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Scalar Radiation in Interaction of Cosmic String with Point Charge

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Abstract: We consider the scalar bremsstrahlung of the spinless relativistic particle, which interacts with infinitely thin cosmic string by linearized gravity. With the iterational scheme, based on the Perturbaion Theory with respect to the Newtonian constant, we compute the radiation amplitude and the emitted energy due to collision. The general phenomenon of mutual cancellation of the leading terms on the local and non-local amplitude, known in the ultrarelativistic regime for several types of collision, also takes place here. Remarkably, this cancellation (destructive interference) is complete, and takes place for any particle's velocity. We compute the spectral and angular distributions of the emitted waves. Particular attention is paid to the ultrarelativistic case. Due to the radiation emission, a string may lose its energy and decrease the tension; it may affect all field effects, including the vacuum polarization and the Casimir effect, in terms of physical problems with the real cosmic strings.

Keywords: cosmic string; point charge; gravitational interaction; bremsstrahlung; destructive interference; perturbation theory

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1. Introduction

Over the past forty years, various cosmological scenarios have been proposed for the explanation of the stages of the Early Universe. All these models try to explain the observable data, including the Cosmic Microwave Background and (since 2016) the gravitational wave detections. Most contemporary theories of the Universe's evolution imply inflation at the very early stages [1,2].

Among them are the cosmological models, which admit the existence of relativistic gravitating objects with non-trivial codimensionality (strings, domain walls) [3–6] and braneworld scenarios, which consider the four-dimensional Universe as being embedded into some higher-dimensional spacetime with non-trivial topology [7,8]. In addition, the spontaneous symmetry breaking implies phase transitions [9–11], where some topological defects may be created [12].

The cosmic string is a topological defect, which might appear in phase transitions in the Early Universe and may well survive during the Universe's evolution [4–6]. It is supposed that the discovery of cosmic strings may become the first evidence of the validity of string theory [13,14]. Some recent attempts to identify the observable data with the existence of cosmic strings may be found in [15,16].

The research on the theory of cosmic strings has been concentrated on the field effects of the curved background, generated by a string (gravitational lensing, vacuum polarization, self-action, gravitational radiation [17–23] etc.), and with the dynamics of the strings themselves, including the radiation due to the collision of the strings [23,24].

The length of a cosmic string is estimated to be comparable with the Universe's size. If so, some inelastic processes involving the cosmic string are important for the proper description of the energy balance during the evolution of the Universe.

In this work, we consider the inelastic encounter of the spinless particle—charged by the massless scalar field—with the infinite cosmic string. Due to the particle being the only charged object, the interaction is purely gravitational. Therefore, we continue the work initiated in [25], extending our consideration from the elastic scattering to the radiation emission. Though there is no hard evidence of the contemporary existence of strings, they might have existed in the past. Therefore, our work may contribute to the study of observable electromagnetic and gravitational cosmic radiation.

Radiation emission represents a collective effect of the system "string–particle", where the string may lose its energy. It decreases the tension, hence it may change the quantitative estimates of the field effects, which contain the string's tension as the field-strength coupling [17–20,23] including the vacuum polarization [26,27] and the Casimir effects [21,22,26], in diverse physical problems with cosmic strings.

The metric of the cosmic string, co-directed with the Cartesian x-axis, in cylindric coordinates (t, x, ϱ, φ) is given by [28]

$$ds^2 = dt^2 - dx^2 - d\varrho^2 - \beta^2\varrho^2 d\varphi^2\,, \tag{1}$$

where $0 < \beta \leqslant 1$. The dimensionless parameter β is connected with energy scale η of the Universe's phase transition by relation $\eta^2 = (1-\beta^2)/8\pi G$. The factor $\beta' \equiv 1-\beta$ represents the relative angular deficit and takes very small values (for $\eta = \eta_{\rm GUT} \sim 10^{16}$ GeV it is of order 10^{-5}) [12,18]. Then $\beta' \simeq 4\pi G\eta^2$.

For the description of scattering, we introduced the Cartesian coordinates [25]. With the change $\varrho \to r$ of the radial variable by relation $\beta\varrho = R_0(r/R_0)^\beta$, where R_0 is an arbitrary parameter with dimensionality of length, the metric becomes (1)

$$ds^2 = dt^2 - dx^2 - \mathrm{e}^{-2\beta' \ln(r/R_0)}(dy^2 + dz^2)\,, \tag{2}$$

(where $r^2 = y^2 + z^2$), that is, conformally-Euclidean (on the hyperplane with fixed x).

The metric (1) and (2) is valid for the infinitely-thin cosmic string. The strings with a finite width were considered in [4,19,29]. However, in practice, the exterior metric coincides with the conical metric (1), while the string's width [4] is estimated to be η^{-1}. For $\eta = \eta_{\rm GUT}$, the string's diameter is of order $d \sim 10^{-29}$ cm [10], which is (presumably) much less than that of the impact parameters supposed for the scattering. Hence, we expect that by preserving the string's linear energy density in the thin-string limit, the string's width may be neglected, with the corresponding restriction on the available impact parameters. Therefore, we consider an infinitely-thin cosmic string; for the mathematical description of which we will use the Polyakov's form.

Concerning the dynamics of cosmic strings, the interaction of cosmic strings, the propagation of strings in the expanding Universe, the scattering on the string, and so forth [5,6,30,31], including the self interactions of cosmic strings [32,33], have previously been considered. In [25], we computed the back-effect of the particle on the string. Though it turned out to be tiny (the gravitational deflection of a string due to a single spinless particle is of the order of its Schwarzschild radius), one might expect the amplification of effects in the inelastic stage, due to the possible multiplicity in a gas of similar particles.

Here, we consider the energy emitted due to collision of a spinless point-like particle in the plane transverse to the unperturbed string. In order to minimize the technical computational routine, we compute the scalar radiation instead of the gravitational radiation, which allows us to not involve polarizations and so forth. As the basis of this substitution, we can refer to an analogy that happened for the bremsstrahlung due to the ultrarelativistic gravitational collisions of a point-like charge: scalar bremsstrahlung on point mass [34] versus pure gravitational bremsstrahlung [35] and scalar bremsstrahlung on the domain wall [36] versus pure gravitational bremsstrahlung [37]. In both analogies, the estimate of the total emitted energy in the gravitational bremsstrahlung can be obtained by the formal substitution of $f \to \sqrt{G}m$ (where f is a scalar charge of the point mass m, G is the Newtonian constant) in the expression for the total emitted scalar energy.

Furthermore, the scalar bremsstrahlung can be an adequate representative of the electromagnetic radiation of electric charge e, with the corresponding replacement of couplings $f \to e$. For the gravitational interaction, we refer to [38], for the non-gravitational interaction we refer to [39,40] and we refer to the scalar and vector Larmor formulae directly.

This paper is organized as follows: after the Introduction, in Section 2 we consider the setup of the gravitational interaction problem for the particle–string system and introduce the iteration scheme we will use. Section 3 is devoted to the computation of the radiation amplitude. In Section 4, we compute the total emitted energy and the particular spectral and angular distributions. Finally, in the Conclusion we will discuss the results and point out some prospects of the presented work. The momentum–space integrals used in the text are computed in the single Appendix A.

