

Topological Effects of a Spiral Dislocation on Quantum Revivals

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Abstract: We analyse the influence of spiral dislocation topology on the revival time for the harmonic oscillator, for a particle confined to one-dimensional quantum ring, and a two-dimensional quantum ring. We first investigate the effects of a cut-off point that stems from the topology of this defect on the harmonic oscillator. Then, we show that the influence of spiral dislocation topology on the harmonic oscillator gives rise to a non-null revival time related to the radial quantum number. In the case of the two-dimensional quantum ring, we show that the revival times related to the radial quantum number and the angular momentum quantum number are influenced by the spiral dislocation topology.

Keywords: spiral dislocation; linear topological defects; harmonic oscillator; analytical solutions; quantum revivals; quantum rings

1. Introduction

In recent decades, the connection between the description of linear topological defects in gravitation and solids has been the focus of a great deal of work [1–33]. This connection is made in an elegant way via the Katanaev–Volovich approach [34]. This model allows us to describe linear topological defects through the Riemann–Cartan geometry [35]. Examples of the use of the Katanaev–Volovich approach [34] are the studies of the influence of a disclination in materials described by the Dirac equation, such as graphene [36–39] and fullerene [40]. Based on nonrelativistic wave equations, several studies have shown the influence of linear topological defects on the electronic properties of solids [41–48]. Linear topological defects have also been studied in quantum rings [49–52], for an electron subject to the deformed Kratzer potential [53] and subject to a uniform magnetic field [54–57]. The most promising perspective in the studies of linear topological defects is the appearance of Aharonov–Bohm-type effects [58–61].

In this work, we study the influence of spiral dislocation topology on the revival time for the harmonic oscillator, a particle confined to one-dimensional quantum ring, and a two-dimensional quantum ring. We start by raising a discussion about a cut-off point that arises from the topology of the spiral dislocation. Then, we study the influence of this cut-off point on the harmonic oscillator. We show that the influence of this cut-off point modifies the spectrum of energy of the harmonic oscillator. From the eigenvalues of energy, we show that a non-null revival time [62] related to the radial quantum number can be obtained due to the influence of this cut-off point on the harmonic oscillator. Later, we extend our discussion about the influence of the spiral dislocation topology on quantum revivals [62] to a particle confined to a one-dimensional quantum ring and a two-dimensional quantum ring.

The structure of this paper is as follows. In Section 2, we show that a cut-off point arises from the topology of the spiral dislocation, then we study the influence of this cut-off point on the harmonic oscillator. We discuss the effects of the topology on the spiral dislocation on the revival time [62–65] in the two-dimensional harmonic oscillator system; in Section 3, we discuss the influence of the spiral dislocation topology on the quantum revivals for a



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particle confined to a one-dimensional quantum ring; in Section 4, we extend the discussion about the influence of the spiral dislocation topology on the quantum revivals for a particle confined to a two-dimensional quantum ring; and in Section 5, we present our conclusions.

2. Effects of a Cut-Off Point Yield by the Spiral Dislocation Topology on the Harmonic Oscillator

Let us introduce the spiral dislocation that corresponds to the distortion of a circle into a spiral [21–23,45]. Based on the Katanaev–Volovich approach [34], the spiral dislocation is described by the line element:

$$ds^2 = dr^2 + 2\beta dr d\varphi + (\beta^2 + r^2)d\varphi^2 + dz^2. \tag{1}$$

The parameter β is a constant and it is related to the Burger vector \vec{b} through the relation $\beta = |\vec{b}|/2\pi$ [45]. In this case, the Burger vector is parallel to the plane $z = 0$. In addition, we see that $0 < r < \infty, 0 \leq \varphi \leq 2\pi$ and $-\infty < z < \infty$. Recently, effects of the spiral dislocation topology have been studied in relativistic quantum systems in Refs. [66–68].

