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Effect of Turbulence on Line Shapes in Astrophysical and Fusion Plasmas

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Abstract: We look at the effect of wave collapse turbulence on a hydrogen line shape in plasma. An atom immersed in plasma affected by strong Langmuir turbulence may be perturbed by a sequence of wave packets with a maximum electric field magnitude that is larger than the Holtsmark field. For such conditions, we propose to calculate the shape of the hydrogen Lyman α Lyman β and Balmer α lines with a numerical integration of the Schrödinger equation coupled to a simulation of a sequence of electric fields modeling the effects of the Langmuir wave. We present and discuss several line profiles of Lyman and Balmer lines.

Keywords: line shape; wave collapse; electric field solitons; plasma turbulence

1. Introduction

The problem of plasma turbulence is of interest both from a theoretical point of view and from an experimental one for laboratory, fusion, and astrophysical plasmas. Plasma turbulence affects the transport and radiation properties of many kinds of plasma. In magnetic fusion studies, the quality of the plasma confinement is strongly dependent on the level of turbulent fluctuations. The first observations of turbulent fluctuations have been made in astrophysics on line shapes dominated by the Doppler effect [1,2]. If nonthermal movements take place on the line of sight, the line shape no longer corresponds to a Maxwellian velocity distribution at the emitter temperature. In the simplest models, a nonthermal velocity is defined as one that allows a quantitative measure of turbulence. The study of line shapes may then provide valuable information on the nature of turbulence, and this has been used in astrophysical and laboratory plasmas [3]. In this work, we are mainly interested in the contribution of Stark effect to the line shapes of hydrogen plasmas. The turbulent fluctuations of the plasma are created by the instabilities appearing in the different types of plasma studied [3].

One kind of plasma turbulence suspected to be present in astrophysical and fusion plasma is driven by plasma waves and electromagnetic waves. We studied the case of nonlinear wave collapse turbulence, a phenomenon occurring in the presence of an external source of energy, and coupling nonlinearly to the Langmuir waves with ion sound and electromagnetic waves. Due to this coupling, the density fluctuations associated with ion sound waves refracts the Langmuir waves in regions of low densities. Coherent wave packets localize in such regions, and experience a cycle driven by the ponderomotive force, which decreases them to shorter scales and enhances their intensity (wave collapse). In such conditions, numerous wave collapse sites are present in the plasma, which change its radiative properties. We proposed a model for calculating the change in the line shape of atoms submitted to the electric field of a nearby wave collapse [3,4]. Our model uses the numerical solution of the emitter Schrödinger equation submitted to an electric field taken as a sequence of envelope solitons oscillating at the plasma frequency. We used the results of numerical simulations of wave collapse [5] to sample the lifetime of each soliton as well as the probability density function for the magnitude of the electric field. We will present the changes expected on a line shape of hydrogen for plasma conditions of interest in astrophysical and fusion plasmas.

The aim of this work is to study the effect of wave collapse on spectral line shapes of Lyman α, Lyman β, and Balmer α emitted by hydrogen atoms. In the following, we consider an atom submitted to a sequence of an electric field modulated by an envelope soliton, and we calculate numerically the atomic dipole autocorrelation function in the single presence of such solitons. Plasmas submitted to an energetic beam of particles or to a strong radiation can be found in many situations. In an astrophysical context, active galactic nuclei [6], pulsar radio sources [7], planetary foreshocks [8], or solar type III radio bursts [9] are possible candidates. Relevant laboratory plasmas are laser plasmas [10], radio experiments [11], or possibly also magnetic fusion plasma, since such plasmas can be affected by the energetic beams of runaway electrons [12].

The paper is organized as follows: in Section 2, we recall the main properties of turbulent Langmuir fields, and we propose a model for computing the dipole autocorrelation function (DAF) and the line shape in Section 3. We present and discuss our results on the hydrogen lines in Section 4.

2. Wave Collapse and Strong Turbulence

The physics of Langmuir turbulence have been studied in detail, since it is necessary to the understanding many radiative and transport properties of plasma. Langmuir turbulence describes a plasma state affected by a high level of excited Langmuir waves [13]. Using the Zakharov equations [14], it is possible to distinguish between weak and strong Langmuir turbulence. A useful quantity for making such a distinction is the ratio W of the wave energy density to the thermal energy density:

$$W = \varepsilon_0 E_L^2 / 4 N_e k_B T, \tag{1}$$

with T and N_e representing the hydrogen plasma temperature and density, E_L the magnitude of the wave, k_B the Boltzmann constant, and ε_0 the permittivity of free space. Beyond a critical value of W, which depends on the plasma conditions, the Langmuir waves couple with ion sound and electromagnetic waves, resulting in a strong turbulence regime. Strong Langmuir turbulence occurs in plasma submitted to an external source of energy, which may be coupled to the plasma waves, thus increasing their intensity, and allowing the start of nonlinear processes such as wave-wave interactions [15]. In this strong turbulence regime, the energy density of the waves can exceed the plasma energy density, and a large amount of energy is available. The physical process at work is the creation of low density regions by the coupling of the density fluctuations associated to the ion sound wave with the Langmuir wave. Wave packets are refracted in regions of low density, which are also regions of high refractive index. The nonlinear ponderomotive force then moves part of the plasma out of the region of maximum field value, thus starting a dynamic process where coherent wave packets evolve to shorter scales and higher intensities reaching several hundred times the average microfield $E_0 = 1/\left(4\pi\varepsilon_0 r_0^2\right)$, where r_0 is the average distance between particles defined by $r_0^3 = 3/(4\pi N_e)$. Plasma computer simulations reveal the existence of a wave packet cycle with a collapse arrested by dissipation, and a nucleation mechanism allowing the creation of new wave packets [5]. The electric field of such wave packets oscillates at a frequency close to the plasma frequency $\omega_p = \sqrt{N_e e^2 / m\varepsilon_0}$, where m is the electron mass, and the wave packet is modulated by an envelope soliton with a Gaussian or Lorentzian shape. The average duration of a cycle is an estimate for the characteristic time of strong turbulence. It can be obtained from plasma simulations such as particle in cells codes and scales as much as 40 times the inverse of the average of W [5], using units of the inverse plasma frequency. Taking account of this relation and of the expression of W given by Equation (1), the choice of a value of W also determines the values of the electric field modulus and of the average duration of a cycle for given background plasma conditions. If energy is supplied from an external source, localized wave packets may be created at a high rate, and the plasma will contain many of those coexisting wave packets. The localized wave packets appear to be densely packed, so that a large number of emitters experience the field of a wave packet.

3. Line Shape Model for Wave Collapse

In this work, we are interested in studying the effect of nonlinear wave collapse on a Stark spectral line shape of hydrogen atoms. Following our description of wave packet collapse, we assume that a large number of hydrogen atoms are submitted to the electric field of a wave packet. We propose to model the electric field of the wave by a sequence of solitons using a renewal process. The maximum magnitude of the electric field is sampled with a probability density function (PDF) that we assume to be half-normal. We jump from one soliton to the next using an exponential waiting time distribution ν exp(-νt), with the jumping frequency ν chosen as the inverse of the average duration of a cycle [3,4]. We call such a sequence of envelope solitons a single electric field history with a duration of the order of the line shape time of interest (inverse of the line width). This time is also the decorrelation time of dipole autocorrelation function (DAF), a quantity $C(t)$ defined by [16]:

$$C(t) = Tr\langle \vec{D} \cdot U^{+}(t)\vec{D}U(t)\rho\rangle, \quad (2)$$

where the trace is over the atomic states, $\vec{D}$ and U are the atomic dipole and evolution operators, ρ is the density matrix, and the angle brackets imply an average over the configurations of the perturbation. The atom perturbation dynamics are obtained by numerically solving the Schrödinger equation for the evolution operator $U(t)$ for each history. The DAF is obtained as an average over a large number (10^4) of independent electric field histories. The sampling of the stochastic variables may be done on the computer with pseudorandom number algorithms, associated to numerical techniques such as transformation or rejection methods [17,18]. The line shape is also obtained numerically using a Fourier transform of the DAF:

$$I(\omega) = \frac{1}{\pi} Re \int_0^\infty \exp(i\omega t) C(t) dt. \quad (3)$$

The calculation presented in the following concerns the hydrogen Lyman α (L_α), Lyman β (L_β), and Balmer α (H_α) lines, neglecting fine structure in order to obtain a fast numerical evaluation, and using the spherical quantum number n, l, m. For our calculations of the Balmer lines, we neglect the broadening of the lower states of the transitions. Using our simulation, it is possible to calculate the effect of Langmuir solitons alone. It is possible to compare to Stark broadening (impact approximation or ab initio) in a hydrogen plasma in equilibrium [16]. Using a convolution, it is also possible to calculate a profile taking into account both equilibrium Stark broadening and Langmuir solitons.

4. Results

We first compute the solution of the Schrödinger equation in the presence of the sequence of solitons alone. We calculate the dipole autocorrelation function $C(t)$ and the line profile for L_α, L_β, and H_α, for a density of 10^{19} m^{-3} and for a temperature of 10^5 K, conditions which can be found in the edge of a tokamak plasma. Solitons are generated for an average value of $W \approx 1.1$, resulting in a jumping frequency of $\nu \approx \omega_p/37$ and an average peak value of $150E_0$ for the electric field. After a study of the shape effect of the shape of the envelope soliton [3], and with the help of plasma simulations, we chose a soliton shape with a width equal to 20% of the wave cycle duration, a value which minimizes the broadening effect of the soliton sequence [3].

