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First-Year Mathematics and Its Application to Science: Evidence of Transfer of Learning to Physics and Engineering

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Abstract: Transfer of mathematical learning to science is seen as critical to the development of education and industrial societies, yet it is rarely interrogated in applied research. We present here research looking for evidence of transfer from university mathematics learning in semester one to second semester sciences/engineering courses (n = 1125). A transfer index measure was derived from extant university assessments, calculated on content-matched questions requiring mathematical concepts and skills in students' completed mathematics and science/engineering exams. We found that transfer could be measured in this way, and present path models of how transfer is associated with educational attainment and other factors. However, transfer was observed only in physics (n = 258) and engineering (n = 426); biology and molecular bioscience assessments did not provide opportunities for students to demonstrate their mathematical learning. In physics and engineering courses, mathematical attainment had a strong, direct, positive effect, with transfer of learning providing an additional direct and mediating effect upon students' performance in these subjects. In physics and engineering, transfer was also associated with higher levels of educational attainment in general. This new, applied approach to examining transfer trialed here may provide opportunities for analysing, evaluating, and improving cross-disciplinary transfer of learning within universities.

Keywords: transfer; learning; quantitative; science; mathematics; interdisciplinary; university; physics; engineering; STEM

1. Introduction

The importance of mathematics and science in schools and universities is widely acknowledged, and needed in order to sustain and advance modern and innovative economies. However, some societies face serious challenges, such as the poor mathematical preparedness of students, and declines in academic performance in STEM subjects. These concerns, the so-called "STEM crises", are recognised in international reports in Australia [1,2], Canada [3], India [4], Japan [5], the U.K. [6] and the U.S. [7] and it is suggested that, "Without a strong foundation in math and science from elementary and secondary school, students may find themselves unprepared to train for and pursue careers in STEM fields" [8] (p. 8). Also highlighted in these reports [1–8] is the importance of interdisciplinary enhancement of students' abilities in mathematics and science. Interdisciplinarity is seen as indispensable to fostering future scientists and engineers, further economic development, advancement in environmental issues, and scientific literacy for modern citizenship. This perspective is reflected in institutional arrangements, and innovative, highly-ranked universities promote interdisciplinary learning as a key goal (e.g., [9]).

The U.S.'s recent 2026 STEM vision paper [10] highlights the importance of (1) interdisciplinary learning; (2) innovative measures of learning; and (3) the development of collaborative communities of practice in STEM. This project makes a research contribution to all three of these issues by (1) examining interdisciplinary learning, by focusing on transfer of learning from mathematics to science/engineering courses at university; (2) presenting a new assessment model for evaluating interdisciplinary transfer of learning; and (3) considering the implications of these approaches in relation to building communication and collaboration for communities of practice among STEM academics and students.

While the relationship between mathematics and science is bidirectional and complex, the transfer of learning between learning disciplines and contexts is a central factor in how STEM learning can be promoted. By measuring transfer of learning from mathematics to science, for example, we can evaluate and inform effective teaching and learning practices. Thus, in this paper we explore and attempt to measure and evaluate the transfer of mathematical learning to learning in science and engineering at an Australian university. This approach presented some challenges, but was able to observe transfer, and we hope it presents an approach that can be refined and more broadly applied in research examining transfer and interdisciplinary learning, in a range of educational settings.

2. Literature Review

Transfer refers to the application of prior learning to the subsequent learning [11,12]. This study uses this definition, due to its versatility and simplicity, although we acknowledge that researchers dispute the definitions of transfer and that a wide range of definitions exist.

Transfer has attracted educational and psychological research attention for more than 100 years (see for example, [13]). However, despite the ubiquity of transfer, there are some gaps in the literature, including a scarcity of applied studies. Although there are insightful sociocultural perspectives with qualitative accounts of transfer [14,15], to date research measuring transfer has been dominated by experimental approaches, controlled conditions, and customised research assessment tools [16–19]. This study, by contrast, provides a naturalistic measure of transfer, using standard university assessments (details to follow). By naturalistic, we mean that the transfer has been measured in natural and authentic learning contexts—in this case, classroom tests and end-of-semester exams that form part of university assessment routines.

The transfer examined in this paper can be conceived of using Barnett and Ceci's [20] taxonomy for transfer (see Table 1), which outlines the contexts possible for transfer. In terms of this study, we examine transfer of mathematical learning to science and engineering in the following contexts: knowledge domain (mathematics/science), physical context (different rooms at university), temporal context (one semester later), functional context (both clearly academic), social context (both individual), and modality (both written exams or tests). Applying this typology we can describe the transfer of learning examined here as near transfer.

Table 1. Barnett & Ceci's Taxonomy of transfer.

Context: When and Where Transferred from and to					
	Near ←——————————————————————→ **Far**				
Knowledge domain	Mouse vs. rat	Biology vs. botany	Biology vs. economics	Science vs. history	Science vs. art
Physical context	Same room at school	Different room at school	School vs. research lab	School vs. home	School vs. the beach

Table 1. *Cont.*

Context: When and Where Transferred from and to					
	Near ←——————————————————————→ **Far**				
Temporal context	Same session	Next day	Weeks later	Months later	Years later
Functional context	Both clearly academic	Both academic but one nonevaluative	Academic vs. filling in tax forms	Academic vs. informal questionnaire	Academic vs. at play
Social context	Both individual	Individual vs. pair	Individual vs. small group	Individual vs. large group	Individual vs. society
Modality	Both written, same format	Both written, multiple choice vs. essay	Book learning vs. oral exam	Lecture vs. wine testing	Lecture vs. wood carving

Note: Source: [20] (p. 621).

