde{w}_{c_1}^h, \ldots, \tilde{w}_{c_m}^h)$, respectively.

3.4. Information Aggregation

The process of information aggregation aggregates all experts' transformed information involved $(\tilde{X}^h, \tilde{W}^h)$ into a group one, which contains all of the experts' wisdom involved and is used for further computations.

The following aggregation equations are employed, i.e.,

(1) *Aggregation of the transformed information* $\tilde{X}^h$

The transformed information, $\tilde{X}^h = [\tilde{x}_{ij}^h]_{n\times m}$, in which $\tilde{x}_{ij}^h = (\tilde{x}_{ij}^{h1}, \tilde{x}_{ij}^{h2}, \tilde{x}_{ij}^{h3}, \tilde{x}_{ij}^{h4})$ of expert e_h is aggregated into $X = (x_{ij})_{n\times m}$, in which $x_{ij} = (x_{ij}^1, x_{ij}^2, x_{ij}^3, x_{ij}^4)$ by Equation (24):

$$\begin{array}{ll} x_{ij}^1 = \min\limits_h\{\tilde{x}_{ij}^{h1}\}, & x_{ij}^2 = \frac{1}{H}\sum_{h=1}^H \tilde{x}_{ij}^{h2} \\ x_{ij}^3 = \frac{1}{H}\sum_{h=1}^H \tilde{x}_{ij}^{h3}, & x_{ij}^4 = \max\limits_h\{\tilde{x}_{ij}^{h4}\} \end{array} \tag{24}$$

where $h = 1,2,\ldots,H$; $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$.

(2) *Aggregation of the transformed criteria importance* $\tilde{W}^h$

Similarly, the transformed fuzzy criteria weights, $\tilde{W}^h = \{\tilde{w}_{c_1}^h, \tilde{w}_{c_2}^h, \ldots, \tilde{w}_{c_m}^h\}$, in which $\tilde{w}_{c_j}^h = (\tilde{w}_{c_j}^{h1}, \tilde{w}_{c_j}^{h2}, \tilde{w}_{c_j}^{h3}, \tilde{w}_{c_j}^{h4})$ of expert e_h is aggregated into $W = \{w_{c_1}, w_{c_2}, \ldots, w_{c_m}\}$, in which $w_{c_j} = (w_{c_j}^1, w_{c_j}^2, w_{c_j}^3, w_{c_j}^4)$ by Equation (25):

$$\begin{array}{ll} w_{c_j}^1 = \min\limits_h\{\tilde{w}_{c_j}^{h1}\}, & w_{c_j}^2 = \frac{1}{H}\sum_{h=1}^H \tilde{w}_{c_j}^{h2} \\ w_{c_j}^3 = \frac{1}{H}\sum_{h=1}^H \tilde{w}_{c_j}^{h3}, & w_{c_j}^4 = \max\limits_h\{\tilde{w}_{c_j}^{h4}\} \end{array} \tag{25}$$

where $h = 1,2,\ldots,H$, $j = 1,2,\ldots,m$.

The superiority and advantages of the aggregation methods presented in Equations (24) and (25) include (1) containing all experts' wisdom in the decision process, (2) avoiding the loss of fuzzy information as much as possible, and (3) ease of understanding and computing.

3.5. Fuzzy TOPSIS Method Based on Alpha-Level Sets

Since the aggregated fuzzy information $X = (x_{ij})_{n\times m}$ and fuzzy criteria weights $W = \{w_{c_1}, w_{c_2}, \ldots, w_{c_m}\}$ are normalized, it is not necessary to conduct the normalization process. Step 1 introduced in Section 2.3 can be skipped; different from Steps 4 and 5 of the fuzzy TOPSIS method based on alpha-level sets introduced in Section 2.3, the proposed early warning method provides two different options to handle the early warning results, i.e.,

(1) The fuzzy relative closeness at each alpha-level set, $(RC_i)_\alpha$, is an interval value, i.e., $(RC_i)_\alpha = [(RC_i)_\alpha^L, (RC_i)_\alpha^U]$. To identify the status of early warning objects easily and clearly, the status results ϕ_z = {Very dangerous (VD), Dangerous (D), Fairly dangerous (FD), Fairly safety (FS), Safety (S)} ($\phi_z, z = 1,2,\ldots,5$) related to fuzzy relative closeness $\tilde{RC}_i$ are defined in Table 2.

Table 2. Status of early warning objects related to fuzzy relative closeness.

Status (ϕ_z)	Very Dangerous (ϕ_1)	Dangerous (ϕ_2)	Fairly Dangerous (ϕ_3)	Fairly Safety (ϕ_4)	Safety (ϕ_5)
$\tilde{RC}_i$	[0, 0.2)	[0.2, 0.4)	[0.4, 0.6)	[0.6, 0.8)	[0.8, 1]

Since $(RC_i)_\alpha$ and $\tilde{RC}_i$ are interval values, the dominance degree of two interval values [49] is employed to rank the p_i. The dominance degree $P(\tilde{RC}_i > (RC_i)_\alpha)$ of interval values $\tilde{RC}_i$ over $(RC_i)_\alpha = [(RC_i)_\alpha^L, (RC_i)_\alpha^U]$ shown in Table 2 can be computed as follows:

$$P(\tilde{RC}_i > (RC_i)_\alpha) = \frac{max[0, (\tilde{RC}_i)^U - (RC_i)_\alpha^L] - max[0, ((\tilde{RC}_i)^L) - (RC_i)_\alpha^U]}{[(RC_i)_\alpha^U - (RC_i)_\alpha^L] + [(\tilde{RC}_i)^U - (\tilde{RC}_i)^L]} \tag{26}$$

where $(\tilde{RC}_i)^U$ and $(\tilde{RC}_i)^L$ are the upper and lower bounds of $\tilde{RC}_i$, respectively.

