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Review

Quantum Imprints on CMBR

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Abstract: Quantum cosmology aims to develop a quantum theory of the universe, attempting to answer open questions of physical cosmology, mainly related to the early epochs of the universe. Such a theory aims to unite relativity theory and quantum theory. Here, the whole universe is treated as a quantum mechanical system and is described by a wave function rather than by a classical spacetime. In this review, I shall describe the mathematical structure and primary formulations that form the backbone of quantum cosmology. We know that over a period of time, several approaches were developed to form a quantum theory of gravity. However, in order to decide which approach is the best, we need testable predictions, effects that can be observed in cosmic microwave background radiation (CMBR). I shall discuss the methodologies for generating quantum gravitational corrections to inflationary background leading to testable predictions. Another aspect of finding quantum imprints on CMBR results through the application of resolution of the 'quantum measurement problem' to early universe physics. In this article, I shall also discuss two such promising models explaining the classicalization of inflationary perturbation and are capable of leaving distinct observational imprints on the observables.

Keywords: quantum cosmology; inflation; cosmic microwave background; Wheeler–DeWitt

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Citation: Banerjee, S. Quantum Imprints on CMBR. *Universe* **2023**, *9*, 405. https://doi.org/10.3390/universe9090405

Academic Editor: Joao Marto

Received: 13 June 2023
Revised: 30 August 2023
Accepted: 31 August 2023
Published: 4 September 2023

1. Introduction

Quantum cosmology is an endeavor to develop a quantum theory of the universe, describing the first phases of the universe and also answering open questions. Such a theory aims to integrate relativity with quantum mechanics. To beginwith, here, the whole universe is described by a quantum mechanical wave function. The process of quantization is not straightforward. The main idea leading to this unification was put forward in the inventive work of Bryce DeWitt [1] in 1967. Later, around 1983, James Hartle and Stephen Hawking proposed the well-known 'no-boundary proposal' [2]. Through these works and many more [2–9], the emergence of the universe out of nothing (no matter or space-time) came into the picture. For a more general review of quantum cosmology, one may refer to [10–14], where properties like the problem of time, singularity avoidance, and boundary conditions in the context of quantum cosmology have been discussed in detail.

Apart from many formal and mathematical problems, conceptual problems also form a major obstacle to the final construction of a quantum theory of gravity and its application to cosmology. A detailed discussion of these aspects is given in [15].

The direct quantization of general relativity leads to a non-renormalizable theory at the perturbative level. There are several non-perturbative approaches present. However, they still lack a complete form. Thus, one can instead look for observational hints that could play a role similar to what the Lamb shift played for the development of quantum electrodynamics.

Around 1980 with the advent of the inflationary scenario [16,17], quantum cosmology was also studied in the context of inflation [18–21]. Observables in the context of inflation have been widely constrained by observational surveys like PLANCK, BICEP, KECK, etc.

The current constraints on inflationary observables from Planck TT, TE, EE + lowE + lensing + BK15 [22] can be quoted as

$$r_{0.002} < 0.065; \quad n_s = 0.9639 \pm 0.0044; \quad \alpha_s = dn_s/dlnk = -0.0069 \pm 0.0069 \tag{1}$$

When running of the scalar spectral index (β_s) is allowed to float, the constraints on n_s, α_s, and β_s can be quoted as

$$n_s = 0.9587 \pm 0.0056; \quad \alpha_s = 0.013 \pm 0.012; \quad \beta_s = 0.022 \pm 0.012 \tag{2}$$

We know that over a period of time, several approaches were developed to form a quantum theory of gravity [23–26]. However, in order to decide which approach is correct, we need testable predictions, effects that can be observed in cosmic microwave background radiation (CMBR). Nonetheless, finding such predictions becomes difficult since quantum gravitation effects become significant only around Planck-scale energy. Inflationary cosmology provides the perfect scenario to visualize such effects. This is a conservative approach where the quantum equation, known as the Wheeler–DeWitt (WDW) equation, leads to general relativity in the semi-classical limit. The WDW equation may not yield the ultimate quantum theory of gravity; however, it should provide a reliable and approximately valid picture, at least close to the Planck scale [27].

In the works by Kiefer and his colleagues [28–31], quantum gravitational corrections to the CMBR power spectrum in the context of the WDW equation have been studied extensively.

In Section 2 of this article, I will review the main results obtained in this approach and discuss the presence of possible quantum effects in the anisotropy spectrum of the CMBR. The calculations show that the quantum gravitational correction terms lead to a modification of the anisotropy power spectrum in a way that leads to an enhancement of power at large scales.

One can also compare these results with the predictions from other approaches to quantum gravity. Suppression of power at large scales has been predicted by non-commutative geometry and effects from string theory [23,24,32–34]. A similar investigation for loop quantum cosmology in [35] shows an enhancement of the power at large scales, while in [36] it shows a suppression. Apart from loop quantum cosmology, an application of supersymmetric quantum cosmology [37,38] to such a situation can also be found in [39]. We thus can see that such considerations may be able to discriminate between different approaches to quantum gravity, but there also exists degeneracy among them.

In Section 3, I will review another different aspect of finding quantum imprints on CMBR. This results from treating foundational issues in quantum mechanics, such as the emergence of classical behavior and its application in early universe physics. Considering the inhomogeneities were of quantum origin, the mechanism of generating the primordial inhomogeneities by inflation is essentially evolving the quantum fluctuations of a scalar field. This opens an exciting possibility of directly observing the outcome of a genuine quantum gravitational effect: the generation of quasi-classical fluctuations of quantum fields, i.e., 'particle creation from the vacuum in a background gravitational field' [40]. However, the key problem here is that though the process of creation from the vacuum and the perturbations themselves are purely of a quantum mechanical nature (at least initially), the observed temperature or density fluctuations in the universe are certainly classical [41–43]. Thus, a complete derivation should include some mechanism of quantum-to-classical transition and collapse of the wave function describing the perturbations.

