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Path Integral Action for a Resonant Detector of Gravitational Waves in the Generalized Uncertainty Principle Framework

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Abstract: The Heisenberg uncertainty principle is modified by the introduction of an observer-independent minimal length. In this work, we have considered the resonant gravitational wave detector in the modified uncertainty principle framework, where we have used the position momentum uncertainty relation with a quadratic order correction only. We have then used the path integral approach to calculate an action for the bar detector in the presence of a gravitational wave and then derived the Lagrangian of the system, leading to the equation of motion for the configuration-space position coordinate in one dimension. We then find a perturbative solution for the coordinate of the detector for a circularly polarized gravitational wave, leading to a classical solution of the same for the given initial conditions. Using this classical form of the coordinate of the detector, we finally obtain the classical form of the on-shell action describing the harmonic oscillator–gravitational wave system. Finally, we have obtained the free particle propagator containing the quantum fluctuation term considering gravitational wave interaction.

Keywords: resonant bar detector; gravitational wave; generalized uncertainty principle; path integral

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1. Introduction

Quantum mechanics and general relativity are the two most successful theories explaining the phenomena at the two most fundamental length scales of the universe. While quantum mechanics explains the intricacies of the atomic length scale, general relativity [1,2] sheds light on the large-scale structure of the universe. In order to understand the fundamental mysteries of the universe, we need a quantum theory of gravity, explaining the analytical structure of the gravitational interaction at the quantum length scale. Theories such as loop quantum gravity [3,4], string theory [5,6], and noncommutative geometry [7] have provided a convincing theoretical framework explaining the Planck-scale nature of gravity, but none of them have compelling experimental evidence to support their claim of providing an exact description of the quantum nature of gravity. Meanwhile, all of them prescribe the existence of an observer-independent minimal length, which can be incorporated by the modification of the standard Heisenberg uncertainty principle (HUP), also known as the generalized uncertainty principle (GUP). The first few attempts to improvise an integral relation between minimal length scale and gravity was shown in [8,9], followed by [10]. We also obtain strong evidence of the existence of this fundamental length scale from the various gedanken experiments in quantum gravity phenomenology as well. This GUP framework has been used to investigate several areas of theoretical physics, including black hole physics and its thermodynamics [11–21], various quantum systems, such as particle in a box and simple harmonic oscillators [22,23], optomechanical systems [24–26], and gravitational wave bar detectors [27,28]. There have been several recent studies involving the path integral formalism of a non-relativistic particle moving in an arbitrary potential in the generalized uncertainty principle framework [27,29,30]. The simplest form of the modified HUP can be written in the following form [31]:

$$\Delta q_i \Delta p_i \geq \frac{\hbar}{2}\left[1+\gamma\left(\Delta p^2+\langle p\rangle^2\right)+2\gamma\left(\Delta p_i^2+\langle p_i\rangle^2\right)\right];\ i=1,2,3 \tag{1}$$

where $p^2 = \sum_{k=1}^{3} p_k p_k$ and q_k, p_k are the phase space position and its conjugate momenta. In Equation (1), the GUP parameter γ in terms of the dimensionless parameter γ_0 can be recast as follows:

$$\gamma = \frac{\gamma_0}{m_p^2 c^2} \tag{2}$$

where m_p is the Planck mass and c is the speed of light. It is quite natural to realize that the order of magnitude of the GUP parameter will play a significant role in providing an understanding of the GUP effects. There have been several studies to find a bound on the GUP parameter itself [17,22,28,32–38].

In 1969, the first proposition to detect gravitational waves was made by J. Weber [39], which was followed by a subsequent paper in 1982 by Ferrari et al. [40]. Bar detectors currently have a sensitivity $\frac{\Delta L}{L} \sim 10^{-19}$[41], where ΔL is the fractional variation of the length L ($\sim$1 m) of the bar detector. A historical perspective on these resonant detectors is given in [42]. The detection of gravitational waves by the LIGO [43,44] and Virgo [45] detectors has unveiled a new realm of quantum gravity phenomenology. There have been several recent investigations regarding the traces of quantum gravitational effects in these gravitational wave detectors. A great deal of investigation has been conducted to check if any signature of this fundamental Planck length, whether it is noncommutativity [46–51] or GUP [28,52], is visible in GW bar detectors. We would like to point out that, to date, there has not been a successful detection of gravitational waves in resonant bar detectors. However, there is strong hope that the sensitivity of the detectors will increase in the future, enabling the detection of these waves. The AURIGA (*Antenna Ultracriogenica Risonante per l'Indagine Gravitazionale Astronomica*) detector at INFN, Italy is probably the only functional bar detector. These bar detectors are sensitive to frequencies of the order of 1kHz, along with a strain sensitivity of the order 10^{-19} [53]. In the case of astrophysical events, collapsing and bouncing cores of supernova can be a source of huge intensities of gravitational waves having frequencies in the vicinity of 1–3 kHz. The value of the strain sensitivity can be calculated using Thorne's formula [54]. The strain sensitivity (h), according to this formula, is given by