According to Table 2 and Equation (26), the status result of p_i can be identified as follows:

- Step 1: To judge the intersection relation between $\tilde{RC_i}$ and $(RC_i)_\alpha$.
- Step 2: When $\tilde{RC_i} \bigcap (RC_i)_\alpha \neq \Phi$, if $P(\tilde{RC_i} > (RC_i)_\alpha) > P((RC_i)_\alpha > \tilde{RC_i})$, then the status results of p_i belongs to the corresponding status $min\{z|\phi_{z-1}\}$; if $P(\tilde{RC_i} > (RC_i)_\alpha) < P((RC_i)_\alpha > \tilde{RC_i})$, then the status results of p_i belongs to the corresponding status $min\{z|\phi_z\}$; and if there exists $P(\tilde{RC_i} > (RC_i)_\alpha) > P((RC_i)_\alpha > \tilde{RC_i})$ and $P(\tilde{RC_i} > (RC_i)_\alpha) < P((RC_i)_\alpha > \tilde{RC_i})$, then the status results of p_i belongs to the corresponding status $min\{z|\phi_z\}$.

Such a way can provide more information about early warning objects regarding its status and tendency; thus, a decision maker has more options to identify the status of early warning objects from different perspectives.

(2) If the status of the early warning objects is identified from the comprehensive perspective instead of at each alpha-level set, the status of early warning objects at all alpha-level sets is employed, which can be obtained by Equation (16). Therefore, if $m(RC_i) \in \tilde{RC_i}$, the status result of p_i belongs to the corresponding status ϕ_z of $\tilde{RC_i}$.

4. Illustrative Example and Comparison

In this section, an illustrative example is presented to demonstrate the proposed method. In addition, to highlight the superiority and advantage of the proposed method, a comparison with related studies is presented.

4.1. Illustrative Example

In this example, an early warning problem about landslides is employed to demonstrate the proposed method. A landslide is a kind of natural disaster that occurs especially easily in mountainous areas, and it is usually caused by earthquakes, heavy rainfall, or geological structures. When a landslide takes place, it might cause seriously security risks to human lives, traffic systems, property, and so on.

To avoid or reduce such kinds of losses and the negative impacts, an early warning is an effective way to identify the security risk in advance in a real-world situation. Therefore, an early warning of a landslide adopted from literature [50] is taken as an example.

4.1.1. Problem Definition

During rainy seasons, it is quite easy for a landslide to occur due to continuous rainfall. Suppose that there are six villages (p_1, p_2, p_3, p_4, p_5, and p_6) located in the low-lying areas in a mountain area in southern China. Three experts (e_1, e_2, and e_3) are invited to involve the early warning problem. Seven criteria are considered, presented in Table 3.

For criteria C_5, C_6, and C_7, the linguistic terms S_1 = {None (N), Very Low (VL), Low (L), Medium (M), Fairly High (FH), High (H), Very High (VH)} are employed; the linguistic terms for criteria importance are S_2 = {None (N), Very Low Importance (VLI), Low Importance (LI), Medium Importance (MI), High Importance (HI), Very High Importance (VHI), Absolutely High Importance (AHI)}; and the corresponding fuzzy numbers S_1 and S_2 are shown in Figure 3.

4.1.2. Information Collection

Regarding the given problem, three experts provided their assessments X^h regarding the alternatives p_i over criteria c_j and criteria importance W^h, and the collected information of X^h and W^h are shown in Tables 4 and 5, respectively (note: 'bt' means 'between' in Tables 4 and 5).

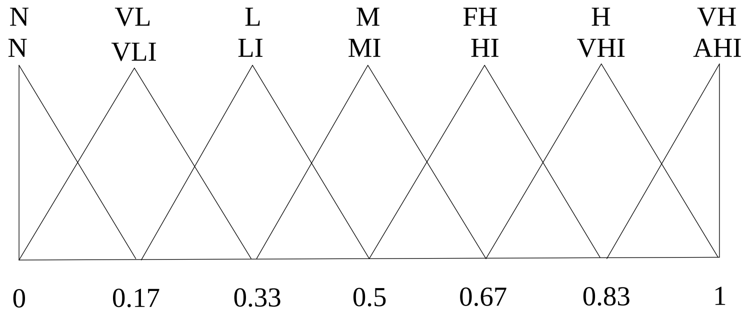

Figure 3. Linguistic terms of S_1 and S_2, and its fuzzy numbers.

Table 3. Description of the criteria considered.