For a spinless particle in an elastic medium with a linear topological defect, such as the spiral dislocation (1), the Laplace–Beltrami operator is given in the form [4,46,55]: $\nabla^2 = \frac{1}{\sqrt{g}} \partial_i (g^{ij} \sqrt{g} \partial_j)$, where g_{ij} is the metric tensor, g^{ij} is the inverse of g_{ij} and $g = \det|g_{ij}|$. In this case, the indices $\{i, j\}$ run over the space coordinates. Hence, when the quantum particle is subject to the two-dimensional harmonic oscillator, the time-independent Schrödinger equation for the harmonic oscillator in the presence of the spiral dislocation is given by [21] (we shall use the units where $\hbar = 1$ and $c = 1$):

$$E\psi = -\frac{1}{2m} \left(1 + \frac{\beta^2}{r^2}\right) \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{2m} \left(\frac{1}{r} - \frac{\beta^2}{r^3}\right) \frac{\partial \psi}{\partial r} + \frac{\beta}{m r^2} \frac{\partial^2 \psi}{\partial r \partial \varphi} - \frac{1}{2m r^2} \frac{\partial^2 \psi}{\partial \varphi^2} - \frac{1}{2m} \frac{\beta}{r^3} \frac{\partial \psi}{\partial \varphi} - \frac{1}{2m} \frac{\partial^2 \psi}{\partial z^2} + \frac{1}{2} m \omega^2 r^2 \psi. \tag{2}$$

The solution to the Schrödinger Equation (2) can be given in the form: $\psi(r, \varphi, z) = \Phi(\varphi) Z(z) u(r)$. After substituting it into Equation (2), we find that $\Phi(\varphi) = e^{i\ell\varphi}$ and $Z(z) = e^{ikz}$, where $\ell = 0, \pm 1, \pm 2, \dots$ and k is a constant. Thereby, we obtain the radial equation:

$$\left(1 + \frac{\beta^2}{r^2}\right) u'' + \left(\frac{1}{r} - \frac{\beta^2}{r^3} - i \frac{2\beta\ell}{r^2}\right) u' - \frac{\ell^2}{r^2} u + i \frac{\beta\ell}{r^3} u - m^2 \omega^2 r^2 u + (2mE - k^2) u = 0. \tag{3}$$

The first aspect to be observed in the radial Equation (3) is that there is no shift in the angular momentum quantum number due to the effects of the spiral dislocation topology. As shown in Refs. [61,69], the effects of the topology of a disclination yield a shift in the angular momentum quantum number given by $\ell \rightarrow \ell/\alpha$, where α is the parameter related to the angular deficit which characterizes the disclination. Moreover, in Refs. [49,60,69], the shift in the angular momentum quantum number is given by $\ell \rightarrow \ell - \bar{\beta}k$, where $\bar{\beta}$ is the parameter related to the Burgers vector which characterizes the screw dislocation (in this case, it corresponds to the distortion of a circular curve into a vertical spiral [45]). These examples of shift of the angular momentum quantum number have shown the influence of the linear topological defects on the eigenvalues of energy that gives rise to Aharonov–Bohm-type effects for bound states [59]. From this perspective, the topology of the spiral dislocation does not yield any shift in the angular momentum quantum number, hence there is no Aharonov–Bohm-type effect for the harmonic oscillator in the presence of a spiral dislocation.

Let us proceed with the solution to the radial Equation (3). It is given in the form [21]:

$$u(r) = \exp\left(i \ell \tan^{-1}\left(\frac{r}{\beta}\right)\right) \times f(r), \tag{4}$$

where $f(r)$ is the solution to the second-order differential equation:

$$\left(1 + \frac{\beta^2}{r^2}\right) f'' + \left(\frac{1}{r} - \frac{\beta^2}{r^3}\right) f' - \frac{\ell^2}{(r^2 + \beta^2)} f - m^2 \omega^2 r^2 f + (2mE - k^2) f = 0. \tag{5}$$