$$h = 2.7 \times 10^{-20}\left[\frac{\Delta E_{GW}}{M_s c^2}\right]^{\frac{1}{2}}\left[\frac{1kHz}{f}\right]^{\frac{1}{2}}\left[\frac{10Mpc}{d}\right] \tag{3}$$

where ΔE_{GW} is the energy converted to gravitational waves, f is the characteristic frequency of the burst, M_s is the solar mass, and d is the distance of the burst source from Earth. A possible value of the fraction of energy converted to gravitational waves for supernova events is around 7×10^{-4}. Now, for $h \sim 3 \times 10^{-19}$ and $f \sim 0.9$ kHz, the distance d has a value around 25 kpc. The occurrence of such a supernova event of the required magnitude at this distance from the Earth would definitely result in the detection of gravitational waves by the bar detectors. An effort to increase the sensitivity of these detectors to $h \sim 10^{-20}$ is presently being carried out, and achieving this sensitivity would increase the distance of the supernova event from the Earth to 250 kpc, which is more likely to occur. The main motivation to work with a gravitational wave bar detector is that it is a very useful and economic alternative to the LIGO/VIRGO detectors.

In this work, we investigate the path integral formalism of a resonant gravitational wave bar detector interacting with the gravitational wave emitted from a distant source in the GUP framework. The incoming gravitational waves interact with the elastic matter in the resonant bar detector, causing tiny vibrations called phonons. Physically, we can describe these detectors as a quantum mechanical gravitational wave–harmonic oscillator (GW-HO) system, because we call these vibrations the quantum mechanical forced harmonic oscillator. To calculate the perturbative solution to the system, we use the gravitational wave and generalized uncertainty modifications as perturbations. Our study presents a path integral approach to look at such a system and is the first work using a path integral. The advantage of working with path integrals is that the effective action describing the system can be easily read off from the structure of the configuration space path integral [55].

2. The Gravitational Wave Resonant Detector Interaction Model

To begin the discussion, we need to present the Hamiltonian for the resonant bar detector in the presence of a gravitational wave in the generalized uncertainty principle framework. The modified commutation relation following from Equation (1) takes the following form [31]:

$$[\hat{q}_i, \hat{p}_j] = i\hbar\left[\delta_{ij} + \gamma\delta_{ij}\hat{p}^2 + 2\gamma\hat{p}_i\hat{p}_j\right] \tag{4}$$

where $i, j = 1, 2, 3$. The modified position and momentum operators $\hat{q}_i$ and $\hat{p}_i$ in terms of the usual variables $\hat{q}_{0i}$ and $\hat{p}_{0i}$ read

$$\hat{q}_i = \hat{q}_{0i}\ ,\ \hat{p}_i = \hat{p}_{0i}\left(1 + \gamma\hat{p}_0^2\right)\ . \tag{5}$$

Here, $\hat{p}_0^2 = \sum_{k=1}^{3}\hat{p}_{0k}\hat{p}_{0k}$ and $[\hat{q}_{0i}, \hat{p}_{0j}] = i\hbar\delta_{ij}$. In order to write the Hamiltonian of the system, we start by analyzing the background metric as a superposition of a small perturbation on the flat background metric. The background metric is taken as follows:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{6}$$

where $\eta_{\mu\nu} = \mathrm{diag}\{1, -1, -1, -1\}$ and $|h_{\mu\nu}| \ll 1$. We now consider a two-dimensional harmonic oscillator with mass m and intrinsic frequency ϖ. The geodesic deviation equation for the aforementioned system in the proper detector frame is given as follows [56]:

$$\begin{aligned} m\ddot{q}^k &= -mR^k{}_{0l0}q^l - m\varpi^2 q^k \\ \implies \ddot{q}^k &= \frac{d\Gamma^k{}_{0l}}{dt}q^l - \varpi^2 q^k\ ;\ k = 1, 2 \end{aligned} \tag{7}$$

where $R^k{}_{0l0}$ in terms of the background perturbation is given by

$$R^k{}_{0l0} = -\frac{d\Gamma^k{}_{0l}}{dt} = -\frac{\ddot{h}_{kl}}{2}\ . \tag{8}$$

Note that, here, we are using the transverse traceless gauge to eliminate the unphysical degrees of freedom. The Lagrangian from which Equation (8) can be obtained reads

$$L = \frac{1}{2}m\dot{q}_k^2 - m\Gamma^k{}_{0l}\dot{q}_k q^l - \frac{1}{2}m\varpi^2 q_k^2\ . \tag{9}$$

The Hamiltonian corresponding to the Lagrangian in Equation (9) reads

$$H = \frac{1}{2m}\left(p_k + m\Gamma^k{}_{0l}q^l\right)^2 + \frac{1}{2}m\varpi^2 q_l^2\ . \tag{10}$$

To write the Hamiltonian in Equation (10) in quantum mechanical description, we simply elevate q and p to the operator prescription. Therefore, the Hamiltonian in terms of the position and momentum operators can be expressed as follows:

$$\hat{H} = \frac{1}{2m}\left(\hat{p}_k + m\Gamma^k{}_{0l}\hat{q}^l\right)^2 + \frac{1}{2}m\varpi^2\hat{q}_l^2\ . \tag{11}$$