We use $\hbar = c = 1$ units and the spacetime metric $g_{\mu\nu}$ with signature $(+ - - -)$. The Greek indices $\mu, \nu, \ldots$ run over the values 0, 1, 2, 3. The Riemann and Ricci tensors are defined as $R^{\mu}{}_{\nu\lambda\rho} \equiv \partial_\lambda \Gamma^{\mu}_{\nu\rho} - \ldots$, $R_{\mu\nu} \equiv R^{\lambda}{}_{\mu\lambda\nu}$. The string's inner coordinates σ on the worldsheet have initial small Latin indices, which take the values 0 and 1.

2. The Setup

The pointlike spinless particle with mass m and scalar charge f moves across the worldline with coordinates $Z^\mu(s)$ parametrized by the affine parameter s; $\dot{Z}^\mu \equiv \partial Z^\mu / \partial s$ stands for the tangent vector to the worldline. The string propagates by its worldsheet $\mathcal{V}_2 \subset \mathcal{M}_4$ with inner coordinates (τ, σ), which define the inner induced metric γ_{ab} with signature $(+-)$.

The interaction is assumed to be gravitational, described by General Relativity with no cosmological constant. The real massless scalar field is presented as well.

Full action for the interacting system "particle–string" reads

$$S = S_{\rm sc} + S_{\rm gr}\,, \tag{3}$$

where

$$\begin{aligned} S_{\rm sc} &= \frac{1}{2}\int g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\,\sqrt{|g|}\,d^4x - f\int \phi\sqrt{g_{\mu\nu}\dot{Z}^\mu\dot{Z}^\nu}ds, \\ S_{\rm gr} &= -\frac{\mu}{2}\int X^\mu_a X^\nu_b g_{\mu\nu}\gamma^{ab}\sqrt{\tilde{\gamma}}\,d^2\sigma - m\int\sqrt{g_{\mu\nu}\dot{Z}^\mu\dot{Z}^\nu}ds - \frac{1}{\varkappa^2}\int R\,\sqrt{|g|}\,d^4x\,. \end{aligned} \tag{4}$$

Here, μ is the string tension, X^μ is the worldsheet embedding coordinates, $X^\mu_a \equiv \partial X^\mu / \partial\sigma^a$ the tangent vectors to the worldsheet, and γ^{ab} is the inverse induced metric. Additionally, $\tilde{\gamma} \equiv \big|\det\|\gamma_{ab}\|\big|$ and $\varkappa^2 = 16\pi G$.

Varying (3) over X^μ, one obtains the string's equation of motion in covariant form:

$$\partial_a\left(X^\nu_b g_{\mu\nu}\gamma^{ab}\sqrt{\tilde{\gamma}}\right) = \frac{1}{2}\,g_{\nu\lambda,\mu}X^\nu_a X^\lambda_b\gamma^{ab}\sqrt{\tilde{\gamma}}\,, \tag{5}$$

while the variation with respect to γ^{ab} yields the constraint

$$\left(X^\mu_a X^\nu_b - \frac{1}{2}\,\gamma_{ab}\gamma^{cd}X^\mu_c X^\nu_d\right)g_{\mu\nu} = 0\,, \tag{6}$$

whose solution defines γ_{ab} as an induced metric on $\mathcal{V}_2$: $\gamma_{ab} = X^\mu_a X^\nu_b g_{\mu\nu}\big|_{x=X}$.

Varying S over coordinates Z^μ, we obtain the particle's equation of motion:

$$(m + f\phi)\ddot{Z}^\mu = -m\Gamma^\mu_{\nu\lambda}\dot{Z}^\nu\dot{Z}^\lambda + f\,\Pi^{\mu\nu}\phi_{,\nu}\,, \tag{7}$$

where we impose the natural parametrization of the particle's worldline ($g_{\mu\nu}\dot{Z}^\mu\dot{Z}^\nu = 1$). Here, $\Pi^{\mu\nu} \equiv g^{\mu\nu} - (g_{\lambda\rho}\dot{Z}^\lambda\dot{Z}^\rho)^{-1/2}\dot{Z}^\mu\dot{Z}^\nu$ stands for the projector onto a plane transverse to the instantaneous 4-velocity.

Finally, variations over ϕ and over full metric $g_{\mu\nu}$ yield the field equations

$$g^{\mu\nu}\nabla_\mu\nabla_\nu\phi = -j \tag{8}$$

$$R^{\mu\nu} - \frac{R}{2}\,g^{\mu\nu} = \frac{\varkappa^2}{2}\left(T^{\mu\nu} + \bar{T}^{\mu\nu} + T^{\mu\nu}_{\rm sc}\right), \tag{9}$$

where

$$\begin{aligned}
T^{\mu\nu} &= \mu\int X^\mu_a X^\nu_b \gamma^{ab}\,\frac{\delta^4\big(x - X(\sigma)\big)}{\sqrt{|g|}}\,\sqrt{\tilde{\gamma}}\,d^2\sigma \\
\bar{T}^{\mu\nu} &= m\int \frac{\dot{Z}^\mu \dot{Z}^\nu \delta^4(x - Z(s))}{\big(g_{\lambda\rho}\dot{Z}^\lambda \dot{Z}^\rho\big)^{1/2}\sqrt{|g|}}\,ds \\
T^{\mu\nu}_{\rm sc} &= \partial^\mu\phi\,\partial^\nu\phi - \frac{1}{2}\,g^{\mu\nu}\partial_\lambda\phi\,\partial^\lambda\phi
\end{aligned} \tag{10}$$

represent the energy–momentum tensor of the string, particle and scalar field, respectively, and

$$j = f\int\sqrt{g_{\mu\nu}\dot{Z}^\mu\dot{Z}^\nu}\,\frac{\delta^4(x - Z(s))}{\sqrt{|g|}}\,ds \tag{11}$$

is a scalar current.

Approximation Method

We intend to use an approximation technique that relies on the fact that the deviation from the Minkowski metric is small, that is, $|g_{\mu\nu} - \eta_{\mu\nu}| \ll 1$. In what follows:

$$Gm \sim r_g \ll b\,. \tag{12}$$

The possible restrictions due to the charge do not affect the perturbative approximation we use, and the discussion of these is postponed to the Section 5.

We will be solving the equations of motion iteratively. Introducing the exact deviation of the total metric with respect to the Minkowski one, $H_{\mu\nu} \equiv (g_{\mu\nu} - \eta_{\mu\nu})/\varkappa$, we expand $H_{\mu\nu}$ in a gravitational constant. Therefore, all fields and kinematical quantities are to be effectively expanded as follows:

$$\phi = {}^0\phi + {}^1\phi + {}^2\phi + \ldots\,, \tag{13}$$

where ϕ can be $H_{\mu\nu}$, $T^{\mu\nu}$, $\bar{T}^{\mu\nu}$, $T^{\mu\nu}_{\rm sc}$, ϕ, j, Z^μ or their derivatives. Thus, the left superscript is used to mark the order of iteration. Therefore, effectively, we expand all tensors in G. In fact, the parameter of expansion is a dimensionless angular deficit $\beta' \sim G\mu$. Each term of any expansion, starting from $H_{\mu\nu}$, becomes the tensor with respect to the *flat* metric. In what follows, any contraction and the index raising will be carried out with help of the Minkowski metric.

