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Review

Generic Features of Thermodynamics of Horizons in Regular Spherical Space-Times of the Kerr-Schild Class

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Abstract: We present a systematic review of thermodynamics of horizons in regular spherically symmetric spacetimes of the Kerr-Schild class, $ds^2 = g(r)dt^2 - g^{-1}(r)dr^2 - r^2 d\Omega^2$, both asymptotically flat and with a positive background cosmological constant λ. Regular solutions of this class have obligatory de Sitter center. A source term in the Einstein equations satisfies $T_t^t = T_r^r$ and represents an anisotropic vacuum dark fluid ($p_r = -\rho$), defined by the algebraic structure of its stress-energy tensor, which describes a time-evolving and spatially inhomogeneous, distributed or clustering, vacuum dark energy intrinsically related to space-time symmetry. In the case of two vacuum scales it connects smoothly two de Sitter vacua, $8\pi G T_\nu^\mu = \Lambda\delta_\nu^\mu$ as $r \to 0$, $8\pi G T_\nu^\mu = \lambda\delta_\nu^\mu$ as $r \to \infty$ with $\lambda < \Lambda$. In the range of the mass parameter $M_{cr1} \le M \le M_{cr2}$ it describes a regular cosmological black hole directly related to a vacuum dark energy. Space-time has at most three horizons: a cosmological horizon r_c, a black hole horizon $r_b < r_c$, and an internal horizon $r_a < r_b$, which is the cosmological horizon for an observer in the internal R-region asymptotically de Sitter as $r \to 0$. Asymptotically flat regular black holes ($\lambda = 0$) can have at most two horizons, r_b and r_a. We present the basic generic features of thermodynamics of horizons revealed with using the Padmanabhan approach relevant for a multi-horizon space-time with a non-zero pressure. Quantum evaporation of a regular black hole involves a phase transition in which the specific heat capacity is broken and changes sign while a temperature achieves its maximal value, and leaves behind the thermodynamically stable double-horizon ($r_a = r_b$) remnant with zero temperature and positive specific heat. The mass of objects with the de Sitter center is generically related to vacuum dark energy and to breaking of space-time symmetry. In the cosmological context space-time symmetry provides a mechanism for relaxing cosmological constant to a certain non-zero value. We discuss also observational applications of the presented results.

Keywords: horizons thermodynamics; regular black holes; relaxing cosmological constant

1. Introduction

In 1973 Bekenstein introduced a black hole entropy related to the Hawking area theorem [1]. A year later Hawking found that an observer at $r = const$ outside a black hole will detect a stationary flux of particles from a black hole with the thermal spectrum [2,3]. This gave the birth to thermodynamics of black holes [4–6]. In 1976 Gibbons and Hawking found that also cosmological horizon emits thermal radiation [7], and this gave rise to thermodynamics of horizons [8–16]. The basic point is that an observer for whom a horizon prevents seeing the whole space-time, does not have an access to the complete quantum state of the system, and the loss of information about quantum state is responsible for the thermal character of radiation [5,6].

Regular spherical black holes are typically described by the metrics of the Kerr-Schild class [17]

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2 \tag{1}$$

which can be presented as $g_{\mu\nu} = \eta_{\mu\nu} + 2f(r)k_\mu k_\nu$, where $\eta_{\mu\nu}$ is the Minkowski metric, k_μ are principal null congruences, and $f(r) = r\mathcal{M}(r)$; $\mathcal{M}(r) = 4\pi \int_0^r \rho(x)x^2 dx$. They form the special class of algebraically degenerated solutions to the Einstein equations, with $\partial_\mu T^\mu_\nu = 0$ and pseudotensor of gravitational energy $t_{\mu\nu} = 0$ [18], and admit decomposition into (2+1)-dimensional shells with $\partial_\mu T^\mu_\nu = 0$, each of shells treated as a closed system and described by a (2+1) Hamiltonian [19].

Regular solutions of the Kerr-Schild class have obligatory de Sitter center provided that their source terms in the Einstein equations satisfy the weak energy condition [20].

Stress-energy tensors responsible for the Kerr-Schild metrics have the algebraic structure [20–23]

$$T^t_t = T^r_r \ \ (p_r = -\rho) \tag{2}$$

and can be identified as vacuum dark fluid defined by the algebraic structure of its stress-energy tensor [24]. The Einstein cosmological term $\lambda\delta^\mu_\nu$, $\lambda = const$, corresponds to the maximally symmetric de Sitter vacuum $T^\mu_\nu = \rho_{vac}\delta^\mu_\nu$; $\rho_{vac} = (8\pi G)^{-1}\lambda = const$. Algebraic classification of stress-energy tensors implies the opportunity to introduce stress-energy tensors whose symmetry is partially reduced (as compared with the maximally symmetric de Sitter vacuum) to $T^t_t = T^\alpha_\alpha$ $(p_\alpha = -\rho)$ which represents a vacuum dark fluid invariant under the Lorentz boosts in the α-direction(s) [24]. Vacuum dark fluid provides the description of the time-dependent and spatially inhomogeneous vacuum dark energy which is generically related to the space-time symmetry and can be evolving and clustering (for a review [25]). Space-time contains regions of the de Sitter vacuum with the restored maximal symmetry [24,26]; transitions to regions with the reduced vacuum symmetry involve breaking of space-time symmetry from the de Sitter group [20]. In the cosmological context the space-time symmetry provides the mechanism of relaxing of cosmological constant to a certain non-zero value, tightly fixed by dynamics of quantum evaporation of the cosmological horizon, in accordance with the basic sense of the Holographic Principle ([27,28], for a review [25,29]). The key point is that although QFT does not contain an appropriate symmetry to zero out ρ_{vac} or to reduce it to a certain non-zero value, a relevant symmetry does exist in General Relativity as the space-time symmetry which intrinsically involve breaking and restoration of the maximal de Sitter symmetry when needed.