Going by the Copenhagen interpretation [44,45] of quantum mechanics, this means that the wave function describing the perturbations has collapsed to one of its eigenstates (in this case, the eigenstate of the inflaton field) and the CMBR map corresponds to one of its eigenvalues. However, instantly the question arises regarding the process or the act of measurement in the early universe. There seems to be a missing link between the quantum early universe and the classical structures. This then leads us to the problem of understanding quantum to classical transition in the cosmological context, which can be thought of as a more serious form of the so-called 'quantum measurement problem' [46–48] already present in conventional laboratory situations [49].

This issue has been studied widely for a better understanding of this problem leading to suitable solutions. In Section 3, I will discuss two of such approaches, decoherence and continuous spontaneous collapse, leading to an explanation of quantum to classical transition in the context of the early universe. It was found that such scenarios are also capable of leaving distinct signatures on inflationary power spectra and observables. For example, in the presence of a collapse mechanism, we find that independent accurate measurements of r and n_T would give us a direct handle on the model parameter, leading to its falsification. Since the next-generation CMBR experiments are expected to shed more light on the constraints on r, single-field consistency relation, etc., such models thus provide promising testing scenarios.

2. Quantum Gravitational Corrections in Quantum Cosmology

Here, I will mainly discuss the works by Kiefer and his colleagues [28–31]. For this, one needs to analyze the WDW equation in the context of quantum field theory in curved spacetime. This is achieved in a Born–Oppenheimer type of approximation scheme, as described in [50]. This scheme is then realized by expanding the wave function with respect to the Planck mass [28,29].

The advantage of this approximation is that at consecutive orders, one retrieves first the dynamics of the background spacetime (classical), then a Schr´odinger equation for the perturbations propagating on this background, and finally, quantum gravitational corrections to it containing correction terms [28]. Their interpretation in terms of Feynman diagrams and generalization to supergravity can be found in [51,52]. Earlier, this approximation was already used to derive such corrections from the Planck mass expansion [28–31,53,54] and from an alternative expansion [55–58].

In order to describe the mathematical structure one needs to begin with the usual decomposition of the Hamiltonian into a perpendicular component and three components tangential to the spatial hypersurfaces with all four components being constrained to vanish. If $\Psi(h_{ab})$ is the wave functional of the universe with h_{ab} denoting the three-metric, the four constraints can be written as

$$\widehat{\mathcal{H}}_a \Psi(h_{ab}) = 0\,, \qquad \widehat{\mathcal{H}}_\perp \Psi(h_{ab}) = 0\,, \tag{3}$$

The first equation is the Hamiltonian constraint, popularly known as the Wheeler–DeWitt (WDW) equation. The other equations are known as diffeomorphism (or momentum) constraints.

It is important to note that this equation is timeless; it does not contain any external time parameter [59]. However, the WDW equation should be valid at least as an effective equation, because it gives the correct semi-classical limit.

In order to quantize them, we use the following quantum operators

$$\widehat{h}_{ab}\Psi(h_{ab}) = h_{ab}(\mathbf{x})\cdot\Psi(h_{ab})\,, \qquad \widehat{p}_{cd}\Psi(h_{ab}) = \frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{cd}(\mathbf{x})}(h_{ab})\,, \tag{4}$$

Using these operator forms, the constraint equations now read as

$$\left(-16\pi G\hbar^2 G_{abcd}\frac{\delta^2}{\delta h_{ab}\delta h_{cd}} - \frac{\sqrt{h}}{16\pi G}\left({}^{(3)}R - 2\Lambda\right)\right)\Psi = 0\,, \tag{5}$$

$$-2D_b h_{ac}\frac{\hbar}{\mathrm{i}}\frac{\delta\Psi}{\delta h_{bc}} = 0\,. \tag{6}$$

Here, G_{abcd} is the DeWitt metric, Λ is the cosmological constant, and D_b is the covariant derivative of the three-metric. The first term in the first equation has the structure of kinetic energy, while the second term represents the potential. The second equation has the form similar to the Gauss constraint in electrodynamics.

Now, we consider a homogeneous, isotropic FLRW universe filled with a perfect cosmological fluid with curvature index $K = \pm 1, 0$. This is an example of a mini-superspace model [60,61], an approximation of the otherwise infinite-dimensional phase space of a field theory. The WDW equation becomes

$$\begin{aligned} \hat{\mathcal{H}}\Psi &= \Bigg(\frac{2\pi G\hbar^2}{3}\frac{\partial^2}{\partial\zeta^2} - \frac{\hbar^2}{2}\frac{\partial^2}{\partial\phi^2} \\ &\quad + e^{6\zeta}\left(\mathcal{V}(\phi) + \frac{\Lambda}{8\pi G}\right) - 3e^{4\zeta}\frac{k}{8\pi G}\Bigg)\Psi(\zeta,\phi) = 0\,, \end{aligned} \tag{7}$$

where $\zeta = \ln a$, with a being the scale factor. This convention is used only to represent the equation in a convenient form. Here, we have introduced a massive minimally coupled scalar field ϕ, playing the role of the inflaton, with potential $\mathcal{V}(\phi)$.