Zeroth order. To the zeroth order, one expects the flat space with no fields in it:

$${}^0H_{\mu\nu} = 0\,, \qquad\qquad {}^0\phi = 0\,. \tag{14}$$

In what follows, to this order, the particle moves freely:

$${}^0\ddot{Z}^\mu = 0\,, \tag{15}$$

with constant velocity ${}^0\dot{Z}^\mu \equiv u^\mu$. We will be working in the Lorentz frame, where the unperturbed string is at rest and aligned with the x-axis; the unperturbed particle moves along the z-axis while the vector of closest proximity b^μ between the particle and string is chosen to coincide with the y-axis. Finally, we set the moment of collision to $t = 0$. In what follows,

$$u^\mu = \gamma(1,0,0,v)\,, \qquad {}^0Z^\mu = {}^0u^\mu s + b^\mu\,, \qquad b^\mu = (0,b,0,0)\,. \tag{16}$$

Thus, $\gamma = (1-v^2)^{-1/2}$ represents the Lorentz factor of collision, and $b > 0$ represents the impact parameter of this scattering.

Therefore, the unperturbed string's worldsheet is a plane spanned by t- and x-axes: $\sigma^0 \equiv \tau = t$, $\sigma^1 \equiv \sigma = x$, thus

$${}^0X^\mu = \delta^\mu_a \sigma^a \qquad {}^0\gamma_{ab} = \eta_{ab}\,. \tag{17}$$

Finally, the scalar and gravitational sources are given by

$$\begin{aligned} {}^0j(t,\boldsymbol{x}) &= \frac{f}{\gamma}\,\delta(x)\,\delta(y-b)\,\delta(z-vt) \\ {}^0\bar{T}^{\mu\nu}(t,\boldsymbol{x}) &= \frac{m}{\gamma}\,u^\mu u^\nu\,\delta(x)\,\delta(y-b)\,\delta(z-vt) \\ {}^0T^{\mu\nu}(t,\boldsymbol{x}) &= \mu\,\delta^\mu_a\delta^\nu_b\eta^{ab}\;\delta(y)\,\delta(z) = \mu\,\delta(y)\,\delta(z)\,\mathrm{diag}(1,-1,0,0)\,, \end{aligned} \tag{18}$$

while ${}^0T^{\mu\nu}_{\mathrm{sc}}(t,\boldsymbol{x})$ vanishes due to ${}^0\phi = 0$.

First order. The zeroth-order sources produce the corresponding first-order fields. Namely, from the Einstein Equation (9), one expects to obtain the equation for ${}^1H_{\mu\nu}$.

Setting the flat de Donder gauge, $2\partial_\nu H^{\mu\nu} = H^{,\mu}$, the first-order Einstein equations are given by

$$\Box\,{}^1H^{\mu\nu} = -\varkappa\left[\left({}^0T^{\mu\nu} + {}^0\bar{T}^{\mu\nu}\right) - \frac{\eta^{\mu\nu}}{2}\left({}^0T + {}^0\bar{T}\right)\right], \tag{19}$$

(where $\Box = \eta^{\lambda\rho}\partial_\lambda\partial_\rho$ is a flat d'Alembertian, ${}^0T \equiv \eta_{\lambda\rho}\,{}^0T^{\lambda\rho}$ is a flat trace). Thus, we can split the first-order $H^{\mu\nu}$ as ${}^1H^{\mu\nu} = {}^1h^{\mu\nu} + {}^1\bar{h}^{\mu\nu}$ and consider the separate fields ${}^1h^{\mu\nu}$, ${}^1\bar{h}^{\mu\nu}$, which satisfy

$$\Box\,{}^1h^{\mu\nu} = -\varkappa\left({}^0T^{\mu\nu} - \eta^{\mu\nu}\frac{{}^0T}{2}\right), \tag{20}$$

plus the same equation for the 1st-order field $\bar{h}^{\mu\nu}$, due to a particle. These solutions are well-known: the linearized field created by the string reads

$${}^1h_{\mu\nu}(x) = \frac{\varkappa\mu}{2\pi}\,\Sigma_{\mu\nu}\,\ln\frac{r}{R_0}\,, \qquad \Sigma_{\mu\nu} \equiv \mathrm{diag}\,(0,0,1,1), \tag{21}$$

where $r \equiv \sqrt{y^2+z^2}$ is the Euclidean distance to the string, and $R_0 > 0$ is an arbitrary positive lengthy parameter. Expanding (2) over β' and making the comparison with (21), we find that the correspondence between the string's tension and the relative angular deficit reads $\beta' = 4G\mu$. The corresponding linearized solution for the particle is nothing but the Newton potential boosted in the z-direction:

$${}^1\bar{h}_{\mu\nu}(x) = -\frac{\varkappa m}{4\pi}\left(u_\mu u_\nu - \frac{1}{2}\,\eta_{\mu\nu}\right)\left[\gamma^2(z-vt)^2 + x^2 + (y-b)^2\right]^{-1/2}. \tag{22}$$

Furthermore, the first order of (8) reads

$$\Box\,{}^1\phi = -\,{}^0j \tag{23}$$

as also a d'Alembert equation, with solution

$$ {}^{1}\phi(x) = -\frac{f}{4\pi}\frac{1}{\sqrt{\gamma^2(z-vt)^2+x^2+(y-b)^2}}\,. \tag{24}$$

Now, consider the first-order equation of motion for the charge: making use of (7), one derives the formal expression for the scalar part of a force, acting on the charge:

$$ m\,{}^{1}\ddot{Z}^{\mu}_{\rm sc} = f\,{}^{0}\Pi^{\mu\nu}\,{}^{1}\phi_{,\nu}\,, \tag{25}$$

where ${}^{1}\phi_{,\nu}$ is a total first-order scalar field. To exclude the self-action, one has to consider only the *external* scalar field acting on the charge. However, the total scalar field to this order is the one (24) created by the charge itself. In what follows, we have to omit ${}^{1}\ddot{Z}^{\mu}_{\rm sc}$ and this also concerns the gravitational self-action in the gravitational contribution to the total acceleration.

In what follows, to the first order, a particle moves along the geodesics created by the gravitational field produced by the string, and vice versa.

