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Circular Polarimetry of Hard X-rays with Rayleigh Scattering

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Abstract: We present a theoretical investigation of the elastic Rayleigh scattering of X-rays by atomic targets. Special attention is paid to the question of how the polarization of the scattered photons is affected if the incident light is itself polarized. In particular, we found that the circular polarization of the incoming X-rays may lead to a remarkable modification of the linear polarization of the scattered photons. Based on this 'circular-to-linear-polarization-transfer' and on the fact that the linear polarization of X-rays can be conveniently observed by solid-state Compton detectors, we argue that Rayleigh scattering may be used as a tool for circular polarimetry of hard X-rays. To illustrate our proposal, we performed detailed calculations of 145 and 500 keV circularly polarized photons scattered by lead atoms. Based on these calculations, we found that the photon scattering under large angles with respect to the incident beam direction is most favorable for the circular polarimetry of hard X-rays. In particular, for 500 keV photon energy and scattering angles around 70 deg we found a remarkable modification of the linear polarization of scattered light for the case when the incident radiation is circularly polarized.

Keywords: elastic scattering; X-rays; Rayleigh; circular polarization; polarimetry; Compton detector

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1. Introduction

Recently, great progress has been achieved in the development of polarization-sensitive X-ray detectors. In particular, state-of-the-art solid-state Compton scatterers allow for a highly precise determination of the linear polarization of photons in the energy range between about 60 keV up to 1 MeV [1,2]. It was shown that the uncertainty of the polarization reconstruction using such polarimeters is mainly limited by the statistics of registered Compton scattering events in the detector and can be reduced to below 1% [3]. These polarimeters were successfully used to investigate fundamental processes that occur in collisions of highly charged ions with photons, electrons and atoms. In particular, the polarization properties of X-rays emitted in radiative electron capture, dielectronic recombination, bremsstrahlung and Rayleigh scattering were analyzed using Compton detectors [4–9]. These polarization measurements gained valuable information about the physics of electronic systems in the presence of strong electromagnetic fields.

In contrast to linear polarization, much less progress has been made so far in the development of circular X-ray polarimetry techniques. Some proposals based on Compton scattering off polarized electrons were made in the past [10–15] but the drawbacks of this method, such as the need for magnetized targets and the rivalry between solid-angle coverage and angular resolution, were not yet overcome [16]. Recently, another proposal was made, based on the spin transfer from the photon to the electron during Compton scattering [16] but this proposal was not yet realized in practice.

In this contribution, we propose yet another approach to measure the circular polarization of X-rays and discuss its feasibility. This approach is based on the application of the elastic Rayleigh scattering of X-rays by atomic targets. During the recent years, Rayleigh scattering has been studied in detail for the energy range from 17 to 889 keV and for various target atoms both in low-Z and high-Z domains [17–20]. These studies focused on the linear polarization of the elastically scattered photons which were measured by means of Compton detectors. In the present work we show that Rayleigh scattering may also lead to the partial transfer of the circular polarization of incident X-rays to linear polarization of the scattered photons, where the latter can be measured by conventional Compton detectors.

The article is organized as follows. In Section 2, we first introduce the geometry of the process. Then, we discuss the Rayleigh scattering amplitudes and introduce the Stokes parameters in order to describe the polarization transfer between incident and outgoing photons. In Section 3, we use the derived formulas to analyze the scattering of 145 and 500 keV photons by a lead atom. We show, in particular, that the linear polarization of the scattered photons can be remarkably affected if the incoming light is circularly polarized and that this effect can be observed with current Compton scatterers. A brief summary of our results is given finally in Section 4. Atomic units ($\hbar = e = m_e = 1$) are used throughout this article unless stated otherwise.