In [30,31,53], this semi-classical Planck mass expansion scheme was implemented in the context of WDW equation for gauge invariant scalar and tensor inflationary perturbations. Below, I have highlighted some of the results of this approach. The perturbed action for the scalar perturbations in terms of the Mukhanov–Sasaki variable v can be written as

$$\frac{1}{2}\,\delta^2 S = \frac{1}{2}\int d\tau\, d^3\mathbf{x}\left[\left(v'\right)^2 - \delta^{ij}\,\partial_i v\,\partial_j v + \frac{z''}{z}\,v^2\right]. \tag{8}$$

where $z = a\sqrt{\epsilon}$ with ϵ being the first slow-roll parameter. Performing similar analysis for tensor part and then going over to the Fourier space, one can write the complete WDW equation (along with the background equation given by Equation (7)) for scalar and tensor perturbations each mode $\mathbf{k}$ as,

$$\frac{1}{2}\left\{e^{-2\zeta}\left[\frac{1}{m_{\mathrm{P}}^2}\frac{\partial^2}{\partial\zeta^2} - \frac{\partial^2}{\partial\phi^2} + 2\,e^{6\zeta}\,\mathcal{V}(\phi)\right] - \frac{\partial^2}{\partial v_{\mathbf{k}}^2} + \omega_{\mathbf{k}}^2(\tau)\,v_{\mathbf{k}}^2\right\}\Psi_{\mathbf{k}}(\zeta,\phi,v_{\mathbf{k}}) = 0\,. \tag{9}$$

where the frequencies ${}^{\mathrm{S}}\omega$ and ${}^{\mathrm{T}}\omega$ corresponding to the scalar and tensor parts are, respectively, given by

$${}^{\mathrm{S}}\omega_{\mathbf{k}}^2(\tau) := k^2 - \frac{z''}{z}\,, \quad {}^{\mathrm{T}}\omega_{\mathbf{k}}^2(\tau) := k^2 - \frac{a''}{a}\,, \tag{10}$$

with $z := a\,\phi'/\mathcal{H}$.

Here, m_{P} represents the rescaled Planck mass with respect to which the semi-classical expansion is performed. It reads as

$$m_{\mathrm{P}}^2 := \frac{3}{4\pi G}\,. \tag{11}$$

Next, we apply the semi-classical approximation in this context. For this, we use the following WKB-type ansatz for the wave functional

$$\Psi_{\mathbf{k}}(q^A, v_{\mathbf{k}}) = \mathrm{e}^{\mathrm{i}\,U(\zeta,\tilde{\phi},\psi_{\mathbf{k}})}\,, \tag{12}$$

with $U(\zeta,\tilde{\phi},\psi_{\mathbf{k}})$ again expanded in powers of m_{P}^2 as

$$U(\zeta,\tilde{\phi},\psi_{\mathbf{k}}) = m_{\mathrm{P}}^2\,U_0 + m_{\mathrm{P}}^0\,U_1 + m_{\mathrm{P}}^{-2}\,U_2 + \ldots\,. \tag{13}$$

where U_0 and U_1 represent the wave function at the corresponding orders.

Using this ansatz in the WDW, at order $m_{\rm P}^2$, one obtains the Hamilton–Jacobi equation of the mini-superspace background.

$$-\left(\frac{\partial U_0}{\partial \zeta}\right)^2 + m_{\rm P}^2\left(\frac{\partial U_0}{\partial \phi}\right)^2 + \frac{2\,e^{6\zeta}}{m_{\rm P}^2}\,\mathcal{V}(\phi) = 0\,. \tag{14}$$

At the order, $m_{\rm P}^0$, one retrieves the Schrödinger equation

$$\mathcal{H}_{\mathbf{k}}\psi_{\mathbf{k}}^{(0)} = i\frac{\partial}{\partial\tau}\psi_{\mathbf{k}}^{(0)}, \tag{15}$$

where τ is the conformal time defined as $\frac{\partial}{\partial\tau} = e^{-2\zeta}\left[-\frac{\partial U_0}{\partial\zeta}\frac{\partial}{\partial\zeta} + m_{\rm P}^2\frac{\partial U_0}{\partial\phi}\frac{\partial}{\partial\zeta}\right]$, with the perturbative Hamiltonian operator for each mode, given as

$$\mathcal{H}_{\mathbf{k}} := -\frac{1}{2}\frac{\partial^2}{\partial v_{\mathbf{k}}^2} + \frac{1}{2}\,\omega_{\mathbf{k}}^2(\tau)\,v_{\mathbf{k}}^2. \tag{16}$$

Finally, at the next order, $m_{\rm P}^{-2}$, one obtains the effects of quantum gravitational correction in the form of the corrected Schrödinger equation:

$$i\frac{\partial}{\partial\tau}\psi_{\mathbf{k}}^{(1)} = \mathcal{H}_{\mathbf{k}}\psi_{\mathbf{k}}^{(1)} - \frac{\psi_{\mathbf{k}}^{(1)}}{2\,m_{\rm P}^2\,\psi_{\mathbf{k}}^{(0)}}\quad\left[\frac{\left(\mathcal{H}_{\mathbf{k}}\right)^2}{V}\,\psi_{\mathbf{k}}^{(0)} + i\,\frac{\partial}{\partial\tau}\left(\frac{\mathcal{H}_{\mathbf{k}}}{V}\right)\psi_{\mathbf{k}}^{(0)}\right]. \tag{17}$$

In [31], the authors analyze these equations in the context of gauge invariant inflationary perturbations-both tensor and scalar. The primary results of their work are encoded in the following modified power spectra for gauge-invariant scalar and tensor modes:

$$\mathcal{P}_{\rm s}^{(1)}(k) = \mathcal{P}_{\rm s}^{(0)}(k)\left\{1+\delta_{\rm s}\right\}, \tag{18}$$

$$\mathcal{P}_{\rm T}^{(1)}(k) = \mathcal{P}_{\rm T}^{(0)}(k)\left\{1+\delta_{\rm T}\right\}, \tag{19}$$