Figures 1–3 plot the DAF of L_α, L_β, and H_α for the pure Stark effect using an impact approximation (dashed line), pure soliton effect (solid line), and the product of the two preceding DAFs (dotted line). In Figure 1, a strong decay is observed on the soliton DAF of L_α (solid line) for times shorter than the average duration of a wave packet cycle. This strong initial decay can be attributed to the large magnitude of the soliton electric field. A similar behavior of the DAF for short times is also observed for L_β (Figure 2, solid line) and H_α (Figure 3, solid line). For intermediate times, the soliton DAF of L_α has a weaker decay than the Stark DAF, but the two curves are similar for long times. For times longer than the wave packet cycle, we observe for L_β and H_α a weaker decay of the soliton than the impact DAF. Small amplitude oscillations are seen on these DAF, and are in phase with the plasma frequency. In all cases, the decay of the product DAF (dotted line) is significantly larger than for the Stark DAF.

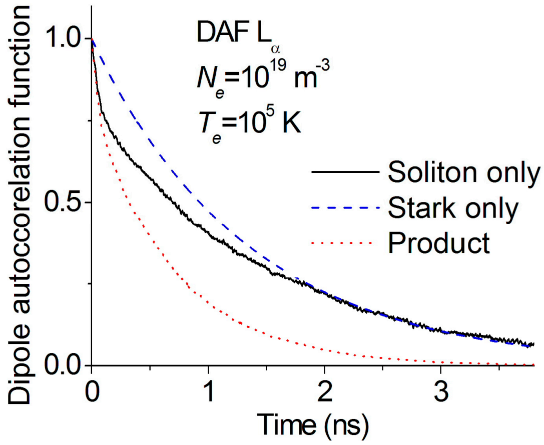

Figure 1. Dipole autocorrelation functions (DAF) for L_α submitted to 10^4 solitons sequences calculated for $W = 1.1$ (solid line) for 10,000 histories, and compared to the Stark DAF (dashed line) calculated with an impact approximation, and the product DAF (dotted line) in plasma with a density of $N_e = 10^{19}$ m^{-3} and temperature of $T_e = 10^5$ K.

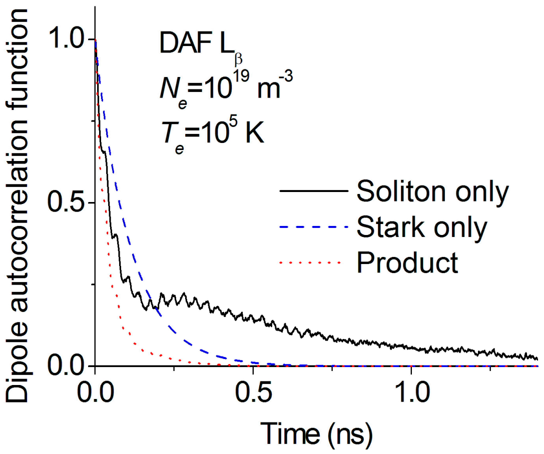

Figure 2. Same as Figure 1 for L_β.

In Figures 4–6, we present the line shapes of L_α, L_β, and H_α for the same plasma conditions. Although being especially studied for high density plasmas, detailed Stark line shapes are also needed for low density plasmas, since they enter in the modeling of radiative transfer together with Doppler broadening [19]. We calculated the line shape profile of L_α, L_β, and H_α by a Fourier transform of the dipole autocorrelation functions already discussed. In Figure 4, we found that the line profile which is only affected by solitons (solid line) is similar to the Stark profile (dashed line) for L_α. The convolution profile (dotted line) is, however, about 2.1 broader than the Stark profile. For L_β (Figure 5) and H_α (Figure 6), the profile only affected by solitons (solid line) is narrower than the pure Stark profile.

The convolution profile (dotted line) is broader by a factor 2 than the Stark profile of L_β, and broader by a factor 1.7 for H_α.