Thus, only the gravitational part of the total force (7) survives and the total first-order equation-of-motion represents a motion in the external linearized gravitational field, and reads

$$ {}^{1}\ddot{Z}^{\mu} = -\varkappa\,{}^{0}\Pi^{\mu\nu}\left({}^{1}h_{\nu\lambda,\rho} - \frac{1}{2}\,{}^{1}h_{\lambda\rho,\nu}\right)u^{\lambda}u^{\rho}\,. \tag{26}$$

The solution to it, with initial conditions ${}^{1}Z^{\mu} = 0$ and ${}^{1}\dot{Z}^{\mu} = 0$ in the Cartesian coordinates is given by [25]

$$\begin{aligned} {}^{1}Z^{0} &= {}^{1}Z^{x} = 0 \\ {}^{1}Z^{y} &= -\beta'\left[\gamma vs\arctan\frac{\gamma vs}{b} - \frac{b}{2}\,\ln\frac{b^2+\gamma^2v^2s^2}{b^2}\right] \\ {}^{1}Z^{z} &= \beta'\left[\frac{\gamma vs}{2}\left(\ln\frac{b^2+\gamma^2v^2s^2}{b^2} - 2\right) + b\arctan\frac{\gamma vs}{b}\right]. \end{aligned} \tag{27}$$

Therefore, the trajectory is determined by the zeroth order for Z^z and by the first order for Z^y:

$$ y(z) = b - \beta'\left[z\arctan\frac{z}{b} - \frac{b}{2}\,\ln\frac{b^2+z^2}{b^2}\right]. \tag{28}$$

The corresponding gravitational deformations of the string may be found in [25]; here, we will not use them.

Second order equation for $\boldsymbol{\phi}$-radiation. The generic formula for the total scalar radiation due to collision reads

$$ \frac{dE}{d\omega d\Omega} = \frac{\omega^2}{16\pi^3}\,|j(k)|^2, \qquad\qquad \omega = k^0 = |\boldsymbol{k}|, \tag{29}$$

where $j(k)$ stands for the Fourier-transform of the total source. This formula is well-known and may be obtained in the assumption of the proper decay rate of the fields involved in the process.[1]

The solution (24) of linear Equation (23) is a field generated by a uniformly moving charge and represents the boosted "Coulomb" field; hence, it does not contribute to radiation. In four dimensions, it explicitly follows from the Larmor-like formula for the scalar radiation by an accelerated charge.

The second order of our scheme leads to the radiation. For the scalar emission in the bremsstrahlung process, it is enough to consider only the correction to scalar field ${}^2\phi$ and its source.

Taking the next order of (8), one obtains

$$\Box\, {}^2\phi = -\,{}^1j(x)\,, \qquad\qquad {}^1j(x) \equiv \rho(x) + \sigma(x)\,, \tag{30}$$

where

$$\rho(x) = -f\int {}^1Z^\mu(s)\,\partial_\mu\delta^4\big(x - {}^0Z(s)\big)\,ds \tag{31}$$

$$\sigma(x) = \varkappa\,\partial_\mu\Big({}^1h^{\mu\nu}(x)\,\partial_\nu{}^1\phi(x) - \frac{1}{2}\,{}^1h(x)\,\eta^{\mu\nu}\partial_\mu{}^1\phi(x)\Big)\,, \tag{32}$$

respectively.

We will refer to the first term as the *local* term, since it is fixed on the unperturbed trajectory of charge f (due to delta-function), while the second term will be referred to as the *non-local* current. The latter comes from the left-hand side of (8) and represents the non-linear terms of the d'Alembertian in the presence of gravity.

3. Radiation Amplitudes

For the first non-vanishing contribution of the emitted energy we have to take

$$j = {}^0j + {}^1j \tag{33}$$

and plug it into (29) in the momentum space with the on-shell condition $k^2 = 0$. The wave vector of the emitted scalar field is parametrized as

$$k^\mu = \omega\big(1, \sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta\big)\,. \tag{34}$$

However, we encounter terms ${}^0j(k)\,{}^0j^*(k)$, ${}^0j(k)\,{}^1j^*(k)$ and ${}^0j^*(k)\,{}^1j(k)$ with at least one zeroth-order j. Each of them corresponds to the uniform motion and does not contribute to radiation. Therefore, we are left with the ${}^1j^*(k)\,{}^1j(k)$-term, and reduce our problem to the computation of the two constituents of ${}^1j(k)$.

Local amplitude. The Fourier transform of (31) is given by[2]

$${}^1\rho(k) = if\,\mathrm{e}^{i(kb)}\int ds\, e^{i(ku)s}\,k\cdot{}^1Z(s)\,, \tag{35}$$

where "$\cdot$" denotes the Minkowskian scalar product ($a\cdot b \equiv \eta_{\mu\nu}a^\mu b^\nu$).

It will be useful for us to work with momentum representations for all quantities involved. For ${}^1Z^\mu(s)$, we need the linearized $h_{\mu\nu}$ in the Fourier space. From (21), we deduce:

$${}^1h_{\mu\nu}(q) = -\frac{(2\pi)^2\varkappa\mu\,\delta(q^0)\delta(q^1)}{\delta_{\alpha\beta}q^\alpha q^\beta}\,\Sigma_{\mu\nu}\,. \tag{36}$$

Substituting $h_{\mu\nu}$, the particle's acceleration reads

$${}^1\ddot{Z}^\mu(s) = \frac{i\varkappa^2\mu\gamma^2v^2}{(2\pi)^2}\int d^4q\,\frac{\delta^2(q^0,q^x)}{q^2}\,\mathrm{e}^{-i(qb)}\mathrm{e}^{-i(qu)s}\Big(-\eta^{\mu z}q^z - \frac{1}{2}\,q^\mu\Big)\,. \tag{37}$$

Double integration with initial conditions ${}^1Z^\mu = 0$ and ${}^1\dot{Z}^\mu = 0$ yields

$${}^1Z^\mu(s) = -\frac{i\varkappa^2\mu\gamma^2v^2}{(2\pi)^2}\int d^4q\,\frac{\delta^2(q^0,q^x)}{q^2\,(qu)^2}\,\mathrm{e}^{-i(qb)}\Big[\mathrm{e}^{-i(qu)s} - 1 + i(qu)s\Big]\Big(-\eta^{\mu z}q^z - \frac{1}{2}\,q^\mu\Big)\,. \tag{38}$$

Thus, substituting it into (35), one obtains

$$^1\rho(k) = \frac{\varkappa^2 f\mu\gamma^2 v^2}{2(2\pi)}\, e^{i(kb)} \int d^4q \frac{\delta^2(q^0, q^x)\delta(k\cdot u - q\cdot u)}{q^2\,(qu)^2}\, e^{-i(qb)} \Big(k^y q^y - k^z q^z \Big)\,. \tag{39}$$

Introducing the following vectorial Fourier-integrals,

$$I^\mu = \int \frac{\delta(q^0)\delta(q^x)\,\delta(k\cdot u - q\cdot u)\, e^{-i(qb)}}{q^2}\, q^\mu\, d^4q\,, \tag{40}$$

the local source due to the particle reads

$$^1\rho(k) = \frac{\varkappa^2 f\mu\gamma^2 v^2\omega}{4\pi(ku)^2}\, e^{i(kb)} \Big(\sin\vartheta \sin\varphi I^y - \cos\vartheta I^z \Big). \tag{41}$$

The computation is provided in the single Appendix A. Substituting I^μ (A3), one obtains

$$^1\rho(k) = -\frac{\varkappa^2 f\mu v}{4\omega\gamma\psi^2}\, e^{i(kb)} \exp\Big(-\frac{(ku)b}{\gamma v} \Big) \Big(\cos\vartheta + i\sin\vartheta\sin\varphi \Big)\,, \tag{42}$$

where $\psi \equiv 1 - v\cos\vartheta$.

