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# Frame-Dragging in Extrasolar Circumbinary Planetary Systems

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**Abstract:** Extrasolar circumbinary planets are so called because they orbit two stars instead of just one; to date, an increasing number of such planets have been discovered with a variety of techniques. If the orbital frequency of the hosting stellar pair is much higher than the planetary one, the tight stellar binary can be considered as a matter ring current generating its own post-Newtonian stationary gravitomagnetic field through its orbital angular momentum. It affects the orbital motion of a relatively distant planet with Lense-Thirring-type precessional effects which, under certain circumstances, may amount to a significant fraction of the static, gravitoelectric ones, analogous to the well known Einstein perihelion precession of Mercury, depending only on the masses of the system's bodies. Instead, when the gravitomagnetic field is due solely to the spin of each of the central star(s), the Lense-Thirring shifts are several orders of magnitude smaller than the gravitoelectric ones. In view of the growing interest in the scientific community about the detection of general relativistic effects in exoplanets, the perspectives of finding new scenarios for testing such a further manifestation of general relativity might be deemed worth of further investigations.

**Keywords:** experimental studies of gravity; experimental tests of gravitational theories; extrasolar planetary systems

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## 1. Introduction

According to the General Theory of Relativity [1] (GTR), the deformed spacetime generated by a localized, non-static distribution of matter-energy such as a rotating star affects the orbital motion of a nearby test particle like, e.g., a planet p in such a way that its trajectory is not closed, as in the case of the unchanging Keplerian ellipse of the Newtonian mechanics. Among other things, there are two types of resulting secular effects to the first post-Newtonian (1pN) order: the static "gravitoelectric" (GE) Einstein precession [2] of the pericenter $\omega_{\rm p}$ due solely to the total mass $M$ of the system, and the stationary "gravitomagnetic" Lense-Thirring (LT) precessions [3] of the longitude of the ascending node $\Omega_{\rm p}$, and the pericenter caused by the proper angular momentum $\boldsymbol{J}$ of the spinning central object (see, e.g., [4,5]. In general, the pN gravitomagnetic field is generated by mass-energy currents); for the concept of gravitoelectromagnetism within general relativity, see, e.g., [6–18], and references therein. The gravitoelectric and gravitomagnetic net shifts per orbit of the planet's pericentre $\omega_{\rm p}$ are [19–21]

$$\Delta\omega_{\rm p}^{\rm GE} = \frac{6\,\pi\,G\,M}{c^2\,a_{\rm p}\left(1-e_{\rm p}^2\right)}, \tag{1}$$

$$\Delta\omega_{\rm p}^{\rm LT} = -\frac{4\,\pi\,G\,\boldsymbol{J}\cdot\left(2\,\hat{\boldsymbol{h}}_{\rm p}+\cot I_{\rm p}\,\hat{\boldsymbol{m}}_{\rm p}\right)}{c^2\,n_{\rm p}\,a_{\rm p}^3\left(1-e_{\rm p}^2\right)^{3/2}}, \tag{2}$$

where $G$ is the Newtonian constant of gravitation, $c$ is the speed of light in vacuum, $a_{\rm p}$ is the planet's semimajor axis, $e_{\rm p}$ is the planet's eccentricity, $I_{\rm p}$ is the inclination of the planetary orbital plane to the reference $\{x,\, y\}$ plane, customarily identified with the plane of the sky,

$$n_{\rm p} \doteq \sqrt{\frac{G\,M}{a_{\rm p}^3}} \tag{3}$$

is the planet's Keplerian mean motion,

$$\hat{\boldsymbol{h}}_{\rm p} = \left\{\sin I_{\rm p}\,\sin\Omega_{\rm p},\, -\sin I_{\rm p}\,\cos\Omega_{\rm p},\, \cos I_{\rm p}\right\} \tag{4}$$

is a unit vector directed along the planet's orbital angular momentum, and

$$\hat{\boldsymbol{m}}_{\rm p} = \left\{-\cos I_{\rm p}\,\sin\Omega_{\rm p},\, \cos I_{\rm p}\,\cos\Omega_{\rm p},\, \sin I_{\rm p}\right\} \tag{5}$$

is a unit vector in the planetary orbital plane perpendicular to the line of the nodes, which is the intersection of the orbital plane with the reference $\{x, y\}$ plane [4,5]. For the sake of completeness, also the Lense-Thirring node precession is mentioned: its net shift per orbit is [19,21]

$$\Delta\Omega_{\rm p}^{\rm LT} = \frac{4\,\pi\,G\,\csc I_{\rm p}\,\boldsymbol{J}\cdot\hat{\boldsymbol{m}}_{\rm p}}{c^2\,n_{\rm p}\,a_{\rm p}^3\left(1-e_{\rm p}^2\right)^{3/2}}. \tag{6}$$