Criteria	Description	Information Type
Rainfall (mm) (C_1)	The higher the rainfall, the more easily a landslide occurs	I
Coverage rate of the forest (%) (C_2)	The lower the coverage rate of the forest, the more easily a landslide occurs	N
Saturated water content of soil (%) (C_3)	The higher the saturated water content of the soil, the more easily a landslide occurs	N
Slope (∘) (C_4)	The higher the slope, the more easily a landslide occurs	N
Influence Degree of Earthquake (C_5)	The higher the influence degree of an earthquake, the more easily a landslide occurs	s_i, S_{II}
Degree of human activity (C_6)	The higher the degree of human activity, the more easily a landslide occurs	s_i, S_{II}
Stability of Geological Structure (C_7)	The lower the stability of a geological structure, the more easily a landslide occurs	s_i, S_{II}

Table 4. Collected information of X^h provided by experts.

Experts	Objects	Criteria						
		C_1	C_2	C_3	C_4	C_5	C_6	C_7
e_1	p_1	[35,55]	62	33	35	M	L	H
	p_2	[25,38]	45	26	29	L	L	M
	p_3	[20,35]	33	40	38	VL	H	H
	p_4	[30,46]	65	38	40	H	H	M
	p_5	[18,35]	55	29	33	M	bt M and FH	M
	p_6	[23,34]	42	35	28	L	M	H
e_2	p_1	[40,50]	58	35	33	bt L and M	M	FH
	p_2	[22,35]	50	28	36	M	FH	M
	p_3	[25,40]	44	38	32	L	L	M
	p_4	[20,38]	53	44	37	M	H	bt M and FH
	p_5	[22,38]	48	36	31	H	bt FH and H	H
	p_6	[21,44]	39	29	30	At most H	M	H
e_3	p_1	[33,45]	66	29	38	L	L	M
	p_2	[18,36]	53	33	41	FH	L	L
	p_3	[18,24]	48	42	26	M	M	FH
	p_4	[23,38]	50	41	33	L	At most H	M
	p_5	[16,24]	49	39	29	H	FH	FH
	p_6	[20,34]	45	34	34	H	L	FH

Table 5. Collected information of criteria importance W^h provided by experts.

Experts	Criteria Importance						
	C_1	C_2	C_3	C_4	C_5	C_6	C_7
e_1	VHI	HI	bt MI and HI	HI	MI	LI	HI
e_2	At least HI	HI	MI	VHI	LI	VLI	MI
e_3	VHI	MI	HI	bt HI and VHI	LI	LI	VHI

4.1.3. Information Transformation

According to Tables 4 and 5, the related transformation can be conducted based on Equations (17)–(23), and the transformed information $\tilde{X}^h$ and $\tilde{W}^h$ regarding X^h and W^h are shown in Tables 6–9, respectively.

Table 6. Transformed information of $\tilde{X}^h$.

Experts	Objects	Criteria			
		C_1	C_2	C_3	C_4
e_1	p_1	(0.4872,0.4872,1.0000,1.0000)	(0.9394,0.9394,0.9394,0.9394)	(0.7500,0.7500,0.7500,0.7500)	(0.8537,0.8537,0.8537,0.8537)
	p_2	(0.2308,0.2308,0.5641,0.5641)	(0.6818,0.6818,0.6818,0.6818)	(0.5909,0.5909,0.5909,0.5909)	(0.7073,0.7073,0.7073,0.7073)
	p_3	(0.1026,0.1026,0.4872,0.4872)	(0.5000,0.5000,0.5000,0.5000)	(0.9091,0.9091,0.9091,0.9091)	(0.9268,0.9268,0.9268,0.9268)
	p_4	(0.3590,0.3590,0.7692,0.7692)	(0.9848,0.9848,0.9848,0.9848)	(0.8636,0.8636,0.8636,0.8636)	(0.9756,0.9756,0.9756,0.9756)
	p_5	(0.0513,0.0513,0.4872,0.4872)	(0.8333,0.8333,0.8333,0.8333)	(0.6591,0.6591,0.6591,0.6591)	(0.8049,0.8049,0.8049,0.8049)
	p_6	(0.1795,0.1795,0.4615,0.4615)	(0.6364,0.6364,0.6364,0.6364)	(0.7955,0.7955,0.7955,0.7955)	(0.6829,0.6829,0.6829,0.6829)
e_2	p_1	(0.6154,0.6154,0.8718,0.8718)	(0.8788,0.8788,0.8788,0.8788)	(0.7955,0.7955,0.7955,0.7955)	(0.8049,0.8049,0.8049,0.8049)
	p_2	(0.1538,0.1538,0.4872,0.4872)	(0.7576,0.7576,0.7576,0.7576)	(0.6364,0.6364,0.6364,0.6364)	(0.8780,0.8780,0.8780,0.8780)
	p_3	(0.2308,0.2308,0.6154,0.6154)	(0.6667,0.6667,0.6667,0.6667)	(0.8636,0.8636,0.8636,0.8636)	(0.7805,0.7805,0.7805,0.7805)
	p_4	(0.1026,0.1026,0.5641,0.5641)	(0.8030,0.8030,0.8030,0.8030)	(1.0000,1.0000,1.0000,1.0000)	(1.9024,0.9024,0.9024,0.9024)
	p_5	(0.1538,0.1538,0.5641,0.5641)	(0.7273,0.7273,0.7273,0.7273)	(0.8182,0.8182,0.8182,0.8182)	(0.7561,0.7561,0.7561,0.7561)
	p_6	(0.1282,0.1282,0.7179,0.7179)	(0.5909,0.5909,0.5909,0.5909)	(0.6591,0.6591,0.6591,0.6591)	(0.7317,0.7317,0.7317,0.7317)
e_3	p_1	(0.4359,0.4359,0.7436,0.7436)	(1.0000,1.0000,1.0000,1.0000)	(0.6591,0.6591,0.6591,0.6591)	(0.9268,0.9268,0.9268,0.9268)
	p_2	(0.0513,0.0513,0.5128,0.5128)	(0.8030,0.8030,0.8030,0.8030)	(0.7500,0.7500,0.7500,0.7500)	(0.0000,1.0000,1.0000,1.0000)
	p_3	(0.0513,0.0513,0.2051,0.2051)	(0.7273,0.7273,0.7273,0.7273)	(0.9545,0.9545,0.9545,0.9545)	(0.6341,0.6341,0.6341,0.6341)
	p_4	(0.1795,0.1795,0.5641,0.5641)	(0.7576,0.7576,0.7576,0.7576)	(0.9318,0.9318,0.9318,0.9318)	(0.8049,0.8049,0.8049,0.8049)
	p_5	(0.0000,0.0000,0.2051,0.2051)	(0.7424,0.7424,0.7424,0.7424)	(0.8864,0.8864,0.8864,0.8864)	(0.7073,0.7073,0.7073,0.7073)
	p_6	(0.1026,0.1026,0.4615,0.4615)	(0.6818,0.6818,0.6818,0.6818)	(0.7727,0.7727,0.7727,0.7727)	(0.8293,0.8293,0.8293,0.8293)