2.1. Transfer in Science/Engineering Educational Research

Given the acknowledged importance of transfer of learning, we argue that there has been a relative paucity of research examining it in recent decades. This is especially the case in relation to transfer between mathematics and science [21], and surprising given the substantial policy and investment focus recently directed to STEM education. However, within the relatively slim literature, several different aspects of transfer have been investigated, with some studies exploring quantitative skills in university science education (see [22–24]), and others looking at the transfer of mathematics more generally, with both quantitative and qualitative methodological approaches (see for example, [14–19]).

According to Becker and Park's [25] meta-analysis, the integration of all four STEM domains together has the largest effect size on student achievement. However, other research suggests that while learning outcomes are best predicted by prior learning in that subject area, mathematics learning is consistently the most influential interdisciplinary predictor of those outcomes [26,27].

Mathematics is foundational to science and engineering, and it has been argued that "the best 'practical' approach to mathematics is to understand it as a language for describing physical and chemical laws" [28] (p. 145). Sazhin [28] also emphasises the need for balance between practical application and in-depth understanding of mathematical equations. Given the ever-increasing dependency of science/engineering on the mathematical sciences, this need for a balanced approach takes on even more importance [7].

To date, there has been little research examining transfer of mathematics to science. This is surprising, as worldwide many college and university programs are built upon the assumption that learning in one area is transferred into the primary disciplinary learning area. For example, university "service courses" in mathematics are offered by Mathematics Department for students of non-mathematics majors, such as science and engineering. These courses cover mathematical contents (e.g., differential and integral calculus), which can be applied to learning in disciplines other than mathematics. In this, what is learned in the service courses is assumed to contribute to the learning in other disciplinary areas, such as science and engineering. Having a measure of transfer of learning from these courses to others would be a practical aid in evaluating and innovating to improve that interdisciplinary learning. There are a handful of studies measuring transfer, which can help inform development of a measure for evaluating interdisciplinary learning. We review these, and subsequently develop a measure of transfer from mathematics to science/engineering at university. Such a measure will enable us to test the assumption that mathematics learning is transferred to science/engineering, and provide a tool to understand this process more, so that it can be explored and improved.

2.2. Quantitative Measures of Transfer

There are a wide range of approaches used to quantitatively measure transfer, and much of the broader transfer literature highlights the difficulties of empirical study, including problems in demonstrating examples [29–33]. Furthermore, Potgieter et al. [34] remind us of the difficulty and the subsequent disappointment faced by researchers in demonstrating transfer, and of their tendency to assume that the mathematics has first been learnt, but this may not always be the case. Thus, attempting to measure mathematical transfer in university is a difficult but important endeavor.

Within the scant literature specifically on mathematics transfer, there are two different formulae for quantitative measurement of students' transfer of mathematical learning to science (see Table 2). First, Britton et al. [35] made an instrument consisting of two parts: mathematics components and non-mathematics components, such as physics. They used the *transfer rating* to give a relative score of transfer, based on the comparison of scores in mathematics components with scores of its application, such as in the physics components. However, this transfer rating (No. 1) had a problem: the student with the lowest mathematics score had the highest transfer rating.

Table 2. Quantitative measures of transfer.

No	Formulae to Measure Transfer
1	*Transfer rating* = z-score for first attempted component − z-score for mathematics
2	*Transfer index* = the sum of transfer scores ÷ the number of paired questions × 50

Note: The index is multiplied by 50 to convert it to an easily interpreted scale of 0 to 100. Source: [35] (p. 7); [36] (p. 434).

To overcome this problem, a second mathematics *transfer index* (No. 2) was developed by Roberts et al. [36]. Their index was calculated by summing transfer scores calculated for pairs of questions with mathematically-matched content, reflecting the degree of transfer of learning. Using this index, researchers showed that transfer was associated with Universities Admission Index and university marks in mathematics and science (with Spearman's rank correlation coefficient 0.58 ($n = 36$, $p < 0.01$), 0.62 ($n = 47$, $p < 0.01$) and 0.61 ($n = 43$, $p < 0.01$) respectively). However, like Britton et al. [35], this study also used a customised exam to assess transfer, and the sample consisted of student volunteers. The test was composed of two sections, i.e., mathematics and non-mathematics (biology). There were seven pairs of questions, consisting of mathematics and non-mathematics components, and paired questions were matched up in terms of transfer of mathematics. In other words, students were required to apply mathematical skills and knowledge from a mathematics question (i.e., exponential and logarithmic functions) into its corresponding non-mathematics question (i.e., exponential decay in the context of biology). For each pair of questions, the transfer score was calculated, and a score of 0, 1 or 2 was allocated to participants. Four patterns to allocate transfer scores were as follows:

(i) A student gave the correct answers in both sections. His or her transfer score was 2, and it is assumed that transfer of learning occurred;
(ii) A student gave the wrong answer in a mathematics question; however, he or she answered correctly on its corresponding non-mathematics question. A score of 1 was given, as it was considered that to some extent, transfer of learning had occurred;
(iii) If students gave a right answer in a mathematics section, but did not get the corresponding question in non-mathematics section, a score of 0 was awarded;
(iv) If students answered incorrectly in both questions, a score of 0 was given.