2. Theoretical Background

2.1. Geometry

To provide a description of the polarization transfer in Rayleigh scattering, we first discuss the geometry of the process. For the theoretical analysis, it is convenient to define the so-called scattering (xz) plane, spanned by the wave vectors $\boldsymbol{k}_i$ and $\boldsymbol{k}_f$ of the incoming and outgoing photons, see Figure 1. The propagation direction of the initial photon is chosen parallel to the z axis and the scattered photon is emitted under the polar angle θ. In our study, we focus on the case when the incident light is circularly polarized, while the linear polarization of the scattered radiation is detected. The linear polarization vector of the outgoing photon does not necessarily lie in the scattering plane. It can be tilted with respect to this plane by an angle χ.

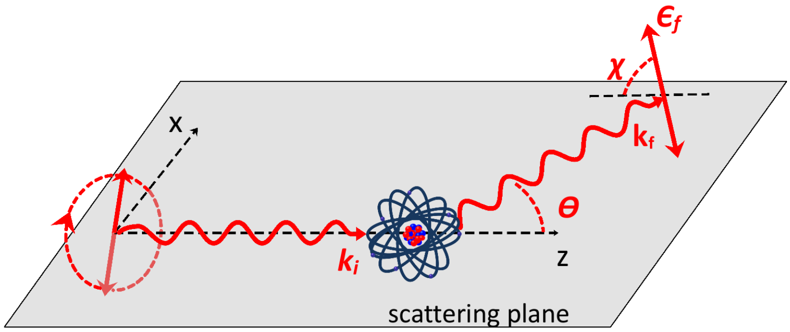

Figure 1. The geometry of the elastic photon-atom scattering. The scattering plane, chosen as xz plane, is defined by the wave vectors of the incoming and outgoing photons. The propagation direction of the incident photon is taken along the z axis. We are interested especially in the case when incident light is circularly polarized, and the linear polarization of the scattered radiation is detected.

2.2. Scattering Amplitudes

The main 'building block' to describe the Rayleigh scattering is the scattering amplitude. Within the framework of second-order perturbation theory, this amplitude can be written as:

$$\begin{aligned} A_{fi}(\boldsymbol{\epsilon}_i, \boldsymbol{\epsilon}_f) = \alpha^2 \sum\!\!\!\!\!\!\int_\nu \Bigg\{ & \frac{\left\langle \psi_f \middle| \sum_{j=1}^N \boldsymbol{\alpha}_j \boldsymbol{\epsilon}_f e^{-i\boldsymbol{k}_f \boldsymbol{r}_j} \middle| \psi_\nu \right\rangle \left\langle \psi_\nu \middle| \sum_{j=1}^N \boldsymbol{\alpha}_j \boldsymbol{\epsilon}_i e^{i\boldsymbol{k}_i \boldsymbol{r}_j} \middle| \psi_i \right\rangle}{E_i + \omega_i - E_\nu} \\ & + \frac{\left\langle \psi_f \middle| \sum_{j=1}^N \boldsymbol{\alpha}_j \boldsymbol{\epsilon}_i e^{i\boldsymbol{k}_i \boldsymbol{r}_j} \middle| \psi_\nu \right\rangle \left\langle \psi_\nu \middle| \sum_{j=1}^N \boldsymbol{\alpha}_j \boldsymbol{\epsilon}_f e^{-i\boldsymbol{k}_f \boldsymbol{r}_j} \middle| \psi_i \right\rangle}{E_i - \omega_f - E_\nu} \Bigg\}, \end{aligned} \tag{1}$$

where α is the fine structure constant and $\boldsymbol{\alpha}_j$ and $\boldsymbol{r}_j$ are the vector of Dirac matrixes and the coordinate vector of the jth electron in a target atom [21]. The polarization vectors and energies of the incoming and outgoing photon are denoted as $\boldsymbol{\epsilon}_{k,f}$ and $\omega_i = \omega_f$. The initial, intermediate and final atomic states $|\psi\rangle$ and their energies E are identified by the indices i, ν and f, respectively.

