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Editorial

Editorial of *Modified Theories of Gravity and Cosmological Applications*

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General Relativity is a theory of gravity that describes some of the effects of gravity with high accuracy, such as solar system tests, gravitational lensing, gravitational waves, black holes, deflection angle, etc., in a definite framework of an homogeneous and isotropic space–time.

However, taking into account the abundance and nature of dark energy and dark matter, the nature of inflation, cosmological tensions such as the H0 and S8, the possible values of local anisotropy in the evolution of the universe, as well as the theoretical problems of the cosmological constant and of nonrenormalizability, the validity range of general relativity might be restricted.

Modified theories of gravity extend the framework of general relativity through various methods, leading to different field equations and thus to different cosmological implications. They play an essential role in and contribute to modern cosmology, providing a foundation for the current understanding of physical phenomena of the Universe.

We would like to thank all the valued authors, who contributed to the success of this Special Issue, "*Modified Theories of Gravity and Cosmological Applications*". Their research has promoted the topics of Modified Theories of Gravity, General Relativity and Cosmology. Here, we will briefly cite the main results of the contributors.

Yu-Peng Zhang, Shao-Wen Wei and Yu-Xiao Liu, in their paper "*Spinning Test Particle in Four-Dimensional Einstein-Gauss-Bonnet Black Holes*" [1], investigated the motion of a spinning test particle in a background of a spherically symmetric black hole based on the novel four-dimensional Einstein–Gauss–Bonnet gravity. They successfully found an interesting result: that the innermost stable circular orbit (ISCO) of the spinning test particle has similar behavior as the case of a spinning test particle in GR.

Jianhui Qiu and Changjun Gao, in the paper "*Constructing Higher-Dimensional Exact Black Holes in Einstein-Maxwell-Scalar Theory* [2], constructed higher-dimensional and exact black holes in Einstein–Maxwell–scalar theory. They investigated black hole thermodynamics in connection with the generalized Smarr formula and the first law of thermodynamics. They also provided interesting results for the transition from small black holes to medium and finally to large black holes, by using Hawking temperature.

Thomas Berry, Alex Simpson and Matt Visser, in their paper *Photon Spheres, ISCOs, and OSCOs: Astrophysical Observables for Regular Black Holes with Asymptotically Minkowski Cores* [3], calculated physically observable quantities for a recently proposed regular black hole with an asymptotically Minkowski core. They studied the manner in which the photon sphere and the extremal stable timelike circular orbit (ESCO) relate to the presence (or absence) of horizons. The authors also investigated different situations of photon spheres and ESCO, which is extended to horizonless compact massive objects providing interesting results.

Panayiotis Stavrinos and Sergiu I. Vacaru, in the paper *Broken Scale Invariance, Gravity Mass, and Dark Energy in Modified Einstein Gravity with Two Measure Finsler-Like Variables* [4], studied new classes of generic off-diagonal and diagonal cosmological solutions for effective Einstein equations in modified gravity theories (MGTs), with modified dispersion relations (MDRs), and encoding possible violations of (local) Lorentz invariance (LIVs). Effective potentials for the scalar field provide an interesting unified description of locally anisotropic and/or isotropic early universe inflation related to acceleration cosmology and dark energy. The authors also describe "emergent universes" by off-diagonal and diagonal solutions for certain nonholonomic phases and parametric cosmological evolution resulting in various inflationary phases.

Sergey Il'ich Kruglov, in his paper *New Model of 4D Einstein-Gauss-Bonnet Gravity Coupled with Nonlinear Electrodynamics* [5], obtained an exact spherically symmetric and magnetized Black Hole solution in 4D Einstein–Gauss–Bonnet gravity coupled with nonlinear electrodynamics. He also investigated the black hole thermodynamics, entropy, shadow, energy emission rate and quasinormal modes of black holes, providing interesting results.