The number of horizons is determined by the number of vacuum scales, $N_{horizons} \leq (2N_{vacuum\ scales} - 1)$ [26]. In the case of two vacuum scales a stress-energy tensor (2) connects two de Sitter vacua, at the center and at infinity,

$$T^\mu_{\nu\,(deSitter)} = (8\pi G)^{-1}\Lambda\delta^\mu_\nu \ \Longleftarrow \ T^\mu_\nu \ \Longrightarrow T^\mu_{\nu\,(deSitter)} = (8\pi G)^{-1}\lambda\delta^\mu_\nu \tag{3}$$

and represents a spherical anisotropic vacuum, $p_r = -\rho$; $p_\perp = -\rho - r\rho'/2$ [20,21]. Space-time can have at most 3 horizons [30]. The metric (1) evolves between two de Sitter asymptotics with $\lambda < \Lambda$.

Asymptotically flat space-time with the de Sitter center has not more than 2 horizons [20].

The early proposals of replacing a black hole singularity with the de Sitter vacuum were based on the hypotheses of self-regulation of geometry by vacuum polarization effects [31], of the existence of the limiting curvature [32], and of the symmetry restoration at the GUT scale in the course of the gravitational collapse [21,33]. The first solution describing regular black hole with the de Sitter interior [21] was obtained with the density profile and corresponding mass function

$$\rho(r) = \rho_0 e^{-r^3/r_0^2 r_g}; \ \ \mathcal{M}(r) = M\left(1 - e^{-r^3/r_0^2 r_g}\right); \ \ r_g = 2GM; \ \ r_0^2 = \frac{3}{8\pi G\rho_0}; \ \ \rho_0 = \rho(r \to 0) \tag{4}$$

which we applied for presented below pictures illustrating generic behavior in the thermodynamics of horizons. It describes vacuum polarization effects, which contribute all together to vacuum polarization in the gravitational field [21], in the simple semiclassical model $\rho \propto \exp\left(-F_{cr}/F\right)$, where the gravitational (tidal) force $F \propto r_g/r^3$, and the critical (de Sitter) force $F_{cr} \propto 1/r_\Lambda^2$ [20,34].

Later a loop quantum gravity and noncommutative geometry provided arguments in favor of a regular black hole with the de Sitter interior [35–38].

Generic analysis of the Hawking evaporation from both horizons of an asymptotically flat regular black hole presented in [34] has shown that the quantum temperature drops to zero at the double horizon and vanishes at infinity (following the Schwarzschild limit), so that it should have a maximum, where the specific heat capacity is broken and changes its sign testifying for a second-order phase transition [34]. Later generic evolution was studied for evaporation of three horizons in a space-time with two vacuum scales and the phase transition in the course of evaporation was found for a regular cosmological black hole [39]. In both cases quantum evaporation of horizons leaves behind a thermodynamically stable double-horizon remnant (an extremal black hole) with zero temperature and positive specific heat ([34,39], for a review [40]).

Presented in the literature results obtained for particular asymptotically flat regular black hole solutions described by metrics of the Kerr-Schild class, confirm the appearance of a temperature maximum and phase transition during evaporation for magnetically charged regular black holes [41–43], for the minimal model proposed by Hayward [44,45], for the Hayward and noncommutative [46] regular black holes [47], for charged AdS black holes with a global monopole [48], for the Bardeen black hole [49] which represents the nonlinear magnetic monopole [50], for the Hayward, Ayon-Beato and Garcia [51] black holes [52,53], and for regular black hole in quadratic gravity [52,54].

In the paper [45] a specific heat capacity was calculated as vanishing at the double horizon by identifying the thermodynamical energy with a black hole mass M, which is true for the Schwarzschild black hole but cannot be applied in the case of two horizons and non-zero pressure [39]. Identifying a black hole mass M with the thermodynamical energy leads also to appearance of inconsistency between the first law of black hole thermodynamics and relation of the black hole entropy with the Bekenstein-Hawking area law ([53,55] and references therein).

The inconsistency between the first and second laws of black hole thermodynamics and uncertainties concerning endpoint of evaporation are directly related to the problem of a unique definition of thermodynamical parameters in a multi-horizon space-time with a non-zero pressure. A general approach to defining thermodynamical variables in this case for spherical space-times with the Kerr-Schild metrics (1) was developed by Padmanabhan [10–12] who considered a canonical ensemble of space-time metrics (1) at the constant temperature of a horizon determined by the periodicity of the Euclidean time in the Euclidean continuation of the Einstein action [10]. In this approach the first law of thermodynamics for black holes described by metrics (1) is obtained in agreement with the second law as a consequence of the Einstein equations [10–12]. A thorough overview of the concept of honrizon is give in [56].

In this paper we present the basic generic features of thermodynamics of horizons in regular space-times with the metrics (1) revealed with using the Padmanabhan approach.

The paper is organized as follows. In Section 2 we outline the regular spherical space-times with the Kerr-Schild metrics and introduce the Padmanabhan approach to thermodynamics of their horizons. In Section 3 we present the basic generic features of horizon thermodynamics including the cosmological model for relaxing cosmological constant, distinguished by the Holographic principle as the only stable state in quantum evolution of a certain class of one-horizon space-times, with the finite non-zero value of vacuum dark energy tightly fixed by evaporation of cosmological horizon. In Section 4 we summarize the results and discuss observational applications.