We thus go further by performing the change of variables: $x = m\omega(r^2 + \beta^2)$ [21]. However, in contrast to Ref. [21], let us take a solution well-behaved when $x \rightarrow \infty$. Therefore, in terms of the dimensionless parameter x , the solution to Equation (5) is given by

$$f(x) = e^{-\frac{x}{2}} x^{\frac{|\ell|}{2}} U\left(\frac{|\ell|}{2} + \frac{1}{2} - \lambda, |\ell| + 1; x\right), \tag{6}$$

where $U\left(\frac{|\ell|}{2} + \frac{1}{2} - \lambda, |\ell| + 1; x\right)$ is regular at $x \rightarrow \infty$ and it is known as the confluent hypergeometric function of second kind [70]. The parameter λ , in turn, is defined as

$$\lambda = \frac{1}{4m\omega} (2mE + m^2\omega^2\beta^2 - k^2). \tag{7}$$

Recently, we have studied the harmonic oscillator under the influence of the spiral dislocation (1) by assuming that β^2 is very small [21]. This assumption has allowed us to analyse the asymptotic behaviour of the radial wave function when $r \rightarrow 0$, as we can consider $x \rightarrow 0$ [23]. In the present work, by contrast, we do not assume that β is very small. Therefore, we cannot assume that $x \rightarrow 0$ when $r \rightarrow 0$. In this case, when $r \rightarrow 0$, then $x \rightarrow m\omega\beta^2$. This gives us a new view of the system: the point $r_0 = \beta$ can be considered as a cut-off point analogous to that considered in Refs. [20,71–75], where the attractive inverse-square potential is dealt with. Hence, the spiral dislocation topology imposes a lower limit on x given by $x_0 = m\omega r_0 = m\omega\beta^2$. Thereby, the wave function (6) is normalized in the range: $x_0 \leq x < \infty$. As a consequence, we have the boundary condition:

$$f(x_0) = 0. \tag{8}$$

Henceforth, we focus on a particular case with the purpose of obtain the eigenvalues of energy explicitly. For x_0 and $B = |\ell| + 1$ fixed, and for large $|A|$ ($A = \frac{|\ell|}{2} + \frac{1}{2} - \lambda$), then, the function $U(A, B; x_0)$ can be written in the form [70]: $U(A, B; x_0) \propto \cos\left(\sqrt{(2B - 4A)x_0} - \frac{B\pi}{2} + A\pi + \frac{\pi}{4}\right)$. Hence, from Equation (6) and the boundary condition (8), we obtain the eigenvalues of energy:

$$E_n = -\omega \left[2n + \frac{1}{2} + \frac{m\omega\beta^2}{2}\right] + \frac{4m\omega^2\beta^2}{\pi^2} \left[1 \pm \sqrt{1 - \frac{\pi^2}{4m\omega\beta^2}(4n + 1)}\right] + \frac{k^2}{2m}, \tag{9}$$

where $n = 0, 1, 2, 3, \dots$ is the radial quantum number.

Hence, the allowed energies (9) stem from the influence of the cut-off point yielded by the topology of the spiral dislocation on the harmonic oscillator. As we have not considered the parameter β to be very small, the effects of the spiral dislocation topology have imposed a lower limit on the range where the wave function must be normalized. Additionally, the energy levels do not depend on the angular momentum quantum number ℓ . Therefore, each energy level E_n has infinity degeneracy. The effects of the spiral dislocation topology also yield the contributions given by the terms proportional to β^2 . In addition, the radial quantum number possesses an upper limit given by

$$n_{\max} < \frac{m\omega\beta^2}{\pi^2} - \frac{1}{4}. \tag{10}$$

This upper limit is the maximum value of the radial quantum number n_{\max} (maximal integer). Hence, the radial quantum number n possesses values from zero to the upper limit given in Equation (10), otherwise, the energy levels (9) would have an imaginary term. Therefore, the upper limit of the radial quantum number (10) is influenced by the spiral dislocation topology. Finally, the last term of the energy levels (9) is the translational kinetic energy that stems from the free motion of the particle in the z -direction. Hence, the bound states occur in two-dimensions even though the system is unconfined in the third dimension in agreement with Refs. [49,76,77].