The crucial factor in (42), with respect to both frequency and angles, is the exponential

$$\frac{(ku)b}{\gamma v} = \frac{\omega b}{v}\,\psi\,. \tag{43}$$

For the local amplitudes in the related problems, the behavior of the expressions with this exponential is well-known. Such an expression may arise, in particular, in purely electromagnetic bremsstrahlung of two charges. The exponential restricts the valuable phase–volume domain as

$$\omega b\psi \lesssim v\,. \tag{44}$$

Due to the phase–volume factors $\omega^2 \sin\vartheta$, the dominant spectral–angular domain is apparently revealed in the ultrarelativistic case:

$$\omega \sim \frac{\gamma^2}{b}\,, \qquad \vartheta \sim \frac{1}{\gamma} \tag{45}$$

(the z-domain). It is this domain that could be characteristic in the absence of a non-local part; however, we have the latter.

Non-local amplitude. The non-local amplitude (32) in the momentum representation reads

$$\sigma(k) = -\frac{\varkappa}{(2\pi)^4} \int k_\mu \Big[{}^1h^{\mu\nu}(q)\,(k_\nu - q_\nu)\,{}^1\phi(k-q) - \frac{1}{2}\,{}^1h(q)\,(k^\mu - q^\mu)\,{}^1\phi(k-q) \Big]\,.$$

Substituting here the string's gravity (36) and the corresponding Fourier-transform of the scalar field (24),

$$^1\phi(q) = -\frac{2\pi f}{q^2}\, e^{iqb}\delta(qu)\,, \tag{46}$$

we obtain the non-local amplitude in the Fourier space:

$$\sigma(k) = \frac{\varkappa^2 f\mu}{2\pi}\, e^{i(kb)} \int \frac{\delta^2(q^0, q^x)\,\delta(k\cdot u - q\cdot u)\, e^{-i(qb)}}{q^2(k-q)^2} \Big[\Sigma_{\mu\nu}k^\mu k^\nu - \Sigma_{\mu\nu}k^\mu q^\nu + k^2 - k\cdot q \Big]\, d^4q\,. \tag{47}$$

In the presence of $\delta^2(q^0, q^x)$, a scalar product $a \cdot q$ (with arbitrary vector a^μ) equals $-\Sigma_{\mu\nu} a^\mu q^\nu$. Thus, the 2nd and 4th terms in the parentheses mutually cancel, while the 3rd term vanishes on-shell. Therefore, we are left with the 1st term alone, which is $(k^0)^2 - (k^x)^2$ and does not depend upon the integration variables; thus we rewrite:

$$\sigma(k) = \frac{\varkappa^2 f \mu \omega^2}{2\pi}\, \mathrm{e}^{i(kb)} \left(1 - \sin^2\vartheta \cos^2\varphi\right) J\,, \tag{48}$$

where we introduce

$$J \equiv \int \frac{\delta(q^0)\, \delta(q^x)\, \delta(k\cdot u - q\cdot u)\, \mathrm{e}^{-i(qb)}}{q^2 (k-q)^2}\, d^4 q\,. \tag{49}$$

This two-propagator momentum–space integral is computed in the Appendix A. Substituting its value (A5), one finally obtains:

$$\begin{aligned}\sigma(k) =& \frac{\varkappa^2 f \mu}{4\gamma v \omega} \left(1 - \sin^2\vartheta \cos^2\varphi\right) \left[\frac{v^2 \mathrm{e}^{ik\cdot b} \exp(-\omega b \psi / v)}{\psi^2 (\cos\vartheta - i \sin\vartheta \sin\varphi)} - \right. \\ & \left. - \frac{\exp(-\omega b R)}{R\, (\psi \cos\vartheta / v - \sin^2\vartheta \sin^2\varphi - i R \sin\vartheta \sin\varphi)} \right],\end{aligned} \tag{50}$$

where $R \equiv \sqrt{\gamma^{-2} v^{-2} + \sin^2\vartheta \cos^2\varphi}$.

Now we have the term with the exponential similar to that from the local source, and also the term with the crucial factor $\mathrm{e}^{-\omega b R}$. The valuable phase–volume domain for the latter is given by $R \lesssim 1/\omega b$. It implies simultaneous

$$\omega b \sin\vartheta |\cos\varphi| \lesssim 1\,, \qquad\qquad \omega b \lesssim \gamma v\,. \tag{51}$$

Thus, the characteristic spectral–angular region for this term drastically depends upon velocity.

Therefore, depending on the ratio between the sum of the two $\mathrm{e}^{-\omega b \psi / v}$-terms and the single $\mathrm{e}^{-\omega b R}$-term, we could expect different behavior of the whole amplitude.

Total amplitude. Combining the local amplitude (42) and the non-local amplitude (50), we notice that the two $\mathrm{e}^{-\omega b \psi / v}$-terms mutually annihilate, and we are left with

$$ {}^{1}\!j(k) = -\frac{\varkappa^2 f \mu}{4\gamma \omega R} \frac{1 - \sin^2\vartheta \cos^2\varphi}{\psi \cos\vartheta - v \sin^2\vartheta \sin^2\varphi - i v R \sin\vartheta \sin\varphi} \exp(-\omega b R)\,. \tag{52}$$

In general, such a phenomenon is well-known, and is called "destructive interference". It is proper for gravitational interaction, but known for the ultrarelativistic collisions. Usually [34–38], the non-local $\mathrm{e}^{-\omega b \psi / v}$-term kills the two leading terms of the corresponding local amplitude (in $1/\gamma$-expansion in the frequency-angular domain (45)), but the resulting sum may even dominate over the remaining non-local term (especially in higher-dimensional models with large-sized extra dimensions).

Thus, it is more surprising that, in the problem at hand, the cancellation is (i) complete, and (ii) valid for any velocity not necessarily in the UR regime.

4. Emitted Energy

Now we compute the total emitted energy of the inelastic gravitational collision. Plugging the total amplitude (52) into the generic formula (29), one obtains

$$\begin{aligned}\frac{dE}{d\omega d\Omega} &= \frac{\varkappa^4 f^2 \mu^2}{2^8 \pi^3 \gamma^2 R^2 \psi^2} \frac{(1 - \sin^2\vartheta \cos^2\varphi)^2}{\cos^2\vartheta + \sin^2\vartheta \sin^2\varphi} \exp(-2\omega b R) \\ &= \frac{G^2 f^2 \mu^2 v^2}{\pi \psi^2} \frac{1 - \sin^2\vartheta \cos^2\varphi}{1 + \gamma^2 v^2 \sin^2\vartheta \cos^2\varphi} \exp(-2\omega b R)\,.\end{aligned} \tag{53}$$

Integrating (53) over frequencies, one obtains the angular distribution

$$\frac{dE}{d\Omega} = \frac{G^2 f^2 \mu^2 \gamma v^3}{2\pi b} \frac{1 - \sin^2\vartheta \cos^2\varphi}{\psi^2 (1 + \gamma^2 v^2 \sin^2\vartheta \cos^2\varphi)^{3/2}} . \tag{54}$$

Integrating (54) over ϑ [41], we obtain the φ-distribution:

$$\frac{dE}{d\varphi} = \frac{G^2 f^2 \mu^2}{\pi b} \frac{\gamma^3 v}{\sin^2\varphi \tan^2\varphi} \left[2 + v^2 \tan^2\varphi + \frac{\cos^2\varphi}{1 - v^2 \sin^2\varphi} - \left(3v + \frac{2 + \cos^2\varphi}{\gamma^2 v} \right) \frac{\mathrm{Arth}\,(v \sin\varphi)}{\sin\varphi} \right], \tag{55}$$

where Arth stands for the inverse hyperbolic tangent.