2. Basic Equations

2.1. Basic Features of Regular Spherical Space-Times with Vacuum Dark Fluid

Eigenvectors of a stress-energy tensor related to its eigenvalues, $T^{\nu}_{\mu} = diag[\rho, -p_1, -p_2, -p_3]$, form a comoving reference frame with a time-like eigenvector representing a velocity. The maximally symmetric de Sitter vacuum $T^{\mu}_{\nu} = \delta^{\mu}_{\nu}\rho$ with the equation of state $p = -\rho$ and $\rho = const$, is denoted in the algebraic classification of stress-energy tensors by [(IIII)]—all eigenvalues equal. It was identified as a vacuum [57] since it has an infinite set of comoving reference frames which makes impossible to fix a velocity with respect to it which is the most general intrinsic property of a vacuum [58].

Algebraic structure of a stress-energy tensor is generically related, via the Einstein equations, to the space-time symmetry. T^{ν}_{μ} with [(IIII)] generates the maximally symmetric de Sitter space-time for any underlying particular model of T^{ν}_{μ}.

A maximal symmetry [(IIII)] can be reduced to [(II)II] or [(III)I)] (more information in [20,21,24])

$$T^t_t = T^{\alpha}_{\alpha} \quad (p_{\alpha} = -\rho). \tag{5}$$

Invariance of (5) under the Lorentz boosts in the α-direction(s) still makes impossible to single out a preferred comoving reference frame and thus to fix the velocity with respect to a medium specified by (5), but makes density and pressures time- and spatially-dependent. Vacuum dark fluid describes vacuum dark energy which can be associated with a variable cosmological term $\Lambda^{\mu}_{\nu} = (8\pi G)^{-1}T^{\mu}_{\nu};\ \Lambda \Longrightarrow \Lambda^t_t = (8\pi G)^{-1}T^t_t$ [22].

Stress-energy tensors for vacuum dark energy generate space-times which contain regions of maximally symmetric de Sitter vacuum, regions with the reduced vacuum symmetry, and transitions between them involving breaking of space-time symmetry. Dependently on the coordinate mapping, solutions to the Einstein equations with the source terms (3) describe cosmological models with several vacuum scales if needed ([26] and references therein), or compact objects with the de Sitter vacuum interiors (for a review [25]).

A spherically symmetric vacuum dark energy defined by $T^t_t = T^r_r$ ($p_r = -\rho$) [21], generates the metrics of the Kerr-Schild class (1) where [23,59]

$$g(r) = 1 - \frac{2G}{r}\mathcal{M}(r) - \frac{\lambda}{3}r^2;\ \ \mathcal{M}(r) = \int_0^r \rho(x)x^2 dx. \tag{6}$$

when the weak energy condition (non-negative density as measured by any observer on a time-like curve) is satisfied, a density component of T^{ν}_{μ} is monotonically decreasing [20] and can be written as

$$T^t_t(r) = \rho(r) + (8\pi G)^{-1}\lambda; \quad \rho(r) \to \rho_0 = (8\pi G)^{-1}\Lambda \quad as\ r \to 0. \tag{7}$$

It includes a background vacuum density $\rho_{\lambda} = (8\pi G)^{-1}\lambda$ and the dynamical density ρ which should vanish at $r \to \infty$ quickly enough to ensure the finiteness of the ADM mass for a compact object

$$M = 4\pi \int_0^{\infty} \rho(r) r^2 dr. \tag{8}$$

In the asymptotically flat case ($\lambda = 0$), the metric is asymptotically de Sitter for $r \to 0$ and asymptotically Schwarzschild for large r. It has two horizons (r_-, r_+ in Figure 1 (right)) which coincide for a certain mass M_{cr} corresponding to an extremal black hole [20,34]. Typical behavior of the metric function is shown in Figure 1 (left). The parameter m refers to the mass M normalized to M_{cr}. The compact objects without horizons ($m < 1$), replacing naked singularities, correspond to the gravitational solitons G-lumps [20,34] defined in the spirit of Coleman lumps [60] as non-singular non-dissipative objects keeping themselves together by their own self-interaction.

In Figure 1 (right) we show two horizons, r_-, r_+, of asymptotically flat space-time and two characteristic surfaces of geometry with the de Sitter center. For any such a geometry there exists zero gravity surface $r = r_c$ defined by $p_\perp(r_c) = 0$ [22,34], beyond which the strong energy condition ($\rho + \sum p_k \geq 0$) is violated and gravitational attraction becomes gravitational repulsion. For geometries satisfying the weak energy condition, there exists also surface of zero scalar curvature $r = r_s$ defined by $\mathcal{R}(r_s) = 0$ [34] which can be essential for details of evaporation dynamics [20,33].