Table 7. Transformed information of $\tilde{X}^h$-continued.

Experts	Objects	Criteria		
		C_5	C_6	C_7
e_1	p_1	(0.3300,0.5000,0.5000,0.6700)	(0.1700,0.3300,0.3300,0.5000)	(0.6700,0.8300,0.8300,1.0000)
	p_2	(0.1700,0.3300,0.3300,0.5000)	(0.1700,0.3300,0.3300,0.5000)	(0.3300,0.5000,0.5000,0.6700)
	p_3	(0.0000,0.1700,0.1700,0.3300)	(0.6700,0.8300,0.8300,1.0000)	(0.6700,0.8300,0.8300,1.0000)
	p_4	(0.6700,0.8300,0.8300,1.0000)	(0.6700,0.8300,0.8300,1.0000)	(0.3300,0.5000,0.5000,0.6700)
	p_5	(0.3300,0.5000,0.5000,0.6700)	(0.3400,0.5000,0.6700,0.8400)	(0.3300,0.5000,0.5000,0.6700)
	p_6	(0.1700,0.3300,0.3300,0.5000)	(0.3300,0.5000,0.5000,0.6700)	(0.6700,0.8300,0.8300,1.0000)
e_2	p_1	(0.1700,0.3400,0.5000,0.6700)	(0.3300,0.5000,0.5000,0.6700)	(0.5000,0.6700,0.6700,0.8300)
	p_2	(0.3300,0.5000,0.5000,0.6700)	(0.5000,0.6700,0.6700,0.8300)	(0.3300,0.5000,0.5000,0.6700)
	p_3	(0.1700,0.3300,0.3300,0.5000)	(0.1700,0.3300,0.3300,0.5000)	(0.3300,0.5000,0.5000,0.6700)
	p_4	(0.3300,0.5000,0.5000,0.6700)	(0.6700,0.8300,0.8300,1.0000)	(0.3400,0.5000,0.6700,0.8400)
	p_5	(0.6700,0.8300,0.8300,1.0000)	(0.5000,0.6700,0.8300,1.0000)	(0.6700,0.8300,0.8300,1.0000)
	p_6	(0.0000,0.0000,0.5900,0.8400)	(0.3300,0.5000,0.5000,0.6700)	(0.6700,0.8300,0.8300,1.0000)
e_3	p_1	(0.1700,0.3300,0.3300,0.5000)	(0.1700,0.3300,0.3300,0.5000)	(0.3300,0.5000,0.5000,0.6700)
	p_2	(0.5000,0.6700,0.6700,0.8300)	(0.5000,0.6700,0.6700,0.8300)	(0.1700,0.3300,0.3300,0.5000)
	p_3	(0.3300,0.5000,0.5000,0.6700)	(0.3300,0.5000,0.5000,0.6700)	(0.5000,0.6700,0.6700,0.8300)
	p_4	(0.1700,0.3300,0.3300,0.5000)	(0.0000,0.0000,0.5900,0.8400)	(0.3300,0.5000,0.5000,0.6700)
	p_5	(0.6700,0.8300,0.8300,1.0000)	(0.5000,0.6700,0.6700,0.8300)	(0.5000,0.6700,0.6700,0.8300)
	p_6	(0.6700,0.8300,0.8300,1.0000)	(0.1700,0.3300,0.3300,0.5000)	(0.5000,0.6700,0.6700,0.8300)

Table 8. Transformed information of fuzzy criteria weights $\tilde{W}^h$.

Experts	Transformed Criteria Importance			
	C_1	C_2	C_3	C_4
e_1	(0.6700,0.8300,0.8300,1.0000)	(0.5000,0.6700,0.6700,0.8300)	(0.3400,0.5000,0.6700,0.8400)	(0.5000,0.6700,0.6700,0.8300)
e_2	(0.5000,0.8600,0.8600,1.0000)	(0.5000,0.6700,0.6700,0.8300)	(0.3300,0.5000,0.5000,0.6700)	(0.6700,0.8300,0.8300,1.0000)
e_3	(0.6700,0.8300,0.8300,1.0000)	(0.3300,0.5000,0.5000,0.6700)	(0.5000,0.3300,0.6700,0.8300)	(0.5000,0.6700,0.8300,1.0000)

Table 9. Transformed information of fuzzy criteria weights $\tilde{W}^h$-continued.

