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Influence of Helical Trajectories of Perturbers on Stark Line Shapes in Magnetized Plasmas

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Abstract: In plasmas subject to a strong magnetic field, the dynamical properties of the microfield are affected by the cyclotron motion, which can alter Stark-broadened lines. We illustrate this effect through calculations of the hydrogen Lyman α line in an ideal one-component plasma. A focus is put on the central Zeeman component. It is shown that the atomic dipole autocorrelation function decreases more slowly if the cyclotron motion is retained. In the frequency domain, this denotes a reduction of the line broadening. A discussion based on numerical simulations and analytical estimates is done.

Keywords: Stark broadening; magnetized plasmas; numerical simulations; cyclotron motion

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In strongly magnetized plasmas, the Larmor radius can be smaller than the Debye length, so that an atomic emitter "feels" the modulation of the microscopic electric field occurring at the cyclotron time period $2\pi/\omega_c = 2\pi m/eB$ (see Figure 1). Examples of such plasmas can be found in magnetic fusion experiments or in magnetized white dwarfs (B attains values up to 10 kT in some white dwarfs [1,2]). There has been only a few investigations of the role of cyclotron motion in Stark broadening models. An early work using a collision operator model [3–5] has indicated the possibility for an alteration of the line broadening. This issue has also been considered recently in [6] using another collision operator model and, last year, a set of standardized cases was devoted to this issue at the fourth edition of the Spectral Line Shape in Plasmas (SLSP) Code Comparison Workshop (see http://plasma-gate.weizmann.ac.il/projects/slsp/slsp4/ and References [7,8] for an overview of this workshop). An essential feature of the electric field modulation due to cyclotron motion is the reduction of the effective duration of the perturbation and, in turn, an overall reduction of the Stark effect. Accordingly, the dipole autocorrelation function has a slower decrease and the resulting line shape is narrower than in the absence of a magnetic field. We report here on calculations aimed at illustrating this effect. The Zeeman central component of Lyman α has been considered within an adiabatic model, suitable for strong magnetic field regimes where interactions between $\Delta m = \pm 1$ levels are negligible (e.g., [9–11]). The dipole autocorrelation function that enters the spectral line shape function through the Fourier transform reads

$$C(t) = \left\{ \cos \left[\frac{d_z}{\hbar} \int_0^t dt' E_z(t') \right] \right\}, \tag{1}$$

and it does not involve quantum mechanical operators, which simplifies the numerical calculations. The brackets $\{...\}$ denote statistical average over the initial positions and velocities of the perturbers, $d_z \equiv 3ea_0$ stands for the dipole matrix element relative to the spherical states $|200\rangle$ and $|210\rangle$, and E_z is the component of the total microfield projected onto the magnetic field direction ($\vec{B}//\vec{e}_z$ is implied). We have evaluated the autocorrelation function by numerically simulating perturbers moving along helical trajectories at the vicinity of an emitter. A rectangular box with periodic boundary conditions has been considered, i.e., a particle leaving the box is reinjected at the opposite face with the same velocity. The size along the (x, y) plane has been set equal to $3\lambda_D$ (with λ_D being the Debye length), while the size along the z axis has been set much larger, namely, of the order of $v \times t_i$ (with v, t_i being the thermal velocity and the time of interest, respectively) in order to get convergence. For the sake of clarity in discussions, only electrons have been retained in the simulations, i.e., all other line broadening mechanisms have not been considered. Correlation effects have been retained through a quasiparticle model; in this framework, the electrons are described as a set of independent particles producing a Debye electric field at the emitter's location. Figure 2 shows an example of result. Conditions relevant to magnetized white dwarf atmospheres have been considered: $N_e = 10^{17}$ cm^{-3}, $T_e = 1$ eV, and $B = 2$ kT. The figure presents a simulation result (the dipole autocorrelation function and the corresponding line shape are shown in Figure 2a,b, respectively) accounting for the helical trajectories ('+' symbol) and one assuming straight lines ('$\circ$'), i.e., formally setting $B = 0$. Two analytical estimates based on collision operators are also shown for comparison purposes. The Griem-Kolb-Shen model [12] (dashed line), applicable to unmagnetized plasmas, serves as a cross-check for the simulation at $B = 0$. The other collision operator model (solid line) accounts for the cyclotron motion in a heuristic way, technically, through the use of an upper cut-off at the Larmor radius $b_L = v/\omega_c$ instead of the Debye length in the Coulomb logarithm. As can be seen in the figure, both the simulation accounting for helical trajectories and the modified collision operator yield a weaker decrease of the autocorrelation function. This is interpretable in terms of the electric field modulation due to cyclotron motion. The duration of the Stark perturbation relative to each single perturber is reduced to a time of the order of the inverse cyclotron frequency and, hence, the dipole decorrelates more slowly and the overall line broadening is reduced. This trend was not clearly established in previous studies.