In particular, the contrast between pattern (ii) and the patterns (i) and (iii) was important, in terms of considering the degree to which transfer of learning has occurred. We adopted this approach for calculating transfer scores, although we acknowledge that it has limitations, particularly in relation to allocation of points for (ii) and (iii). Where students get questions wrong in exams, we know that this may not be due to a lack of learning, but in fact may be a product of exam conditions and performance factors in assessment. In measuring transfer, these factors are important and apply to two different sets of assessments, done under differing conditions.

Roberts et al.'s [36] transfer index has strong content validity, because mathematicians and scientists cooperated to develop the customised transfer tasks, i.e., they designed science questions specifically requiring transfer of mathematical skills and knowledge. However, this strength is offset by other challenges, in terms of limited external and ecological validity. The development of the transfer index used only mathematics and biology questions in the instrument, but the students volunteered from a range of degree programs, and some had no experience in biology. The representativeness of the self-selecting samples of student volunteers is unknown and open to question, especially as it relies on volunteers happy to sit an additional transfer exam. The customised nature of the tasks also means that its relationship with the actual teaching, learning, and assessment that go on in universities can be questioned. We extend the work of Britton, Roberts, and colleagues, by adapting and applying their approach to extant university assessment data, thus building ecological validity and exploring transfer in a full university cohort.

From the outset, it is important to note that because we use existing university exams, there are some constraints on transparency, and we are not able to provide all the details of the exam questions. Instead we provide some synthetic examples. As educational researchers, we worked closely with mathematics and science faculty staff to conduct the study, but we had no influence over the exam content, and needed to respect the confidentiality of the university exam system.

We developed two transfer-of-learning indices, and provided a path model to explain attainments in science, based upon data measuring prior attainment in mathematics, Australian Tertiary Admission Rank (ATAR, a university entrance rank), and the students' transfer of mathematics. In doing so, we tested the feasibility of assessing transfer within an applied university context, and evaluated the premise that transfer of learning contributes to overall attainment.

We asked the question: what is the measurable transfer of learning from mathematics university service courses to biology, molecular bioscience, engineering, and physics? More specifically:

1. Can transfer of mathematics learning be observed in the biology, molecular bioscience, engineering, and physics exam performances?
2. How is transfer related to overall attainment in mathematics and science/engineering courses?
3. What are the relationships between general educational attainment (university entrance rank), mathematics attainment, science/engineering attainment, and the transfer of learning between mathematics and science/engineering?

We hope that our approach can be replicated and used by academics to evaluate and improve teaching for interdisciplinary transfer. Such a measure could also be used to empirically test the assumption that interdisciplinary learning occurs within university classes, across a wide range of disciplines.

3. Materials and Methods

The data collected was related to the first-year university cohort in an Australian university. The research was complied with national standards for ethical research, and steps were taken to keep data anonymous, confidential, and secure. Data were accessed from the university databases, and included academic performance, such as final marks in mathematics, science, and engineering, and demographic information, such as age, gender, and socioeconomic status (SES).

3.1. The Mathematics and Science Courses Examined and Their Assessment

First year mathematics service courses are compulsory for students studying science or engineering in the studied university. We focused on learning shared between first-year mathematical service courses offered in the first semester, and science and engineering courses in the subsequent semester. It is true that learning in science also impacts upon learning in mathematics; however, our study did not have the scope to explore transfer in that direction. The courses and sample sizes examined in this study are listed in Table 3. With the various and multiple mathematics and science combinations possible, proliferation of estimates from small sample sizes became a challenge. This was addressed by using sequential Bonferroni corrections for the *p* values.

Table 3. Student numbers in the university mathematics and science/engineering courses examined.

	Semester 1 Course Codes & Names	MATH1901	MATH1001
Semester 2 Course Codes & Names		Differential Calculus (Advanced)	Differential Calculus
PHYS1902	Physics 1B (Advanced)	67	27
PHYS1003	Physics 1 (Regular)	28	136
ENGG1802	Engineering Mechanics	44	382
MBLG1901	Molecular Biology and Genetics (Advanced)	33	72
MBLG1001	Molecular Biology and Genetics	53	190
BIOL1902	Living Systems (Advanced)	12	20
BIOL1002	Living Systems	6	55
Total Enrolment for Combination of Two Courses		243	882

3.2. *Operationalisation of the Transfer of Mathematics*

This study looked at mathematics and science exams, as these accounted for the largest proportion of the assessment in the courses examined. The exams were conducted under routine circumstances, and the papers were marked by faculty staff.

As education researchers, we collaborated with mathematics and science academics to access the exam papers. We also worked with these disciplinary academics to explore the possibilities for transfer evident in the assessment. The first step was to find any questions involving mathematics in science and engineering exams. Next, we examined the mathematical exams, to see if they assessed the mathematics we had observed in the science exams. When the same mathematical content was found in both assessments, we highlighted these as *matched questions.* These matched questions were used for measurement for the transfer of mathematical learning. In the science and engineering exams, these turned out to be questions requiring partial derivative of functions, such as exponential or trigonometric functions. Unexpectedly, exams in molecular biology and bioscience did not contain any such questions with mathematical content. Consequently, it was not possible to examine mathematical transfer to biology and bioscience in any further analyses.

Transfer of mathematics was quantitatively measured, using an adaption of the transfer index (see Table 1) developed by Roberts et al. [36], and we expanded upon this by providing an ability-adjusted index, using the students' university entrance rank, which we called the *ATAR-adjusted transfer index* (details forthcoming). The former index was calculated as the sum of transfer scores divided by the number of paired, content-matched questions. The calculation of scores was based on the matrix shown in Table 4.