Here, $\mathcal{P}_{\rm s}^{(0)}(k)$ and $\mathcal{P}_{\rm T}^{(0)}(k)$ correspond to the standard power spectra derived for fields propagating on fixed cosmological backgrounds [62]. $\delta_{\rm s}$ and $\delta_{\rm T}$ represent the quantum gravitational corrections on inflationary power spectra given by

$$\delta_{\rm s} = \frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k}\right)^3\left(0.988 + 3.14\,\epsilon - 2.56\,\eta\right);\delta_{\rm T} = \frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k}\right)^3\left(0.988 + 0.58\,\epsilon\right). \tag{20}$$

Here, ϵ and η are the slow-roll parameters.
The corrected inflationary observables are now given by the following expressions

$$n_{\rm s} - 1 = 2\eta - 4\epsilon - \frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k_*}\right)^3(2.96 + 11.40\,\epsilon - 7.68\,\eta), \tag{21}$$

$$n_{\rm T} = -2\epsilon - \frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k_*}\right)^3(2.96 + 3.72\,\epsilon) \tag{22}$$

$$\alpha_{\rm s} \approx 4\epsilon(\eta-\epsilon) - 2\theta + \frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k_*}\right)^3(8.89 + 40.12\,\epsilon - 23.04\,\eta) \tag{23}$$

$$\alpha_{\rm T} \approx 4\epsilon(\eta-\epsilon) + \frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k_*}\right)^3(8.89 + 17.08\,\epsilon) \tag{24}$$

$$r^{(1)} \approx 16\epsilon\left(1 + 2.56\,\frac{H_k^2}{m_{\rm P}^2}\left(\frac{\bar{k}}{k}\right)^3(\eta-\epsilon)\right) \tag{25}$$

with θ being the second-order slow-roll parameter defined as $\theta := \frac{\dot{\epsilon}-\dot{\eta}}{H}$

Relating $\bar{k}$ to the largest scale that could influence the CMBR ($\bar{k} \approx 10^{-4}\,\mathrm{Mpc}^{-1}$) and using experimental constraints on the spectral indices and slow-roll parameters [63], one obtains an estimate for the upper limits of the quantum gravity correction for scalar and tensor perturbations.

$$|\delta_{\mathrm{s}}| \quad \lesssim \quad 2 \times 10^{-10}, \quad |\delta_{\mathrm{T}}| \lesssim 2 \times 10^{-10}. \tag{26}$$

From this, one obtains the corrections to the spectral indices and scalar tensor ratio (evaluated at $k = \bar{k}$) are significantly smaller than the statistical uncertainty in the Planck data

$$\left[n_{\mathrm{s}}^{(1)} - n_{\mathrm{s}}^{(0)}\right]_{k=\bar{k}} \quad \approx \quad -5 \times 10^{-10} \tag{27}$$

$$\left[n_{\mathrm{T}}^{(1)} - n_{\mathrm{T}}^{(0)}\right]_{k=\bar{k}} \quad \approx \quad -5 \times 10^{-10} \tag{28}$$

$$\left[\frac{r^{(1)} - r^{(0)}}{r^{(0)}}\right]_{k=\bar{k}} \quad \approx \quad -4 \times 10^{-12} \tag{29}$$

An effect of the order of 10^{-10} cannot be detected by CBMR experiments due to the presence of cosmic variance at large scales. As a result, the corrections to all the observable power spectra are well inside the current experimental error bars. More precision measurements in the future are required to detect/falsify such quantum gravitational corrections arising from semi-classical approximations, or one can test other inflationary models for larger quantum corrections.

As the authors mention, there are still certain unresolved issues remaining from the above analysis. Firstly, the quantum gravitational correction term is proportional to k^{-3}, thus mildly violating scale invariance. Secondly is the presence of a volume $1/\bar{k}^3$ to regularize the spatial integral in the action due to which a length scale needs to be considered, and the power spectrum becomes dependent on it. Fixing it with proper physical understanding becomes difficult.

Apart from these subtleties, there still remain some open questions. One is yet to test such corrections in the presence of a non-Bunch-Davies vacuum as an initial condition. Such corrections are shown to modify the tensor modes also, leading to suppression of the tensor-to-scalar ratio and modified consistency relations. One can find detailed analysis based on such modified initial conditions in [64–66] and the references therein. Studies showing other quantum gravity effects on tensor modes can also be found in [52,67] and the references therein. One also needs to check whether such corrections are present in situations where cosmic variance is not present, like in galaxy–galaxy correlation functions. Another avenue for improving the strength of quantum gravitational corrections might be to apply the above analysis in the context of Starobinsky inflation, non-local theories, or other modified gravity theories like $f(R)$, $f(G)$, to name a few. Predictions of such scenarios are also highly model-dependent, commonly seen in any inflationary and reheating analysis. The actual realization of the inflaton is highly model dependent; even for a single inflaton field, there are a lot of valid possible potentials. How the energy of the inflaton obtained is converted into radiation at the end of inflation is also highly model dependent. The outcome, therefore, may lead to degeneracy among quantum gravity effects coming from different approaches (as discussed in Section 1).

