

## Article

# How Macro Transactions Describe the Evolution and Fluctuation of Financial Variables

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**Abstract:** The description of the dynamics and fluctuations of macro variables remains one of the most exciting problems of financial economics. This paper models macro variables via the description of transactions between agents. We use risk ratings  $x$  of agents as their coordinates in the economic space. Transactions like buy–sell, investment, credits, etc., between agents change their extensive financial and economic variables. Aggregates of transactions between all agents with risk ratings  $x$  and  $y$  define the macro transactions between points  $x$  and  $y$ . Macro transactions determine the evolution of macro variables. Interactions between different transactions outline their dynamics and fluctuations. We model macro transactions and the interactions between them by economic hydrodynamic-like equations in the economic space. As an example, for simple model interactions between credit–loans and loans–repayment transactions we derive economic hydrodynamic-like equations and wave equations for near perturbations of macro transactions and study simple wave solutions and their consequences. Waves of macro transactions in the economic space propagate from high to low risk agents or vice versa and define the fluctuations of macro financial variables. The existence and diversity of waves and fluctuations of macro transactions in simple models clarifies the importance of wave processes for macro financial modeling and forecasting.

**Keywords:** macro transactions; macro finance; financial waves; risk ratings; economic space

**JEL Classification:** C00; E00; F00; G00

## 1. Introduction

The origin and description of the evolution and fluctuations of economic and financial variables outline the core problems of financial economics modeling. Different types of economic and financial modeling and descriptions of financial fluctuations for years remain matters of intensive research (Schumpeter 1939; Kydland and Prescott 1980, 1982; Hall 1988; King et al. 1991; Stiglitz 1999; Rebelo 2005; Kim 2009; Quadrini 2011; Brunnermeier and Sannikov 2014, 2016; Bauer and Rudebusch 2017; Cochrane 2016). These subjects are very complex and their solutions are far from complete. Studies of financial fluctuations have the goal of explaining the origin and relations between fluctuations of different variables, the origin of market volatility and its influence on returns and asset pricing models. The influence of risks, macroeconomic factors and financial policy on the fluctuation of financial variables determines the core problems of these studies. These problems affect almost all major subjects of finance, such as assets pricing (Sharpe 1964; Campbell 1999; Constantinides et al. 2013; Brunnermeier and Sannikov 2016; Cochrane 2016), volatility modeling (King et al. 1991; Justiniano and Primiceri 2008; Tauchen 2011), market regulation (Bator 1958; Fama 1969; Christiano et al. 2010; Mishkin and Serletis 2011), risk management (Alexander 1999; Engle 2003; McNeil et al. 2005; Diebold 2012), economic and time-series analysis and forecasting (Stock and Watson 1988; Montgomery et al. 2015), and business cycles and financial policy (Schumpeter 1939; Scheinkman 1939; Kydland and Prescott 1980, 1982; Lucas 1980, 1995; Black 1982; Scheinkman 1984;

[Zarnowitz 1992](#); [Rebelo 2005](#); [Kiyotaki 2011](#); [Jorda et al. 2016](#)). These problems cover almost all areas of economic and financial theory and the amount of publications is almost infinite.

Current models of financial fluctuations are mostly based on the description of mutual dependences between the time-series of variables. For example, [Kydland and Prescott \(1982\)](#) state: “A thesis of this essay is that the assumption of multiple-periodic construction is crucial for explaining aggregate fluctuations. A general equilibrium model is developed and fitted to U.S. quarterly data for the post-war period. The co-movements of the fluctuations for the fitted model are quantitatively consistent with the corresponding co-movements for U.S. data.”

Our contribution to the description of economic and financial fluctuations is as follows: first, we propose that the complexities of financial and economic processes require a corresponding complexity of financial modeling, but the intricacy of the description of macro financial time-series is not sufficient for adequate financial modeling. Thus, we propose that macro variables should depend not only on the time but also on certain the economic space variables. As such economic variables, we suggest the risk ratings of economic agents. We explain how risk assessments of economic agents allow defining macro financial variables as functions of time and coordinates in the economic space ([Olkhov 2016a, 2016b, 2017a](#)). Second, we argue that evolutions of extensive financial and economic variables are governed by transactions between agents. We explain why and how transactions between agents can be described in the economic space of risk ratings by economic hydrodynamic-like equations (Sections 3 and 4). Third, we show that the evolution and interactions between macro transactions can induce the generation and propagation of macro transaction waves on the economic space. Such waves can cause waves and time fluctuations of macro variables. Thus, the origins of and interactions between the time fluctuations of macro financial variables can be consequences of hidden wave processes in the economic space. We argue that identical time fluctuations of macro financial variables can be induced by different transaction waves. Therefore different financial policy and management is required for an adequate response to the observed financial fluctuations. To provide a reasonable management, financial and economic policy market, financial authorities should track the state and dynamics of economic and financial variables in the economic space of risk ratings. That requires the development of a risk assessment methodology and the enhancement of available data. Up now there are no data sufficient to develop econometrics models in the economic space. Thus, our model remains purely theoretical and cannot be verified by existing econometric data. However no fundamental obstacles exist that prohibit the required development of risk assessments. We assume that the complexity of the proposed model corresponds to an internal complexity of macro financial processes. Below, we explain the main issues with more details. We develop a model of the dynamics and fluctuations of economic and financial variables that is based on description of transactions between agents. Our approach is as follows: all financial or economic macro variables like investment, assets, credits, loans, etc., are determined by aggregate amounts of the corresponding variables of agents. For example, the sum (without doubling) of investment of all agents defines the macro investment of the entire economics. Transactions between agents change their variables. Transactions from agent *A* that provide credit to agent *B* changes the total credits provided by agent *A* and total loans received by agent *B*. The description of the transaction dynamics between agents can model the evolution of the agent’s variables and hence the model dynamics and fluctuations of macro variables of the entire macro finance. The rest of the paper explains the above considerations, develops a quantitative model of transaction dynamics and fluctuations and explains how transactions define the evolution and fluctuation of macro variables.

We propose that widespread macro financial fluctuations are treated as time oscillations of macro variables reflecting the tip of a concealed economic and financial wave process. To describe economic and financial waves one should model macro variables in a certain space. To do that we propose to use the risk ratings of economic agents as their coordinates in the economic space ([Olkhov 2016a, 2016b, 2017a](#)), which in turn allows us to describe a wide range of economic and financial wave processes. All macro variables are composed of the corresponding variables of economic agents.

For example, macro assets and investment are defined by total amounts of an agent's assets and investment. Aggregates of agent's variables, like assets, investment, credits or loans near point  $x$  in the economic space define assets, investment, credits or loans as functions of time  $t$  and coordinates  $x$  on the economic space. The integral of such variables over the economic space defines the corresponding macro variable of the entire economics as a function of time  $t$  only. For example, macro assets  $A(t)$  equals the "correct" (without doubling) sum of the assets of all economic agents of the entire economics and that equals the integral of assets as the function  $A(t,x)$  by  $dx$  over the economic space. To describe the dynamics and mutual dependence between variables, like investment or assets, profits or loans, credits or loans let us take into account economic and financial transactions between agents. Transactions between agent  $A$  and agent  $B$  describe change of investment from  $A$  to  $B$ , the growth or decline of the profits of  $A$  received from  $B$ , the growth of credit provided from  $A$  to  $B$ , etc. Transactions between agents define the evolution of all economic and financial variables. Thus, fluctuations of transactions between agents  $A$  and  $B$  should induce fluctuations of the macro variables. For example, fluctuations of the transactions that describe the allocation of credits from  $A$  to  $B$  can describe the fluctuation of total loans received by agent  $B$  and the fluctuation of total credits issued by agent  $A$ .