The matrix in Table 4 develops an ordinal measure of transfer, although the scale sensitivity varies depending on the number and the nature of marks allocated to particular questions. The breakdown and sensitivity to these marks was evident in the marking guidelines, and varied in different subject combinations. It is also important to remember that students may have been able to gain marks, and in turn transfer marks, by guessing at questions. Furthermore, it is also possible that mathematics demonstrated in science/engineering course exams had been learnt elsewhere, beyond the mathematics service courses. However, the immediate temporal precedence of the mathematics learning courses suggests that demonstrated abilities are more likely to have been learnt in those service courses than elsewhere. From the outset though, we need to recognise this limitation, which is intrinsic to all measures of transfer.

Table 4. Allocation of transfer scores.

Math score	1	0	1	0
Non-math score	1	1	0	0
Transfer score	2	1	0	0

Note: Source: [36].

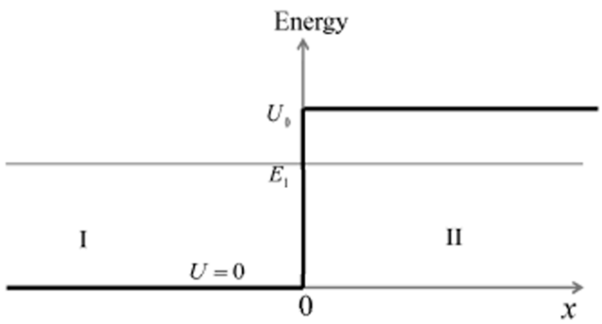
3.3. Demonstration of Calculation of the Transfer Scores and the Transfer Index

As an example, one question in mathematics and related questions in physics are shown in Figure 1. Both these questions require students to differentiate exponential functions. The actual questions used in our study are not shown here, as they are held confidential and secure by the university. We have provided here modified questions that are equivalent to the actual questions used.

Mathematics question

Let be $f(x,y) = -3x^2e^{-5iy} - 5\,y^5e^{-3ix}$. Calculate the mixed partial derivative $f_{yx}(1,2)$.

Physics question



An electron with energy E_1 is free to move in x direction. It is located in a region of space with the potential energy configuration shown in the figure. In the region $x < 0$ (denoted region I) the potential energy is $U = 0$, and in the region $x > 0$ (denoted region II) the potential energy is $U = U_0 > E_1$. This situation may be interpreted physically as describing an electron incident from the left on a step barrier potential of height U_0. The stationary state of the electron is described by the $1-D$ time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x).$$

(a) The stationary state wave function of the electron in region I may be written

$$\psi_{\mathrm{I}}(x) = Ae^{ikx} + Be^{-ikx}.$$

Using the Schrödinger equation, determine the value of k.

(b) The stationary state wave function of the electron in region II may be written

$$\psi_{\mathrm{II}}(x) = Ce^{-ax} + De^{ax}.$$

Explain why $D = 0$ is required, and using the Schrödinger equation, determine the value of a.

Figure 1. Example for calculation of transfer scores.

In order to calculate the transfer score, raw marks of mathematics and physics questions were used. In the mathematics question, a zero mark was given for an incorrect answer and one mark for a correct one. In physics, either zero, one, or two marks for each sub-question is given. Therefore, students' marks in physics varied from zero to four. In this example, the total marks in physics were divided by four, to make the full mark equal to one, which is consistent with the calculation of transfer scores in Table 4. As a consequence of this modification, the transfer scores are expressed as decimals between zero and two. For example, if a student had correct answers in math and the first question in physics, but not for the second physics question, they received one mark for mathematics and two out of four marks in physics, and the calculated transfer score is 1.5.

A transfer index (see Table 2) was calculated by summing the transfer scores (see Table 4). For example, if there were three pairs of mathematics and physics questions, and the transfer scores were 1.5, 0.8, and 1.3, respectively, the transfer index is 60, which is calculated as the sum of 1.5, 0.8, and 1.3, divided by three and multiplied by 50.

3.4. ATAR-Adjusted Transfer Index

As the importance of using multiple measures of transfer was emphasised in previous research, and previous studies have shown the association between transfer and attainment, another transfer index was also calculated, which attempts to account for student attainment using a tertiary admission rank. The Australian Tertiary Admission Rank (ATAR) is produced on the basis of two factors: academic performance in school coursework and Higher School Certificate (HSC) exam results. ATAR can be viewed as an indicator of general educational attainment, to some extent, because its calculation involves academic performance within different disciplinary subjects. Our intention was to create a transfer index that controlled for the influence of such general educational attainment, and we adopted an approach similar to that used in creating age-adjusted scores. In this case, we created an ATAR-adjusted transfer index.

When constructing a regression model to predict transfer, using ATAR as an independent variable, the residuals of the model showed variance which was not explained by the model. To create ATAR-adjusted index, the transfer indices were regressed on ATAR, and their residuals, plus a constant (in this case the group mean), were used as ATAR-adjusted scores. The adjusted index scores show the degree of transfer attained by a student, beyond that which is associated with and predicted by their general educational attainment.

3.5. Modeling the Relationships between Transfer and Attainment in Mathematics and Science/Engineering

In addition to measuring transfer, this study constructed models to explain attainment in science/engineering by mathematics attainment, transfer, ATAR, and demographic variables, such as socioeconomic status (SES), gender, and age. Path analysis was conducted with AMOS [37], using listwise deletion for missing ATAR data, which was not available for most international students. As a result, the sample sizes were reduced in these path analyses. The path analyses are based upon a full longitudinal model (see Figure 2), in which prior learning (ATAR, then first-semester Math) and background factors (age, gender, and SES) predict both transfer and attainment in physics and engineering; in addition, the model also indicates that transfer of mathematics scores may predict performance in physics and engineering.