Using the representation of the position and momentum operators in Equation (5), the Hamiltonian (11) of the GW-HO system in the presence of GUP can be written as follows:

$$\hat{H} = \left(\frac{\hat{p}_{0k}^2}{2m} + \frac{1}{2}m\varpi^2\hat{q}_{0k}^2\right) + \frac{\gamma}{m}\hat{p}_{0k}^2\hat{p}_0^2 + \frac{1}{2}\Gamma^k{}_{0l}\left(\hat{p}_{0k}\hat{q}^{0l} + \hat{q}^{0l}\hat{p}_{0k}\right) + \frac{\gamma}{2}\Gamma^k{}_{0l}\left(\hat{p}_{0k}\hat{p}_0^2\hat{q}^{0l} + \hat{q}^{0l}\hat{p}_{0k}\hat{p}_0^2\right). \tag{12}$$

Now, a typical bar is a cylinder of length $L \equiv 3$ m and radius $R \equiv 30$ cm [56]. Hence, in a first approximation, we can treat the GW detector in the presence of GUP as a one-dimensional HO. The Hamiltonian in Equation (12) can be recast in one dimension as follows:

$$\hat{H} = \frac{p^2}{2m} + \frac{1}{2}m\varpi^2q^2 + \gamma\frac{p^4}{m} + \frac{1}{2}\Gamma^1{}_{01}(pq + qp) + \frac{\gamma}{2}\Gamma^1{}_{01}(p^3q + qp^3) \tag{13}$$

where, for notational simplicity, we have used $\hat{p}_{01} = p$ and $\hat{q}_{01} = q$. In the next section, we will proceed to construct the path integral formalism of the GW-HO system in the presence of the GUP and calculate the propagation kernel for that system.

3. Path Integral and the Propagation Kernel

In this section, we will use the Hamiltonian in Equation (13) to calculate the propagation kernel via the path integral approach. We consider the initial and the final state of the Hamiltonian in Equation (13) at initial time t_i and final time t_f as $|q_i, t_i\rangle$ and $\left|q_f, t_f\right\rangle$, respectively. The general form of the propagation kernel can be written as follows:

$$\begin{aligned}\left\langle q_f, t_f|q_i, t_i\right\rangle &= \lim_{N\to\infty}\int_{-\infty}^{+\infty} dq_{N-1}\dots dq_1\left\langle q_f, t_f|q_{N-1}, t_{N-1}\right\rangle\langle q_{N-1}, t_{N-1}|q_{N-2}, t_{N-2}\rangle\dots\langle q_1, t_1|q_i, t_i\rangle \\ &= \lim_{N\to\infty}\int_{-\infty}^{+\infty}\prod_{\alpha=1}^{N-1} dq_\alpha\left\langle q_f\right|e^{-\frac{i\hat{H}(t_f - t_{N-1})}{\hbar}}|q_{N-1}\rangle\dots\langle q_1|e^{-\frac{i\hat{H}(t_1 - t_i)}{\hbar}}|q_i\rangle \\ &= \lim_{N\to\infty}\int_{-\infty}^{+\infty}\prod_{\alpha=1}^{N-1} dq_\alpha\prod_{\beta=0}^{N-1}\langle q_{\beta+1}|e^{-\frac{i\hat{H}(t_{\beta+1}-t_\beta)}{\hbar}}|q_\beta\rangle\end{aligned} \tag{14}$$

where $t_f = t_N$, $t_i = t_0$ and $t_N - t_{N-1} = \Delta t$. Now, we will introduce the complete set of momentum eigenstates $\left(\int_{-\infty}^{+\infty} dp|p\rangle\langle p| = 1\right)$ in the following way:

$$\begin{aligned}\left\langle q_f, t_f|q_i, t_i\right\rangle &= \lim_{N\to\infty}\int\prod_{\alpha=1}^{N-1} dq_\alpha\prod_{\beta=0}^{N-1}\int dp_\beta\langle q_{\beta+1}|p_\beta\rangle\langle p_\beta|q_\beta\rangle\exp\left(-\frac{iH(q_\beta, p_\beta)(t_{\beta+1}-t_\beta)}{\hbar}\right) \\ &= \lim_{N\to\infty}\int_{-\infty}^{+\infty}\prod_{\alpha=1}^{N-1} dq_\alpha\prod_{\beta=0}^{N-1}\int_{-\infty}^{+\infty}\frac{dp_\beta}{2\pi\hbar}\exp\left[\frac{i\Delta t}{\hbar}\sum_{\beta=0}^{N-1}\left[\frac{p_\beta(q_{\beta+1}-q_\beta)}{\Delta t} - \left(\frac{p_\beta^2}{2m} + \frac{1}{2}m\varpi^2q_\beta^2 + \frac{\gamma p_\beta^4}{m}\right.\right.\right. \\ &\quad\left.\left.\left. + \frac{p_\beta q_\beta(h_{\beta+1}-h_\beta)}{2\Delta t} + \frac{\gamma p_\beta^3 q_\beta(h_{\beta+1}-h_\beta)}{2\Delta t}\right)\right]\right]\end{aligned} \tag{15}$$