In our study, we will focus on scattering off closed-shell atoms. The scattering amplitudes for such systems provide a crucial symmetry property: The entire scattering process can be described by only two linearly independent amplitudes. Due to historical reasons, the amplitudes for parallel and perpendicular linear polarizations with respect to the scattering plane are used [17]. They are usually denoted as $A_\parallel = A_{fi}(\boldsymbol{\epsilon}_i \parallel xz \text{ plane}, \boldsymbol{\epsilon}_f \parallel xz \text{ plane})$ and $A_\perp = A_{fi}(\boldsymbol{\epsilon}_i \perp xz \text{ plane}, \boldsymbol{\epsilon}_f \perp xz \text{ plane})$. The ratio of these two amplitudes $S = A_\parallel / A_\perp$ is a very practical quantity to describe many properties of the process see for instance Ref. [22] for further details.

For many-electron systems, the evaluation of the scattering amplitude (1) is a very complicated task. In our calculations, we approach this issue using the independent particle approximation (IPA) which is known to work very well in the high energy regime [23]. In the IPA, we calculate first the single-electron amplitudes assuming that only one electron interacts with the photon while the others remain 'frozen'. The many-electron amplitude is then the sum over the single-electron amplitudes, including all occupied electron shells. We do not discuss the further details for the evaluation of the scattering amplitude here as it was described in our previous works [22,24] as well as in the literature [17,25–27].

2.3. Stokes Parameters

With the help of the scattering amplitude (1), we are able to investigate all the properties of the scattering process. In this work in particular, we are interested to study how the polarizations of incoming and outgoing photons are related to each other. Such a polarization transfer can be described most conveniently with the Stokes parameters. For linear polarization of light, the Stokes parameters are defined as

$$P_1 = \frac{I(0^\circ) - I(90^\circ)}{I(0^\circ) + I(90^\circ)}, \tag{2}$$

$$P_2 = \frac{I(45^\circ) - I(135^\circ)}{I(45^\circ) + I(135^\circ)}. \tag{3}$$

In these equations, $I(\chi)$ describes the intensity of light linearly polarized under a certain angle χ with respect to the xz plane. Similarly, we can define the degree of circular polarization

$$P_3 = \frac{I(\lambda = +1) - I(\lambda = -1)}{I(\lambda = +1) + I(\lambda = -1)}, \tag{4}$$

with I being the intensity of right- ($\lambda = +1$) or left-handed ($\lambda = -1$) circularly polarized light. In our recent study [24], we derived equations which relate the Stokes parameters of the incoming light (denoted by index i below) to their counterparts of the scattered light (index f):

$$P_{1f} = \frac{|S|^2(P_{1i} + 1) - (1 - P_{1i})}{|S|^2(P_{1i} + 1) + (1 - P_{1i})}, \tag{5}$$

$$P_{2f} = \frac{2P_{2i}\mathrm{Re}(S) + 2P_{3i}\mathrm{Im}(S)}{|S|^2(P_{1i} + 1) + (1 - P_{1i})}, \tag{6}$$

where S is the ratio of the scattering amplitudes as mentioned above. Here we restricted our analysis to the parameters P_{1f} and P_{2f} which describe the linear polarization of the scattered light. We did not include a consideration of the circular polarization of the scattered light (P_{3f}), as modern Compton polarimeters are insensitive to this parameter.

As seen from Equations (5) and (6), the Stokes parameters of the scattered light P_{1f} and P_{2f} exhibit a qualitatively different sensitivity to the circular polarization of the incident light. While P_{1f} is independent on P_{3i}, the second Stokes parameter is proportional to the circular polarization component (if $P_{2i} = 0$). As we will see below, this sensitivity $P_{2f}(P_{3i})$ opens a way for circular polarimetry with the help of Rayleigh scattering.