Experts	Transformed Criteria Importance		
	C_5	C_6	C_7
e_1	(0.3300,0.5000,0.5000,0.6700)	(0.1700,0.3300,0.3300,0.5000)	(0.5000,0.6700,0.6700,0.8300)
e_2	(0.1700,0.3300,0.3300,0.5000)	(0.0000,0.1700,0.1700,0.6700)	(0.3300,0.5000,0.5000,0.6700)
e_3	(0.1700,0.3300,0.3300,0.5000)	(0.1700,0.3300,0.3300,0.5000)	(0.6700,0.8300,0.8300,1.0000)

For clarity, the following calculations were conducted to demonstrate the transformation process.

For criterion C_1, taking $x_{11}^1 = [35, 55]$ as an example, the specific domain of C_1, $[\eta^L, \eta^U]$, is $[16, 55]$. Therefore, based on Equation (20), the normalization can be conducted as $\underline{\xi} = \frac{35-16}{55-35} = 0.4872$, $\overline{\xi} = \frac{55-16}{55-16} = 1.0000$. On such a basis, based on Equation (21), transformation function T_I is used to transform the interval values $[0.4872, 1.0000]$ into related trapezoidal fuzzy numbers, i.e., $(0.4872, 0.4872, 1.0000, 1.0000)$. The rest interval values can be transformed as the same process.

For criterion C_2, since the $x^* = \max\limits_h \{x_{i2}^h\} = 66$, taking $x_{12}^1 = 62$ as an example. Based on Equation (17), $\theta = \frac{62}{66} = 0.9394$. Therefore, based on Equation (18), transformation function T_N is used to transform the numerical values 0.9394 into related trapezoidal fuzzy numbers, i.e., $(0.9394, 0.9394, 0.9394, 0.9394)$. The same computations can be applied to criteria C_3 and C_4.

For criterion C_5, take $x_{15}^1 = M$ as an example. Based on Figure 3, it can be seen that the linguistic term M is associated with related triangular fuzzy number $(0.33, 0.5, 0.67)$. Since the triangular fuzzy number is a special case of trapezoidal fuzzy numbers, based on Equation (22), the linguistic term M can be expressed by the related trapezoidal fuzzy number $(0.33, 0.5, 0.5, 0.67)$. The same operations can be conducted to the other single linguistic terms.

For the linguistic expressions, take $x_{15}^2 = bt\ L$ and M as an example. Based on Figure 3, it can be seen that the linguistic term from L to M is associated with related triangular fuzzy number $(0.17, 0.33, 0.5)$ and $(0.33, 0.5, 0.67)$, respectively. According to Definitions 3 and 4, based on Equation (23), the trapezoidal fuzzy number of $x_{15}^2 = bt\ L\ and\ M$ is (0.17,0.34,0.5,0.67). Similarly, the related linguistic expressions can be also transformed into trapezoidal fuzzy numbers.

4.1.4. Information Aggregation

Based on the transformed information of fuzzy information $\tilde{X}^h$ and fuzzy criteria importance $\tilde{W}^h$ shown in Tables 6–9, respectively, the aggregated information of X and W can be obtained based on Equations (24) and (25), which are shown in Tables 10 and 11.

Table 10. Aggregated information of fuzzy information X and fuzzy criteria weights W.

Aggregated Information	Criteria			
	C_1	C_2	C_3	C_4
X	(0.4359,0.5128,0.8718,1.0000)	(0.8788,0.9394,0.9394,1.0000)	(0.6591,0.7348,0.7348,0.7955)	(0.8049,0.8618,0.8618,0.9268)
	(0.0513,0.1453,0.5214,0.5641)	(0.6818,0.7475,0.7475,0.8030)	(0.5909,0.6591,0.6591,0.7500)	(0.7073,0.8618,0.8618,1.0000)
	(0.0513,0.1282,0.4359,0.6154)	(0.5000,0.6313,0.6313,0.7273)	(0.8636,0.9091,0.9091,0.9545)	(0.6341,0.7805,0.7805,0.9268)
	(0.1026,0.2137,0.6325,0.7692)	(0.7576,0.8485,0.8485,0.9848)	(0.8636,0.9318,0.9318,1.0000)	(0.8049,0.8943,0.8943,0.9756)
	(0.0000,0.0684,0.4188,0.5641)	(0.7273,0.7677,0.7677,0.8333)	(0.6591,0.7879,0.7879,0.8864)	(0.7073,0.7561,0.7561,0.8049)
	(0.1026,0.1368,0.5470,0.7179)	(0.5909,0.6364,0.6364,0.6818)	(0.6591,0.7424,0.7424,0.7955)	(0.6829,0.7480,0.7480,0.8293)
W	(0.5000,0.8400,0.8400,1.0000)	(0.3300,0.6133,0.6133,0.8300)	(0.3300,0.5567,0.6133,0.8400)	(0.5000,0.7233,0.7767,1.0000)

Table 11. Aggregated information of fuzzy information X and fuzzy criteria weights W-continued.