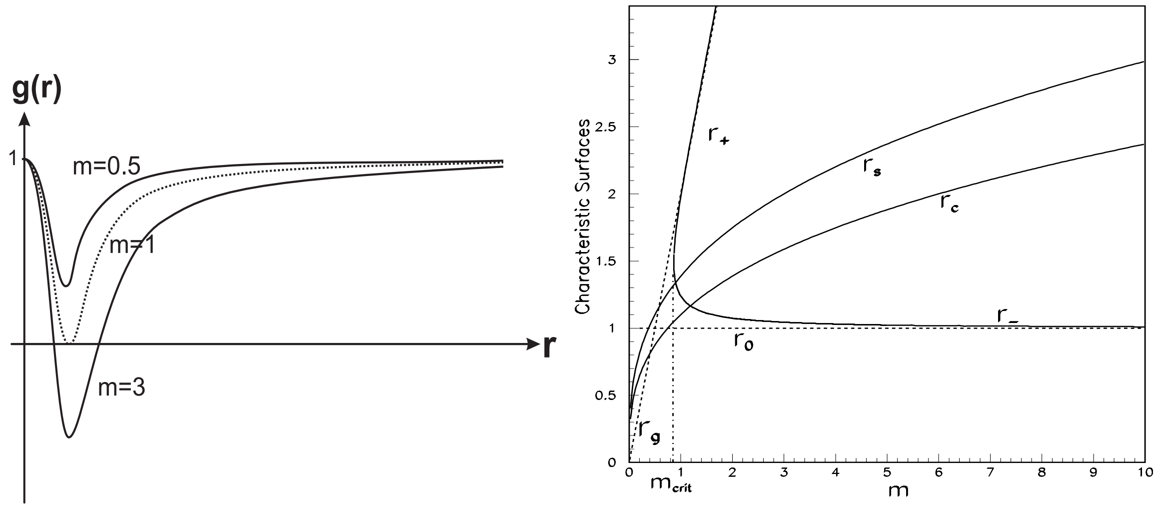

Figure 1. Typical behavior of the metric function $g(r)$ for the asymptotically flat case (**left**); horizons r_+, r_- and surfaces r_s upper, $r = r_c$ lower (**right**).

In the case of two vacuum scales space-time can have at most 3 horizons and describes five possible configurations shown in Figure 2: regular cosmological black hole within the mass range $M_{cr1} < M < M_{cr2}$, which is the T_+–region between the event horizon r_b and the internal Cauchy horizon r_a in the universe with the cosmological horizon r_c (Figure 2 (left)); two extremal double-horizon states, $r_a = r_b(M = M_{cr1})$ and $r_b = r_c(M = M_{cr2})$; and two one-horizon states (Figure 2 (right)) [59].

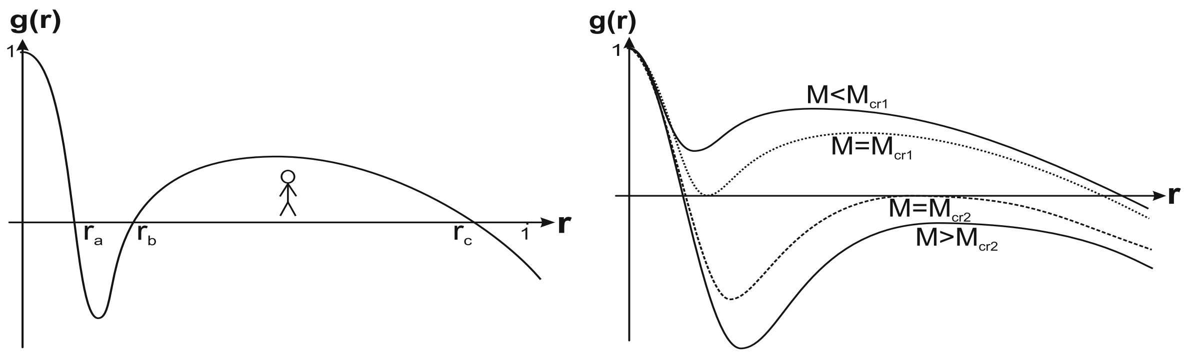

Figure 2. Typical behavior of the metric function $g(r)$ for the case of two vacuum scales.

Space-time has three characteristic length scales and is characterized by the dimensionless parameter q relating vacuum densities in the regular center ($\rho_0 = (8\pi G)^{-1}\Lambda$) and at infinity

$$r_g = 2GM; \; r_0 = \sqrt{\frac{3}{8\pi G\rho_0}}; \; r_\lambda = \sqrt{\frac{3}{8\pi G\rho_\lambda}}; \;\; q = \sqrt{\frac{\Lambda}{\lambda}} = \sqrt{\frac{\rho_0}{\rho_\lambda}}. \tag{9}$$

Space-time without the event horizon (the upper curve in Figure 2 (right)) represents a vacuum gravitational soliton G-lump in the background de Sitter space with $\rho_\lambda = (8\pi G)^{-1}\lambda$.

2.2. Basic Formulae for Thermodynamics of Horizons

For the class of metrics (1) the Gibbons-Hawking quantum temperature of each horizon r_h is defined by a surface gravity κ_h and reads [7]

$$kT_h = \frac{\hbar}{2\pi c}\kappa_h = \frac{\hbar}{4\pi c}|g'(r_h)| \tag{10}$$

where g' is the derivative of the metric function $g(r)$. Quantum temperature (10) corresponds to the periodicity of the Euclidean quantum field theory in the Euclidean time $t_E = it$ [7]. The dynamic surface gravity was studied by Hayward in the elegant approach using the Kodama vector and trapping horizon [61].