Damianos Iosifidis, Nurgissa Myrzakulov and Ratbay Myrzakulov, in their paper *Metric-Affine Version of Myrzakulov* $F(R, T, \mathcal{Q}, \mathcal{T})$ *Gravity and Cosmological Applications* [6], derived the full set of field equations of the class of theories whose gravitational part of the Lagrangian is given by $F(R, T, \mathcal{Q}, \mathcal{T}, \mathcal{D})$. They generalized the family of theories to those also including the divergence of the dilation current obtaining interesting results. In their theory, they also derived the Friedmann equations and examined under what circumstances the presence of torsion can have an accelerating affect on cosmological evolution.

Andronikos Paliathanasis, in his paper *New Anisotropic Exact Solution in Multifield Cosmology* [7], investigated the existence of inflationary solutions on multifield cosmology with a homogeneous locally rotational spacetimes (LRS) anisotropic background space. He also provided an interesting exact solution to describe anisotropic inflation with a Kantowski–Sachs geometry.

Felipe J. Llanes-Estrada, in the paper *Elongated Gravity Sources as an Analytical Limit for Flat Galaxy Rotation Curves* [8], showed that galactic rotation curves are natural in the analytic limit in which the gravitational source is cylindrical, receiving interesting results.

John W. Moffat and Viktor Toth, in the paper *Scalar-Tensor-Vector Modified Gravity in Light of the Planck 2018 Data* [9], extended a calculation that was used previously to demonstrate compatibility between the Scalar–Tensor–Vector–Gravity (STVG) theory. They also found the very interesting result that within the limits of this approximation, the theory accurately reproduces the features of the angular power spectrum.

Gabriele U. Varieschi in the paper *Relativistic Fractional-Dimension Gravity* [10], showed that a relativistic version can be derived from the mathematical theory for spaces with non-integer dimensions, the extended Euler–Lagrange equations for scalar fields, and the existing methods for scalar–tensor models of gravity, multi-scale spacetimes and fractional gravity theories with applications to the FLRW metric of standard cosmology. It was also shown that it is straightforward to extend the standard Friedmann equations and to solve them numerically for different choices of parameters.

Andronikos Paliathanasis, in the paper *Dynamical Analysis and Cosmological Evolution in Weyl Integrable Gravity* [11], investigated the cosmological evolution for the physical parameters in Weyl integrable gravity in a Friedmann–Lemaître–Robertson–Walker universe with zero spatial curvature. He calculated the stationary points for the field equations and he studied their stability properties. He also successfully solved the inverse problem for the case of an ideal gas and proved that the gravitational field equations can follow from the variation of a Lagrangian function.

Joshua Baines, Thomas Berry, Alex Simpson and Matt Visser, in the paper *Killing Tensor and Carter Constant for Painlevé-Gullstrand Form of Lense-Thirring Spacetime* [12], showed that the Painlevé–Gullstrand variant of the Lense–Thirring spacetime possesses a nontrivial Killing tensor, implying separability of the Hamilton–Jacobi equation. They also successfully proved that the Klein–Gordon equation is also separable on this spacetime and they extracted the Carter constant which allowed the geodesic equations to become integrable.

Sergey Paston and Taisiia Zaitseva, in the paper *Nontrivial Isometric Embeddings for Flat Spaces* [13], used an interesting method of sequential surface deformations for the construction of unfolded embeddings to successfully construct such embeddings of flat Euclidean three-dimensional space and Minkowski space, which can be used to analyze the equations of motion of embedding gravity. This method can also be used to build new multidimensional embeddings based on already known embeddings with a small value of the embedding class.

Hongxing Zhang, Naying Zhou, Wenfang Liu and Xin Wu in their paper, *Charged Particle Motions near Non-Schwarzschild Black Holes with External Magnetic Fields in Modified Theories of Gravity* [14], introduced a metric deformation to the Schwarzschild spacetime. They also discussed orbital dynamical properties and successfully proved that the deformation perturbation metric can be changed into a Kerr-like black hole metric via some appropriate coordinate transformation. Finally, they used one of the obtained time-transformed explicit symplectic integrators combined with the techniques of Poincaré sections and FLIs to show how small changes of the parameters affect the dynamical transitions from order to chaos.