Aggregated Information	Criteria		
	C_5	C_6	C_7
X	(0.1700,0.3900,0.4433,0.6700)	(0.1700,0.3867,0.3867,0.6700)	(0.3300,0.6667,0.6667,1.0000)
	(0.1700,0.5000,0.5000,0.8300)	(0.1700,0.4433,0.4433,0.8300)	(0.1700,0.4433,0.4433,0.6700)
	(0.0000,0.3333,0.3333,0.6700)	(0.1700,0.5533,0.5533,1.0000)	(0.3300,0.6667,0.6667,1.0000)
	(0.1700,0.5533,0.5533,1.0000)	(0.0000,0.5533,0.7500,1.0000)	(0.3300,0.5000,0.5567,0.8400)
	(0.3300,0.7200,0.7200,1.0000)	(0.3400,0.6133,0.7233,1.0000)	(0.3300,0.6667,0.6667,1.0000)
	(0.0000,0.3867,0.5833,1.0000)	(0.1700,0.4433,0.4433,0.6700)	(0.5000,0.7767,0.7767,1.0000)
W	(0.1700,0.3867,0.3867,0.6700)	(0.0000,0.2767,0.2767,0.5000)	(0.3300,0.6667,0.6667,1.0000)

For clarity, take the transformed fuzzy criteria weight of C_1 shown in Table 8 as an example to demonstrate the aggregation process. Based on Equation (25), the aggregated fuzzy criteria weight of C_1, $w_{c_1} = (w_{c_1}^1, w_{c_1}^2, w_{c_1}^3, w_{c_1}^4)$, can be conducted as follows, i.e., $w_{c_1}^1 = \min_h\{\tilde{w}_{c_1}^{h1}\} = \min_h\{0.6700, 0.5000, 0.6700\} = 0.5000$, $w_{c_1}^2 = \frac{1}{H}\sum_{h=1}^{H}\tilde{w}_{c_j}^{h2} = \frac{1}{3}(0.8300 + 0.8600 + 0.8300) = 0.8400$. Similarly, $w_{c_1}^3$ and $w_{c_1}^4$ can be also obtained. Therefore, the aggregated fuzzy criteria weight of C_1 is $(0.5000, 0.8400, 0.8400, 1.0000)$. For the remaining criteria, the same aggregation process can be used.

4.1.5. Fuzzy TOPSIS Method Based on Alpha-Level Sets

According to the aggregated information of fuzzy information X and fuzzy criteria weights W shown in Tables 10 and 11, 11 alpha levels (i.e., $\alpha = \{0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1\}$) are set to compute the corresponding fuzzy relative closeness RC_i at different alpha levels. Based on Section 3.5, the fuzzy relative closeness RC_i and related status results are presented in Tables 12 and 13.

For clarity, based on Table 12, take p_1 at alpha = 0 as an example to show the process of how to obtain the status result. According to Table 12, Table 2, and Equation (26), since $(RC_1)_0 = [0.4027, 0.9478]$, $\tilde{RC_i} \bigcap (RC_1)_0 \neq \Phi, i = 3,4,5$. Due to $P(\tilde{RC_3} > (RC_1)_0) = 0.2648$, $P((RC_1)_0 > \tilde{RC_3}) = 0.7352$, $P(\tilde{RC_4} > (RC_1)_0) = 0.5332$, $P((RC_1)_0 > \tilde{RC_4}) = 0.4668$, $P(\tilde{RC_5} > (RC_1)_0) = 0.8016$, $P((RC_1)_0 > \tilde{RC_5}) = 0.1984$. Since $P(\tilde{RC_3} > (RC_1)_0) = 0.2648 < P((RC_1)_0 > \tilde{RC_3}) = 0.7352$, $P(\tilde{RC_4} > (RC_1)_0) = 0.5332 > P((RC_1)_0 > \tilde{RC_4}) = 0.4668$, and $P(\tilde{RC_5} > (RC_1)_0) = 0.8016 > P((RC_1)_0 > \tilde{RC_5}) = 0.1984$, the status result of p_i is $min\{z|\phi_3, \phi_4, \phi_5\} = \phi_3$, i.e., $\phi_3 = FD$.

Table 12. The fuzzy relative closeness RC_i and related status results.

Alpha	p_1		p_2		p_3	
	$(RC_1)_\alpha$	Status	$(RC_2)_\alpha$	Status	$(RC_3)_\alpha$	Status
0	[0.4027,0.9478]	FD	[0.2613,0.8143]	D	[0.2732,0.8792]	D
0.1	[0.4348,0.9357]	FD	[0.2879,0.7930]	D	[0.2989,0.8589]	D
0.2	[0.4634,0.9190]	FD	[0.3146,0.7760]	D	[0.3253,0.8357]	D
0.3	[0.4903,0.9012]	FD	[0.3412,0.7586]	D	[0.3519,0.8106]	D
0.4	[0.5169,0.8857]	FD	[0.3675,0.7403]	D	[0.3786,0.7845]	D
0.5	[0.5431,0.8685]	FD	[0.3936,0.7212]	D	[0.4053,0.7578]	FD
0.6	[0.5688,0.8495]	FD	[0.4192,0.7014]	FD	[0.4317,0.7307]	FD
0.7	[0.5939,0.8291]	FD	[0.4443,0.6814]	FD	[0.4577,0.7040]	FD
0.8	[0.6182,0.8074]	FS	[0.4688,0.6611]	FD	[0.4834,0.6772]	FD
0.9	[0.6416,0.7845]	FS	[0.4925,0.6400]	FD	[0.5085,0.6501]	FD
1	[0.6647,0.7600]	FS	[0.5160,0.6182]	FD	[0.5332,0.6231]	FD
$m(RC_i)$	0.7012	FS	0.5551	FD	0.5800	FD

Table 13. The fuzzy relative closeness RC_i and related status results continued.