Quantum Revivals

According to Refs. [62–65], quantum revivals occur when the wave function recovers its initial shape at a time called the revival. Quantum revivals have drawn attention in recent years, where it is worth citing the studies of quantum revivals in the infinite square well [64,65,78–80], quantum pendulum [81], position-dependent mass systems [82], Rydberg atoms [83–85] and graphene [86,87]. Based on Refs. [62,63,88–90], when a quantum system has one quantum number ν , the energy eigenvalues can be expanded about central value ν_1 of this quantum number. Thereby, the energy can be expanded in Taylor series as [62,63]:

$$E_\nu \approx E_{\nu_1} + \left(\frac{dE}{d\nu}\right)_{\nu=\nu_1} (\nu - \nu_1) + \frac{1}{2} \left(\frac{d^2E}{d\nu^2}\right)_{\nu=\nu_1} (\nu - \nu_1)^2 + \dots \tag{11}$$

Therefore, there are distinct time scales. The classical period is given by

$$T_{\text{cl}} = \frac{2\pi\hbar}{\left|\left(\frac{dE}{d\nu}\right)_{\nu=\nu_1}\right|}, \tag{12}$$

while the revival time is defined by [62,63]

$$\tau = \frac{4\pi\hbar}{\left|\left(\frac{d^2E}{d\nu^2}\right)_{\nu=\nu_1}\right|}. \tag{13}$$

Although we have studied a bidimensional system in the previous section, the energy levels (9) depend on just one quantum number, i.e, the allowed energies are determined only by the radial quantum number. In this way, both the classical period (12) and the revival time (13) are defined in terms of the radial quantum number n . With respect to the revival time (13), for a given value of n , we have

$$\tau = \frac{4\pi}{\left|\frac{d^2E_n}{dn^2}\right|} = \frac{4m\beta^2}{\pi} \times \left[1 - \frac{\pi^2}{4m\omega\beta^2}(4n + 1)\right]^{3/2}. \tag{14}$$

Hence, we have obtained a non-null revival time in contrast to what is shown in Ref. [62], where there is no revival time in the one-dimensional harmonic oscillator system. However, in the two-dimensional case (cylindrical symmetry), the energy levels of the harmonic oscillator are proportional to n and $|\ell|$ [21]. By dealing with the two-dimensional harmonic oscillator as in Refs. [62,63,88–90], the quantum revivals are obtained with respect to the quantum numbers $\{n, \ell\}$. Therefore, there is no revival time related to the radial quantum number, but we would have a non-null revival time related to the angular momentum quantum number due to contribution that stems from the term proportional to $|\ell|$. Returning to the allowed energies (9), they do not depend on ℓ , so there is no revival time associated with the angular momentum quantum number. The existence of

the revival time associated with the radial quantum number given in Equation (14) is due to the influence of the cut-off point that stems from the spiral dislocation topology on the harmonic oscillator.

3. Effects of the Spiral Dislocation Topology on Quantum Revivals in a One-Dimensional Ring

Recently, one of the authors studied the effects of the spiral dislocation on a particle confined to a one-dimensional quantum ring [52]. With r_a as the radius of the one-dimensional quantum ring, the corresponding spectrum of energy is [52]

$$E_\ell \approx \frac{\ell^2}{2m(r_a^2 + \beta^2)} - \frac{1}{8m(r_a^2 + \beta^2)}, \tag{15}$$

where the effects of the spiral dislocation topology give rise to the presence of the effective radius $x_a = \sqrt{r_a^2 + \beta^2}$ and the analogue of the Costa term [49,91], which is given by the last term of the right-hand side of Equation (15). Note that $\ell = 0, \pm 1, \pm 2, \pm 3, \dots$ is the angular momentum quantum number.

