Supplementary Material 3: Improving Future Estimation of Cheliff-Mactaa-Tafna Stream-flow via an Ensemble of Bias Correction Approaches

Climate model bias: Definition

Model bias is defined as a systematic distortion of statistical findings from the expected value. According to this definition, climate model biases describe systematic climate model errors only. It should, however, be noted that the term bias in the context of climate change impact studies is often misleadingly used to describe model errors in general (i.e., a combination of both systematic and random error).

Quantile mapping

In general, the QM methods implement statistical transformations for post-processing of climate modeling outputs. The statistical transformations involve transforming the distribution functions of the modeled variables into the observed ones using a mathematical function, which can be mathematically expressed as (Piani et al., 2010):

$$x^o = f(x^m) \qquad 1$$

in which x^o = observed variable; x^m = modeled variable; and f() = transformation function.

Given that the QM methods use the quantile-quantile relation to converge the simulated variables' distribution function to the observed one, one should note that with the CDFs of both observed and simulated variables' time series, their quantile relation can also be determined, as shown below (Ringard et al., 2017):

$$x^r = F_r^{-1}[F_m(x^m)] \qquad 2$$

where $F_m(x^m)$ = CDF of x^m; and F_r^{-1} [] = inverse form of the CDF of x^r, which is technically referred to as the quantile function.

Scaled Distribution Mapping

A new bias correction methodology, called SDM, is proposed by Switanek et al., (2017). The conceptual framework of the method is quite similar to QDM. The SDM method makes no assumption of stationarity. It scales the observed distribution by raw model projected changes in magnitude, rain-day frequency (for precipitation), and likelihood of events. The scaling changes as a function of the bias correction period. The SDM methodology for precipitation is now can be expressed mathematically as.

$$f(x; k, \theta) = \frac{x^{k-1}\left(-\frac{x}{\theta}\right)}{\theta^k \, \Gamma(k)} \qquad 3$$

where $k(> 0)$ is the shape parameter, $\theta(> 0)$ is the scale parameter, $x(> 0)$ is the precipitation amount, and $\Gamma(k)$ is the gamma function evaluated at k. Next, use the fitted shape and scale parameters to find the corresponding CDF values of the positive precipitation events in the three time series (Switanek et al., 2017).

Delta Quantile Mapping

The QDM process for bias correction is based on four steps (Cannon et al., 2015). In the first, QDM uses a time-dependent cumulative density function (CDF) of the model-projected (prospective run) series $x_{m,p}$, (Equation 4). The CDF is estimated by a time window given by time t. $\tau_{m,p}$ is the non exceedance probability associated at time t.

$$\tau_{m,p}(t) = F_{m,p}^{(t)}\left[x_{m,p}(t)\right], \tau_{m,p}(t) \in \{0, 1\} \qquad 4$$

The corresponding modelled $\tau_{m,p}$ quantile in the historical period can be found by entering this value into the historical inverse CDF $F_{m,p}^{-1}$. The relative change in quantiles between the historical period and t is given by Equation 5.

$$\Delta_m(t) = \frac{F_{m,p}^{(t)-1}\left[\tau_{m,p}(t)\right]}{F_{m,h}^{-1}\left[\tau_{m,p}(t)\right]} = \frac{x_{m,p}(t)}{F_{m,h}^{-1}\left[\tau_{m,p}(t)\right]} \qquad 5$$

The modelled quantile $\tau_{m,p}$ at any value of t can be corrected by applying the inverse CDF estimated from observed values $x_{mo,h}$ over the historical period (Equation 6). Finally, the bias correction in future projection at any time t is obtained by applying relative change $\Delta_m(t)$ multiplicative to historical bias-corrected value (Equation 7).

$$\hat{x}_{o:m,h:p}(t) = F_{o,h}^{-1}\left[\tau_{m,p}(t)\right] \qquad 6$$

$$\hat{x}_{m,p}(t) = \hat{x}_{o:m,h:p}(t)\Delta_m(t) \qquad 7$$

As shown in Cannon et al., (2015), the QDM is related to the equidistant CDF matching algorithm proposed by Li, et al., (2014), and when compared to Quantile mapping algorithms, it is less prone to issues such as inflating relative trends in precipitation extremes.

Reference

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