Alpha	p_4		p_5		p_6	
	$(RC_4)_\alpha$	Status	$(RC_5)_\alpha$	Status	$(RC_6)_\alpha$	Status
0	[0.3280,0.9328]	D	[0.3202,0.8591]	D	[0.2880,0.8686]	D
0.1	[0.3544,0.9176]	D	[0.3409,0.8404]	D	[0.3126,0.8495]	D
0.2	[0.3808,0.8992]	D	[0.3619,0.8193]	D	[0.3378,0.8277]	D
0.3	[0.4073,0.8802]	FD	[0.3832,0.7984]	D	[0.3632,0.8046]	D
0.4	[0.4337,0.8609]	FD	[0.4049,0.7775]	FD	[0.3886,0.7837]	D
0.5	[0.4599,0.8405]	FD	[0.4267,0.7567]	FD	[0.4137,0.7630]	FD
0.6	[0.4855,0.8188]	FD	[0.4488,0.7350]	FD	[0.4384,0.7428]	FD
0.7	[0.5106,0.7961]	FD	[0.4711,0.7128]	FD	[0.4625,0.7229]	FD
0.8	[0.5349,0.7724]	FD	[0.4935,0.6902]	FD	[0.4860,0.7025]	FD
0.9	[0.5584,0.7478]	FD	[0.5159,0.6672]	FD	[0.5087,0.6815]	FD
1	[0.5811,0.7225]	FD	[0.5384,0.6441]	FD	[0.5306,0.6600]	FD
$m(RC_i)$	0.6465	FS	0.5912	FD	0.5880	FD

From Tables 12 and 13, it can be seen clearly that the status results of each early warning object p_i at different alpha levels are different; such differences are supported by the superiority and advantage of handling fuzzy information. The status results of p_i from a comprehensive perspective $m(RC_i)$ are presented in the last row of Tables 12 and 13.

According to Tables 12 and 13, a decision maker can analyze the safety status of each early warning object p_i from different perspectives based on the given problem and individual preference. However, such options seldom appear in existing early warning studies.

4.1.6. Sensitivity Analysis

To illustrate the feasibility and validity of the proposed method, a sensitivity analysis is conducted, in which the fuzzy criteria weights of C_5 are taken as an example.

Suppose that three experts providing the criteria importance regarding C_5 are $w^1_{c_5}$ = VHI, $w^2_{c_5}$ = VHI, and $w^3_{c_5}$ = HI (the rest of the information remains the same as shown in Section 4.1). The same transformation and aggregation processes can be carried out as shown in Sections 4.1.3 and 4.1.4, respectively; therefore, the aggregated fuzzy criterion weight of C_5 is $w_{c_5} = (0.5000, 0.7767, 0.7767, 1.0000)$. The fuzzy relative closeness and related status results can be obtained as the same processes presented in Section 4.1, and the sensitivity analysis results of RC_i and related status results are presented in Tables 14 and 15.

Table 14. Sensitivity analysis results of RC_i and related status results.

Alpha	p_1		p_2		p_3	
	$(RC_1)_\alpha$	Status	$(RC_2)_\alpha$	Status	$(RC_3)_\alpha$	Status
0	[0.3696,0.9093]	D	[0.2512,0.8169]	D	[0.2453,0.8606]	D
0.1	[0.3992,0.8952]	D	[0.2781,0.7936]	D	[0.2696,0.8376]	D
0.2	[0.4282,0.8783]	FD	[0.3053,0.7749]	D	[0.295,0.812]	D
0.3	[0.4564,0.8595]	FD	[0.3324,0.7557]	D	[0.3213,0.7847]	D
0.4	[0.4817,0.8395]	FD	[0.3594,0.7355]	D	[0.3481,0.7565]	D
0.5	[0.5065,0.8186]	FD	[0.3863,0.7145]	D	[0.3752,0.7276]	D
0.6	[0.5307,0.797]	FD	[0.4128,0.693]	FD	[0.4024,0.6985]	FD
0.7	[0.5539,0.7756]	FD	[0.4389,0.6714]	FD	[0.4294,0.6693]	FD
0.8	[0.577,0.7535]	FD	[0.4645,0.649]	FD	[0.4563,0.6405]	FD
0.9	[0.6001,0.7309]	FD	[0.4894,0.6259]	FD	[0.4831,0.6119]	FD
1	[0.6238,0.7074]	FD	[0.5143,0.6021]	FD	[0.5097,0.5828]	FD
$m(RC_i)$	0.6587	FS	0.5484	FD	0.5508	FD

Table 15. Sensitivity analysis results of RC_i and related status results-continued.