Economic agent  $A$  at point  $x$  (with risk rating  $x$ ) can buy or sell assets, invest or provide credits to any economic agent  $B$  at point  $y$  (with risk rating  $y$ ). Transactions between agents at different points  $x$  and  $y$  on the economic space describe complex relations between macro variables. For example, transactions from agent  $A$  at point  $x$  can describe investment into the assets of agents at point  $y$ . The aggregation of the transactions from agent  $A$  at  $x$  to all agents at point  $y$  defines investment transactions from agent  $A$  at  $x$  into assets at point  $y$  on the economic space. The aggregation of the investment transactions of all agents at point  $x$  to all agents at point  $y$  defines macro investment transactions from point  $x$  into point  $y$  as function of time  $t$  and two variables  $(x,y)$ . Aggregates of the economic and financial transactions between agents with coordinates  $x$  and  $y$  in the economic space define the corresponding macro transactions between points  $x$  and  $y$ . Macro transactions between points  $x$  and  $y$  of the economic space define the evolution and fluctuation of macro variables at points  $x$  and  $y$ . This paper shows that the waves and fluctuations of the transaction's disturbances in the economic space can induce waves and fluctuations of the macro variables. In other words, we propose that the observed fluctuations of macro variables like assets or investment, credits or stock prices, etc., can be induced by hidden dynamics and waves of macro transactions in the economic space. Thus, modeling transactions might be very important for the description of macro financial fluctuations.

The evolution of the macro transactions in the economic space depends on interactions between them. Indeed, transactions of a particular agent depend on information about the transactions of other agents. The aggregation of transactions of all agents at point  $x$  to all agents at point  $y$  leads to certain interactions of macro transactions as functions of  $(x, y)$ . Below, we study simple models that describe how a particular transaction  $F$  depends on other transactions  $G_1, G_2, \dots, G_n$ . As we state that the evolution of all macro variables is determined by corresponding transactions, it is reasonable to assume that any particular transaction  $F$  depends only on other transactions. Thus, interactions between different transactions define models of possible economic evolution. To describe the evolution of macro transactions and interactions between them we derive hydrodynamic-like equations. These hydrodynamic-like equations allow us to derive wave equations for perturbations of macro transactions and to obtain simple wave solutions. The generation and propagation of macro transaction waves uncovers hidden processes that can induce waves and time fluctuations of macro variables. For example, investment transaction waves can induce time fluctuations in the investment of entire economics. Thus, waves of transactions in the economic space can be an origin of fluctuations of macro variables and in particular can govern the fluctuations of asset prices. The diversity of macro transactions and numerous wave equations in simple models uncovers the complexity and importance of wave generation, propagation and interaction for financial economics.

It is very interesting that descriptions of transactions between agents as a ground for modeling economic evolution are already widespread in economics. Similar models were developed eighty

years ago by Leontief (Leontief 1936, 1973; Miller and Blair 2009; Horowitz and Planting 2009) and are well known as input–output analysis or inter-industry tables frameworks. Leontief allocated economic agents by industry sectors and described transactions as an exchange of resources between industries in such a way that the output from one industry becomes the input to another. The difference between Leontief’s inter-industry tables framework and our approach is very simple. We replace Leontief’s allocation of agents by industries with the allocation of agents by their risk ratings  $x$  as coordinates in the economic space. That allows us to replace transactions between industries by transactions between points  $x$  and  $y$  in the economic space. The modifications are small but the advantages are high. The main issues are as follows: Leontief’s partition of the entire economy by industry sector does not define any linear space. Measured input–output coefficients between industries define rigid relations between industries and that prevent adequate economic evolution modeling. Contrary to Leontief’s approach, we describe economics in a linear economic space. The allocation of agents by their risk rating  $x$  as coordinates lets us define the time evolution of macro transactions and macro variables by methods similar to mathematical physics. That allows us to describe the time evolution of transactions and hence the timing evolution of economic and financial variables. This “simple” replacement of agent distribution by industry with distribution by risk ratings  $x$  in the economic space makes it possible to model macro finance in a completely different manner.

The rest of the paper is organized as follows. In Section 2 we discuss the model setup and present definitions of the economic space, economic and financial variables and macro transactions. In Section 3 we describe the dynamics of macro transactions via hydrodynamic-like equations. To demonstrate advantages of our approach, we present simple model interactions between credit–loans and loans–repayment transactions. In Section 4—for hydrodynamic-like equations that describe model interactions between credit–loans and loans–repayment transactions—we obtain wave equations for macro transaction disturbances and study a simple example of the fluctuation of macro variables that were induced by transaction waves. Further discussion and conclusions are provided in Section 5.

## 2. Model Setup

In this section, we define the economic space and explain the transition from a description of economic and financial variables of agents at point  $x$  to a description of macro variables as functions of the  $x$  coordinates in the economic space. Then we introduce transactions between agents at points  $x$  and  $y$  in the economic space. Further, we explain the transition from the description of transactions between two agents at points  $x$  and  $y$  to the description of macro transactions between points  $x$  and  $y$ . Our approach is based on widespread notions of economic agents and risk ratings. We do not study behavioral and decisions-making descriptions of agents (Simon 1959; Cramer et al. 2004; Tesfatsion and Judd 2005) but propose to regard agents as simple units of macro finance each described by numerous variables like investment and assets, capital and credits, profits and demand, etc. Let us replace the question: “Why do agents take certain decisions?” with a different one: “How do the agent’s variables and interactions between them describe macro finance?” Numerous economic agents can be treated like a kind of “economic gas”. We believe that no direct parallels between modeling macro finance as a multi-agent system and description of physical multi-particle systems exist. Nevertheless, certain resemblances and parallels between financial economics treated as multi agent systems and multi-particle systems in physics allows us to develop a model of macro finance in a new manner using the language of theoretical physics.

### 2.1. Definition of the Economic Space

Let us assume that it is possible to make a risk rating assessments for all agents of the entire macro financial system. Let us treat the risk rating  $x$  of each agent as its coordinates in the economic space. The international rating agencies (Fitch 2006; S&P 2012; Moody’s 2007) estimate the risk ratings of huge corporations and banks. Risk ratings take values of risk grades and are noted as AAA, BB, C and so on. Let us treat risk grades like AAA, BB, C as points  $x_1, x_2, \dots, x_m$  of a discreet space. We propose that risk

assessment methodologies can be extended to estimate risk ratings for all agents of the entire economy. That will distribute all agents over points of finite discrete space determined by a set of risk grades  $x_1, x_2, \dots, x_m$ . Many risks impact macro finance. Let us regard the grades of single risk as points of one-dimensional space and simultaneous assessments of  $n$  different risks as the agent's coordinates in  $n$ -dimensional space. Let us assume that risk assessment methodologies can be generalized in such a way that risk grades can take continuous values and the risk grades of  $n$  different risks establish the space  $R^n$ .

We define (Olkhov 2016a, 2016b, 2017a, 2017b, 2017c) the economic space as any mathematical space that maps agents by their risk rating  $x$  as space coordinates. The number  $n$  of risks ratings measured simultaneously determines the dimension  $n$  of the economic space. Let us put positive a direction along each risk axis as the risk growth direction. Let us assume that econometric data provide sufficient information about the risk ratings and variables of each agent. These assumptions require significant development of current econometrics and statistics. The quality, accuracy and granularity of the current U.S. National Income and Product Accounts system (Fox et al. 2014) give us confidence that all these problems can be solved.