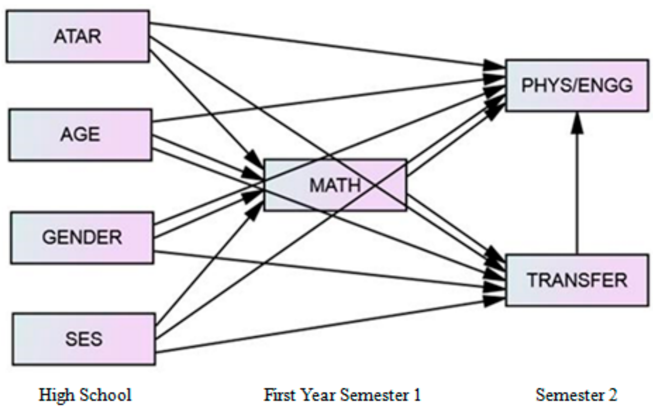


Figure 2. Full path model of the relationships between transfer and ATAR, attainments in mathematics and science/engineering, and demographic variables.

4. Results

We present our findings to address each of the three research questions.

4.1. Can Transfer of Mathematics Learning be Observed in Biology, Molecular Bioscience, Engineering and Physics Exam Performance?

In terms of observing transfer of mathematical learning, we identified matched questions only in mathematics and physics/engineering exams. Similar to matched questions shown in Figure 1, this study explored the transfer of mathematical skills and knowledge (i.e., derivative of exponential, trigonometric, logarithmic, and power functions) to electromagnetics (e.g., electromagnetic induction), quantum physics (e.g., the blackbody spectrum), and engineering mechanics (e.g., equilibrium equation). As previously mentioned, we cannot provide the detailed information relating to the exam questions, due to the confidentiality required by the university. Transfer scores were calculated for each pair of matched questions in mathematics and physics/engineering questions (see Table 4), and the transfer index was calculated on the basis of the transfer scores according to the formula (see Table 2).

Transfer index scores were calculated for a total of six mathematics and physics/engineering combinations. There were varying numbers of physics/engineering questions that required mathematics; therefore, the scores allocated for these mathematics based questions also varied (see Table 5).

Table 5. Descriptive statistics of the transfer index.

Transfer Index		Min	Max	n	Mean	Mode	SD	SE of Mean
MATH1901 Differential Calculus (Advanced)	PHYS1902 Physics 1B (Advanced)	2.5	95.0	67	48.69	26.25/92.50 *	28.29	3.46
	PHYS1003 Physics 1 (Regular)	0.0	95.0	28	47.02	43.50/68.50 *	26.30	4.97
	ENGG1802 Engineering Mechanics	22.5	100.0	44	67.28	70.00	20.93	3.16
MATH1001 Differential Calculus	PHYS1902 Physics 1B (Advanced)	7.5	85.0	27	50.49	69.17	19.41	3.74
	PHYS1003 Physics 1 (Regular)	0.0	100.0	136	30.15	0.00	28.76	2.47
	ENGG1802 Engineering Mechanics	0.0	100.0	382	74.79	77.50	18.18	0.93

Note: * Bimodal distributions.

Table 5 shows that some transfer was observed in all six math/physics and math/engineering combinations. Overall, individual scores of the transfer index ranged from zero (no transfer) to 100 (fully successful transfer).

The observed means varied according to each combination of mathematics and physics/engineering courses. These means are not directly comparable, as they are developed from assessments which varied in format (multiple-choice to 10-point extended response), number of opportunities for students to transfer their learning (differed according to level), and also difficulty of the assessments (higher in advanced courses, but also varying across disciplines). However, the means do provide a baseline assessment, which reflects the opportunities for, and assessment of, interdisciplinary learning in each course.

4.2. How Is Transfer Related to Overall Attainment in Mathematics and Physics/Engineering Courses?

There were four findings related to association (see Table 6). First, bivariate relationships are shown in Table 6. Demographic variables *gender*, *SES* and *age* were not significant in bivariate correlations, and are not reported in this table.

Unsurprisingly, the majority of transfer indices were significantly correlated with attainment in physics/engineering, because these subjects are considered mathematically-cognate disciplines. The significant relationships between transfer and ATAR suggests that transfer and general educational attainment are also intertwined. By contrast, the ATAR-adjusted transfer index had non-significant correlation with attainment in mathematics, except for a significant correlation with a combination of normal mathematics MATH1001 and engineering. This might suggest the importance of general educational attainment for transfer of learning between mathematics and physics/engineering, because when we controlled for the general educational attainment, the significance of the relationship receded. Alternatively, this may be due to the small sample size and reduced statistical power, as ATAR data was not available for the international students in the class. Furthermore, the small sample of students undertaking these subject combinations prohibited further analyses, including the multivariate path modelling.

Table 6. Spearman's rank correlation coefficient between the transfer indices and final marks in mathematics, science, and ATAR.