Alpha	p_4		p_5		p_6	
	$(RC_4)_\alpha$	**Status**	$(RC_5)_\alpha$	**Status**	$(RC_6)_\alpha$	**Status**
0	[0.3075,0.9376]	D	[0.3211,0.8704]	D	[0.2571,0.8827]	D
0.1	[0.3364,0.9217]	D	[0.3434,0.8530]	D	[0.2807,0.8640]	D
0.2	[0.3655,0.9006]	D	[0.3663,0.8326]	D	[0.3057,0.8411]	D
0.3	[0.3929,0.8789]	D	[0.3896,0.8106]	D	[0.3315,0.816]	D
0.4	[0.4205,0.8569]	FD	[0.4133,0.7889]	FD	[0.3579,0.7908]	D
0.5	[0.4479,0.8333]	FD	[0.4372,0.7676]	FD	[0.3845,0.7669]	D
0.6	[0.4750,0.8082]	FD	[0.4614,0.7458]	FD	[0.4110,0.7438]	FD
0.7	[0.5017,0.7819]	FD	[0.4857,0.7234]	FD	[0.4373,0.7211]	FD
0.8	[0.5279,0.7546]	FD	[0.5099,0.7005]	FD	[0.4632,0.6979]	FD
0.9	[0.5533,0.7264]	FD	[0.5342,0.6772]	FD	[0.4885,0.6738]	FD
1	[0.5779,0.6975]	FD	[0.5583,0.6536]	FD	[0.5132,0.6490]	FD
$m(RC_i)$	0.6366	FS	0.602	FS	0.5763	FD

According to Tables 14 and 15, it can be seen clearly that the results are different from that presented in Tables 12 and 13. The change in fuzzy criteria weight C_5 obviously affects the status results of early warning objects at different alpha levels; this is because the proposed method handles the fuzzy information in a reasonable and proper way, in which the tendency and slight changes can be clearly reflected. However, from a comprehensive perspective, only the status result of early warning object p_5 is changed, and it is clearly seen that the changes are more sensitive at different alpha levels than that from a comprehensive perspective due to the comprehensive perspective being obtained by the average level cuts.

From the sensitivity analysis, the superiority and advantages of the proposed method in handling fuzzy information is clearly shown again.

4.2. Comparisons with Existing Studies

Due to the fact that non-homogeneous information is not considered in existing early warning studies, it is unfair to conduct computation comparisons with existing studies to highlight the difference and superiority of the proposed method. Therefore, a descriptive comparison is provided to highlight the difference and superiority of the proposed method, which is shown in Table 16.

Table 16. Descriptive comparison with existing studies.

Literature	**Information Type**	**Hesitant Information Considered**
literature [14,21]	Numerical values	No
literature [22]	Interval values	No
literature [23]	Linguistic terms	No
Our proposal	Numerical values, interval values, linguistic terms, hesitant fuzzy linguistic informaiton	Yes

The specific results obtained from Table 16 show the difference between existing early warning studies and the proposed method. It clearly points out the superiority and advantages of the proposed method in handling non-homogeneous information and hesitant information consideration; the consideration of these types of information is neglected in extant early warning studies. The proposed method enriches the ability to consider information in extant early warning studies and provides a reference for future studies.

5. Discussion

From the illustrative example and comparisons presented in Section 4, the main superiority and advantages of the proposed method can be summarized as follows:

(1) The proposed early warning method provides a novel perspective for considering and handling non-homogeneous information, which describes the early warning problem from both quantitative and qualitative contexts, together with considering experts' hesitant information. This is the significant difference between existing studies and the proposed method.

(2) The fuzzy TOPSIS method based on alpha-level sets is employed in the proposed method, due to its capacity to retain as much information as possible during the computation process. In addition, more options are provided for a decision maker to analyze the status of early warning objects, and this extends methods for handling early warning results. This is a new perspective that has not been considered in existing early warning studies.

Except for the aforementioned superiority and advantages, the proposed method has limitations in its current version, i.e., it does not consider the experts' bounded rational and judgments during the early warning process. Actually, bounded rational and judgments [51,52] are quite common in real-world situations, particularly under uncertain and complex environments. Although this is a limitation in the proposed method, it is a promising future field of research, which will allow early warning studies to better represent real-world situations.

6. Conclusions and Future Work

Early warnings play important roles in emergency management and have drawn great attention due to their importance in avoiding or reducing the losses and negative impacts caused by emergency events in advance. Information is a critical element in the early warning process and should be considered. There are various information types related to the early warning problem in the real world; however, existing early warning studies just employ a single information type to describe the early warning problem and might not describe the problem comprehensively and properly. To fill this gap, this study considered non-homogeneous information, including numerical values, interval values, linguistic terms, and hesitant fuzzy linguistic term sets with a consideration of experts' hesitation. The fuzzy TOPSIS method based on alpha-level sets was employed to handle non-homogeneous information properly and reasonably; in addition, two options were provided to analyze the status results of early warning objects. An illustrative example and a comparison were presented to demonstrate the novelty and superiority of the proposed method. It is hoped that the proposed method has potential application in the near future.

For future work, some promising research directions include the following: (1) the development of an early warning system, in which different early warning models and methods are implemented to facilitate real-world early warnings; (2) consideration of experts' bounded and rational judgments in the early warning process, which is an inevitable issue in our daily life and must be considered. Those future researches can enable early warning studies to better represent real-world situations and to be easily understood and accepted.

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