A lot of economic and financial risks have an impact on agents. It is impossible to take into account all possible risks. To develop a reasonable model of financial economics, one should select the major risks and neglect minor risks. The definition of the economic space  $R^n$  requires the choice of  $n$  risks with a major impact on macro finance and its agents. These  $n$  major risks define the initial state of the economic space  $R^n$ . The selection of the most valuable risks requires procedures that allow measuring and comparing the influence of different risks on the state and evolution of financial economics. The assessment and comparison of different risks and their influence on economics and finance establishes tough problems, and such models should be developed. Risk assessment methodologies and procedures and the comparison of risk influence on the performance of agents can establish procedures similar to measurement theory in physics and improve financial modeling and forecasting.

Economic and financial risks have a random nature and can unexpectedly arise and then vanish. Thus, some current risks that define the initial representation of the economic space  $R^n$  can accidentally disappear and other risks may come into play. Economic and financial forecasting in timeframe  $T$  requires the prediction of  $m$  main risks that may play a major role in a particular timeframe and may define the economic space  $R^m$ . Such a set of  $m$  risks determines a target state of the economic space  $R^m$ . To describe the transition from the initial economic space  $R^n$  to a target economic space  $R^m$ , one should describe how the action of an initial set of  $n$  risks dissipates and how the action of new  $m$  risks develop.

Current general equilibrium models (Starr 2011; Cardenete et al. 2012) describe relations between macro variables—such as investment and assets, GDP and labor, credits and debts, model asset prices and their fluctuations—each treated as a function of time  $t$ . The introduction of the economic space provides ground for the description of macro variables as functions of time  $t$  and coordinates  $x$ . This small step uncovers a hidden complexity of economic and financial processes and provides a fresh view of financial modeling.

## 2.2. Transition from Agent's Variables to Macro Variables in the Economic Space

This subsection explains the transition from the description of the agent's variables to the description of the macro variables as functions of time  $t$  and coordinates  $x$  in the economic space (Olkhov 2016a, 2016b, 2017a, 2017c). This transition has parallels to the transition from the description of multi-particle system in physics that takes into account the granularity of separate particles to the hydrodynamic approximation of continuous media. Indeed, the risk rating  $x$  of separate agents changed under the action of economic and financial processes and transactions between agents. Thus, economic agents can move in the economic space like economic particles of "economic gas" and this motion induces changes in the agent's variables. For example, the random motion of the agent in the economic space due to random changes of the agent's risk ratings can induce random changes



of the agent's investment and assets, consumption and demand, etc. Let us describe the agents and their variables by probability distributions. Averaging an agent's economic and financial variables by probability distributions allows us to describe financial economics as "*financial fluids*". In such an approximation we neglect the granularity of variables like assets or capital that belong to separate agents at point  $x$  and describe assets or capital as a function of  $x$  in the economic space similarly to "*assets fluids*" or "*capital fluids*" in hydrodynamics.

Below, we present the definition of macro variables as functions of coordinates in the economic space (Olkhov 2016a, 2016b, 2017a). For brevity we further call economic agents economic particles or e-particles and the economic space the e-space. Each e-particle has many variables, like assets and debts, investment and savings, credits and loans, etc. Let us call e-particles "independent" if the sum of extensive (additive) variables of any group of e-particles equals same variable for the entire group. For example, the sum of assets of  $n$  e-particles equals the assets of the entire group. Let us assume that all e-particles are "independent" and that any extensive macro variable equals the sum of the corresponding variables of the agents. The sum of the assets of e-particles with coordinates  $x$  in the e-space define assets as a function of time  $t$  and  $x$ . The integral of the assets by  $dx$  over the e-space equals the assets of the entire economy as a function of time  $t$  only. The coordinates of the e-particles represent their risk ratings and hence they are under random motion in the e-space. Thus the sum of the assets of the e-particles near point  $x$  is also random. To obtain regular values for macro variables like the assets at point  $x$ , let us average the assets at point  $x$  by probability distribution  $f$ . Distribution  $f$  defines the probability to observe  $N(x)$  e-particles with the value of assets equal to  $a_1, \dots, a_{N(x)}$ . That determines a density of assets at point  $x$  in the e-space (Equation (3) below). Macro assets as a function of time  $t$  and coordinate  $x$  behave like an assets fluid. To describe the motion of the assets fluid (Olkhov 2016a, 2016b, 2017a), we need to define the velocity of such a fluid. Let us mention that the velocities of e-particles are not additive variables and their sum does not define the velocity of asset motion. To define the velocity of assets fluids correctly one should define "*asset impulses*" as the product of assets  $a_j$  of a particular  $j$ -e-particle and its velocity  $v_j$  (Equation (4) below). Such "*asset impulses*"  $a_j v_j$  are additive variables and the sum of "*asset impulses*" can be averaged by a similar probability distribution  $f$ . The densities of the assets and densities of the asset impulses permit the definition of the velocities of "*asset fluids*" (Equation (5) below). Different "*financial fluids*" can flow with different velocities. For example, the flow of capital in the e-space can have a velocity higher than the flow of assets, nevertheless they are determined by the motion of the same e-particles. Let us present these issues in a more formal way.

We assume that each e-particle in the e-space  $R^n$  at moment  $t$  is described by extensive variables  $(u_1, \dots, u_l)$ . Extensive variables are additive and admit averaging by probability distributions. Intensive variables, like prices or interest rates, cannot be averaged directly. Enormous numbers of extensive variables like capital and credits, investment and assets, profits and savings, etc., make financial modeling very complex. As usual, macro variables are defined as aggregates of the corresponding values of all e-particles of the entire economy. For example, macro investment equals the total investment of all e-particles and macro assets can be calculated as the cumulative assets of all e-particles. Let us define macro variables as functions of time  $t$  and coordinates  $x$  in the e-space in a more formal way.

Let us assume that there are  $N(x)$  e-particles at point  $x$ . Let us take the velocities of the e-particles at point  $x$  equal  $v = (v_1, \dots, v_{N(x)})$ . Velocities  $v = (v_1, \dots, v_n)$  describe a change of e-particle ratings  $x$  during the timeframe  $dt$ . Each e-particle has  $l$  extensive variables  $(u_1, \dots, u_l)$ . Assuming that the values of variables equal  $u = (u_{1i}, \dots, u_{li}), i = 1, \dots, N(x)$ , each extensive variable at point  $x$  defines the macro variable  $U_j$  as a sum of variables  $u_{ji}$  of  $N(x)$  e-particles at point  $x$

$$U_j = \sum_i u_{ji}; \quad j = 1, \dots, l; \quad i = 1, \dots, N(x)$$

To describe the motion of variable  $U_j$ , let us establish an additive variable *like an impulse* in physics. For e-particle  $i$  let us define impulses  $p_{ji}$  as product of variable  $u_j$  that takes the value  $u_{ji}$  and its velocity  $v_i$ :

$$p_{ji} = u_{ji}v_i \quad (1)$$

For example, if the assets  $a$  of e-particle  $i$  take value  $a_i$  and velocity of e-particle  $i$  equals  $v_i$  then the impulse  $p_{ai}$  of the assets of e-particle  $i$  equals  $p_{ai} = a_i v_i$ . Thus, if the e-particle has  $l$  extensive variables  $(u_1, \dots, u_l)$  and velocity  $v$  then it has  $l$  impulses  $(p_1, \dots, p_l) = (u_1 v, \dots, u_l v)$ . Let us define impulse  $P_j$  of variable  $U_j$  as a sum of impulses of e-particles at point  $x$ :

$$P_j = \sum_i u_{ji} \cdot v_i; \quad j = 1, \dots, l; \quad i = 1, \dots, N(x) \quad (2)$$