where we have used $h_{11} = h$. The final form of Equation (15) in the $\Delta t \to 0$ limit can be recast as follows:

$$\left\langle q_f, t_f|q_i, t_i\right\rangle = \int\mathcal{D}q\mathcal{D}p\exp\left(\frac{i}{\hbar}\mathcal{S}\right) \tag{16}$$

where $\mathcal{S}$ is the phase space action. The phase space action is given as follows:

$$\mathcal{S} = \int_{t_i}^{t_f} dt\left[p\dot{q} - \left(\frac{p^2}{2m} + \frac{\dot{h}_{11}}{2}pq + \frac{1}{2}m\varpi^2q^2 + \frac{\gamma p^4}{m} + \frac{\gamma\dot{h}_{11}}{2}p^3q\right)\right]. \tag{17}$$

To obtain the configuration space Lagrangian, we will simplify Equation (15) as follows:

$$\left\langle q_f, t_f | q_i, t_i \right\rangle \cong \lim_{N\to\infty} \int_{-\infty}^{+\infty} \prod_{\alpha=1}^{N-1} dq_\alpha \prod_{\beta=0}^{N-1} \int_{-\infty}^{+\infty} \frac{dp_\beta}{2\pi\hbar} \left[1 - \frac{i\gamma\Delta t}{m\hbar}\left(p_\beta^4 + \frac{h_{\beta+1}-h_\beta}{2\Delta t} p_\beta^3 q_\beta\right) + \mathcal{O}(\gamma^2)\right] \tag{18}$$

$$\times \exp\left[\frac{i\Delta t m}{2\hbar}\left[\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{h_{\beta+1}-h_\beta}{4\Delta t} q_\beta\right)^2 - \omega^2 q_\beta^2\right]\right] \exp\left[-\frac{i\Delta t}{2m\hbar}\left[p_\beta - \left(\frac{m(q_{\beta+1}-q_\beta)}{\Delta t} - \frac{m(h_{\beta+1}-h_\beta)q_\beta}{4\Delta t}\right)\right]^2\right].$$

To perform the momentum integral for each β value, we shall perform the following coordinate transformation:

$$\bar{p}_\beta = p_\beta - \left(\frac{m(q_{\beta+1}-q_\beta)}{\Delta t} - \frac{m(h_{\beta+1}-h_\beta)q_\beta}{4\Delta t}\right). \tag{19}$$

Using Equation (19) in Equation (18), the propagation kernel up to $\sim \gamma, h$ can be recast as

$$\begin{aligned}\left\langle q_f, t_f | q_i, t_i \right\rangle \cong \lim_{N\to\infty} \int_{-\infty}^{+\infty} \prod_{\alpha=1}^{N-1} dq_\alpha \prod_{\beta=0}^{N-1} \int_{-\infty}^{+\infty} \frac{d\bar{p}_\beta}{2\pi\hbar} \left[1 - \frac{i\gamma\Delta t}{m\hbar}\left[\left(\bar{p}_\beta + \left(\frac{m(q_{\beta+1}-q_\beta)}{\Delta t} - \frac{m(h_{\beta+1}-h_\beta)q_\beta}{4\Delta t}\right)\right)^4 + \right.\right.\\ \left.\left.\frac{h_{\beta+1}-h_\beta}{2\Delta t}\left(\bar{p}_\beta + \left(\frac{m(q_{\beta+1}-q_\beta)}{\Delta t} - \frac{m(h_{\beta+1}-h_\beta)q_\beta}{4\Delta t}\right)\right)^3 q_\beta\right] + \mathcal{O}(\gamma^2)\right] \exp\left[-\frac{i\Delta t}{2m\hbar}\bar{p}_\beta^2\right]\\ \times \exp\left[\frac{i\Delta t m}{2\hbar}\left[\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{h_{\beta+1}-h_\beta}{4} q_\beta\right)^2 - \omega^2 q_\beta^2\right]\right].\end{aligned} \tag{20}$$

The momentum integral in Equation (20) can be obtained as follows:

$$\begin{aligned}\langle q_{\beta+1}, t_{\beta+1} | q_\beta, t_\beta \rangle \cong \sqrt{\frac{m}{2\pi i\hbar\Delta t}}\left\{1 - 6\gamma m^2\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{h_{\beta+1}-h_\beta}{4\Delta t} q_\beta\right)^2 - \frac{3\gamma m^2(h_{\beta+1}-h-\beta)}{2\Delta t}\left(\frac{q_{\beta+1}-q_\beta}{\Delta t}\right.\right.\\ \left.\left. - \frac{(h_{\beta+1}-h_\beta)}{4\Delta t} q_\beta\right) q_\beta + \frac{3i\gamma m\hbar}{\Delta t}\right\} \exp\left[\frac{im\Delta t}{2\hbar}\left[\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{(h_{\beta+1}-h_\beta)q_\beta}{4}\right)^2 - 2\gamma m^2\right.\right.\\ \left.\left.\times\left[\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{(h_{\beta+1}-h_\beta)q_\beta}{4}\right)^4 + \frac{(h_{\beta+1}-h_\beta)q_\beta}{2}\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{(h_{\beta+1}-h_\beta)q_\beta}{4}\right)^3\right] - \omega^2 q_\beta^2\right]\right].\end{aligned} \tag{21}$$