The expression (55) is invariant with respect to the symmetries (i) $\varphi \to -\varphi$ and (ii) $\cos\varphi \to -\cos\varphi$. They originate from the total spectral–angular distribution (53), the dependence of which upon φ is reduced to the dependence on $\cos^2\varphi$. With the definition of φ (the ray $\varphi = 0$ is parallel to the string), the symmetry (ii) is just the symmetry with respect to reflection in the scattering plane. The symmetry (i) is more surprising. It means that if we look along the particle's unperturbed trajectory, the radiation towards the string (angles $-\pi < \varphi < 0$) equals the total energy emitted outwards from the string (angles $0 < \varphi < \pi$). This happens because we consider the total emitted radiation. The radiation amplitude represents the Fourier-transform of the corresponding coordinate-space amplitude, that is, the source averaged with respect to the total collision time. The perturbed trajectory ${}^1Z^\mu(s)$ (27) is an even function of the proper time s. Hence, the expected local (instantaneous, at fixed time s) left–right asymmetry of the radiation, being averaged over the whole time on the symmetric trajectory, vanishes. Most likely, this left–right symmetry is restored at the level of the direct sum of the local radiation at proper time $s = -|s_0|$ (before the collision) and the corresponding radiation at $s = +|s_0|$ (after the collision).

Plots of the φ-distribution for different velocities in the relativistic regime are shown in Figure 1. One can see that, for any velocity, the plot has a minimum of $\varphi = 0$ and a maximum of $\varphi = \pi/2$.

Thus, we define the φ-anisotropy coefficient δ_φ as

$$\delta_\varphi \equiv \frac{dE/d\varphi\,(\varphi = \pi/2)}{dE/d\varphi\,(\varphi = 0)} . \tag{56}$$

From Equation (55), we deduce:

$$\delta_\varphi = \frac{15}{5 + v^2}, \qquad \frac{5}{2} < \delta_\varphi \leqslant 3 . \tag{57}$$

Thus, apart from the overall normalizing coefficient, qualitatively the φ-distribution varies slowly from the non-relativistic regime to the ultra-relativistic one.

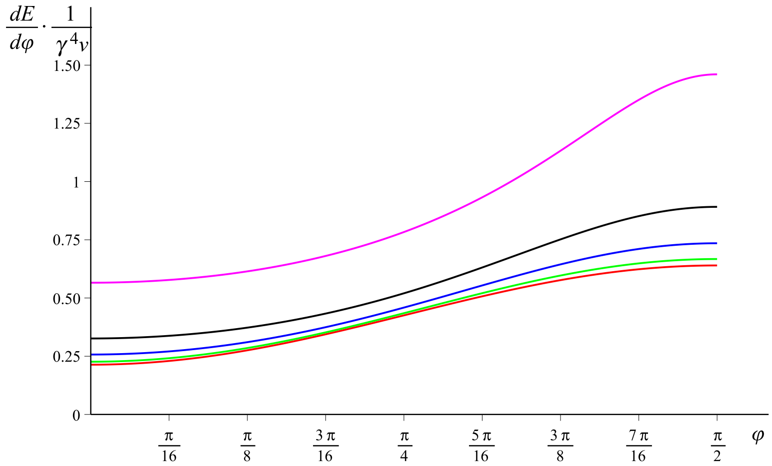

Figure 1. Plots of the φ-distribution of the scalar bremsstrahlung, normalized by the overall factor $1/\gamma^4 v$, in units $G = \mu = b = f = 1$: $v = 0.1$ (red), $v = 0.3$ (green), $v = 0.5$ (blue), $v = 0.7$ (black), $v = 0.9$ (magenta).

Integration of (54) over φ [42] gives the ϑ-distribution:

$$\frac{dE}{d\vartheta} = \frac{2G^2 f^2 \mu^2 \gamma v}{\pi b} \frac{\sin\vartheta}{\psi^2 \sqrt{1+\gamma^2 v^2 \sin^2\vartheta}} \left[\boldsymbol{E}\left(\frac{\gamma v \sin\vartheta}{\sqrt{1+\gamma^2 v^2 \sin^2\vartheta}} \right) - \frac{1}{\gamma^2} \boldsymbol{K}\left(\frac{\gamma v \sin\vartheta}{\sqrt{1+\gamma^2 v^2 \sin^2\vartheta}} \right) \right], \tag{58}$$

where $\boldsymbol{K}$ and $\boldsymbol{E}$ are complete elliptic integrals of the 1st and 2nd kind, respectively. Plots of the ϑ-distribution for different velocities are shown in Figure 2. Amongst the other aspects, they differ in terms of the position of the global maximum and its value. From (58), one notices that the crucial factor in the behavior of $dE/d\vartheta$ is $\psi^{-2}\sin\vartheta$, since the elliptic integrals are of order $\mathcal{O}(1)$.[3] Such a factor is common for angular distribution in the diverse problems of classical electrodynamics. For the typical integrands (with respect to subsequent integration over ϑ) $\psi^{-m}\sin^n\vartheta$, the case $2m > n+1$ corresponds to a beaming, with the most valuable ϑ-domain determined by relation

$$\cos\vartheta \sim v\,. \tag{59}$$

Therefore, for higher velocities most of the contribution to the total emitted energy comes from smaller values ϑ. Figure 2 confirms our estimates. In what follows, we shall consider the ultra-relativistic regime separately and in more detail.

The red plot in Figure 2 ($v = 0.1$, $\gamma \approx 1.005$) reveals some features proper to the non-relativistic motion. Namely, the domain $\pi/2 < \vartheta < \pi$ (backward direction, with respect to the particle) becomes of the same importance as the forward-direction region and, in the limit $v \to 0$, the plot is expected to be symmetric according to the symmetry of collision.

The plots of the frequency distribution for several values of velocity in the relativistic regime are shown in Figure 3. Any plot has a finite zero-frequency limit (ZFL). Here, one notices that the mean value of the frequency (or the value of half-ZFL) shifts to ultraviolet with an increase of the particle's speed.