Transfer Indices		MATH Final Marks	n	PHYS/ENGG Final Marks	n	ATAR	n
MATH1001 (Norm) & PHYS1003(Reg)	TI	0.477 *	136	0.447 *	136	0.423 *	100
	ATAR Adj TI	0.186	100	0.147	100		
MATH1001 (Norm) & PHYS1902 (Adv)	TI	0.759 *	27	0.753 *	27	0.355	22
	ATAR Adj TI	0.483	22	0.619 *	22		
MATH1001 (Norm) & ENGG1802	TI	0.505 *	382	0.711 *	382	0.361 *	255
	ATAR Adj TI	0.239 *	255	0.479 *	255		
MATH1901 (Adv) & PHYS1003 (Reg)	TI	0.495	28	0.706 *	28	0.494	24
	ATAR Adj TI	0.032	24	0.489	24		
MATH1901 (Adv) & PHYS1902 (Adv)	TI	0.497 *	67	0.537 *	67	0.438 *	57
	ATAR Adj TI	0.368	57	0.474 *	57		
MATH1901(Adv) & ENGG1802	TI	0.488 *	44	0.541 *	44	0.315	39
	ATAR Adj TI	0.400	39	0.500 *	39		

Note: * Significance according to Sequential Bonferroni Correction. Transfer Index (TI); ATAR-Adjusted Transfer Index (ATAR Adj TI).

4.3. What Are the Relationships between General Educational Attainment (University Entrance Rank), Mathematics Attainment, Physics/Engineering Attainment, and the Transfer of Learning between Mathematics and Physics/Engineering?

Three path models (see Figure 3) examine how the transfer of learning relates to performance in two physics units of study, (a) regular physics and (b) advanced physics, as well as in (c) an engineering unit, and how that performance interacts with the other variables. These units were the only ones in which transfer was able to be calculated, and which had a substantial sample size suitable for modelling ((a) $n = 57$, (b) $n = 100$, and (c) $n = 255$, respectively).

The models show direct effects of independent variables on dependent variables (e.g., ATAR to PHY1003), as well as indirect effects of independent variables on those dependent variables (e.g., ATAR to MATH1001 to PHY1003). Path coefficients shown in Figure 3 are standardised beta coefficients, showing the direct effect of one variable on another in the path model, and comparing the strength of the effect of each individual independent variable to the dependent variable. The greater the absolute value of the beta coefficient, the stronger the effect.

Model fit was examined using a range of indices (chi-square, RMSEA, CFI and GFI, see [38]) and these indicated that the models were a good fit. Chi-squared values for the three models all show a significant fit ($p > 0.05$). In addition, the other indices for the models ranged from <0.000 to 0.047 for RMSEA, 0.993 to 1.000 for CFI, and 0.960 to 0.992 for GFI. The benchmark figures are <0.050 for RMSEA and >0.950 for CFI and GFI. In addition, due to non-normality, standard errors for regression weights were calculated using bootstrap. Accordingly, p values for the estimation of regression weights were recalculated based on the new standard errors, including the $p = 0.056$ path shown in Figure 3.

Figure 3 suggests three substantive findings. First, mathematics is central for learning in these courses; attainment in mathematics is highly related to attainment in physics/engineering. With every increase of one standard deviation in mathematics, final marks in engineering rose by 0.42 standard deviations, assuming that the other variables were held constant. This is consistent with research showing how mathematics is predictive of science attainment across disciplines [27], and is well established in more specific models of attainment in physics [39,40].

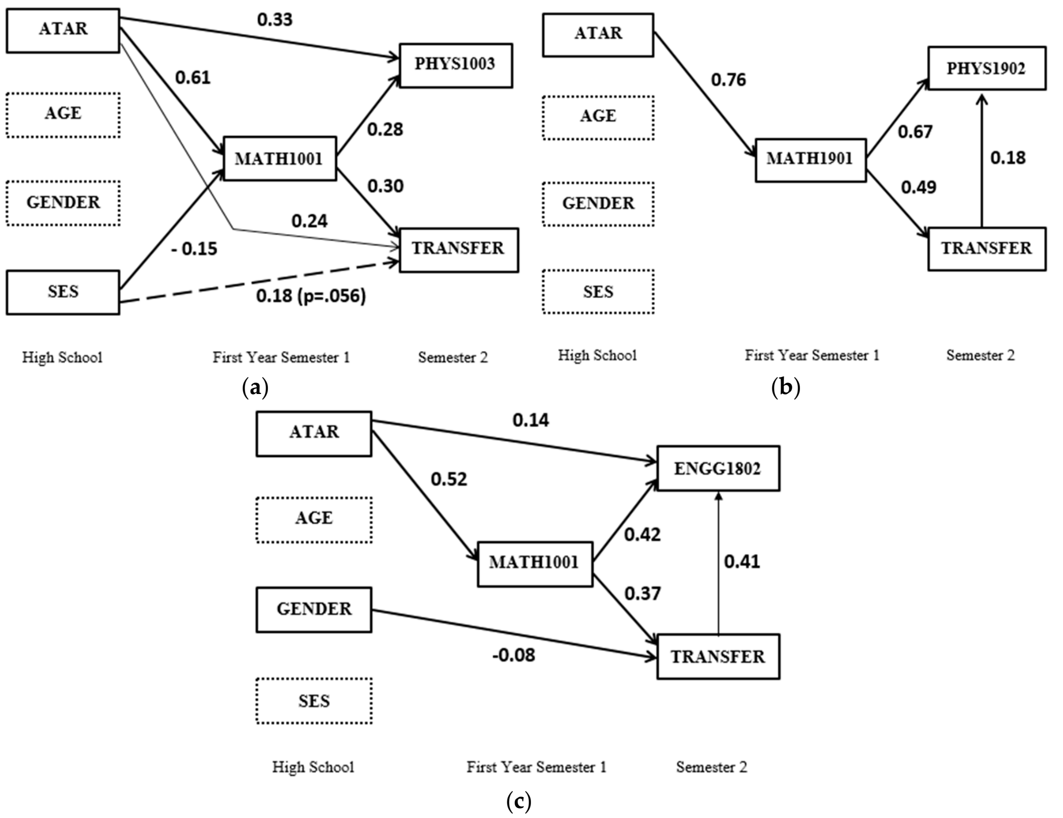


Figure 3. Path models showing the relationship among transfer, ATAR, and attainments in mathematics and physics/engineering. (**a**) normal mathematics and regular physics, (**b**) advanced mathematics and advanced physics, (**c**) normal mathematics and engineering