The basic fact concerning the metrics of the class (1) is that they allow for a positive definite continuation to the Euclidean time $t_E = it$ and require it to be treated as periodic, with the period which can be directly related to a finite temperature of a horizon [11]. As a result the exact partition function can be introduced as the path integral sum for a canonical ensemble of space-time metrics (1) at the constant temperature of the horizon, determined by the periodicity of the Euclidean time in the Euclidean continuation of the Einstein action [10,11]

$$Z(kT_h) = \sum \exp\left(-\frac{1}{16\pi}\int_0^{(kT_h)^{-1}} dt_E \int d^3x\sqrt{g_E}R_E[g(r)]\right). \tag{11}$$

which gives (here and in what follows we use the geometric units $c = \hbar = G = 1$) and results in

$$Z(kT_h) = Z_0 \exp\left[\frac{1}{4}\left(4\pi r_h^2\right) - \frac{1}{kT_h}\left(\frac{|g'|}{g'}\frac{r_h}{2}\right)\right] \propto \exp\left[S(r_h) - \frac{E(r_h)}{kT_h}\right] \tag{12}$$

which leads to the identification [10,11]

$$S_h = \frac{1}{4}\left(4\pi r_h^2\right) = \frac{1}{4}A_h;\;\; E_h = \frac{|g'|}{g'}\frac{r_h}{2} = \left(\frac{A_h}{16\pi}\right)^{1/2};\;\; F_h = E_h - T_h S_h \tag{13}$$

where S_h is the entropy, E_h is the thermodynamical energy, $F_h(r_h)$ is the free energy, and A_h is the horizon area. A specific heat capacity, $C_h = dE_h/dT_h$, is calculated from $C_h^{-1} = \frac{dT_h}{dr_h}\frac{dr_h}{dE_h}$ and can be written in the form [39]

$$C_h = \frac{2\pi r_h}{g'(r_h) + g''(r_h)r_h}. \tag{14}$$

On a double horizon ($g' \to 0$) it reduces to

$$C_h = \frac{2\pi}{g''(r_h)} \tag{15}$$

which allows to identify a product of evaporation as thermodynamically stable either unstable [39].

3. Thermodynamics of Horizons in Spherical Spacetimes with De Sitter Center

The basic problem concerning evaporation of a singular black hole is which remnant (if any) it leaves. Generalized uncertainty principle requires the existence of a remnant of the Planck mass [62]. On the other hand, an evident symmetry or quantum number preventing complete evaporation is not found [63,64], while complete evaporation creates the problems related to a causal structure of space-time [16,65]: A transition from a space-time with a distinguished center (distinguished by a singularity), with the Schwarzschild-de Sitter metric function $g(r)_{Schw-deS} = 1 - \frac{2GM}{r} - \frac{\lambda}{3}r^2$

(the curve 3 in Figure 3 (left)) to the final stage of evaporation which is maximally symmetric de Sitter space with the metric $g(r)_{deS} = 1 - \frac{\lambda}{3}r^2$ (Figure 3 (central)) and the global structure of the de Sitter space (Figure 3 (right)) would not only involve a dramatic change in the global structure but also creates the problem of how to evaporate a singularity [65].

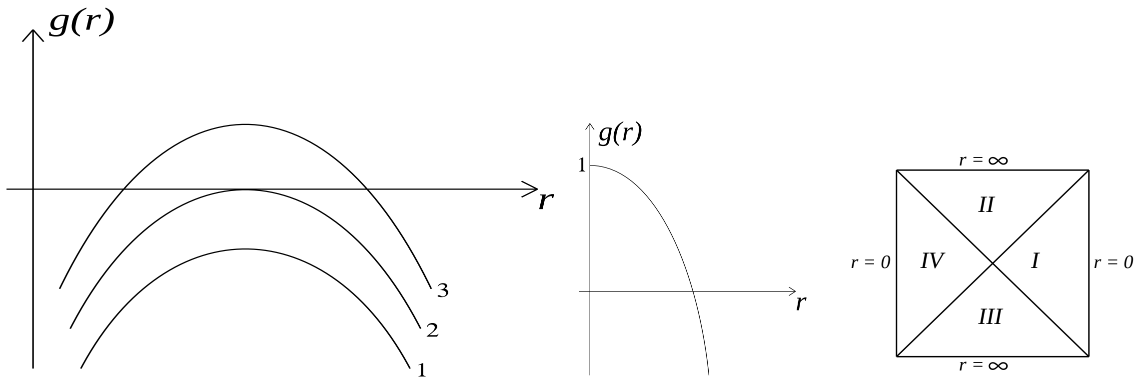

Figure 3. Evolution of the Schwarzschild-de Sitter black hole in the case of complete evaporation.

In the case of a regular black hole the end-product of evaporation is defined uniquely by quantum dynamics of evaporation which excludes the option of the complete evaporation.

3.1. Evolution during Evaporation

Applying the Padmanabhan approach [10] for a regular spherical space-time with the metric (1), we get on the horizons $r = r_h$ the temperature and specific heat capacity [39]

$$kT_h = \frac{1}{4\pi}\left|\frac{1}{r_h} - \lambda r_h - 8\pi\rho(r_h)r_h\right|; \;\; r_h = r_a, r_b, r_c \tag{16}$$

$$C_h^{-1} = -\frac{1}{2\pi}\left[8\pi\rho'(r_h)r_h + 8\pi\rho(r_h) + \lambda + \frac{1}{r_h^2}\right]. \tag{17}$$

Following Teitelboim [14] we can take derivative of the relation $g(r_h, M) = 0$ keeping λ and Λ constant. It gives on the horizons [14,39]

$$\frac{dr_h}{dM} = -\frac{\partial g}{\partial M}\frac{1}{g'(r_h)}. \tag{18}$$

To study the derivative $\partial g/\partial M$ on horizons, we normalize r on a some characteristic scale r_* which we define from the formula for the mass (8) where we normalize $\rho(r)$ on the central density ρ_0, so that we have $\rho(r) = \rho_0\tilde{\rho}(r/r_*)$ and $M = 4\pi\rho_0 r_*^3 \int_0^\infty \tilde{\rho}(y)y^2dy$. With taking into account (9) this gives

$$r_* = \frac{1}{3\beta}(r_0^2 r_g)^{1/3}; \;\; \beta = \int_0^\infty \tilde{\rho}(y)y^2dy \tag{19}$$