Another important observable of the Rayleigh scattering process is the differential cross section. If the outgoing polarization is not observed, it is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}(|A_{\parallel}|^2 + |A_{\perp}|^2) + \frac{1}{4}P_{1i}(|A_{\parallel}|^2 - |A_{\perp}|^2), \tag{7}$$

and is independent on the circular polarization of the incoming photon [25].

2.4. Competing Processes

In experiments, Rayleigh scattering cannot be separated from the two other elastic channels, Delbrück and nuclear Compton scattering. Nevertheless, in this work we will not consider these two competing processes, since their contribution to the cross section is negligible for the parameter space considered here. For example, using the Delbrück amplitudes obtained in Refs. [28,29] and the low-energy ω^2 scaling, we estimate that the Delbrück contribution to the cross section is in the order of less than 0.1% for 145 keV and approximately 1% for 500 keV. Moreover, using analytical formulas for nuclear Compton scattering [30], we find that its contribution is smaller than 0.01%.

3. Results

By making use of Equations (5)–(7), we are ready to investigate the Rayleigh scattering of circularly polarized X-rays by an atomic target. In this work, we performed calculations for the scattering of photons with energies of 145 keV and 500 keV by a lead atom. This particular choice of energies and target was proposed by previous theoretical and experimental studies [6,17,31]. To find the scattering amplitudes and hence, the amplitude ratio S, we employed the IPA as discussed in Section 2.2 and restricted our calculations to the scattering from K and L shells only. Such an approximation is well justified since the scattering by higher shells is known to contribute only for very small scattering angles which are typically not approached in modern experiments [6,27,32].

In Figure 2, we display the differential cross sections as well as the Stokes parameters P_{1f} and P_{2f} for the case when the incident light is completely right-handed circularly polarized ($P_{3i} = 1$). As we mentioned already above, only the second Stokes parameter P_{2f} is sensitive to the incident circular polarization and, hence, can be used to measure P_{3i}. As seen from the figure, P_{2f} reaches the maximum value 0.112 for the energy 145 keV and 0.142 for 500 keV photons. To observe these rather remarkable values of P_{2f}, one needs to place the Compton detectors at the angles 85° and 70°, respectively. As seen from the left panel of Figure 2, the Rayleigh scattering cross section reaches 0.16 barn and 6×10^{-3} barn at these angles and for the energies 145 keV and 500 keV. These relatively large cross sections make the measurement of the polarization Stokes parameter P_{2f} feasible. Please note that for higher photon energies, larger values of the second Stokes parameter are achieved, and this might even partially compensate for the reduced statistics due to the lower differential cross section.

To study the sensitivity of P_{2f} to the initial circular polarization, we show in Figure 3 the second Stokes parameter of the outgoing photons for $P_{3i} = 1, 0.7, 0.4$. As seen from the figure, we observe a strong dependence $P_{2f}(P_{3i})$, for example if P_{3i} is varied from 1 to 0.4, the Stokes parameter P_{2f} changes from 0.142 to 0.05. This modification can be measured by state-of-the-art Compton polarimeters which makes the Rayleigh scattering a promising tool for circular polarimetry, when taking also the theoretically calculated amplitude ratio into account.