Let us introduce economic distribution function  $f = f(t, x; U_1, \dots, U_l, P_1, \dots, P_l)$ , which determines the probability to observe variables  $U_j$  and impulses  $P_j$  at point  $x$  at time  $t$ .  $U_j$  and  $P_j$  are determined by the corresponding values of e-particles that have the coordinates  $x$  at time  $t$ . They take random values at point  $x$  due to the random motion of e-particles in the e-space. Averaging the  $U_j$  and  $P_j$  within distribution function  $f$  allows the establishment of a transition from the approximation that takes into account the variables of separate e-particles to an “economic fluid” similar to a hydrodynamic approximation that neglects e-particles granularity and describes averaged macro variables as functions of time and coordinates in the e-space. Let us define the density functions

$$U_j(t, x) = \int U_j f(t, x, U_1, \dots, U_l, P_1, \dots, P_l) dU_1 \dots dU_l dP_1 \dots dP_l \quad (3)$$

and impulse density functions  $P_j(t, x)$

$$P_j(t, x) = \int P_j f(t, x, U_1, \dots, U_l, P_1, \dots, P_l) dU_1 \dots dU_l dP_1 \dots dP_l \quad (4)$$

This allows the definition of the e-space velocities  $v_j(t, x)$  of the densities  $U_j(t, x)$  as

$$U_j(t, x) v_j(t, x) = P_j(t, x) \quad (5)$$

Densities  $U_j(t, x)$  and impulses  $P_j(t, x)$  are determined to be the mean values of the aggregates of the corresponding variables of separate e-particles with coordinates  $x$ . The functions  $U_j(t, x)$  can describe the densities of investment and loans, assets and debts and so on.

To describe the evolution of variables like investment and loans, assets and debts, etc., it is important to highlight that they are composed of corresponding variables of e-particles (Equations (3)–(5)). However the assets of e-particle 1 at point  $x$  are determined by numerous buy or sell transactions with e-particles at any point  $y$  in the e-space. To describe the evolution of macro variables, let us define and describe macro transactions in the e-space.

### 2.3. Transition from Transactions between Agents to Macro Transactions between Points

To change its assets, an e-particle should buy or sell them. The value of the assets of an e-particle can change due to variations in market prices determined by market buy–sell transactions performed by other e-particles. Any e-particle at point  $x$  may carry out transactions with e-particles at any point  $y$  in the e-space.

Macro variables like assets, investments or credits, etc., have important properties. For example, macro investment at moment  $t$  determines investment made during a certain timeframe  $T$  that may be equal to a minute, day, quarter, year, etc. Thus, any variable at time  $t$  is determined by factor  $T$ , which indicates the timeframe of the accumulation of that variable. The same parameter  $T$  defines the duration of the transaction. Let us further treat any transactions as the rate or speed of change

of the corresponding variable. For example, we treat transactions by investment at moment  $t$  as an investment made during time  $dt$ .

Transactions between e-particles are the only tools that implement economic and financial interactions and processes. In his Nobel lecture, [Leontief \(1973\)](#) indicated that: “Direct interdependence between two processes arises whenever the output of one becomes an input of the other: coal, the output of the coal mining industry, is an input of the electric power generating sector”. Let us call the economic and financial variables of two e-particles mutual if “the output of one becomes an input of the other”. For example, credits as the output of banks are mutual to loans as the input of borrowers. Assets as output of investors are mutual to debts as the input of debtors. Any exchange between e-particles by mutual variables is carried out by a corresponding transaction. Transactions between two e-particles at points  $x$  and  $y$  by assets, loans, capital, investment, etc., define transactions as a function of time  $t$  and variables  $(x, y)$ . Different transactions define the evolution of different couples of mutual variables. Let us repeat that the above treatment has parallels to Leontief’s framework. We replace Leontief’s specification by industry by mapping economics in the e-space. Thus, we replace transactions between industry sectors—inter-industry tables—with transactions between points in the e-space. The most important distinction is that inter-industry tables do not allow the development of a time evolution of the entire economy, because the matrix coefficients between different industries are not constant and are not described by Leontief’s framework. As we show below, using the economic space gives grounds for modeling the economic and financial evolution in time via a description of macro transaction dynamics using hydrodynamic-like equations.

Transactions between e-particle 1 at point  $x$  and e-particle 2 at point  $y$  determine the transactions  $a_{1,2}(x, y)$ , which describe the exchange of variables  $B_{out}(1, x)$  and  $B_{in}(2, y)$  at moment  $t$  during timeframe  $dt$ . Let  $a_{1,2}(x, y)$  be equal to output variable  $B_{out}(1, x)$  of e-particle 1 to e-particle 2 and equal to the input of variable  $B_{in}(2, y)$  of e-particle 2 from e-particle 1 at moment  $t$  during timeframe  $dt$ . Thus,  $a_{1,2}(x, y)$  describes the speed of change of variable  $B_{out}(1, x)$  of e-particle 1 at point  $x$  due to the exchange with e-particle 2 at point  $y$ . At the same time,  $a_{1,2}(x, y)$  describes the speed of change of variable  $B_{in}(2, y)$  of e-particle 2 at point  $y$  due to the exchange with e-particle 1. Thus, variable  $B_{out}(1, x)$  of e-particle 1 at point  $x$  changes due to the action of transactions  $a_{1,2}(x, y)$  with all e-particles at point  $y$  as follows:

$$dB_{out}(1, x) = \sum_i a_{1,i}(x, y) dt \quad (6)$$

and vice versa

$$dB_{in}(2, y) = \sum_i a_{i,2}(x, y) dt \quad (7)$$

For example, credit–loan transactions may describe credits (output) from e-particle 1 to e-particle 2. In such a case,  $B_{in}(2)$  equals loans received by e-particle 2 and  $B_{out}(1)$  equals credits issued by e-particle 1 during a certain timeframe  $T$ . The sum of transactions over all input e-particles equals the speed of change of the output variable  $B_{out}(1)$  of e-particle 1.

Let us assume that all extensive variables of e-particles can be presented as pairs of mutual variables or can be described by mutual variables. Otherwise there should be macro variables that do not depend on any economic or financial transactions, and do not depend on markets, investment, etc. We assume that any economic or financial variable of e-particles depends on certain transactions between e-particles. For example, the value of an e-particle (value of a corporation or bank) does not take part in transactions, but is determined by buy–sell transactions that define the stock price of the corresponding bank, or by economic variables like assets and loans, credits and loans, sales and purchases, etc. Let us assume that all extensive variables can be described by Equations (6) and (7) or through other mutual variables. Thus, transactions describe the dynamics of all extensive variables of e-particles and hence determine the evolution of entire financial economics.

Now let us explain the transition from the description of transactions between the e-particle to the description of macro transactions between points in the e-space. We assume that transactions between



e-particles at point  $x$  and e-particles at point  $y$  are determined by the exchange of mutual variables like assets and loans, credits and loans, buy and sell, etc. Different transactions describe exchanges by different mutual variables. For example credit–loan ( $cl$ ) transactions at time  $t$  describe a case when e-particle “one” at point  $x$  during time  $dt$  issues credit (output) of amount  $cl$  to e-particle “two” at point  $y$  and e-particle “two” at point  $y$  at moment  $t$  during time  $dt$  receives a loan (input) of amount  $cl$  from e-particle “one” at point  $x$ . We use an example of credit–loan transactions to provide a formal definition of macro transactions.