where $y = r/r^*$ and β is a numerical coefficient. For the regular black hole with the density profile and mass function given by (4), $\beta = 1/3$ and $r_* = (r_0^2 r_g)^{1/3}$ which is the characteristic scale for a Schwarzschild-de Sitter transition at $r \to 0$ [31]. The mass function takes the form [39]

$$\mathcal{M}(r) = 3M\phi(y); \;\; \phi(y) = \int_0^y \tilde{\rho}(y)y^2dy, \tag{20}$$

the metric function reduces to

$$g = 1 - M^{2/3}\frac{1}{r_0^{2/3}}\left[\left(\frac{3}{2^{1/3}}\right)\frac{\phi(y)}{y} + \frac{(2^{2/3})}{q^2}y^2\right] \tag{21}$$

and on the horizons we get [39]

$$\frac{\partial g}{\partial M} = -\frac{2}{3M}. \tag{22}$$

Then it follows from (18) that

$$\frac{dr_a}{dM} < 0; \quad \frac{dr_b}{dM} > 0; \quad \frac{dr_c}{dM} < 0. \tag{23}$$

Generic form of the horizons-mass diagram is shown in Figure 4 (left) [40].

For an observer in the R-region $0 \le r \le r_a$, the horizon $r = r_a$ is his cosmological horizon, which is the boundary of his manifold, so that the second law of thermodynamics reads $dS_a \ge 0$. As a result, the evolution of r_a as governed by the second law gives $dr_a \ge 0$. It follows then from (23) that when r_a increases, mass M decreases, as a result black hole event horizon shrinks, $dr_b < 0$, and cosmological horizon moves outward, $dr_c > 0$. Quantum evaporation proceeds with decreasing mass M, and goes towards a formation of the double horizon $r_a = r_b$ [39].

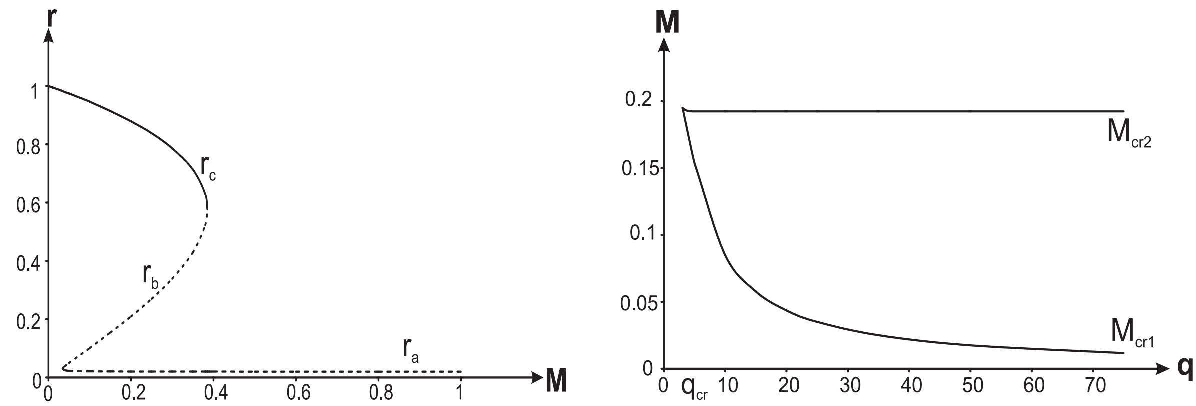

Figure 4. Generic form of the horizons-mass diagram (**left**) and dependence of critical masses on the parameter q (**right**).

Two double-horizon states $r_a = r_b$ and $r_b = r_c$, correspond to the values M_{cr1} and M_{cr2}, respectively. In Figure 4 (right) [40] we show dependence of the double horizon $r_b = r_a$ (line denoted by M_{cr1}) and of the double horizon $r_c = r_b$ (line M_{cr2}) dependently on the parameter q. For the certain values of q_{cr} and M_{cr}, three horizons coincide at the triple horizon r_t (Figure 4 (right), the point where two lines meet). The values of q_{cr}, M_{cr} and r_t are uniquely defined by three algebraic equations defining the triple horizon, $g(r_t) = 0$, $g'(r_t) = 0$ and $g''(r_t) = 0$ [27,40].

3.2. Thermodynamics of a Regular Black Hole

An asymptotically flat black hole evaporates from both horizons, and generic asymptotic behavior of the metric function $g(r)$ defines dynamics of evaporation. Temperature drops to zero at the double horizon and follows the Schwarzschild limit for a large values of $r_b \to r_g$, as a result it should have a maximum, so that evaporation process involves a phase transition where a specific heat is broken and changes its sign [20,34,45] (more references in Introduction, p. 3). A regular asymptotically flat black hole leaves behind a thermodynamically stable double-horizon remnant with zero temperature and the positive specific heat.