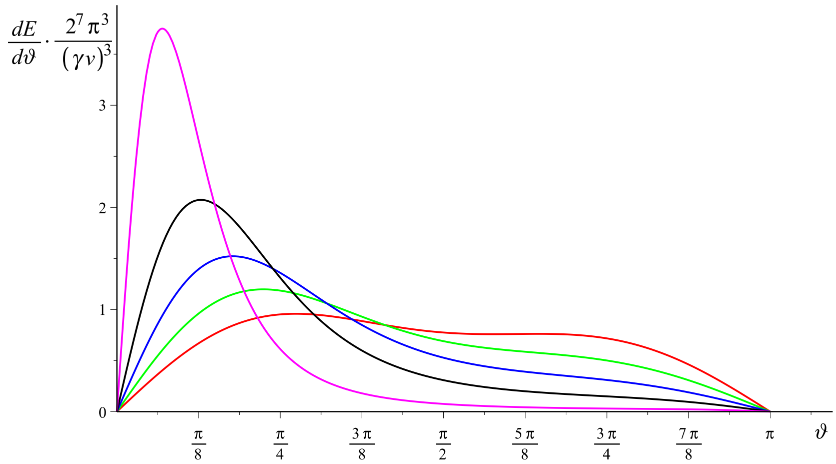

Figure 2. Plots of the ϑ-distribution of the scalar bremsstrahlung, normalized by the overall factor $2^7(\pi/\gamma v)^3$, in units $G = \mu = b = f = 1$: $v = 0.1$ (red), $v = 0.3$ (green), $v = 0.5$ (blue), $v = 0.7$ (black), $v = 0.9$ (magenta).

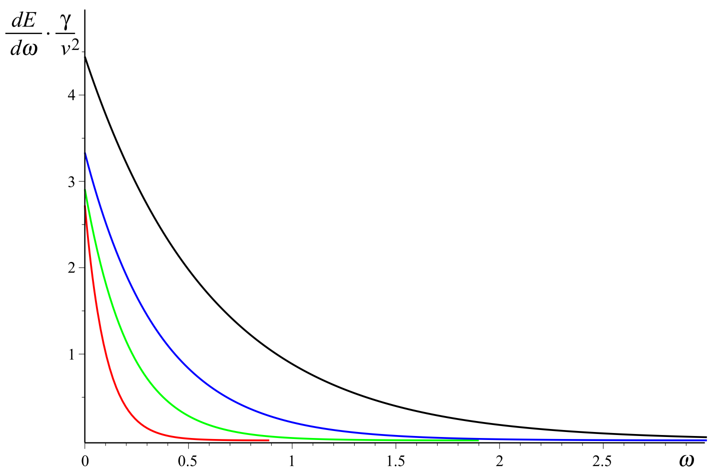

Figure 3. Plots of the frequency distribution of the scalar bremsstrahlung, normalized by the overall factor γ/v^2, in units $G = \mu = b = f = 1$: $v = 0.2$ (red), $v = 0.4$ (green), $v = 0.6$ (blue), $v = 0.8$ (black).

Qualitatively, in the spectral–angular distribution (53) the only factor, involving ω, is the exponential $\exp(-2\omega bR)$. After the integration over all angles, the pre-factor becomes some average value times 4π, while

$$\langle \sin\vartheta \rangle \sim \gamma^{-1}\,, \qquad \langle \cos\varphi \rangle \sim 1\,. \tag{60}$$

In what follows, the average value of R is estimated as

$$\langle R \rangle \sim (\gamma v)^{-1}. \tag{61}$$

Hence, the frequency distribution approximates as

$$\frac{dE}{d\omega} \sim \left(\frac{dE}{d\omega}\right)_{\omega=0} \mathrm{e}^{-2b\langle R \rangle \omega}, \tag{62}$$

with characteristic frequency

$$\langle \omega \rangle \sim \frac{\gamma v}{b}\,,$$

times a coefficient of order $\mathcal{O}(1)$.

The total energy emitted due to collision may be integrated numerically. We consider the particular case of ultrarelativistic collision, which is of the most significance.

Ultrarelativistic Regime

Due to $\gamma \gg 1$ in the UR regime, we are interested in the leading contribution to the emitted energy and can drop all subleading terms. From the estimate (60), it follows that the 2nd term in the numerator of angular fraction in (54) is suppressed by γ^2 with respect to the 1st one in the UR regime. Thus, it may be neglected. Furthermore, we can put $v = 1$ for the overall factor (but not for ψ):

$$\frac{dE}{d\Omega} = \frac{G^2 f^2 \mu^2 \gamma}{2\pi b} \frac{1}{\psi^2 (1 + \gamma^2 \sin^2\vartheta \cos^2\varphi)^{3/2}} \,. \tag{63}$$

Now we integrate with respect to ϑ: introducing the new variable $\chi = \gamma\vartheta$, we expand

$$\sin\vartheta \simeq \frac{\chi}{\gamma} \,, \qquad \psi \simeq \frac{1+\chi^2}{2\gamma^2} \,. \tag{64}$$

These expansions are valid at $0 < \chi < \mathcal{O}(1)$ but, for higher χ, we notice that both the original integrand and the approximated one undergo very fast decay. Thus, the contribution of the domain $\mathcal{O}(1) < \chi < \pi\gamma$ is subleading with respect to the most valuable domain $0 < \chi < \mathcal{O}(1)$. *A fortiori*, it concerns the domain $\pi\gamma < \chi < \infty$. In what follows, we can extend the upper integration limit up to infinity. Performing the integration, one obtains the ultrarelativistic φ-distribution:

$$\frac{dE}{d\varphi} = \frac{G^2 f^2 \mu^2}{\pi b} \frac{\gamma^3}{\sin^5\varphi} \left[\sin\varphi \left(1 + 2\cos^2\varphi\right) - \frac{3}{2}\cos^2\varphi \ln\frac{1+\sin\varphi}{1-\sin\varphi} \right] . \tag{65}$$

Integrating (65), one arrives at

$$E = \frac{3\pi}{8} \frac{G^2 f^2 \mu^2}{b} \gamma^3 \,, \tag{66}$$

plus subleading in γ terms. The characteristic frequency is $\langle\omega\rangle \sim \gamma/b$. Therefore, the destructive interference does not change the characteristic angles but decreases the characteristic frequency by a factor of γ.

5. Conclusions

We have considered the classical scalar bremsstrahlung of the spinless relativistic charge in the process of gravitational collision with an infinitely thin cosmic string.

Effectively, our approximation method is based on the formal perturbation theory over the Newtonian constant. With the impact parameter given in the problem-at-hand, it implies

$$b \gg r_g \,. \tag{67}$$

For, say, protons, the Schwarzschild radius is $r_g \sim 10^{-52}$ cm.

In the first order of our approximation, we computed the radiation amplitude. It consists of two parts—the local part due to direct variation of the scalar current, and the non-local part due to the curved background. In turn, the non-local amplitude splits into two constituents with different spectral–angular behaviors of the characteristic exponentials. One of them reproduces the local-current exponential and thus may be combined with it.

The general phenomenon of mutual cancellation of the leading (at z-domain of the phase volume) terms of the local and non-local amplitude, known in the ultrarelativistic regime for several types of collision, also takes place here. Remarkably, this cancellation (destructive interference) is complete and takes place at any velocity of the particle.