The energy levels (15) depend only on the angular momentum quantum number, hence both the classical period (12) and the revival time (13) are defined in terms of the angular momentum quantum number ℓ . In this way, the revival time (13) is given by

$$\tau = \frac{4\pi}{\left| \frac{d^2 E_\ell}{d\ell^2} \right|} = 4\pi m (r_a^2 + \beta^2), \tag{16}$$

where there is the influence to the spiral dislocation topology on the revival time.

4. Effects of the Spiral Dislocation Topology on Quantum Revivals in a Two-Dimensional Ring

Recently, the effects of the spiral dislocation on a particle confined to a two-dimensional quantum ring have also been studied by one of us in Ref. [52]. In this study, the quantum particle is restricted to move between two fixed radii $r = r_a$ and $r = r_b$ ($r_b > r_a$) in the plane $z = 0$. The corresponding spectrum of energy is given by [52]

$$E_{n,\ell} \approx \frac{n^2 \pi^2}{2m \left[\sqrt{r_b^2 + \beta^2} - \sqrt{r_a^2 + \beta^2} \right]^2} + \frac{4\ell^2 - 1}{8m \sqrt{(r_b^2 + \beta^2)(r_a^2 + \beta^2)}}. \tag{17}$$

In this case, the effects of the spiral dislocation topology have given rise to the presence of the effective radii $x_a = \sqrt{r_a^2 + \beta^2}$ and $x_b = \sqrt{r_b^2 + \beta^2}$ in the energy levels (17). Note that $n = 0, 1, 2, 3, \dots$ is the radial quantum number and $\ell = 0, \pm 1, \pm 2, \pm 3, \dots$ is the angular momentum quantum number.

In this case, the bidimensional system is characterized by having two quantum numbers $\{n, \ell\}$. When a quantum system is characterized by having two or more quantum numbers, we follow Refs. [62,63,88–90] with the purpose of obtaining the quantum revivals. With two quantum numbers ν_1 and ν_2 , we can label the energy eigenvalues as E_{ν_1, ν_2} . Thereby, the energy eigenvalues can be expanded about central values ν'_1 and ν'_2 of these quantum numbers, hence the energy can be expanded in Taylor series as [62,63]:

$$\begin{aligned} E_{\nu_1, \nu_2} &\approx E_{\nu'_1, \nu'_2} + \left(\frac{\partial E}{\partial \nu_1} \right)_{\nu'_1 \nu'_2} (\nu_1 - \nu'_1) + \left(\frac{\partial E}{\partial \nu_2} \right)_{\nu'_1 \nu'_2} (\nu_2 - \nu'_2) \\ &+ \frac{1}{2} \left(\frac{\partial^2 E}{\partial \nu_1^2} \right)_{\nu'_1 \nu'_2} (\nu_1 - \nu'_1)^2 + \frac{1}{2} \left(\frac{\partial^2 E}{\partial \nu_2^2} \right)_{\nu'_1 \nu'_2} (\nu_2 - \nu'_2)^2 \\ &+ \left(\frac{\partial^2 E}{\partial \nu_1 \partial \nu_2} \right)_{\nu'_1 \nu'_2} (\nu_1 - \nu'_1)(\nu_2 - \nu'_2) + \dots \end{aligned} \tag{18}$$

Therefore, the classical periods are given by

$$T_{cl}^{(1)} = \frac{2\pi\hbar}{\left| \left(\frac{\partial E}{\partial v_1} \right)_{v'_1 v'_2} \right|}; \quad T_{cl}^{(2)} = \frac{2\pi\hbar}{\left| \left(\frac{\partial E}{\partial v_2} \right)_{v'_1 v'_2} \right|}, \tag{19}$$

while the revival times are defined by [62,63]

$$\tau^{(1)} = \frac{4\pi\hbar}{\left| \left(\frac{\partial^2 E}{\partial v_1^2} \right)_{v'_1 v'_2} \right|}; \quad \tau^{(2)} = \frac{4\pi\hbar}{\left| \left(\frac{\partial^2 E}{\partial v_2^2} \right)_{v'_1 v'_2} \right|}; \quad \tau^{(12)} = \frac{4\pi\hbar}{\left| \left(\frac{\partial^2 E}{\partial v_1 \partial v_2} \right)_{v'_1 v'_2} \right|}. \tag{20}$$