In the case of two vacuum scales, an observer in the R-region $r_b < r < r_c$ (shown in Figure 2, left) can detect radiation from the black hole horizon r_b and from the cosmological horizon r_c. It is impossible for him (her) to obtain, by one transformation, regularity on both horizons (required for calculation in the Kruskal coordinates of quantum temperature corresponding to periodicity of the Euclidean quantum field theory in the Euclidean time), so that an observer would detect the mixture of radiations from horizons [7]. A common (global) temperature can be defined only in the case when the relation of surface gravities on horizons r_b and r_c is the rational number, $\kappa_b/\kappa_c = n_b/n_c$ where n_b and n_c are prime integers, and the common temperature is given by [66,67]

$$kT_{com} = \frac{\hbar}{2\pi c}\frac{(\kappa_b + \kappa_c)}{(n_b + n_c)} = \frac{\hbar}{2\pi c}\frac{\kappa_b}{n_b} = \frac{\hbar}{2\pi c}\frac{\kappa_c}{n_c}. \tag{24}$$

Derivative of the metric function $g(r)$ in (16) is positive on a black hole event horizon $r_h = r_b$ which gives

$$T_b = \frac{1}{4\pi}\left(\frac{1}{r_b} - \frac{3r_b}{l^2} - 8\pi\rho(r_b)r_b\right); \quad E_b = \frac{r_b}{2}. \tag{25}$$

Behavior of temperature of the event horizon ($r = r_{\rm b}$ here), shown in Figure 5 (left), plotted with the value of $q = 50$ [39], is generic for black holes with the de Sitter center and dictated by the behavior of the metric function $g(r)$ [39]: $g'(r_b = r_c) = 0$; $g'(r_a = r_b) = 0 \to T_b = 0$ as $r_b \to r_c$, and $T_b = 0$ as $r_a \to r_b$. In the maximum of T_b the specific heat capacity (shown in Figure 5 (right), plotted with $q = 10$ [39]) is broken and changes sign, hence a second order phase transition occurs which is followed by the quantum cooling [34,39,45]. At the phase transition the temperature T_b acquires its maximal value [39]

$$T_{b\ max} = T_{tr} = -\frac{1}{4\pi}g''(r_b)r_b. \tag{26}$$

In the case of the density profile (4) $T_{tr} \simeq \alpha T_{Pl}\sqrt{\rho_0/\rho_{Pl}}$. For $\rho_0 = \rho_{GUT}$ and $M_{GUT} \simeq 10^{15}$ GeV, temperature at the phase transition $T_{tr} \simeq 0.2 \times 10^{11}$ GeV [34,39].

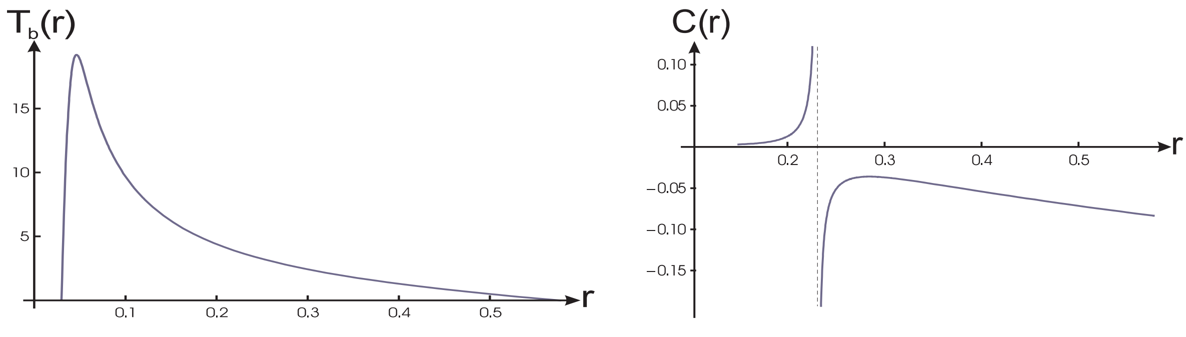

Figure 5. Generic behavior of temperature (**left**) and specific heat (**right**) of a regular cosmological black hole.

Before the transition, $C_b < 0$, hence $dT_b/dr_b < 0$; when r_b decreases, E_b decreases too, temperature increases to a maximum (26) where C_b changes sign, so that after transition we have $dT_b/dr_b > 0$, and thus further decreasing r_b leads to decreasing T_b until it vanishes at the double horizon. At this point the specific heat C_b is positive and takes the value (15), free energy is positive and equal E_b, and energy E_b achieves its minimum, so that an extreme black hole with the double horizon $r_b = r_a$ is the thermodynamically stable end-point of evolution during evaporation.

Derivative of the metric function $g(r)$ in (16) is negative on the cosmological horizon $r_h = r_c$ and the internal horizon $r_h = r_a$, which gives

$$T_{c,a} = \frac{1}{4\pi}\left(8\pi\rho(r_h)r_h + \frac{3r_h}{l^2} - \frac{1}{r_h}\right); \quad E_h = -\frac{r_h}{2} \tag{27}$$

An observer in the R-region $0 \le r < r_a$ can detect the Gibbons-Hawking radiation from the internal horizon r_a which is his cosmological horizon. For the internal horizon $dr_a/dM < 0$, by (23), and r_a grows while M decreases. Since $g' < 0$ on the internal horizon, we have [40]

$$\frac{dT_a}{dr_a} = -\frac{T_a}{r_a} + \frac{1}{4\pi} g''(r_a). \tag{28}$$

Specific heat C_a is positive near $r_a \to r_b$, so that $dT_a/dE_a > 0$ and $dT_a/dr_a < 0$ when r_a approaches the double horizon. Hence T_a decreases with increasing r_a, the mass M decreases too, $dT_a/dM > 0$ and $dT_a/dr_a < 0$, so the growth in r_a leads to monotonic decreasing of the mass M and of the temperature T_a until it vanishes at the double horizon where the energy E_a achieves its minimal value $E_a = -r_a/2$. Temperature of the internal horizon is shown in Figure 6 (left) [40]. It decreases monotonically with increasing r_a, and vanishes for $M = M_{cr1}$ when $r_a = r_b$ [40].