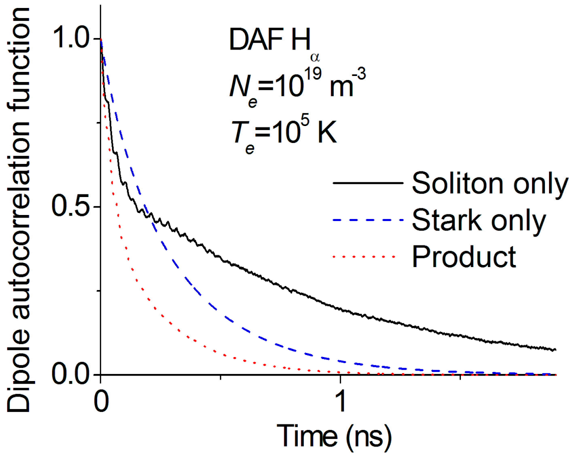

Figure 3. Same as Figure 1 for H_α.

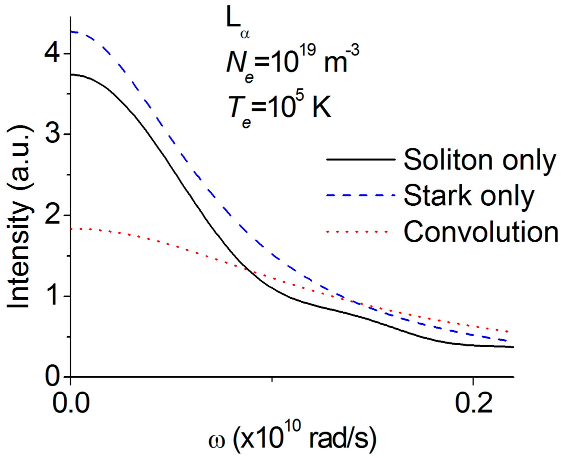

Figure 4. Line shape of L_α for soliton only with W = 1.1 (solid line), for Stark only in the impact approximation (dashed line), and compared to a convolution (dotted line) of the latter two for the plasma condition of Figure 1.

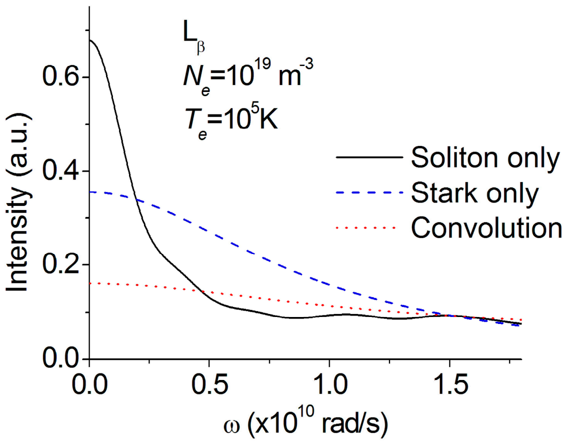

Figure 5. Same as Figure 4 for L_β.

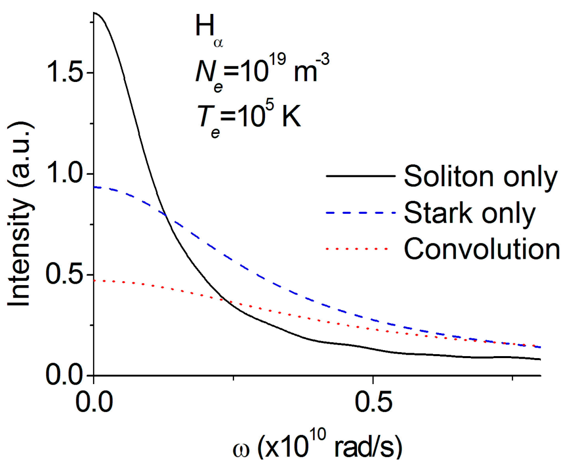

Figure 6. Same as Figure 4 for H_α.

5. Conclusions

The study of Zhakarov equations and numerous numerical simulations has revealed the complex behavior of plasma affected by strong Langmuir turbulence. The nonlinear coupling of the plasma waves creates numerous localized wave packets, subject to collapse. Each wave packet experiences a cycle during which the electric field magnitude grows to values of more than hundred E_0 ($E_0 \approx$ Holtsmark field). Using the main properties of strong Langmuir turbulence obtained from simulation calculations, we proposed a simple stochastic renewal model for the electric field of the wave packets. This model field is well-suited to study the effect of Langmuir turbulence on the line shape emitted in plasma. We used a simulation to generate field histories with a prescribed probability density function and waiting time distribution. The dipole autocorrelation function was obtained by a numerical integration of the Schrödinger equation and an average over 10^4 histories. Our calculations concern the hydrogen L_α, L_β, and H_α lines for plasma conditions for a density equal to 10^{19} m^{-3} and a temperature of 10^5 K. Strong turbulence brings a significant additional broadening to the pure Stark profile for all three lines.

In the future, we will look for other lines, plasma conditions, and wave collapse conditions, and make comparisons with line shapes observed in turbulent plasma.

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