Using Equation (21) in Equation (20), we obtain the form of the propagation kernel up to some constant factor as follows:

$$\begin{aligned}\left\langle q_f, t_f | q_i, t_i \right\rangle = \int_{-\infty}^{+\infty} \prod_{\alpha=1}^{N-1} dq_\alpha \exp\left[\sum_{\beta=0}^{N-1} \frac{im\Delta t}{2\hbar}\left\{\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{(h_{\beta+1}-h_\beta)q_\beta}{4}\right)^2 - 2\gamma m^2\left(\left(\frac{q_{\beta+1}-q_\beta}{\Delta t}\right.\right.\right.\right.\\ \left.\left.\left.\left. - \frac{(h_{\beta+1}-h_\beta)q_\beta}{4}\right)^4 + \frac{(h_{\beta+1}-h_\beta)q_\beta}{2}\left(\frac{q_{\beta+1}-q_\beta}{\Delta t} - \frac{(h_{\beta+1}-h_\beta)q_\beta}{4}\right)^3\right) - \omega^2 q_\beta^2\right\}\right].\end{aligned} \tag{22}$$

Imposing the $\Delta t \to 0$ limit in Equation (22), the final form of the propagation kernel has the usual configuration space path integral structure as follows:

$$\left\langle q_f, t_f | q_i, t_i \right\rangle = \mathcal{N}(T, \gamma, \dot{h}) \int \mathcal{D}q e^{\frac{i}{\hbar}S}. \tag{23}$$

In the above equation, the configuration space structure of the action S is given as follows:

$$\begin{aligned}S &= \int_{t_i}^{t_f} dt\left(\frac{m}{2}\left(\dot{q} - \frac{\dot{h}q}{4}\right)^2 - \frac{1}{2}m\omega^2 q^2 - \gamma m^3\left(\dot{q} - \frac{\dot{h}q}{4}\right)^4 - \frac{\gamma m^3 \dot{h} q}{2}\left(\dot{q} - \frac{\dot{h}q}{4}\right)^3\right)\\ &\cong \int_{t_i}^{t_f} dt\left(\frac{m}{2}\dot{q}^2 - \frac{1}{2}m\omega^2 q^2 - \frac{m\dot{h}\dot{q}q}{4} - \gamma m^3 \dot{q}^4 + \frac{1}{2}m^3\gamma\dot{h}\dot{q}^3 q\right).\end{aligned} \tag{24}$$

In the last line of the above Equation (24), we have kept terms up to $\mathcal{O}(h,\gamma)$. The Lagrangian can be easily read off from Equation (24) as follows:

$$L = \frac{m}{2}\dot{q}^2 - \frac{1}{2}m\varpi^2 q^2 - \frac{m\dot{h}\dot{q}q}{4} - \gamma m^3\dot{q}^4 + \frac{1}{2}\gamma m^3\dot{h}\dot{q}^3 q\,. \tag{25}$$

The equation of motion following from the Lagrangian reads

$$\ddot{q} - \frac{\ddot{h}q}{4} + \varpi^2 q - 12m^2\gamma\ddot{q}\dot{q}^2 + 3\gamma m^2\dot{h}\ddot{q}\dot{q}q + \frac{3}{2}\gamma m^2\ddot{h}\dot{q}^2 q + \gamma m^2\dot{h}\dot{q}^3 = 0\,. \tag{26}$$

In the next section, we calculate the classical solution for the above equation of motion.

4. Obtaining the Classical Solution for a Periodic Circularly Polarized Gravitational Wave

To obtain the classical solution, we shall consider a circularly polarized gravitational wave in the transverse traceless gauge. Now, for a periodic circularly polarized gravitational wave, the perturbation term h containing the polarization information reads

$$h_{kl}(t) = 2f_0\left(\varepsilon_\times(t)\sigma^1_{kl} + \varepsilon_+(t)\sigma^3_{kl}\right)\;;\; k,l = 1,2 \tag{27}$$

where $2f_0$ is the amplitude of the gravitational wave (here, f_0 is very small), and σ^1 and σ^3 are the Pauli spin matrices. In Equation (27), $(\varepsilon_+(t), \varepsilon_\times(t))$ are the two possible polarization states of the gravitational wave satisfying the condition $\varepsilon_+(t)^2 + \varepsilon_\times(t)^2 = 1$. In this particular scenario, the chosen functional forms of the polarization states can be written as follows:

$$\varepsilon_+(t) = \cos(\Omega t)\;,\; \varepsilon_\times(t) = \sin(\Omega t) \tag{28}$$