As it turns out from Equations (1) and (2), the gravitoelectric effect is, in general, larger than the gravitomagnetic one; suffice it to say that in the case of Sun and Mercury [2] it is

$$\left|\frac{\Delta\omega_{\mathrm{☿}}^{\rm LT}}{\Delta\omega_{\mathrm{☿}}^{\rm GE}}\right| \approx 3\times10^{-5}, \tag{7}$$

while for the Earth and, say, the LAGEOS (L) satellite [22], it is

$$\left|\frac{\Delta\omega_{\rm L}^{\rm LT}}{\Delta\omega_{\rm L}^{\rm GE}}\right| \approx 9\times10^{-3}. \tag{8}$$

This is why the gravitoelectric precessions have been known since the observations of the then anomalous Mercury's motion by Le Verrier [23] in the mid-nineteenth century, while the Lense-Thirring effect is still so difficult to measure with both natural and artificial objects [24,25].

Nevertheless, for planets orbiting a binary star [26,27], known as circumbinary planets (CBPs), some of which have already been discovered with different techniques [28–48], certain gravitomagnetic effects may be much larger than expected, amounting to about 10% or so of the gravitoelectric ones. The basic idea is as follows. The CBPs discovered so far can be considered as hierarchical triple systems consisting of an inner binary star b and a distant planet p that orbits the centre of mass of b. Thus, the two inner stars A and B can be approximately considered as a mass current sourcing a gravitomagnetic field much stronger than that due to the individual spins of each star through the binary's orbital angular momentum

$$J_{\rm b} = \mu_{\rm b}\,\sqrt{G\,M_{\rm b}\,a_{\rm b}\left(1-e_{\rm b}^2\right)}, \tag{9}$$

where

$$\mu_{\rm b} \doteq \frac{M_{\rm A}\,M_{\rm B}}{M_{\rm b}} \tag{10}$$

is the binary's reduced mass, $M_{\rm b} \doteq M_{\rm A} + M_{\rm B}$ is the total mass of the binary, $a_{\rm b}$ is the binary's semimajor axis and $e_{\rm b}$ is the binary's eccentricity. Thus, it is expected that in Equation (2), calculated with [3] Equation (9), yields a much larger pericenter precession.

Testing relativistic frame dragging in as much ways as possible is important since it is believed to play important roles in several high-energy astrophysical phenomena in strong-field systems [7,9,49–53]. Extending relativistic gravitomagnetism with confidence to such relatively unknown scenarios, for which no direct access is available, requires that it is corroborated in more than just a single case [25].

## 2. The Gravitomagnetic Precessions Due to a Matter Ring Current

By imposing

$$\Delta\omega_{\rm p}^{\rm LT} = q\,\Delta\omega_{\rm p}^{\rm GE}, \tag{11}$$

with $q > 0$, yields the following condition for the semimajor axis of the planet's orbit about the inner binary

$$a_{\rm p} = \frac{16\,a_{\rm b}\left(1-e_{\rm b}^2\right)M_{\rm A}^2\,M_{\rm B}^2}{9\left(1-e_{\rm p}^2\right)M_{\rm b}\left(M_{\rm b}+M_{\rm p}\right)^3 q^2}. \tag{12}$$

As an example, for a binary of two Sun-like stars ($M_{\rm A} = M_{\rm B} = M_\odot$) in a circular orbit ($e_{\rm b} = 0.0$) and a Jupiter-type ($M_{\rm p} = M_{♃}$) circumbinary planet in a moderately eccentric orbit ($e_{\rm p} = 0.2$), by imposing

$$q = 0.1 \tag{13}$$

one gets

$$a_{\rm p} = 11.5\,a_{\rm b}, \tag{14}$$

which fulfils the assumption that the binary is viewed by the planet as a rotating matter ring. By setting

$$q = 1 \tag{15}$$

in Equation (12), corresponding to

$$\Delta\omega_{\rm p}^{\rm LT} = \Delta\omega_{\rm p}^{\rm GE}, \tag{16}$$

yields

$$a_{\rm p} = 0.1\,a_{\rm b}, \tag{17}$$

which implies that the gravitomagnetic precession cannot be as large as the gravitoelectric one. From Figure 1, it turns out that $a_{\rm p} = a_{\rm b}$, and $P_{\rm p} = P_{\rm b}$, for $q \simeq 0.34$. Thus, for CBPs, this form of Lense-Thirring effect cannot reach the $\simeq$30% of the gravitoelectric one.