We compute the spectral and angular distributions of the emitted waves; the total emitted energy is estimated to be

$$E \sim \frac{G^2\mu^2 f^2}{b}\,\gamma^3. \tag{68}$$

For the estimate of the efficiency of the emitted energy, we note that, in the problem-at-hand, the radiation represents a collective effect: both particle and string (via gravity) take part in the amplitude formation. With our setup of the infinitely long cosmic string, the initial string's energy is infinite and the only thing for us to do is to suppose that only the particle loses its energy. Of course, this drastically restricts the initial energy available for the subsequent radiation during the collision.

Let us suppose this (worst) case; for efficiency, we normalize the emitted energy (basically, we are interested in the UR case) by the mechanical energy $m\gamma$ of the mass m. One estimates:

$$\epsilon_{\rm rad} = \frac{E}{m\gamma} \sim \frac{G^2\mu^2 f^2}{mb}\,\gamma^2 \sim \frac{r_{\rm cl}}{b}\,\beta'^2\gamma^2\,, \tag{69}$$

where $r_{\rm cl} = f^2/m$ is a classical radius of charge f. With temporal values of Lorentz-factors and estimates of the cosmic-string angular deficit, the efficiency may easily be of the order of unity. This implies that the perturbation theory fails, at least in the 2nd order, but the qualitative conclusion remains the same: the charge's loss of energy is comparable with the initial one.

For the real cosmic strings, we have to consider them as finite-sized. The finiteness of the string's width d drastically shifts the restriction on impact parameters:

$$b > d\,, \tag{70}$$

where $d \sim 10^{-29}$ cm. Further, for a cosmic string with finite length L we have to impose the extra restriction

$$\gamma \ll \mu L/m\,, \tag{71}$$

which means that the string's energy is much greater than the total energy of a charge. It also decreases the maximal value of the efficiency $\epsilon_{\rm rad}$ and, moreover, now we have to normalize the emitted energy by the string's emitted energy, which may only decrease the ratio. Some particularly interesting cases may arise in the problem of the collision of a cosmic string with a relativistic gas of charged particles. The string's deflection will vanish on average, but the string is just used as an object of gravitational friction by any single particle. However, these estimates lie beyond the model considered for the problem-at-hand, and demand separate considerations.

Finally, although we have considered only the gravitational interaction, the classical charge's radius has appeared in the final answer, or in the radiation efficiency (69). It can indicate the additional restrictions related to the self-action of a charge in the presence of a curved background [36,43–46].

Looking forward, this paper represents an apparent interest in the bremsstrahlung of particles scattered by a string, not only in the transverse plane. It can help with the solution of the bremsstrahlung problem due to the interaction of a string with a gas of charged particles. It may be significant for the proper analysis of all inelastic processes in the Early Universe. The bremsstrahlung of particles with spin may also be significant.

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Abbreviations

The following abbreviations are used in this manuscript:

ZFL	Zero-Frequency Limit
UR	Ultra-Relativistic

Appendix A. Useful Integrals

In (40) we have introduced the following vectorial integral:

$$I^\mu = \int \frac{\delta(q^0)\,\delta(q^x)\,\delta(k\cdot u - q\cdot u)\,\mathrm{e}^{-i(qb)}}{q^2}\, q^\mu\, d^4q\,. \tag{A1}$$

Let us start with the corresponding scalar integral:

$$I = \int \frac{\delta(q^0)\,\delta(q^x)\,\delta(k\cdot u - q\cdot u)\,\mathrm{e}^{-i(qb)}}{q^2}\, d^4q\,.$$

Integrating over two deltae from the string's gravity source, one has

$$I = -\int \frac{\delta(k\cdot u + q^z\gamma v)\,\mathrm{e}^{iq^y b}}{q_y^2 + q_z^2}\, dq^y\, dq^z\,.$$

Integration over q^z is trivial; integration over q^y is a contour in the upper complex half-plane. The single simple pole at $q^y = i(ku)/\gamma v$ results in

$$I = -\frac{\pi}{(ku)}\, \exp\Big(-\frac{(ku)\,b}{\gamma v}\Big)\,. \tag{A2}$$

Now return to the vectorial integral. In components, we can reduce it to the computed scalar integral I with the help of relations

$$I^x = I^y = 0\,, \qquad I^z = -\frac{(ku)}{\gamma v}\, I\,, \qquad I^y = -i\frac{\partial}{\partial b}\, I\,.$$

Introducing $\lambda^\mu = \big(0, 0, i, -1\big)$, the vectorial integral is given by

$$I^\mu = -\frac{\pi}{\gamma v}\, \exp\Big(-\frac{(ku)b}{\gamma v}\Big)\, \lambda^\mu = \frac{(ku)}{\gamma v}\, I\lambda^\mu\,. \tag{A3}$$

The two-propagator scalar integral J was introduced in (49):

$$J = \int \frac{\delta(q^0)\,\delta(q^x)\,\delta(k\cdot u - q\cdot u)\,\mathrm{e}^{-i(qb)}}{q^2(k-q)^2}\, d^4q\,. \tag{A4}$$

After integrations over q^0, q^x and q^z, through the three delta-functions, we are left with a single-variable integral,

$$J = \frac{1}{\gamma v} \int \frac{\mathrm{e}^{iq^y b}}{(q_y^2 + \omega^2\psi/v^2)[(q^y - k^y)^2 + \omega^2 R^2]}\, dq^y\,,$$

where we introduce

$$\psi = 1 - v\cos\vartheta\,, \qquad R = \sqrt{\gamma^{-2}v^{-2} + \sin^2\vartheta\cos^2\varphi}\,.$$

The integrand has four poles,

$$q_{1,2} = \pm\frac{i\omega\psi}{v}\,, \qquad q_{3,4} = \omega\big(\sin\vartheta\sin\varphi \pm iR\big)\,.$$

Taking the residuals at two upper-halfplane poles and simplifying, one arrives at

$$J = \frac{\pi}{2\gamma v\omega^3}\left[\frac{v^2\exp(-\omega b\psi/v)}{\psi^2\,(\cos\vartheta - i\sin\vartheta\sin\varphi)} - \frac{\mathrm{e}^{-ik\cdot b}\exp(-\omega bR)}{R\,(\psi\cos\vartheta/v - \sin^2\vartheta\sin^2\varphi - iR\sin\vartheta\sin\varphi)}\right]. \tag{A5}$$

Notes

1 In particular, the derivation may be found in [36].

2 Direct Fourier-transform is defined as

$$\mathcal{F}[f(x)](q) \equiv \int d^4x\,\exp\left(i\eta_{\mu\nu}q^\mu x^\nu\right) f(x)\,.$$

3 For the elliptic integral of the 2nd kind it is verified directly: $1 \leqslant \boldsymbol{E}(x) \leqslant \pi/2$ at $x \in [0;1]$. The elliptic integral of the 1st kind satisfies $\boldsymbol{K}(x) \geqslant \pi/2$ and blows up at $x \to 1^-$ as $\boldsymbol{K}(1-z) \simeq (1/2)\ln(z/8)$. However, approaching $z \ll 1$ implies very large Lorentz factors; thus, taking into account the pre-factor γ^{-2}, one verifies that the whole term with $\boldsymbol{K}$ is restricted by $\pi/2$.

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