3. Quantum to Classical Transition of Primordial Perturbations

One facet of explaining the classical nature of the inflationary perturbations is to note that during the course of evolution, the accelerated expansion transforms the coherent vacuum states into strongly squeezed ones [68], thereby making the predictions of the quantum formalism indistinguishable from that of a theory where the fluctuations are assumed to be realizations of a classical stochastic process. This is possible because it has been shown that in the high squeezing limit, the quantum expectations can be well mimicked by the statistical average over a classical stochastic field [69]. In this sense, one can study the evolution of these fluctuations through classical equations and take the fluctuations as classical. Bringing in the phenomenon of decoherence [70–77] helps in diagonalizing the density matrix before recombination [78–81], thereby explaining an important aspect of this problem. However, as we know, even in laboratory experiments, decoherence alone cannot explain the appearance of a single outcome (as it only produces a statistical mixture of states). It requires to be supplemented by the formalism of many worlds (different outcomes exist in different branches of the universe) [82–85] that cannot be falsified. However, the possibility of violations of Bell's inequalities in a cosmological setting was proposed in [86]. The analog of a Bell experiment takes place during inflation. It was shown in [87] that the time and rate of decoherence for inflationary fluctuations put constraints on the parameter space of models that may allow for Bell inequality violations. The phenomenon of decoherence relies heavily on the distinction between system and environment. But as we know neither, we have multiple CMBR maps nor the universe is an open system. Thus, the problem still requires a complete explanation. There have been other attempts dealing with these kinds of issues [88–93], but none of them lead to any new, falsifiable predictions.

Nonetheless, there exists another set of models known as collapse models [49] that are widely used to address the measurement problem in quantum theory. These models have nice features. They modify the Schrödinger equation by adding non-linear stochastic terms leading to the dynamical self-induced collapse of the wave function (thereby avoiding the requirement of observers to perform measurement) into one of its eigenstates. Dynamical collapse happens due to the presence of a background classical stochastic field, filling all space, which couples to the number density operator of the system, thereby reducing it to one of its eigenstates. Moreover, these models have an inbuilt amplification mechanism that has the capability to produce stronger effects for macroscopic objects and milder effects for microscopic ones. But the main advantage of this approach is that it is falsifiable since it leads to predictions different from that of conventional quantum mechanics. As a result, these models have been widely constrained in the laboratories [49,94–96] and also in the cosmological and astrophysical contexts [97–99]. It is therefore interesting to investigate what the collapse theories have to say about the inflationary mechanism. Their role in generating classical density perturbations has been analyzed extensively by [100–106].

3.1. Decoherence

Decoherence corresponds to the absence of quantum coherence as a result of interaction/coupling of the system with the environment. In other words, if we consider a two-state system (say) initially,

$$\psi(t=0) = c_1\psi_1 + c_2\psi_2. \tag{30}$$

Over time, due to interaction with the environment (represented by the states ϕ), this state evolves into the state

$$\Phi(t) = c_1\psi_1\phi_{EA1}(t) + c_2\psi_2\phi_{EA2}(t). \tag{31}$$

Here, $\phi(t)_{EA1}$ and $\phi(t)_{EA2}$ denote macroscopically distinguishable entangled states of the apparatus and the environment. During measurement, due to the presence of the decoherence process, the inner product

$$\langle\phi(t)_{EA1}|\phi(t)_{EA2}\rangle \to 0. \tag{32}$$

The decay occurs exponentially with time at a rate given by the decoherence rate $\Gamma_{decoherence}$. For a more extensive study of this phenomenon, one may refer to [70–77]. It is widely believed that decoherence [74,107,108] could have played an important role in the process of classicalization of inflationary perturbations [46,79–81,87,109–122]. As was discussed in [119], cosmological fluctuations couple (at least gravitationally) to the other degrees of freedom present in the universe, treated as an open quantum system. Thus, by studying decoherence, one can investigate the role played by these additional degrees of freedom whose evolution can be modeled with a Lindblad equation (modified with additional terms, which suppresses the off-diagonal components for the reduced density matrix). It can thus be expected that this will lead to corrections to observables, such as the power spectrum of curvature fluctuations. In this context, one can write the total Hamiltonian as

$$\hat{H} = \hat{H}_v \otimes \hat{\mathbb{I}}_{\rm env} + \hat{\mathbb{I}}_v \otimes \hat{H}_{\rm env} + g\hat{H}_{\rm int}\,, \tag{33}$$

where $\hat{H}_v$ is the Hamiltonian of the system, $\hat{H}_{\rm env}$ is the Hamiltonian for the environment, g is a dimensionless coupling constant between the system and environment, and $\hat{H}_{\rm int}$ is the interaction Hamiltonian, which can be expressed as

$$\hat{H}_{\rm int}(\tau) = \int d^3\mathbf{x}\,\hat{A}(\tau,\mathbf{x}) \otimes \hat{R}(\tau,\mathbf{x})\,, \tag{34}$$

where $\hat{A}$ denotes the system, and $\hat{R}$ denotes the environment.

It has been shown in [119] that the most general expression of the Lindblad equation for the evolution equation for the density is given as

$$\frac{d\hat{\rho}_v}{d\tau} = -i\big[\hat{H}_v, \hat{\rho}_v\big] - \frac{\Omega}{2}\int d^3\mathbf{x}\,d^3\mathbf{y}\,C_R(\mathbf{x},\mathbf{y})\big[\hat{A}(\mathbf{x}), \big[\hat{A}(\mathbf{y}), \hat{\rho}_v\big]\big]\,, \tag{35}$$

where C_R is the correlation function of the environment, $C_R(\mathbf{x},\mathbf{y}) = \langle \hat{R}(\tau,\mathbf{x})\hat{R}(\tau,\mathbf{y})\rangle$, and the coefficient Ω is related to the coupling constant g and to the auto-correlation time τ_c of $\hat{R}$ [1] as

$$\Omega = 2g^2\tau_c\,. \tag{36}$$

There have been several works assuming different kinds of coupling between the environment and the system, for example, inflaton self-interactions [79,80], coupling to short-wavelength inflaton fluctuations [79], gravitational waves [123], isocurvature or additional fields [81,109,124], entanglement between spatially separated Hubble volumes [114], etc.