with Ω being the frequency of the gravitational wave. In our case, we will consider that the only non-zero polarization state is $\varepsilon_+(t) = \cos(\Omega t)$. Therefore, in one dimension, the perturbation term can be written as $h = 2f_0\cos(\Omega t)$. The equation of motion in Equation (26) up to $\mathcal{O}(f_0,\gamma)$ takes the form as follows:

$$\ddot{q} + \omega^2 q - 12m^2\gamma\ddot{q}\dot{q}^2 = 0 \tag{29}$$

where $\omega^2 = \varpi^2 - \frac{\ddot{h}}{4}$. For the equation of motion in Equation (29), we consider a solution up to $\mathcal{O}(f_0,\gamma)$ as

$$q(t) = q_0(t) + f_0 q_{f_0}(t) + \gamma q_\gamma(t)\,. \tag{30}$$

For the form $q(t)$ in the above equation, we obtain the solution of Equation (26) as a linear combination as $q_0(t), q_{f_0}(t)$ and $q_\gamma(t)$. The analytical forms of $q_0(t), q_{f_0}(t)$ and $q_\gamma(t)$ are given as follows:

$$q_0(t) = \mathcal{A}_1\cos(\varpi t) + \mathcal{A}_2\sin(\varpi t), \tag{31}$$

$$\begin{aligned} q_{f_0}(t) = \;& \mathcal{A}_3\cos(\varpi t) + \mathcal{A}_4\sin(\varpi t) - \tfrac{\Omega}{2(4\varpi^2-\Omega^2)}[\Omega\cos(\Omega t)\{\mathcal{A}_1\cos(\varpi t) + \mathcal{A}_2\sin(\varpi t)\} \\ & -2\varpi\sin(\Omega t)\{\mathcal{A}_2\cos(\varpi t) - \mathcal{A}_1\sin(\varpi t)\}]\,, \end{aligned} \tag{32}$$

$$\begin{aligned} q_\gamma(t) = \;& \mathcal{A}_5\cos(\varpi t) + \mathcal{A}_6\sin(\varpi t) - \tfrac{3m^2\varpi^2}{2}[t\varpi\mathcal{A}_1(\mathcal{A}_1^2 + \mathcal{A}_2^2)\sin(\varpi t) - t\varpi\mathcal{A}_2(\mathcal{A}_1^2 + \mathcal{A}_2^2)\cos(\varpi t) \\ & + \tfrac{\mathcal{A}_1}{4}(\mathcal{A}_1^2 - 3\mathcal{A}_2^2)\cos(3\varpi t) - \tfrac{\mathcal{A}_2}{4}(\mathcal{A}_2^2 - 3\mathcal{A}_1^2)\sin(3\varpi t)] \end{aligned} \tag{33}$$

where $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \mathcal{A}_4, \mathcal{A}_5$ and $\mathcal{A}_6$ are arbitrary constants, which we will calculate for the $q_{cl}(t)$. To obtain the form of the above constants, we will apply the following set of the initial conditions:

$$q(t) = \begin{cases} q_0 & \text{for} \quad t = 0 \\ q_f & \text{for} \quad t = T \end{cases}. \tag{34}$$

Using the initial conditions in Equation (34), the constants can be obtained as follows:

$$\mathcal{A}_1 = q_0\,,\ \mathcal{A}_2 = \frac{q_f - q_0\cos(\varpi T)}{\sin(\varpi T)}\,, \tag{35}$$

$$\mathcal{A}_3 = \frac{\mathcal{A}_1\Omega^2}{2(4\varpi^2-\Omega^2)}\,, \tag{36}$$

$$\mathcal{A}_4 = \frac{\Omega\{\cos(\varpi T)[\Omega\mathcal{A}_1\cos(\Omega T)-2\varpi\mathcal{A}_2\sin(\Omega T)]+\sin(\varpi T)[\Omega\mathcal{A}_2\cos(\Omega T)+2\varpi\mathcal{A}_1\sin(\omega T)]\}}{2(4\varpi^2-\Omega^2)\sin(\varpi T)} - \mathcal{A}_3\cot(\varpi T)\,, \tag{37}$$

$$\mathcal{A}_5 = \tfrac{3}{8}m^2\varpi^2\mathcal{A}_1\left(\mathcal{A}_1^2 - 3\mathcal{A}_2^2\right), \tag{38}$$

$$\mathcal{A}_6 = \frac{3m^2\varpi^2\left[\varpi T(\mathcal{A}_1\sin(\varpi T)-\mathcal{A}_2\cos(\varpi T))(\mathcal{A}_1^2+\mathcal{A}_2^2)+\frac{\mathcal{A}_1(\mathcal{A}_1^2-3\mathcal{A}_2^2)\cos(3\varpi T)}{4}-\frac{\mathcal{A}_2(\mathcal{A}_2^2-3\mathcal{A}_1^2)\sin(3\varpi T)}{4}\right]}{2\sin(\varpi T)} - \mathcal{A}_5\cot(\varpi T). \tag{39}$$