Second, the ATAR (university rank) is also predictive of mathematics and physics/engineering science. This is consistent with the literature, which outlines mathematics' particularly close relationship with general ability [41]. We know, for example, that mathematics attainment is the most predictive school discipline upon *g* factor intelligence [42]. However the relationship between ATAR and physics/engineering appears to vary with subject combinations—in particular, the level of the course, which is also related to cohort ability.

Third, there are significant direct-transfer effects observed in the advanced physics (PHYS1902) and engineering (ENGG1802) cohorts. For example, when transfer increased by one standard deviation, final marks in advanced physics increased by a 0.67 standard deviation. However, in evaluating this effect, we must be aware that the transfer measure was derived from the physics assessment performance, so the measures are not entirely independent; although, given that they are based upon a very small fraction of the overall physics assessment, this relationship shows a surprisingly large magnitude. Transfer shows no direct effect upon attainment in the regular physics (PHYS1003) cohort. This may be a true null finding for regular physics, or the results of limited statistical power in this much smaller sub sample (n = 57).

5. Discussion

We found evidence of transfer in pre-existing university assessments. However, as transfer, learning, and ability are not independent, the relationships between them are complex. At the outset we discuss the evidence for transfer occurrence, as this has been historically controversial in transfer research literature (see, for example, [29–33]), and little research has examined the transfer of mathematics to science. We also discuss the nature of the relationships between transfer and educational attainment, and reflect on the methodological limitations and insights evident in our study.

5.1. Transfer from Mathematics to Science

We were not able to find assessment items that enabled demonstration of transfer of mathematical learning into biology and bioscience units of study. While assessment in these units did draw on some mathematical work, we could find no questions aligning with that assessed in the mathematical services courses. It is possible that content from those service courses was applied and assessed elsewhere in these units, such as lab and large project work, but it was beyond the scope of the study to confirm this.

We found a range in transfer, with more transfer evident between the higher-ability cohorts (advanced mathematics and physics students), but with a range of transfer abilities seen in all the subject mathematics/engineering/physics combinations. However, the nature of the transfer index makes it difficult to compare between the subject combination cohorts, and also difficult to confidently comment on the meaningfulness of the magnitude of the transfer. It should be noted that in some cases, the measures of transfer might not be actual transfer, but rather coincidental, independent learning of a task separately in both mathematics and science.

In particular, the direct effects of transfer and mathematics on later learning in engineering that we measured are substantive (beta 0.41 and 0.42) and larger than the effect of general educational attainment, the ATAR (beta 0.14). However, to compare these effects confidently would require the transfer index to be measured and replicated in research on multiple courses, and with larger sample sizes. The fact that transfer has been observed gives encouragement to this endeavor. Future studies are needed, where exams provide more consistent opportunities that draw on mathematics transfer, and these would need to be marked with a range of marks sensitive to measurement of that transfer.

Our findings are consistent with the studies of Roberts et al. [36], who demonstrated mathematics transfer in volunteer students with customised science tests. To our knowledge, our study is the first to demonstrate the transfer of mathematics learning using pre-existing assessments, and it provides a model of how transfer might be measured and analysed for teaching and learning purposes.

While evidence of transfer in physics and engineering was found in this study, the opportunity to examine transfer is dependent on the level of alignment between the curriculum and assessment of the mathematics and science/engineering courses. Although there was a high level of mathematics content in the science/engineering course assessments we examined, only a small proportion of this content was covered in the mathematics courses of the previous semester. Some may have been related to curriculum in concurrent second-semester mathematics courses, and some of it might have relied on coverage in high school. The alignment between these courses is not fully understood, and is perhaps an area where we could improve opportunities for transfer and promote interdisciplinary learning. Scientists, engineers, and mathematicians could share their lectures and teaching materials, discuss the nexus between mathematics and science/engineering, and highlight the topics for transfer in classes. It may also be good to provide opportunities to discuss transfer with students.

5.2. Relationships between Transfer and Educational Attainment

The limitations of the study are discussed shortly; nevertheless, we make some comment on the relationships between transfer and student attainment and prior learning (as reflected in ATAR rank). The transfer we observed is correlated with student attainment in mathematics (r = 0.48 to 0.76), science (r = 0.45 to 0.75), and more generally with ATAR (r = 0.36 to 0.44). These correlations are large, in educational-research terms, and suggest that between 20 and 57 per cent of the variance in physics attainment is explained by the observable transfer, and that between 29 and 50 per cent of variance in engineering attainment is explained by transfer. These positive correlations are substantial, although we need to consider that these measures are not entirely independent. Transfer is calculated from a small number of questions within the exams, which make a contribution (along with assignments, lab reports, etc.) to the final marks. We calculated the transfer items' contribution to final marks, and this turned out to be trivial (<5%).