For instance, in [119], the coupling between $\hat{v}$ and $\hat{\psi}$ was assumed to be of the form

$$\hat{H}_{\rm int} = \lambda\mu^{4-n-m}\int d^3\mathbf{x}\sqrt{-g}\hat{\phi}^n(\tau,\mathbf{x})\hat{\psi}^m(\tau,\mathbf{x})\,, \tag{37}$$

where μ is a fixed-mass scale parameter, and $\hat{\phi} = \hat{v}/a$. Here, $\hat{v}$ is the Mukhanov–Sasaki variable [125] describing the inflation curvature perturbations and ψ represents the scalar field forming the environment with $M >> H$. The interaction can be both linear or non-linear. For linear interaction in the presence of a heavy test scalar field, the modified Lindblad equation turned out to be

$$\frac{d\hat{\rho}^s_{\mathbf{k}}}{d\tau} = -i\big[\hat{\mathcal{H}}^s_{\mathbf{k}}, \hat{\rho}^s_{\mathbf{k}}\big] - \frac{\Omega}{2}(2\pi)^{3/2}\tilde{C}_R(\mathbf{k})[\hat{v}^s_{\mathbf{k}}, [\hat{v}^s_{\mathbf{k}}, \hat{\rho}^s_{\mathbf{k}}]]\,. \tag{38}$$

The presence of the second term results in the suppression of the off-diagonal components, thus resulting in decoherence.

The corresponding spectrum of curvature perturbations is

$$\mathcal{P}_\zeta = \mathcal{P}_\zeta|_{\rm standard}(1 + \Delta\mathcal{P}_{\mathbf{k}}), \quad \text{with} \quad \Delta\mathcal{P}_{\mathbf{k}} \equiv \frac{\mathcal{J}_{\mathbf{k}}}{|v_{\mathbf{k}}|^2} \tag{39}$$

with
$$\mathcal{J}_{\mathbf{k}}(\tau) \equiv 4(2\pi)^{3/2} \int_{-\infty}^{\tau} d\tau' \Omega(\tau') \tilde{C}_R(\mathbf{k}, \tau') \mathrm{Im}^2\left[v_{\mathbf{k}}(\tau') v_{\mathbf{k}}^*(\tau)\right]. \tag{40}$$

We now have
$$n_s = n_s|_{\text{standard}} - \frac{\frac{\pi}{6}\frac{k_\Omega^2}{k_*^2}}{1 + \frac{\pi}{6}\frac{k_\Omega^2}{k_*^2}}(6m-2)\epsilon. \tag{41}$$

Here, k_Ω is a particular comoving scale appearing in the interaction term used to fix the dimension of $\Omega\tilde{C}_R$.

It was thus shown that the correction to the power spectrum is quasi-scale invariant. In that case, the presence of the environment improves the fit to the data for some inflationary models (such as power-law inflation) and deteriorates it for others (such as natural inflation) while having a negligible effect on others (like Higgs inflation). One obtains the observational constraint on the interaction strength, here parameterized by k_Ω/k_*.

In [87], a similar study of classicalization of inflationary perturbations has been carried out, but here, the coupling occurs between short- and long-wavelength modes. The author finds the evolution of the reduced density matrix for a given long-wavelength fluctuation by tracing out the other short-wavelength modes. It has been shown that inflation produces rapid phase oscillations in the wave functional due to the growth of the interacting part of the Lagrangian, $L_{int} \propto a(t)$, which suppresses off-diagonal components of the reduced density matrix, leaving a diagonal mixture of different classical configurations. The Hubble-scale modes act as the decohering environment. It was found that the decoherence rate scales as the physical volume in Hubble units of the inflating region, $\Gamma_{\text{decoherence}} \propto (aH)^3$. When $\Gamma \approx O(1)$, off-diagonal components of the reduced density matrix are exponentially suppressed, with a decay time of order the Hubble time. Shorter-wavelength modes play the role of an environment for superhorizon modes after they cross the Hubble scale, at which point they are the leading source of decoherence, $k_{\text{environment}} \sim aH$.

3.2. Collapse Model-Csl

The continuous spontaneous localization (CSL) approach to quantum mechanics attempts to solve the quantum measurement question in a general context. In this model, adding new non-linear and stochastic terms to the Schrödinger equation causes the collapse of the wave function. It was developed to provide a phenomenological explanation to the existing conceptual issues with quantum mechanics like the absence of superpositions, distinction between microscopic and macroscopic objects, probabilistic outcome, Born rule, etc. This approach seems to follow the standard strategy followed in physics, where first we consider a linear theory and then, in order to have a more accurate description, consider non-linear corrections. Following the pioneering work by Pearle [126] and its major improvement in the framework of the GRW model [127], the CSL model has been proposed as an upgraded and by far the most advanced version of this model [128,129]. The modified Schrödinger equation in Fourier space, with the pointer basis states being given by the Mukhanov–Sasaki operators, is given by

$$\begin{aligned} d\Psi_{\mathbf{k}}^{\mathrm{R}} &= \Big[-i\hat{\mathcal{H}}_{\mathbf{k}}^{\mathrm{R}} d\tau + \sqrt{\gamma}\Big(\hat{v}_{\mathbf{k}}^{\mathrm{R}} - \langle \hat{v}_{\mathbf{k}}^{\mathrm{R}} \rangle\Big) dW_\tau \\ &\quad - \frac{\gamma}{2}\Big(\hat{v}_{\mathbf{k}}^{\mathrm{R}} - \langle \hat{v}_{\mathbf{k}}^{\mathrm{R}} \rangle\Big)^2 d\tau\Big] \Psi_{\mathbf{k}}^{\mathrm{R}}, \end{aligned} \tag{42}$$