Let us assume that e-particles in the e-space  $R^n$  at moment  $t$  are described by the coordinates  $x = (x_1, \dots, x_n)$  and velocities  $v = (v_1, \dots, v_n)$ . Let us assume that at moment  $t$  there are  $N(x)$  e-particles at point  $x$  and  $N(y)$  e-particles at point  $y$ . Let us state that at moment  $t$  each e-particle at point  $x$  carries out credit–loan transactions  $cl_{ij}(x, y)$  with e-particles  $N(y)$  at point  $y$ . In other words, if e-particle  $i$  at moment  $t$  at point  $x$  provides credits  $cl_{ij}(x, y)$  to e-particle  $j$  at point  $y$  then e-particle  $j$  at point  $y$  at moment  $t$  increases its loans by  $cl_{ij}(x, y)$  ahead of e-particle  $i$ . Let us assume that all e-particles in the e-space are “independent” and thus the sum  $i$  of credit–loan transactions  $cl_{ij}(x, y)$  at point  $x$  in the e-space  $R^n$  at time  $t$  during  $dt$  equals the increase in loans  $l_j(x, y)$  of e-particle  $j$  at point  $y$  ahead of all e-particles at point  $x$  at moment  $t$

$$l_j(x, y) = \sum_i cl_{ij}(x, y) = c_j(x, y); \quad i = 1, \dots, N(x); \quad j = 1, \dots, N(y)$$

and equally an increase  $c_j(x, y)$  of credits at moment  $t$  during  $dt$  of all e-particles at point  $x$  allocated at e-particle  $j$  at point  $y$ . The sum by  $j$  of transactions  $cl_{ij}(x, y)$  at point  $y$  in the e-space  $R^n$  equals the increase  $c_i(x, y)$  of credits of e-particle  $i$  at point  $x$  allocated by all e-particles at point  $y$  time  $t$  during  $dt$

$$c_i(x, y) = \sum_j cl_{ij}(x, y) = l_i(x, y); \quad i = 1, \dots, N(x); \quad j = 1, \dots, N(y)$$

and equals the increase of loans of all e-particles at point  $y$  ahead of e-particle  $i$  at point  $x$ . Let us define transactions  $cl(x, y)$  between points  $x$  and  $y$  as

$$cl(x, y) = \sum_{ij} cl_{ij}(x, y); \quad i = 1, \dots, N(x); \quad j = 1, \dots, N(y) \quad (8)$$

$cl(x, y)$  equals the growth of credits of all e-particles at point  $x$  that are allocated to e-particles at point  $y$  at moment  $t$  and equals the increase in loans of all e-particles at point  $y$  ahead of all e-particles at point  $x$  at moment  $t$ . Transactions (8) between two points in the e-space are random due to random deals between e-particles. To define macro transactions as regular functions and to derive equations that describe the evolution of macro transactions in the e-space let us introduce the equivalent of a “transaction impulse”, similar to Equations (1) and (2) and (Olkhov 2017a, 2017c). To do that, let us define the additional variables  $p_X$  and  $p_Y$  that describe the flux of credits by e-particles along the  $x$  and  $y$  axes. For credit–loan transactions  $cl$  let us define the impulses  $p = (p_X, p_Y)$ :

$$p_X = \sum_{i,j} cl_{ij} \cdot v_i; \quad i = 1, \dots, N(x); \quad j = 1, \dots, N(y) \quad (9)$$

$$p_Y = \sum_{i,j} cl_{ij} \cdot v_j; \quad i = 1, \dots, N(x); \quad j = 1, \dots, N(y) \quad (10)$$

Credit–loan transactions  $cl(t, x, y)$  (8) and the “impulses”  $p_X$  and  $p_Y$  (9) and (10) take random values due to random transactions between e-particles. To obtain regular functions we apply an averaging procedure. Let us introduce the distribution function  $f = f(t; z = (x, y); al; p = (p_X, p_Y))$  in the  $2n$ -dimensional e-space  $R^{2n}$  that determine the probability of observing credit–loan transactions  $cl$  at point  $z = (x, y)$  with impulses  $p = (p_X, p_Y)$  at time  $t$ . The averaging of credit–loan transactions and their “impulses” by distribution function  $f$  determine the “mean” value of the economic hydrodynamic-like approximation of macro transactions as functions of  $z = (x, y)$ . Let us repeat that the main goal of this averaging procedure is the transition from the description of credit–loan transactions as properties of

separate economic agents, to a description of credit–loan transactions as properties of economic space points. Credit–loan macro transactions  $CL(z = (x, y))$  and “impulses”  $P = (P_X, P_Y)$  take form:

$$CL(t, z = (x, y)) = \int cl f(t, x, y; cl, p_X, p_Y) dcl dp_X dp_Y \quad (11)$$

$$P_X(t, z = (x, y)) = \int p_X f(t, x, y; cl, p_X, p_Y) dcl dp_X dp_Y \quad (12)$$

$$P_Y(t, z = (x, y)) = \int p_Y f(t, x, y; cl, p_X, p_Y) dcl dp_X dp_Y \quad (13)$$

This defines the e-space velocity  $v(t, z = (x, y)) = (v_x(t, z), v_y(t, z))$  of macro transaction  $AL(t, z)$ :

$$P_X(t, z) = CL(t, z) v_X(t, z) \quad (14)$$

$$P_Y(t, z) = CL(t, z) v_Y(t, z) \quad (15)$$

Macro transactions may describe many important properties. Credit–loan transactions  $CL(t, z = (x, y))$  describe the rate of change of credits provided from point  $x$  (from agents with risk rating  $x$ ) to point  $y$  (to agents with risk ratings  $y$ ) at moment  $t$  during timeframe  $dt$ . Due to Equation (3), the integral of transactions  $CL(t, x, y)$  by variable  $y$  over e-space  $R^n$  defines the rate of change of credits  $RC(t, x)$  issued from point  $x$ .

$$RC(t, x) = \int dy CL(t, x, y); RL(t, y) = \int dx CL(t, x, y) \quad (16)$$

The integral of  $CL(t, x, y)$  by  $x$  in e-space  $R^n$  determines the rate of change of loans  $RL(t, y)$  received at point  $y$ . The integral of  $CL(t, x, y)$  by variables  $x$  and  $y$  in the e-space describes the rate of change of total credits  $RC(t)$  provided in the economy or the rate of change of total loans  $RL(t)$ .

$$RC(t) = \int dx RC(t, x) = \int dx dy CL(t, x, y) = \int dy RL(t, y) = RL(t) \quad (17)$$

In other words,  $RC(t)$  defines an amount of credits provided in the economy at moment  $t$  during timeframe  $dt$ .  $RL(t)$  defines an amount of loans received in the economy at moment  $t$  during timeframe  $dt$ . Let us underline an important issue, namely that relations (17) define the time-series of financial variables. Due to (17), different functions  $CL(t, x, y)$  can define identical time-series functions of total credits  $RC(t)$  provided and total loans  $RL(t)$  received in the entire economy during timeframe  $dt$ . In other words, the total credits  $RC(t)$  of the entire economy provided at moment  $t$  can be determined by different credit transactions  $CL(t, x, y)$ . Hence, the policy responses of market and financial authorities to fluctuations of macro credits  $RC(t)$  should depend on properties of “hidden” credit transactions  $CL(t, x, y)$ . Thus, most problems that are related to financial fluctuations and market volatility may correspond to proper descriptions of underlying macro transactions in the economic space of risk ratings. This makes the development of reasonable financial policy much more complex.

Macro transactions define financial variables in the e-space. For example, credits–loan transactions  $CL(t, x, y)$  define the total credits  $CL(t, x)$  issued from point  $x$  at moment  $t$  during timeframe  $dt$  as

$$C(t, x) = C(0, x) + \int_0^t d\tau \int dy CL(\tau, x, y) \quad (18)$$

and total loans  $L(t, y)$  received at point  $y$  at moment  $t$  during timeframe  $dt$ :

$$L(t, x) = L(0, x) + \int_0^t d\tau \int dx CL(\tau, x, y) \quad (19)$$

Credit–loan transactions  $CL(t, x, y)$  can determine the position of the maximum creditor at point  $x_C$  and position  $y_B$  of the maximum borrowers of credits and the “risk” distance  $r = y - x$  between

them. This “risk” distance  $r$  between creditors and borrowers can fluctuate in time and reflects different phases of the business cycle. Relations similar to Equations (16)–(19) define the evolution and fluctuations of all economic and financial variables that are determined by macro transactions. These relations form the basis for modeling economic and financial variables via the description of macro transaction dynamics. Below, we derive hydrodynamic-like equations to describe the evolution of credit–loan transactions.