Using Equation (30) along with Equations (35)–(39) in Equation (24) (with h being replaced by $2f_0\cos(\Omega t)$), we obtain the form of the classical action up to $\mathcal{O}(\gamma, f)$ as follows:

$$S_{\mathcal{C}} = S_{\mathcal{C}}^{(0)} + S_{\mathcal{C}}^{(\gamma)} + S_{\mathcal{C}}^{(f_0)} \tag{40}$$

where $S_{\mathcal{C}}^{(0)}$, $S_{\mathcal{C}}^{(\gamma)}$, and $S_{\mathcal{C}}^{(f_0)}$ are given by the following equations:

$$S_{\mathcal{C}}^{(0)} = \frac{m\varpi}{2\sin(\varpi T)}\left((q_0^2+q_f^2)\cos(\varpi T) - 2q_0q_f\right), \tag{41}$$

$$\begin{aligned} S_{\mathcal{C}}^{(\gamma)} = &-\frac{\gamma m^3\varpi^3}{32\sin^4(\varpi T)}\Big[12\varpi T\left(q_f^4+4q_f^2q_0^2+q_0^4\right) - 48q_0q_f\varpi T\cos(\varpi T)(q_f^2+q_0^2) + 24q_0^2q_f^2\varpi T\cos(2\varpi T) \\ &-44q_0q_f\sin(\varpi T)(q_0^2+q_f^2) + 4\sin(2\varpi T)\left(2q_0^4+15q_0^2q_f^2+2q_f^4\right) - 12q_0q_f\sin(3\varpi T)(q_0^2+q_f^2) + \sin(4\varpi T)(q_0^4+q_f^4)\Big], \end{aligned} \tag{42}$$

$$\begin{aligned} S_{\mathcal{C}}^{(f_0)} = &-\frac{f_0 m\varpi\Omega}{2\sin(\varpi T)(4\varpi^2-\Omega^2)}\left[\frac{\varpi\sin(\Omega T)}{\sin(\varpi T)}\left(q_0^2 - 2q_0q_f\cos(\varpi T) + q_f^2\cos(2\varpi T)\right) + 2q_0q_f\Omega\cos^2\left(\frac{\Omega T}{2}\right)\right. \\ &\left.-\Omega\cos(\varpi T)\left(q_0^2+q_f^2\cos(\Omega T)\right)\right]. \end{aligned} \tag{43}$$

Therefore, we now have the final form of the propagator for the resonant bar detector interacting with a gravitational wave as follows:

$$\left\langle q_f, T | q_0, 0\right\rangle = \sqrt{\frac{m\varpi}{2\pi i\hbar\sin(\varpi T)}}\tilde{\mathcal{N}}(T,\gamma,f_0)e^{\frac{i}{\hbar}S_{cl}}. \tag{44}$$

To obtain an overall structure of the fluctuation parameter in the above equation, we consider the free particle structure involving gravitational wave (GW) interaction only. In this case, the infinitesimal propagator considering the particle GW interaction from Equation (15) can be extracted as follows (in the $\varpi \to 0$ limit):

$$\begin{aligned} \langle q_1, \Delta t | q_0, 0\rangle &= \int_{-\infty}^{\infty}\frac{dp_0}{2\pi\hbar}\exp\left[\frac{i\Delta t}{\hbar}\left(p_0\frac{(q_1-q_0)}{\Delta t} - \left(\frac{p_0^2}{2m} + \frac{\gamma p_0^4}{m} + \frac{p_0q_0f_0}{\Delta t}(\cos(\Omega\Delta t)-1)\right)\right)\right] \\ &\simeq \sqrt{\frac{m}{2\pi i\hbar\Delta t}}e^{\frac{im}{2\hbar\Delta t}(q_1-q_0)^2}\left[1 + \frac{3im\gamma\hbar}{\Delta t} - 6\gamma m^2\left(\frac{q_1-q_0}{\Delta t}\right)^2 - \frac{i\gamma m^3(q_1-q_0)^4}{\hbar\Delta t^3}\right. \\ &\quad\left. - \frac{if_0q_0}{\hbar}\left(\frac{m(q_1-q_0)}{\Delta t}\right)(\cos(\Omega\Delta t)-1)\right]. \end{aligned} \tag{45}$$

Now, the total propagator can be written using the set of infinitesimal propagators as follows:

$$\begin{aligned}\left\langle q_f, T|q_0, 0\right\rangle \simeq & \left(\frac{m}{2\pi i\hbar\Delta t}\right)^{\frac{N}{2}}\int dq_1 dq_2\cdots dq_{N-1}e^{\frac{im}{2\hbar\Delta t}\left[(q_1-q_0)^2+(q_2-q_1)^2+\cdots+(q_f-q_{N-1})^2\right]}\left[1+\frac{3i\gamma m\hbar N}{\Delta t}-\frac{6\gamma m^2}{\Delta t^2}((q_1-q_0)^2\right.\\ & +(q_2-q_1)^2+\cdots+(q_f-q_{N-1})^2)-\frac{i\gamma m^3}{\hbar\Delta t^3}((q_1-q_0)^4+(q_2-q_1)^4+\cdots+(q_f-q_{N-1})^4)\\ & \left.-\frac{if_0 m}{\hbar\Delta t^2}\Big[q_0(q_1-q_0)(\cos(\Omega\Delta t)-1)+\cdots+q_{N-1}(q_f-q_{N-1})(\cos(N\Omega\Delta t)-\cos((N-1)\Omega\Delta t))\Big]\right].\end{aligned} \tag{46}$$