Similarly, Roberts et al. [36] had found transfer index scores were correlated with university science and mathematics attainment averaged across courses ($r = 0.61$, $n = 43$, $p < 0.01$; and $r = 0.62$, $n = 47$, $p < 0.01$ respectively). In their study, the assessment of transfer was customized, and did not draw on the exam attainments. Thus, in an earlier assessment of the relationship, approximately 37 per cent of the mathematics and science attainment was explained by transfer. Such proportions are very large in terms of educational effects; we know, for example, that the much-discussed influence of socioeconomic status explains between only 12 and 17 per cent of the variance in Australian 15-year-olds' mathematics performance [43].

The fact that transfer is also correlated with a more general measure of student attainment, the ATAR (student university entrance rank), is also interesting. This is consistent with the notion that transfer is central to all learning. Haskell reminds us that, "all learning is transfer of learning. In short, virtually all learning involves *carrying over* previous learning to new situations" [44] (p. 24). Roberts et al. [36] also found transfer to be correlated with university entrance scores ($r = 0.58$, $n = 36$, $p < 0.01$). Our finding confirms that the relationship between mathematics and physics/engineering is not explained as a composite of disciplinary abilities plus transfer of learning; rather, these two factors are deeply intertwined.

Given that the transfer of learning and academic attainment are so intertwined, it is reasonable to assume that educators and students consider transfer in how they plan and prepare teaching and learning. Indeed, in many schools and universities, this point is so assumed it is often unspoken; mathematical calculations are integrated into science and mathematics service courses, which serve a range of STEM faculties. Further research is needed to examine these assumptions across a range of STEM degree programs. Even in the highly cognate disciplines of mathematics and physics/engineering, we found opportunities to demonstrate that transfer varied according to the curriculum alignment and the timing of curriculum delivery. It could then also be argued that there is opportunity for universities to provide more planning and design in relation to promoting transfer. The development of effective transfer skills may be enhanced by providing additional opportunities to utilise the learning from mathematics service courses. Greater communication and cooperation between mathematics and science/engineering faculties to strengthen alignment between learning objectives and assessments could also promote more effective interdisciplinary learning.

5.3. Strengths, Limitations, and Insights for Future Transfer Research

We found the alignment between mathematics and the curriculum and assessment in science/engineering is substantial, and it enabled us to demonstrate transfer of learning. However, this alignment for transfer of learning may vary from course to course, both across institutions and across the various discipline areas of STEM. We think that further science education research has the potential to provide insight into interdisciplinary STEM learning, particularly relating to mathematics. Physics and engineering education may well be able to provide examples highlighting best practices for promoting transfer of mathematical learning.

The measurement method used here presents a model of how transfer can be assessed within existing educational assessments. More deliberate cooperation between mathematics and science/engineering academics in the preparation of assessment would enable more opportunities for transfer and easier calculation of transfer scores. This, then, would allow for productive reflection and conversations on how teaching and learning can be designed to promote transfer.

We argue that the complexity of transfer as a concept requires multiple observational and measurement approaches. In this study, we calculated two indices for transfer—the first based on Roberts et al. [30], and the second a variation on the first, where the transfer index was adjusted for student prior learning through ATAR. With the second, adjusted index, we were able to examine how transfer may or may not occur, somewhat independently of general educational attainment. Other approaches are needed to measure transfer. We advocate a pluralist approach to understanding transfer. In the larger project [45], we do this by examining teachers' perspectives on transfer, and also students' *think aloud* [46] accounts of how they work through transfer tasks. We would encourage other researchers to take creative and innovative approaches to observing and measuring transfer.

While our research is very applied and has practical benefits, the validity of the transfer measure should be considered. Unfortunately, as there is no gold standard measure for transfer to evaluate against, it was not possible to check for concurrent validity. We rely on the logic of the matched questions, and this provides strong content validity.

Unfortunately, it was not possible to confidently compare levels of transfer between the advanced- and normal-level courses in mathematics and physics/engineering. This was due to a lack of common questions in exams in these courses. So when we consider transfer, each course must be examined as separate and independent.

Our findings in relation to the path model must be treated with caution. The sample used here was relatively small; although we did find good model fit, the non-significance of some pathways may be due to low statistical power. It must be remembered that our data are from samples within a single university, and may not be representative of students across classes and institutions. Furthermore, no measure of delayed or long term effect of transfer of learning, beyond the six-month interval, was examined. Replication studies are needed, and could look at transfer between a range of disciplines and courses, and across a longer time period. The primary aim of the present study was to test the practicality and validity of the approach.

5.4. Implications for Teaching and Learning Science

If we are really interested in interdisciplinary learning in STEM, more concerted efforts for communication and coordination academics in planning and designing curriculum, classes, and assessments, may help promote transfer and the student development of transfer skills. For example, laboratory work and mathematical analysis required in science/engineering could be collaboratively planned and assessed, by both mathematics and science/engineering departments. We have established that transfer can be observed and measured in an applied and naturalistic way; with additional coordination in planning classes and assessment, transfer could be researched and evaluated more within universities. Further, application of our approach could improve upon our efforts here, and set up ongoing monitoring and evaluation of effectiveness of teaching and learning for transfer. We believe that, given the current mandates to develop strong STEM education and interdisciplinary learning, such approaches hold considerable potential. The highly cognate relationship between mathematics and science/engineering education, and the demonstrated transfer of learning between them, suggests that further research and identification of best practices in this area could provide insights for STEM more generally.

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