### 3. Equations for Macro Transactions

Macro transactions between points  $x$  and  $y$  in the e-space determine the evolution of macro variables Equations (16)–(19). To describe the dynamics of macro transactions, let us derive economic hydrodynamic-like equations similar to (Olkhov 2016a, 2017a, 2017c). The economic arguments for the use of hydrodynamic-like equations are very clear and simple. To describe the evolution of transaction  $A(t, z = (x, y))$  and its impulses  $P = (P_x, P_y) = (v_x A, v_y A)$  in a unit volume  $dV$  at point  $z = (x, y)$  in the  $2n$ -dimensional e-space  $R^{2n}$ , one should take into account two factors. The first factor describes the evolution of transaction  $A(t, z)$  in a unit volume due to a change to transaction  $A$  in time as  $\partial A / \partial t$ . The second factor describes a change of transaction  $A(t, z)$  in a unit volume due to the flux  $Av$  of the transaction through the surface of a unit volume. The divergence theorem (Strauss 2008, p. 179) states that the integral of flux through the surface of a unit volume equals the volume integral of divergence. Thus, the second factor is described by the divergence from a unit volume and equals  $\text{div}(A(t, z)v(t, z))$ . Here  $v$  is the velocity of transaction  $A(t, z = (x, y))$  in the  $2n$ -dimensional e-space  $R^{2n}$ . To balance the action of factors describing the change of transaction  $A(t, z)$  in a unit volume, let us take into account the impact of other transactions or any other impacts on transaction  $A$ . This defines the right-hand side of economic hydrodynamic-like equations. Economic hydrodynamic-like equations for impulses  $P = (P_x, P_y) = (v_x A, v_y A)$  of macro transactions have the same meaning. For simplicity, the equations for impulses take the form of the equation of motion for velocity  $v = (v_x, v_y)$  of transaction  $A$ . Below, we present these considerations in a more formal way.

#### 3.1. Economic Hydrodynamic-Like Equations for Macro Transactions

Transaction  $A(t, x, y)$  and impulse  $P(t, x, y)$  are determined in (11)–(15) by averaging the procedures of the aggregates of credit–loan transactions between e-particles at points  $x$  and  $y$ . Similar transactions can describe the exchange by mutual variables, such as investment and debts, buy and sell, etc. Let us define macro transaction  $A(t, x, y)$  between two mutual variables  $A_{out}(x)$  and  $A_{in}(y)$  in e-space  $R^n$ .  $A(t, x, y)$  equals input  $A_{in}(y)$  at  $y$  from  $x$  and equals output  $A_{out}(x)$  from  $x$  to  $y$ . The functions  $A(t, z = (x, y))$  and  $v(t, z = (x, y)) = (v_x(t, z), v_y(t, z))$  are determined in the  $2n$ -dimensional e-space  $R^{2n}$ . Similar to (Olkhov 2016a, 2017a, 2017c), continuous Equation (20) and Equation of motion (21) for transaction  $A(t, z)$  take form:

$$\frac{\partial A}{\partial t} + \text{div}(vA) = Q_1 \quad (20)$$

$$A \left[ \frac{\partial v}{\partial t} + (v \cdot \nabla) v \right] = Q_2 \quad (21)$$

Let us remember the economic meaning of Equations (20) and (21). The left side of Equation (20) describes the change of transaction  $A(t, z)$  in a unit volume on e-space  $R^{2n}$  at point  $z = (x, y)$ . It can change due to variations in time that are described by derivative  $\partial A / \partial t$  and due to flux  $A(t, z)v$  through the surface of the unit volume. Due to the divergence theorem (Strauss 2008, p. 179), the integral of flux through the surface of a unit volume equals the volume integral of divergence  $\text{div}(Av)$ . The right side  $Q_1$  describes any external factors that can change  $A(t, z)$ . The left side of the equations of motion describes the same variation of the transaction impulse  $P(t, z) = A(t, z)v(t, z)$ . If one takes into account continuity equations, then the left side of equations of motion can be simplified as (21). Factor  $Q_2$  describes any factors that can change the left side (21).

Equations (20) and (21) for transaction  $A(t,z)$  and its velocity  $v(t,z)$  are determined by factors  $Q_1$  and  $Q_2$ . Let us assume that only transactions  $B(t,z)$ , those different from  $A(t,z)$ , define  $Q_1$  and  $Q_2$ . Let us propose that transactions  $B(t,z)$  are *conjugate* to transaction  $A(t,z)$  if  $B(t,z)$  or their velocities determine factors  $Q_1$  and  $Q_2$  of economic hydrodynamic-like Equations (20) and (21). Any transaction  $A(t,z)$  can have one, two or many *conjugate* transactions  $B(t,z)$  that determine the right-hand side of (20) and (21). For example, the credit–loan transactions  $CL(t,x,y)$  may depend on loan–repayment transactions, supply–demand or buy–sell transactions. Market buy–sell transactions can be determined by transactions with various assets, etc. Numerous *conjugate* transactions—those determine by the right-hand side of equations like (20) and (21)—make the modeling of real financial processes extremely complex. In this paper, we present the simplest models of financial economics based on economic hydrodynamic-like equations for macro transactions. Let us assume that transaction  $A(t,z)$  has only one *conjugate* transaction  $B(t,z)$  and vice versa. In this case, it is possible to derive equations for  $A(t,z)$  and  $B(t,z)$  and to study their evolution under their mutual interactions in a self-consistent manner. Further, we define loan–repayment transactions and study the simplest model of mutual dependence between two *conjugate* transactions: credit–loan and loans–repayment transactions.

### 3.2. Two Conjugate Transactions Model

To derive Equations (20) and (21) in a closed form, we study the simplest model of mutual dependence between two conjugate transactions, such as credit–loan  $CL(t,z)$  and loan–repayment  $LR(t,z)$ . Let us define loan–repayment  $LR(t,z = (x,y))$  transactions as all payoffs that are made by e-particles at point  $y$  related to credits received from point  $x$  at moment  $t$  during the timeframe  $dt$ . Transactions  $CL(t,z = (x,y))$  describe credits issued from point  $x$  to point  $y$  at moment  $t$  during the timeframe  $dt$ . Credit–loan  $CL(z)$  and loan–repayment  $LR(t,z)$  transactions describe core economic properties. These transactions are responsible for economic growth and financial sustainability and their descriptions are extremely complex. The introduction of the e-space allows us establish and study various models that describe the relations between variables and transactions and model different approximations of real financial processes.