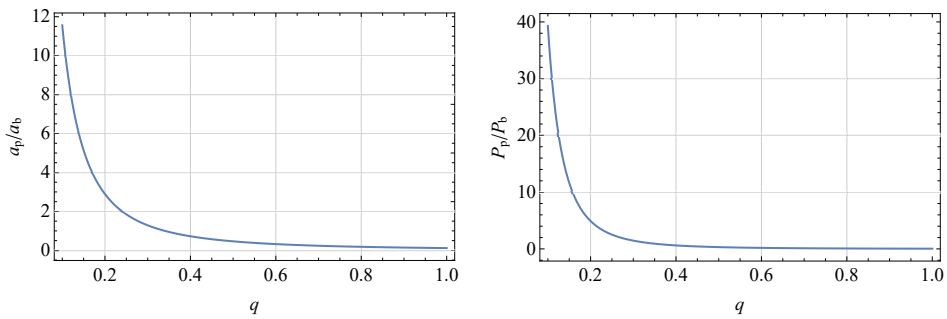


**Figure 1.** Left panel: ratio of the planet's semimajor axis $a_{\rm p}$ to the inner binary's one $a_{\rm b}$ as a function of $q$ according to Equation (12). Right panel: same for the orbital periods of the planet ($P_{\rm p}$) and of the inner binary ($P_{\rm b}$).

According to [54], most of the CBPs exhibit a high degree of coplanarity with the inner binary, i.e., $\boldsymbol{J}_{\rm b}$ and $\boldsymbol{h}_{\rm p}$ are almost aligned. Thus, the pericentre change of Equation (2) can be approximated by

$$\Delta\omega_{\rm p}^{\rm LT} \simeq -\frac{8\,\pi\,G\,J_{\rm b}}{c^2\,n_{\rm p}\,a_{\rm p}^3\left(1-e_{\rm p}^2\right)^{3/2}}, \tag{18}$$

while the node shift of Equation (6) almost vanishes. The precession of Equation (18) is always negative, i.e., the pericentre moves in the opposite direction of the motion of the iner binary. There are some special cases, reported in the literature, where negative orbital plane precessions were reported, e.g., in the presence of a Kerr naked singularity [55] and of a hypothetical gravitomagnetic monopole [56]. Furthermore, ref. [54] remarks that the mass of the primary star varies from 0.69 to 1.53 $M_{\odot}$, with a mass ratio between 1.03 and 3.76 and eccentricity $0.023 \leq e_{\rm b} \leq 0.521$. As far as the CMPs are concerned, their orbital periods are in the range $7.44\,{\rm d} \leq P_{\rm p} \leq 41\,{\rm d}$, with eccentricities $e_{\rm p}$ varying from 0.007 to 0.182 [54]. In order to be stable around the binary host, the planet's orbit must be characterized by $a_{\rm p} = 2 - 4\,a_{\rm b}$, a condition that is fulfilled by the CBPs considered in [54]. However, from the point of view of a possible detection of the sought effect, the issue of the stability of a discovered CBP is not relevant since its lifetime, even if short in astronomical terms, is certainly much longer than any conceivable time span during which observations are collected. Figure 2 displays $\Delta\omega_{\rm p}^{\rm LT}$ and $\Delta\omega_{\rm p}^{\rm GE}$, in arcsec cycle$^{-1}$, as functions of the binary's orbital period $P_{\rm b}$, in d, by imposing the orbital stability condition $a_{\rm p} = 2\,a_{\rm b}$. For the remaining physical and orbital parameters, the values $M_{\rm A} = 0.69\,M_{\odot} = 1.03\,M_{\rm B}$, $e_{\rm b} = 0.023$, $M_{\rm p} = M_{♃}$, $e_{\rm p} = 0.521$ were adopted. Among other things, also the binary's semimajor axis $a_{\rm b}$ and orbital angular momentum $J_{\rm b}$ are shown; it can be noted that $J_{\rm b}$ is about $10^4$ times larger than the spin angular momentum of the Sun which is of the order of $J_{\odot} \simeq 10^{41}$ J s [57].