The revival time $\tau^{(12)}$ is called as the cross-revival time. Due to the fact that the allowed energies (17) are determined by the quantum numbers $\{n, \ell\}$, thus, the classical periods (19) and the revival times (20) are defined in terms of the quantum numbers $\{n, \ell\}$.

As our focus is on the revival times, with respect to the radial quantum number, the revival time is

$$\tau^{(1)} = \frac{4\pi}{\left| \frac{\partial^2 E_{n,\ell}}{\partial n^2} \right|} = \frac{4m}{\pi} \left[\sqrt{r_b^2 + \beta^2} - \sqrt{r_a^2 + \beta^2} \right]^2. \tag{21}$$

On the other hand, with respect to the angular momentum quantum number, the revival time is

$$\tau^{(2)} = \frac{4\pi}{\left| \frac{\partial^2 E_{n,\ell}}{\partial \ell^2} \right|} = 4\pi m \sqrt{(r_b^2 + \beta^2)(r_a^2 + \beta^2)}. \tag{22}$$

Finally, the cross-revival time [63] is given by

$$\tau^{(12)} = \frac{4\pi}{\left| \left(\frac{\partial^2 E_{n,\ell}}{\partial n \partial \ell} \right) \right|} = 0. \tag{23}$$

Therefore, both revival times (21) and (22) are influenced by the topology of the spiral dislocation. However, there is no cross-revival time for a particle confined to a two-dimensional quantum ring in the presence of the spiral dislocation. It is worth observing that the revival time related to the radial quantum number (21) differs from the revival time related to the angular momentum number (23). Furthermore, we can assume that the relation between $\tau^{(1)}$ and $\tau^{(2)}$ is satisfied, i.e., $\frac{\tau^{(1)}}{\tau^{(2)}} = \frac{\bar{a}}{\bar{b}}$, where \bar{a} and \bar{b} are relatively prime integers. This relation also certifies that they are commensurate [62,63]. The effective radii also determine whether these revival times are commensurate.

5. Conclusions

We have analysed the influence of a cut-off point on the harmonic oscillator in an elastic medium with a spiral dislocation. In this case, the cut-off point is yielded by the topology of the spiral dislocation. We have seen that the presence of the cut-off point modifies the spectrum of energy of the harmonic oscillator, where the energy levels are infinitely degenerated with respect to the angular momentum quantum number ℓ . In addition, the effects of the topology of the spiral dislocation yield the contributions to the energy levels given by the terms proportional to β^2 . However, there is no analogue of the Aharonov–Bohm effect for bound states [49,59,60]. In addition, the radial quantum number has an upper limit, which is determined by the angular frequency and the spiral dislocation topology.

We have also analysed the influence of the topology of the spiral dislocation on the revival time in the harmonic oscillator system. We have seen that the revival time is

influenced by the spiral dislocation topology. In this case, the effects of the topology of the spiral dislocation have allowed us to obtain a non-null revival time related to the radial quantum number. On the other hand, no revival time related to the angular momentum quantum number exists.

Finally, we have raised a discussion about the influence of the spiral dislocation topology on the quantum revivals for a particle confined to one-dimensional quantum ring and a two-dimensional quantum ring. We have seen in these cases that the spiral dislocation topology influences the revival times. In the case of the two-dimensional quantum ring, we have obtained different revival times with respect to both radial and angular momentum quantum numbers. However, no cross-revival time exists.

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