Let us start with a simple model and assume that credit–loan transactions  $CL(t,z = (x,y))$  at moment  $t$  depend on loan–repayment  $LR(t,z = (x,y))$  at moment  $t$  only. Our assumptions mean that creditors at point  $x$  take decisions on providing credits to point  $y$  on base of loan–repayments received from point  $y$  to point  $x$  at the same moment  $t$ . We simplify the problem of developing a reasonable model of their mutual interaction. To describe the evolution of credit–loan transactions  $CL(t,z)$ , let us take Equations (20) and (21) and define factors  $Q_1$  and  $Q_2$  using same approach and considerations as (Olkhov 2016a, 2017a). Let us assume that  $Q_1$  on the right-hand side of the continuity equation (20) for credit–loan transactions  $CL(t,z)$  is proportional to the divergence of loan–repayment velocity  $u(z)$  in the e-space  $R^{2n}$ :

$$Q_1 \sim LR(t,z) \nabla \cdot u(t,z) \quad (22)$$

The positive divergence (22) of loans–repayment  $LR(t,z)$  velocity  $u(t,z)$  describes the growth of flux of loans–repayment that may attract creditors at point  $x$  to increase their credits at point  $y$ . The negative divergence of velocity  $u(t,z)$  means that the loan–repayment  $LR(z)$  flow decreases, which may prevent creditors at point  $x$  from approving further loans to point  $y$ . Let us assume that the  $Q_1$  factor that defines the right-hand side of (20) for loan–repayment  $LR(t,z)$  is proportional to the divergence of credit–loan velocity  $v(t,z)$ :

$$Q_1 \sim CL(t,z) \nabla \cdot v(t,z) \quad (23)$$

The positive divergence (23) of credit–loan  $CL(z)$  velocity  $v(t,z)$ , describes the growth of credit–loan flux and that may increase loan–repayment  $LR(t,z)$ . The growth of credits from point  $x$  to point  $y$  in the e-space may induce a growth in payoffs from  $y$  to  $x$ . In addition, the negative divergence of credit–loan  $CL(x,y)$  flux describes the decline of credit flow allocated by point  $x$  at  $y$ , which may reduce payoffs from  $y$  to  $x$ . It is obvious that we neglect the time gap between the allowance of credits

and loan–repayment and other factors that may determine creditor decisions from  $x$  to  $y$  to simplify the model. Let us determine  $Q_2$  factors in the equations of motion (21) for credit–loan  $CL(z = (x, y))$ . We assume that the velocity  $v(t, z)$  of credit–loan  $CL$  depends on the right-hand side factor  $Q_2$ , which is proportional to the gradient of loan–repayment  $LR(t, z)$ :

$$Q_2 \sim \nabla LR(t, z) \quad (24)$$

The relations in (24) propose that credit–loan velocity  $v(t, z)$  grows in the direction of higher loan–repayments. Let us make the same assumptions regarding  $Q_2$ , which determines the equation of motion (21) for loan–repayment velocity  $u(t, z)$ :

$$Q_2 \sim \nabla CL(t, z) \quad (25)$$

The relations in (25) propose that loan–repayment velocity  $u(t, z)$  grows in the direction of higher credit–loans. The assumptions in (22)–(25) define the right-hand side factors and define hydrodynamic-like equations for the two conjugate transactions credit–loan and loans–repayment in a closed form. Continuity Equations:

$$\frac{\partial CL}{\partial t} + \nabla \cdot (vCL) = a_2 LR(t, z) \nabla \cdot u(t, z) \quad (26)$$

$$\frac{\partial LR}{\partial t} + \nabla \cdot (uLR) = a_1 CL(t, z) \nabla \cdot v(t, z) \quad (27)$$

Equations of motion:

$$CL(z) \left[ \frac{\partial v}{\partial t} + (u \cdot \nabla) v \right] = b_2 \nabla LR(t, z) \quad (28)$$

$$LR(z) \left[ \frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = b_1 \nabla CL(t, z) \quad (29)$$

Equations (26)–(29) provide grounds for the derivation of transaction wave equations.

#### 4. Wave Equations for Macro Transactions

Let us derive wave equations for transaction disturbances in a linear approximation. We simplify the problem and assume

$$CL(t, z) = CL + cl(t, z); LR(t, z) = LR + lr(t, z) \quad (30)$$

Let us assume that  $CL$  and  $LR$  are constant or their variations are negligible to compare the variations of the small disturbances  $cl(t, z)$ ,  $lr(t, z)$ ,  $v(t, z)$  and  $u(t, z)$  and let us neglect nonlinear factors in Equations (26)–(29). These assumptions allow us to derive wave equations for disturbances in a linear approximation similar to the derivation of acoustic wave equations (Landau and Lifshitz 1987). Continuity equations for the disturbances take the form:

$$\frac{\partial cl}{\partial t} + CL \nabla \cdot v = \alpha_2 LR \nabla \cdot u; \quad \frac{\partial lr}{\partial t} + LR \nabla \cdot u = \alpha_1 CL \nabla \cdot v \quad (31)$$

Equations of motion for the disturbances take the form:

$$CL \frac{\partial v}{\partial t} = \beta_2 \nabla lr(t, z); LR \frac{\partial u}{\partial t} = \beta_1 \nabla cl(t, z) \quad (32)$$

Equations (30)–(32) allow us to derive equations for  $cl$  and  $lr$

$$\left[ \frac{\partial^4}{\partial t^4} - a \Delta \frac{\partial^2}{\partial t^2} + b \Delta^2 \right] cl(t, z) = 0 \quad (33)$$

$$a = \alpha_1\beta_2 + \alpha_2\beta_1; \quad b = \beta_1\beta_2(\alpha_1\alpha_2 - 1)$$

The derivation of (33) from Equations (31) and (32) is simple and we omit it here. For

$$c_{1,2}^2 = \frac{a \pm \sqrt{a^2 - 4b}}{2} > 0$$

(33) takes the form of a bi-wave equation:

$$\left(\frac{\partial^2}{\partial t^2} - c_1^2\Delta\right)\left(\frac{\partial^2}{\partial t^2} - c_2^2\Delta\right)cl(t, z) = 0 \quad (34)$$

Here  $c_{1,2}$  is the different velocities of transaction disturbance waves propagating in the e-space. It is clear that the Green function of bi-wave Equation (34) equals the convolution of Green functions of common wave equations with wave speeds equal to  $c_1$  and  $c_2$ .

$$\left(\frac{\partial^2}{\partial t^2} - c_1^2\Delta\right)cl(t, z) = 0; \quad \left(\frac{\partial^2}{\partial t^2} - c_2^2\Delta\right)cl(t, z) = 0$$

Thus, even simple  $\delta$ -function shocks induce a complex wave response. Equations (33) or (34) validate the diversity of the wave processes that govern the evolution of macro transactions. Thus, transaction disturbances can induce waves that propagate through the e-space domain from low to high risk areas or vice versa and may cause time fluctuations of macro variables as assets, investments, profits, capital, etc. Let us show that Equation (33) admit wave solutions with amplitudes growing as exponents in time. Let us take  $cl(t, z)$  as:

$$cl(t, z) = \cos(\omega t - \mathbf{k} \cdot \mathbf{z}) \exp(\gamma t); \quad \mathbf{k} = (k_x, k_x) \quad (35)$$

Solution (35) satisfies Equation (33) if:

$$\omega^2 = \gamma^2 + \frac{ak^2}{2}; \quad 4\gamma^2\omega^2 = k^4 \left(b - \frac{a^2}{4}\right) > 0; \quad 4b > a^2$$

$$\gamma^2 = k^2 \frac{\sqrt{4b + 3a^2} - 2a}{8} > 0; \quad \omega^2 = k^2 \frac{\sqrt{4b + 3a^2} + 2a}{8} > 0$$

For the  $\gamma > 0$  wave amplitude increases as  $\exp(\gamma t)$ . The relations in (35) describe simple harmonic waves of credit–loan disturbances  $cl(t, \mathbf{x}, \mathbf{y})$ , with the amplitude increasing in time as an exponent. Due to the definition of the e-space in Section 2, the coordinates of the e-particles define their risk ratings. Thus, for the simplest 1-dimensional e-space  $R$ , the credit–loan transaction  $CL(t, x, y)$  is determined in the e-space  $R^2$ . Let us assume that the risk ratings of e-particles are reduced by minimum  $X_{min}$  and maximum  $X_{max}$  risk grades. For simplicity, let us take the borders of the e-space domain as  $X_{min} = 0$  and  $X_{max} = X$ . Hence, the e-space

$$0 \leq x \leq X \quad (36)$$

Due to (30), the credit–loan transaction  $CL(t, x, y)$  is presented as

$$CL(t, x, y) = CL + cl(t, \mathbf{x}, \mathbf{y}) \quad (37)$$

Due to assumptions (35)–(37) and relations (16), the credits  $RC(t, x)$  provided from point  $x$  at moment  $t$  during the timeframe  $dt$  equal:

$$RC(t, x) = \int dy CL(t, x, y) = CLX - 2 \frac{\exp(\gamma t)}{k_y} \cos\left(\omega t - k_x x - \frac{k_y X}{2}\right) \sin\left(\frac{k_y X}{2}\right) \quad (38)$$