Here, $\hat{v}$ is the M-S operator, which is related to the comoving curvature perturbation ($v = a\phi'\mathcal{R}/\mathcal{H}$) [130]; $\langle \hat{v}_{\mathbf{k}}^{\mathrm{R}} \rangle = \langle \Psi_{\mathbf{k}}^{\mathrm{R}} | \hat{v}_{\mathbf{k}}^{\mathrm{R}} | \Psi_{\mathbf{k}}^{\mathrm{R}} \rangle$ is the expectation value of $\hat{v}$; γ is a new constant of nature, which determines the strength of the collapse, which for a laboratory system is of the form $\gamma = \gamma_0 \frac{m}{m_0}$; m_0 is the mass of the nucleon; and γ_0 measures the collapse strength with a value 10^{-2} m^{-2} s^{-1} [131,132] such that the collapse of the wave function can be explained consistently with all known experimental data. Gravity-induced collapse or continuous localization models have been stringently constrained by several recent laboratory experiments [49,133–135], reducing the parameter space to a very narrow range. W_η is the standard Wiener process (Brownian motion) that encodes the stochastic aspect [49]. There are two non-linear, non-unitary terms in the modified equation. The first one is proportional to $dW(t)$, and the second is proportional to dt, the latter being $(-1/2)$ times the former, forming the well-known martingale structure for the stochastic differential equation. The 'martingale' structure of the non-linear equation preserves the norm during evolution, despite being non-unitary, and is responsible for the emergence of the Born rule [136]. In [49], different forms of background noise in the laboratory framework have been discussed. [2]

In [47], the CSL equation was explored in the context of inflationary perturbation for a constant collapse parameter. It was shown in [47] that the power spectrum for the scalar perturbations for observationally relevant modes in the strong squeezing limits turned out to be

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{16\pi^2 \epsilon M_{\rm Pl}^2 \gamma k_0} e^{-\Delta N}. \tag{43}$$

ΔN is the number of folds the mode has spent outside the horizon after its exit and thus for observationally relevant modes $\Delta N \approx 50 - 60$.

We see that for longer modes, the power is scale-dependent ($\mathcal{P}_{\mathcal{R}}(k) \propto k^3$), which is inconsistent with the observations.

However, in [104–106], the authors assumed the CSL strength parameter to depend on physical scales so as to capture the CSL amplification mechanism with the phenomenological form being given by

$$\gamma = \frac{\gamma_0(k)}{(-k\tau)^\alpha}, \tag{44}$$

with $\gamma_0(k) = \tilde{\gamma}_0 \left(\frac{k}{k_0}\right)^\beta$. Here, $\alpha > 0$ so that CSL effects become dominant as the modes become a superhorizon. It was shown that with $1 < \alpha < 2$ yields a squeezing in the direction of MS field variables.

The power spectrum for the scalar perturbations is now given by

$$\mathcal{P}_{\mathcal{R}} = A_s(k_*)\left(\frac{k}{k_*}\right)^{3+\alpha-\beta+2\eta-4\epsilon} = A_s(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}, \tag{45}$$

As we can see, by setting $\beta = 3 + \alpha$, one can obtain a scale-invariant power spectrum.

It was assumed that the CSL collapse mechanism affects each helicity mode of the gravitons the same way as it affects the inflatons, keeping in tune with the concept of universality of the collapse mechanism. Thus, the CSL-modified dynamics of each helicity mode of the gravitons would be the same as that of the massless inflatons.

The observables in this case are modified to

$$\begin{aligned} n_s - 1 &= \delta + 2\eta - 4\epsilon, \\ n_T &= \delta - 2\epsilon, \qquad (46) \\ r &= -8n_T + 8\delta \qquad (47) \end{aligned}$$

where we have defined $\delta = 3 + \alpha - \beta$. The observation of the scalar spectral index by PLANCK [63] indicates that δ can at best be of the order of slow-roll parameters so that the comoving curvature power spectrum remains to be nearly scale-invariant. Independent accurate measurements of r and n_T would give us a direct handle on δ in this model.

In [106], the above analysis for canonical inflation was further extended to the non-canonical case (k-inflation). The scalar power spectrum for comoving curvature perturbations is now given by

$$\mathcal{P}_{\mathcal{R}} = \frac{k_0^2 c_s^{\alpha+2} H^2}{8\pi^2 \epsilon m_P^2 \tilde{\gamma}_0} \left(\frac{k}{k_0}\right)^{3+\alpha-\beta} e^{-(1+\alpha)\Delta N}, \tag{48}$$

which yields a scale-independent spectrum when $\beta \sim 3+\alpha$. Here, c_s is the 'speed of sound' for inflationary perturbations.

It is then straightforward to determine the scalar spectral index, considering the tilt, as

$$n_s - 1 = \delta - 2\epsilon - \epsilon_1 + (\alpha+2)s, \tag{49}$$

where $\delta \equiv 3+\alpha-\beta$. $\epsilon_1 = \dot{\epsilon}/(H\epsilon)$, s represents the first slow-roll parameter related to c_s.

The observables for k-inflation scenario in the presence of CSL are now given by the following expressions-

$$r = 16\epsilon c_s^{2+\alpha} \tag{50}$$

$$n_s - 1 = \delta - 2\epsilon - \epsilon_1 + (2+\alpha)s \tag{51}$$

$$n_T = \delta - 2\epsilon \tag{52}$$

$$r = -8(n_T - \delta) c_s^{-(2+\alpha)} \tag{53}$$

$$\alpha_s = -2\epsilon\epsilon_1 - \epsilon_1\epsilon_2 + (2+\alpha)ss_1 \tag{54}$$

Here, $s_1 = \dot{s}/(Hs)$.