Stanislav Alexeyev, Daniil Krichevskiy and Boris Latosh, in the paper *Gravity Models with Nonlinear Symmetry Realization* [15], studied three interesting models with particular non-linear conformal symmetry realizations. Two models are found to be equivalent up to a change of coset coordinates. It was found that models contain ghost degrees of freedom that may be excluded by an introduction of an additional symmetry to the target space. One model was also found to be safe in the early universe.

Aleksander Kozak and Aneta Wojnar, in their paper *Interiors of Terrestrial Planets in Metric-Affine Gravity* [16], used modified gravity theory and showed that it affects the internal properties of terrestrial planets, such as the physical characteristics of their core, mantle and core–mantle boundary. They successfully applied these results for modeling a two-layer exoplanet in Palatini $f(R)$ gravity.

Alexei M. Frolov, in his paper *On Maxwell Electrodynamics in Multi-Dimensional Spaces* [17], derived the equations of Maxwell electrodynamics in multi-dimensional spaces from the variational principle of least action, which is applied to the action function of the electromagnetic field, providing interesting results. He also successfully applied methods of scalar electrodynamics to analyze Maxwell equations in the two- and one-dimensional spaces.

Yuri Shtanov, in the paper *On the Conformal Frames in $f(R)$ Gravity* [18], pointed out that the effect of "running units" in the Einstein frame is related to the fact that the explicit and implicit quantum parameters of the Standard Model, such as the Higgs vacuum expectation value and the parameter Λ_{QCD}, are modified by the conformal transformation of the metric and matter fields and become scalaron-dependent. Considering the scalaron of $f(R)$ gravity describing dark matter, he showed that the effect of running units in this case is extremely weak, making two frames practically equivalent. He also focused on the interesting situation that arises in a late-time universe in which the oscillating scalaron plays the role of dark matter.

Dusko Borka, Vesna Borka Jovanovic, Violeta N. Nikolic, Nenad D. Lazarov and Predrag Jovanovic, in their paper *Estimating the Parameters of the Hybrid Palatini Gravity Model with the Schwarzschild Precession of S2, S38 and S55 Stars: Case of Bulk Mass Distribution* [19], estimated the parameters of the Hybrid Palatini gravity model with the Schwarzschild precession of S-stars, specifically of the S2, S38 and S55 stars. They took into account the case of bulk mass distribution near the Galactic Center. Based on this observational fact, they successfully evaluated the parameters of the Hybrid Palatini Gravity model with the Schwarzschild precession of the S2, S38 and S55 stars, and they estimated the range of parameters of the Hybrid Palatini gravity model for which the orbital precession is as in GR for all three stars. They also evaluated the parameters of the Hybrid Palatini Gravity model in the case of different values of bulk mass density distribution of extended matter.

Mahmoud AlHallak, Amer AlRakik, Nidal Chamoun and Moustafa Sayem El-Daher, in their paper *Palatini $f(R)$ Gravity and Variants of k-/Constant Roll/Warm Inflation within Variation of Strong Coupling Scenario* [20], showed that upon applying Palatini $f(R)$ method, characterized by an αR^2, one obtains a quadratic kinetic energy. They investigated in Palatini formalism two extreme cases corresponding first to $(\alpha >> 1)$, which represents a highly non-canonical k-inflation, and second to $(\alpha << 1)$, where they kept terms to the first order and examined a specific type of the k-inflation, namely the constant-roll inflation. They also successfully showed the viability of the model for some choices of the free parameters in regards to the spectral parameters (n_s, r) when compared to the results of Planck 2018.

Conflicts of Interest: The authors declare no conflict of interest.

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