The relations in (38) show that the credits  $RC(t, x)$  have a wave component that propagates along the risk axis  $x$  and the amplitude of this wave increases in time as an exponent. Thus, the credit–loan transaction  $CL(t, x, y)$  waves (35) and (36) cause waves of credits  $RC(t, x)$ , with the amplitude of the perturbations growing in time by exponent. This could cause disturbances of the entire macro financial system. For assumptions (35)–(37) and due to (17), the rate of change of credits  $RC(t)$  at moment  $t$  equals

$$RC(t) = RC_0 + c(t); \quad RC_0 \sim CL X^2$$

$$c(t) = \frac{4 \exp(\gamma t)}{k_x k_y} \cos\left(\frac{k_x + k_y}{2} X - \omega t\right) \sin \frac{k_x}{2} X \sin \frac{k_y}{2} X$$

Here  $c(t)$  is determined by relations (17) for  $cl(t, x, y)$ , defined by (35) and (37) and:

$$c(t) = \int_0^X dx \int_0^X dy cl(t, x, y)$$

Hence, change of total credits  $RC(t)$  follows time oscillations with frequency  $\omega$ . For  $\gamma > 0$ , the amplitude of credit  $RC(t)$  fluctuations may increase with time as  $\exp(\gamma t)$ . At the amplitude  $\gamma < 0$ , credit  $RC(t)$  dissipates and tends to maintain the constant rate  $A_0$ . These examples illustrate the relations between the time oscillations of the change of credit  $RC(t)$  allocation on the one hand and a simple model of interactions between credit–loan  $CL(t, x, y)$  and loans–repayment  $LR(t, x, y)$  transactions and their disturbances waves  $cl(t, x, y)$  and  $lr(t, x, y)$  in the e-space on the other hand. Thus, we show that the relations between the macro variables like investment, assets, credits, etc., when treated as functions of time, can be determined by complex interactions between *conjugate* macro transactions as functions of time and coordinates in the e-space  $R^{2n}$ . Simply speaking, relations between the macro variables at point  $x$  are determined by complex interactions of transactions between agents with risk ratings  $x$  and  $y$  in the e-space. Equations for transaction disturbances admit wave solutions and can describe the exponential growth of wave amplitudes in time.

## 5. Discussion

Financial economics is an extremely complex system and its modeling should correspond to this complexity. This may excuse and explain some of complications of the methods and models we propose for the description of financial processes and fluctuations. Any model should be based on certain approximations and we made many assumptions to simplify the problem. We used the risk ratings of economic agents as their coordinates in the economic space to establish the basis for modeling macro finances, similar to the description of multi-particle systems in physics. The economic space notion is a core issue of our approach to financial modeling, which uncovers the internal complexity of interactions between financial variables and transactions. It helps us to allocate agents by their coordinates in the economic space and to develop a description of “*economic fluids*” or economic hydrodynamic-like approximations. Macro variables like credits and loans, assets and debts, investment and profits, etc., are presented as functions of time  $t$  and coordinates  $x$  in the economic space. The integral of variables over e-space defines the corresponding macro variables as functions of time  $t$  only. For example, the integral of investment over e-space equals the investment of the entire macro finance as a function of time only. We regard transactions between agents only as tools for the implementation of economic and financial processes. Any market regulations and decisions taken by economic and financial authorities come into play only by affecting transactions between agents. Transactions between agents change an agent’s extensive set of variables and that induces changes in the corresponding macro variables. The aggregation of transactions between agents at points  $x$  and  $y$  define the macro transactions between different points  $x$  and  $y$  in the economic space, and we describe the dynamics of such transactions using economic hydrodynamic-like equations. Macro transactions (16)–(19) determine the evolution and fluctuations of extensive economic and financial variables. Macro transactions can depend on numerous *conjugate* transactions, which makes

financial modeling extremely difficult. For example, for simple model interactions between credit–loan and loan–repayment transactions, we obtained economic hydrodynamic-like equations in a closed form. This permitted us to describe the relations between these transactions in a self-consistent manner and to model the evolution of such important variables as credits, loans, loan–repayments. For this simple model, we derived wave equations for transaction disturbances. We showed that disturbances of transactions between economic agents causes disturbances of macro transactions and induces waves that propagate in the economic space from high to low risk rating areas or vice versa. The wave propagation of macro transaction disturbances induces fluctuations in financial variables and might be important for the further modeling of financial fluctuations. The influence of wave processes on financial evolution may help to explain market volatility, asset price dynamics, business cycle fluctuations, the development of financial crises, etc. The fluctuation of financial variables determined by the dynamics of macro transactions due to economic hydrodynamic-like equations can form a basis for modeling business cycles in a completely new manner.

Due to the relations in (17), identical fluctuations in the time-series of macro variables can be induced by different macro transactions. Different macro transactions require different policy responses from macro financial authorities. Thus, the development of adequate macro financial policy requires reasonable econometric data on the state and evolution of macro transactions. This applies to all financial regulation decisions and policy making by macro authorities in response to the fluctuations of extensive macro-financial variables. The management of financial variable fluctuations requires econometric data for the corresponding macro transaction evolution in the economic space of risk ratings.

Our model demonstrates that the relations between macro transactions and small disturbances causes the generation and propagation of transaction waves in the economic space. The amplitudes of the transaction waves (as, for example, in (35)) from low to high risk or vice versa can increase significantly in time. Transaction waves induce waves and fluctuations of corresponding macro financial variables (38) and can cause disturbances of the entire financial system. The existence of numerous transaction waves and waves of corresponding macro financial variables as credits and loans, investment and debts, demand and supply, etc., with wave amplitudes growing in time as an exponent, might cause a lack of equilibrium in the economy. Such a negative conclusion or doubts on the validity of the equilibrium assumptions in economics corresponds to the Occam's razor (Baker 2007) principle that states, in different formulations, that "assumptions should be reduced to their minimum" or "plurality should not be posited without necessity" or "the theory with fewer entities postulated should be the preferred theory". In simple words, a model of macro finance that is not based on an assumption of general equilibrium "should be the preferred". Thus, the description of numerous transaction waves in macro finance might have an impact on the validity of theories based on general equilibrium (Starr 2011), such as asset pricing (Campbell 1999; Cochrane 2016), business cycles (Zarnowitz 1992; Lucas 1995; Kiyotaki 2011), risk management (Alexander 1999; Diebold 2012), etc. In fact, the relation between our description of macro financial fluctuations and equilibrium-based models should be studied further. Our approach to financial modeling requires an econometric foundation. Until there does not exist any econometric data sufficient to compare our theory's predictions with econometric observations. The development of an econometric basis, risk ratings assessment and economic data performance sufficient for the description of interactions between financial variables and macro transactions in the economic space requires the collective efforts of economic regulators, rating agencies and market authorities, businesses and government statistical bureaus, academic and business researchers, etc. The effective development of national accounts (Fox et al. 2014) and the development of Leontief's input–output inter-industry tables (Horowitz and Planting 2009; Miller and Blair 2009) over eighty years prove that such problems can be solved.

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