Upon a numerical estimation and comparison with Planck observations, it was found that the observables like n_s, running of scalar tilt α_s, and running of running of scalar tilt β_s, cannot potentially distinguish a collapse modified inflationary dynamics in the realm of the canonical scalar field and k-inflationary scenarios. The only distinct imprint of the collapse mechanism lies in the observables of the tensor perturbations in the form of modified consistency relation and a blue-tilted tensor spectrum, which is possible only when the collapse parameter δ is non-zero and positive.

Thus, we see that both decoherence and collapse mechanism leaves distinct imprints on the inflationary power spectra and observables. However, which process is responsible for the quantum to classical transition is still debatable. Future constraints on the inflationary observables, especially on the tensor sector, might be able to provide us with more confirmatory answers.

4. Conclusions

Unifying quantum mechanics and gravity is the foremost and central, still open problem in theoretical physics. Among several tempting routes, quantum cosmology is an attempt to develop a quantum theory of the universe. Diverse approaches have been developed, in particular in the last half-century or so, but in order to ultimately decide which one better describes nature, we need realistically testable predictions. In this context, the main pragmatic challenge for quantum cosmology is to make it 'observationally' congruent, fitting with current as well as forthcoming cosmic microwave background radiation (CMBR) data. From observations, we know that the universe is mostly flat, 'mildly' non-Gaussian, and there are severe constraints on some observables based on the CMBR power spectrum. This has been a long-awaited goal pursued by several scientists for several decades. Though there has been progress in this area, we are still far from suggesting any noticeable quantum effect on the CMBR map theoretically and numerically.

In this short article, I have reviewed two such approaches, where the inflationary observables are modified by quantum imprints. In the first approach, making use of the Born–Oppenheimer approximation to the WDW equation, quantum corrections to Schrödinger equations were obtained. A corrected inflationary power spectrum was also obtained. This led to an enhancement of power on the largest scales for both scalar and tensor perturbations. For the mathematical analysis reviewed in this article, a Gaussian ansatz for the wave function was considered. The physical quantities/observables were considered up to linear order in the slow-roll parameters. Terms of higher order in slow-roll parameters have been neglected as their effect will be comparable or sub-dominant compared to the first-order corrections. A major contribution to the quantum gravitational correction comes from the de Sitter term. The slow-roll effect results in a small modification. It will be interesting to see the effect of non-Gaussinities on these corrections. One can surely look into that which will require mathematical analysis. Possibilities are there that the effect will be of the same order or smaller.

In the second approach, I have discussed the possible modifications to inflationary power spectra through the presence of decoherence and collapse mechanisms responsible for the classicalization of inflationary perturbations. This opens another possibility of observing the outcome of another pure quantum gravitational effect. Both processes leave distinct signatures on the inflationary power spectra, especially on the tensor sector. Therefore, detection of the tensor sector by future experiments like BICEP2, PRISM, and COrE will be able to shed more light on this debate. In Table 1, I have quoted the main results of the models I have reviewed.

Table 1. Corrections to inflationary observables from different approaches (here, $\epsilon_2 = \dot{\epsilon_1}/(H\epsilon_1)$).

	r	n_s	n_T	α_s
WDW	16ϵ $+\left(2.56\,\frac{H_k^2}{m_P^2}\left(\frac{\bar{k}}{k}\right)^3(\gamma-\epsilon)\right)$	$-\frac{H_k^2}{m_P^2}\left(\frac{\bar{k}}{k_*}\right)^3(2.96+11.40\,\epsilon)$ $+\frac{H_k^2}{m_P^2}\left(\frac{\bar{k}}{k_*}\right)^3(7.68\,\gamma)+1+2\gamma-4\epsilon$	-2ϵ $-\frac{H_k^2}{m_P^2}\left(\frac{\bar{k}}{k_*}\right)^3(2.96+3.72\,\epsilon)$	$4\epsilon(\gamma-\epsilon)-2\theta+\frac{H_k^2}{m_P^2}\left(\frac{\bar{k}}{k_*}\right)^3(8.89)$ $+\frac{H_k^2}{m_P^2}\left(\frac{\bar{k}}{k_*}\right)^3(40.12\,\epsilon-23.04\,\gamma)$
Decoherence	$\frac{r\vert_{\text{standard}}}{1+\frac{\pi}{6}\frac{k_\Omega^2}{k_\star^2}}$	$n_s\vert_{\text{standard}}-\frac{\frac{\pi}{6}\frac{k_\Omega^2}{k_*^2}}{1+\frac{\pi}{6}\frac{k_\Omega^2}{k_*^2}}(6m-2)\epsilon$	$n_T\vert_{\text{standard}}$	-
CSL-Generic	16ϵ	$1+\delta+2\eta-4\epsilon$	$\delta-2\epsilon$	$-2\epsilon\epsilon_1-\epsilon_1\epsilon_2$
CSL-*k*-inflation	$16\epsilon c_s^{2+\alpha}$	$1+\delta-2\epsilon-\epsilon_1+(2+\alpha)s$	$\delta-2\epsilon$	$-2\epsilon\epsilon_1-\epsilon_1\epsilon_2+(2+\alpha)ss_1$

Funding: This research received no external funding.

Data Availability Statement: Not available.

Conflicts of Interest: The author declares no conflict of interest.

Notes

[1] In continuous spontaneous collapse models, the auto-correlation time is strongly constrained not only by experiments, but also by tests of Bell's inequalities that rule out the existence of hidden parameters (see, e.g., [120,121]). Any hidden self-interaction or correlation is similar to a hidden parameter

[2] A noise can decay if the solution of the equation approaches a steady state. However, such behavior is not consistent with the contextuality of quantum mechanics [137].

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