In the absence of the gravitational wave [29], the form of the propagator in Equation (46) reads

$$\left\langle q_f, T|q_0, 0\right\rangle = \sqrt{\frac{m}{2\pi i\hbar T}}e^{\frac{im}{2\hbar T}(q_f-q_0)^2}\left(1+\frac{3i\gamma m\hbar}{T}-6\gamma m^2\left(\frac{q_f-q_0}{T}\right)^2-\frac{i\gamma m^3}{\hbar T^3}(q_f-q_0)^4\right). \tag{47}$$

In the presence of the gravitational wave, the propagator has the form given as

$$\begin{aligned}\left\langle q_f, T|q_0, 0\right\rangle \simeq & \sqrt{\frac{m}{2\pi i\hbar T}}e^{\frac{im}{2\hbar T}(q_f-q_0)^2}\left(1+\frac{3i\gamma m\hbar}{T}-6\gamma m^2\left(\frac{q_f-q_0}{T}\right)^2-\frac{i\gamma m^3}{\hbar T^3}(q_f-q_0)^4\right.\\ & \left.+\frac{if_0 mT}{\hbar}\left(\frac{(q_f-q_0)}{T}\right)^2[\cos(\Omega T)-1]-\frac{if_0 q_f}{\hbar}\left(\frac{m(q_f-q_0)}{T}\right)[\cos(\Omega T)-1]\right)\\ \simeq & \sqrt{\frac{m}{2\pi i\hbar T}}\tilde{\mathcal{N}}(T,\gamma,f_0)e^{\frac{i}{\hbar}S_{cl}^{(f)}}\end{aligned} \tag{48}$$

where $S_{cl}^{(f)}$ is the classical action involving free particles and gravitational waves given by

$$S_{cl}^{(f)} = \frac{m}{2T}(q_f-q_0)^2-\frac{\gamma m^3}{T^3}(q_f-q_0)^4-\frac{mf_0}{2T}(q_f-q_0)\left[(q_f\cos[\Omega T]-q_0)-(q_f-q_0)\frac{\sin[\Omega T]}{\Omega T}\right] \tag{49}$$

and the form of the fluctuation term is given as follows:

$$\begin{aligned}\tilde{\mathcal{N}}(T,\gamma,f_0) \simeq & 1+\frac{3i\gamma m\hbar}{T}-6\gamma m^2\left[\frac{q_f-q_0}{T}\right]^2+\frac{if_0 mT}{\hbar}\left[\frac{q_f-q_0}{T}\right]^2[\cos(\Omega T)-1]-\frac{if_0 m q_f}{\hbar}\left[\frac{q_f-q_0}{T}\right][\cos(\Omega T)-1]\\ & -\frac{imf_0(q_f-q_0)}{2\hbar T}\left[\frac{(q_f-q_0)\sin(\Omega T)}{\Omega T}-(q_f\cos(\Omega T)-q_0)\right].\end{aligned} \tag{50}$$

5. Summary

In this work, we have constructed the path integral formalism of the propagation kernel for a resonant bar detector in the presence of a gravitational wave in the generalized uncertainty principle framework. In this framework, we have considered only quadratic-order correction in the momentum. We have obtained the configuration space action for this system using the path integral formalism. With the action in hand, we have then obtained the equation of motion of the system. From the equation of motion, we observe that the overall frequency of the resonant detector shifts due to interaction with the gravitational wave. Next, we have used the form of the perturbation term for a circularly polarized gravitational wave to calculate the classical solution of the detector coordinate $q(t)$. Using this form of $q(t)$, we have finally obtained the classical action for a resonant bar detector interacting with a gravitational wave in the generalized uncertainty principle framework. We have then investigated the quantum fluctuation parameter of the bar detector in the presence of a circularly polarized gravitational wave. In order to obtain the final form of the fluctuation, we have considered a free particle interacting with the gravitational wave. The final form of the fluctuation picks up correction terms due to both GUP correction and gravitational wave interaction. In this process, we have neglected cross terms considering both GUP and GW interactions as it would result in a much smaller correction to the fluctuation factor than the other corrections present in the analytical form of the quantum fluctuation. It would also be important to carry out the above analysis in a linear

GUP framework. However, we would like to report this in future. From an observational point of view, the importance of our work lies in the fact that resonant bar detectors have the potential for detecting gravitational waves with their present sensitivity at distances of the order of 10^2 kpc from the Earth. The propagator captures the quantum effects also. Hence, detectability of such quantum effects in resonant bar detectors is also a possibility in the near future. Knowledge of the propagator of the detector coordinates is therefore necessary, if not absolutely essential.

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