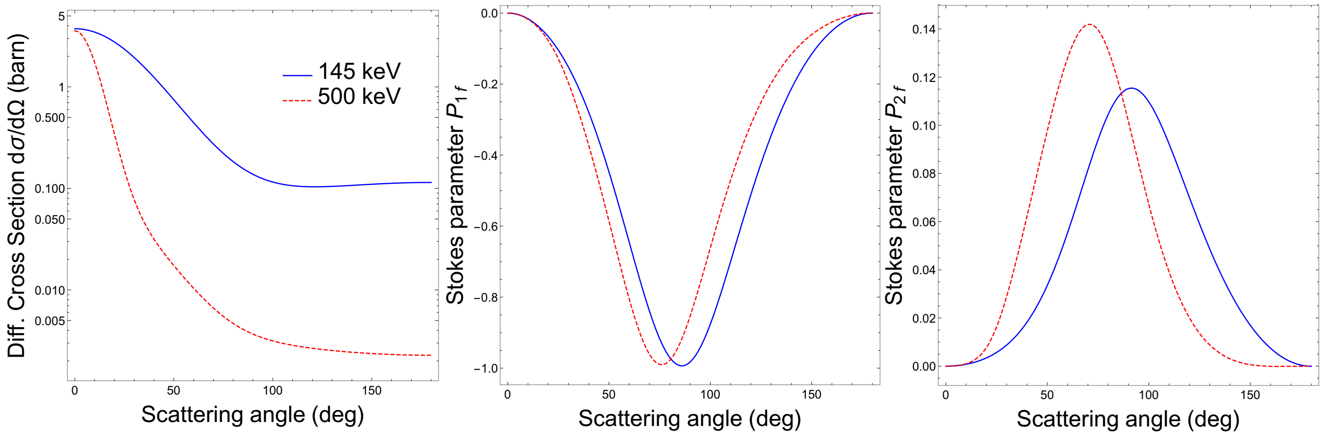

Figure 2. Rows from left to right: Differential cross sections and Stokes parameters P_{1f} and P_{2f} for the Rayleigh scattering of 145 keV (red dashed line) and 500 keV (blue solid line) circularly polarized photons off a lead target atom. The theoretical predictions are presented as a function of the polar photon scattering angle θ.

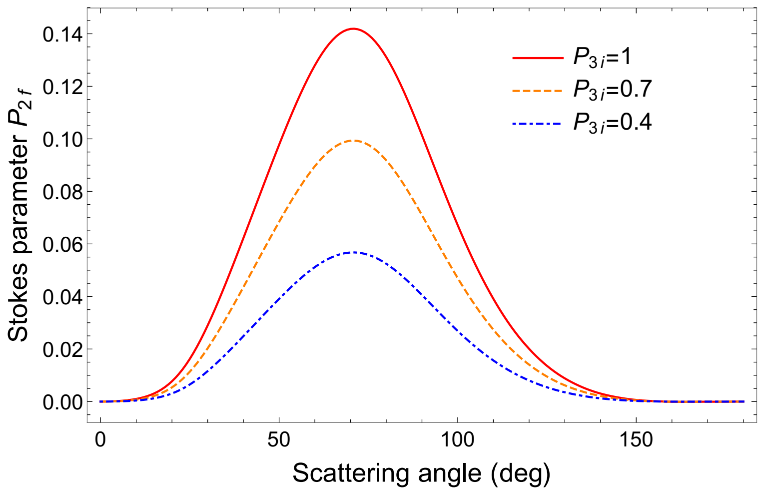

Figure 3. The second Stokes parameter P_{2f} of the outgoing photons for the Rayleigh scattering of circularly polarized 500 keV photons by a lead atom. Calculations were performed for different degrees of circular polarization of the incident light: P_{3i} = 1 (red solid line), P_{3i} = 0.7 (orange dashed line), P_{3i} = 0.4 (blue dash-dotted line).

4. Conclusions

We re-investigated the elastic Rayleigh scattering of X-rays by atomic targets. We paid special attention to the polarization transfer between the incident and the outgoing photons. In particular, we found that the circular polarization of incident light can result in a non-zero linear polarization of the scattered light, orientated in such a way that the polarization vector points out of the scattering plane. To illustrate this 'circular-to-linear-polarization-transfer' effect, we performed calculations for 145 keV and 500 keV photons, scattered by lead atoms. For the case of a fully right-handed circularly polarized 500 keV photon beam, the second Stokes parameter that characterizes the out-of-plane linear polarization of the scattered photons can reach 0.142 which can be detected by modern Compton detectors. Thus, we show that elastic X-ray scattering can be potentially used for circular polarimetry in the previously problematic region of few hundred keV.

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