In summary, we have shown that the helical motion of particles in magnetized plasmas can affect the Stark broadening of atomic lines due to a change of the microfield statistical properties. This issue first concerns the broadening due to electrons, because their Larmor radius is much smaller than that of ions and it can attain values smaller than the Debye length for relatively 'weak' values of B. We have identified relevant plasma conditions in magnetized white dwarf atmospheres and in tokamak experiments. Using numerical simulations and analytical estimates, we have seen that the dipole autocorrelation decreases more slowly than in an unmagnetized plasma where no cyclotron motion is present. An interpretation is that the characteristic duration of the perturbation due to each electron passing near the emitter is reduced to a time of the order of the inverse cyclotron frequency, so that the dipole is globally less perturbed. The calculations discussed here are still preliminary and require a more comprehensive study. One major issue is the design of numerical simulation techniques which are suitable for addressing the electrons in near-impact regime within a reasonable CPU time. In the presence of cyclotron motion, the problem is no longer isotropic; special care needs to be taken in order to set up the geometry correctly (the 'box', which is usually cubic or spherical for applications in isotropic cases, has to be rethought). A possible candidate would be the so-called "collision time technique" where an emphasis is put on relevant perturbers that pass near the emitter during the time of interest, e.g., [13,14]. An adaptation to perturbers moving on helices remains to be done. A complement to this work should also consist in using a realistic atomic model accounting for the Zeeman effect, including both linear and quadratic (diamagnetic) Hamiltonians (e.g., see [15] for a recent discussion in the context of white dwarfs), and accounting for the Stark effect due to the thermal Lorentz electric field [16].

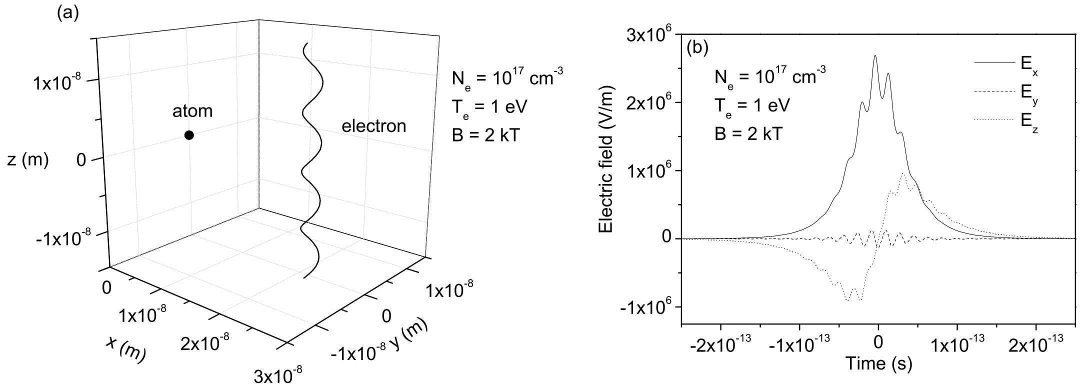

Figure 1. An example of electron helical trajectory is shown in (**a**) and the resulting electric field at an atom's location is shown in (**b**). Conditions relevant to magnetized white dwarf atmospheres have been assumed: given an atomic emitter placed at the origin, the helix axis has been placed at $x_0 = N_e^{-1/3}$ with $N_e = 10^{17}$ cm^{-3}, the velocities $v_\perp$ and $v_{//}$ have both been set equal to $\sqrt{T_e/m_e}$ with $T_e = 1$ eV, and a value of 2 kT has been assumed for the magnetic field. The Debye length $\lambda_D \simeq 2.4 \times 10^{-8}$ m is much larger than the Larmor radius $b_L \simeq 1.2 \times 10^{-9}$ m and, accordingly, the electric field exhibits oscillations.

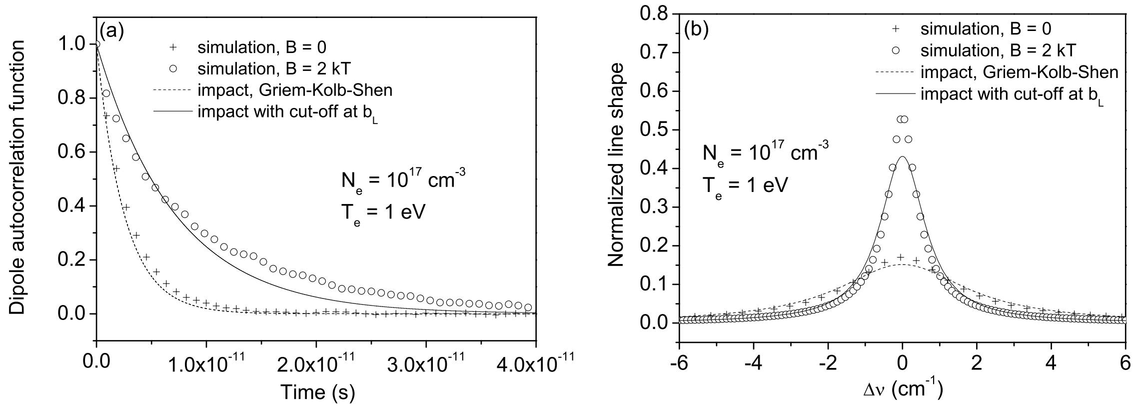

Figure 2. Plot of (**a**) the atomic dipole autocorrelation function and (**b**) the corresponding line shape obtained from a numerical simulation accounting for the helical motion of the electrons. The Lyman α line in magnetized white dwarf atmosphere conditions is considered with focus on the central Zeeman component. The decrease is slower than in the absence of a magnetic field. This trend is also expected from analytical estimates based on collision operators.

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