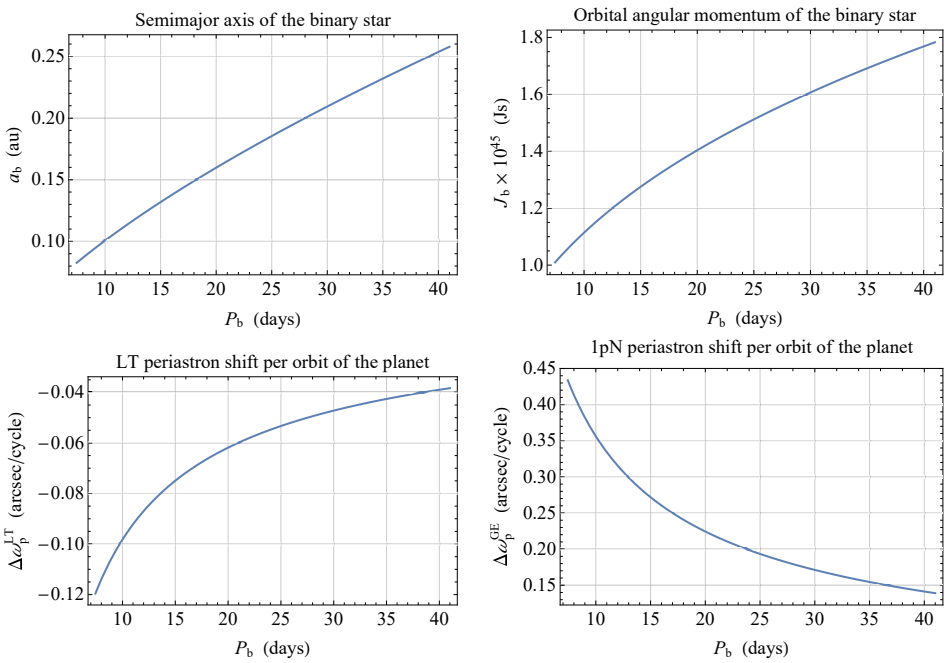


**Figure 2.** Upper row: binary's semimajor axis $a_{\rm b}$, in au, and orbital angular momentum $J_{\rm b}$, in J s, as functions of the binary's orbital period $P_{\rm b}$ ranging from 7.44 to 41 d. Lower row: planet's gravitomagnetic and gravitoelectric net shifts per orbit, in arcsec cycle$^{-1}$, as functions of the binary's orbital period $P_{\rm b}$ ranging from 7.44 to 41 d. The orbital stability condition $a_{\rm p} = 2\,a_{\rm b}$ was adopted along with $M_{\rm A} = 0.69\,M_{\odot} = 1.03\,M_{\rm B}$, $e_{\rm b} = 0.023$, $M_{\rm p} = M_{♃}$, $e_{\rm p} = 0.521$.

The size of the Lense-Thirring shift ranges from 0.12 to 0.04 arcsec cycle$^{-1}$, while the gravitoelectric one is in the range 0.45–0.15 arcsec cycle$^{-1}$.

## 3. Conclusions

So far, a handful of circumbinary planets orbiting different types of stellar pairs, including compact objects as well, have been discovered; although for all of them the matter ring current approximation is substantially valid for their hosting stellar pairs,

they are likely too distant from them to allow for a measurement of relativistic effects. Nonetheless, there may be reasons for being somewhat optimistic.

From the one hand, it is not unrealistic to expect that in a not too far future one or more systems with the right characteristics will be at our disposal.

On the other hand, there is a growing interest in the community of extrasolar planetary scientists about the possibility of extracting general relativistic signatures in such scenarios as well [58–73].

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## Notes

1 For a recent overview, see, e.g., [1] and references therein.

2 In Equations (7) and (8), Equation (2) is computed in a coordinate system whose reference $\{x, y\}$ plane is aligned with the primary's equatorial plane, i.e., $\hat{\boldsymbol{J}} = \{0, 0, 1\}$.

3 In Equations (1)–(3), $M$ is now meant as $M_{\rm b} + M_{\rm p}$, where $M_{\rm p}$ is the planet's mass.

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