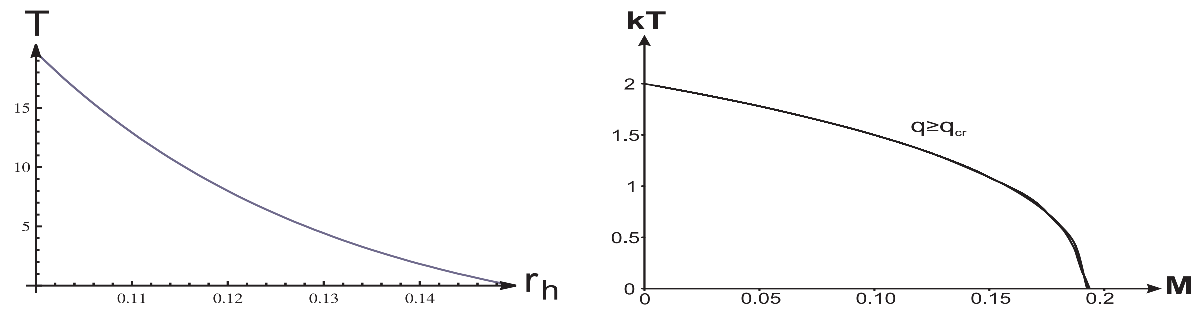

Figure 6. Typical behavior of temperature on the internal (**left**) and cosmological (**right**) horizon.

The cosmological horizon r_c moves outwards, in accordance with (23), similar to the case of r_a, but the essential difference is that the specific heat is negative near the double horizon $r_b = r_c$. Hence $dT_c/dE_c < 0$ and $dT_c/dr_c < 0$; since $dr_c/dM < 0$, it follows that M decreases while r_c increases, and $dT_c/dM < 0$ what we see in Figure 6 (right) where k is the Boltzmann constant and $q_{\rm cr}$ is the value of q at which the triple horizon appears [40]. As a result E_c decreases to its minimum, and specific heat capacity remains negative.

A regular cosmological black hole leaves behind a thermodynamically stable double-horizon remnant generically related to vacuum dark energy through its interior de Sitter vacuum ([39,40] and references therein). Mass of the remnant is given by $M_{remnant} \simeq \beta M_{Pl}\sqrt{\rho_{Pl}/\rho_{int}}$. For the density profile (4) with $\rho_\Lambda = \rho_{GUT}$ and $M_{GUT} \simeq 10^{15}$ GeV, the mass $M_{remnant} \simeq 0.3 \times 10^{11}$ GeV [34].

In the case of one-horizon space-time with the de Sitter global structure and two vacuum scales (initial and final) the evaporation process directed towards decreasing M starts in the state $M > M_{cr2}$ (Figure 2 (right)). Evaporation results in the state $M = M_{cr2}$ (the regularized Nariai solution) with the negative specific heat according to (15), which is thermodynamically unstable.

3.3. Triple-Horizon Spacetime Singled Out by the Holographic Principle

One more possibility is presented by the class of one-horizon solutions with the inflection point instead of the minimum (Figure 7 (left) [40]). In the course of quantum evaporation starting from the state $M > M_{cr}$, cosmological horizon moves outwards in accordance with (23) and goes towards the triple-horizon space-time $r_h = r_t$ ($M = M_{cr}$) [27]. Specific heat capacity of this horizon is always positive and tends to infinity at the triple horizon [40], so that the triple-horizon spacetime is the thermodynamically stable final product of evaporation of the cosmological horizon. Evaporation stops completely at $T_h = 0$ and $C_h \to \infty$. Three algebraic equations which specify the triple-horizon state ($g(r_t) = 0$; $g'(r_t) = 0$; $g''(r_t) = 0$) define uniquely the basic parameters M_{cr}, r_t, and the key parameter

$q_{cr} = \rho_0/\rho_\lambda$, which gives the tightly fixed non-zero final value of a vacuum dark energy density ρ_λ for the given value ρ_0 [27].

The Holographic Principle states that the number of quantum degrees of freedom in a spatial volume is bounded from above by its surface area [68]. It is also formulated as the Conjecture: A physical system can be completely specified by data stored on its boundary [63].

Evolution of one-horizon space-time with the inflection point is governed by quantum dynamics of surrounding it surface (cosmological horizon) and goes towards the triple-horizon space-time, whose basic physical parameters, M_{cr}, r_t and q_{cr}, are entirely defined by the data stored on its boundary (triple-horizon surface)—in agreement with the basic sense of Holographic Principle [27].

Choosing the density profile (4) we obtain $M_{cr} = 2.33 \times 10^{56}$ g; $q_{cr}^2 = 1.37 \times 10^{107}$; $r_t = 5.4 \times 10^{28}$ cm [27]. To evaluate the vacuum dark energy density from $q_{cr}^2 = \rho_0/\rho_\lambda$, we adopt $\rho_0 = \rho_{GUT}$. The Grand Unification scale is estimated as $M_{GUT} \sim 10^{15}$–10^{16} GeV. This gives the value of ρ_λ within the range 1.7×10^{-30} g cm^{-3}–1.7×10^{-26} g cm^{-3}, respectively. The observational value $\rho_{\lambda\ (obs)} \simeq 6.45 \times 10^{-30}$ g cm^{-3} [69] corresponds, in the considered context, to $M_{GUT} \simeq 1.4 \times 10^{15}$ GeV. This gives $\rho_{GUT} = 8.8 \times 10^{77}$ g cm^{-3} and $r_0 = 1.8 \times 10^{-25}$ cm. For this scale q_{cr}^2 gives the value of the present vacuum density ρ_λ in agreement with its observational value [29].

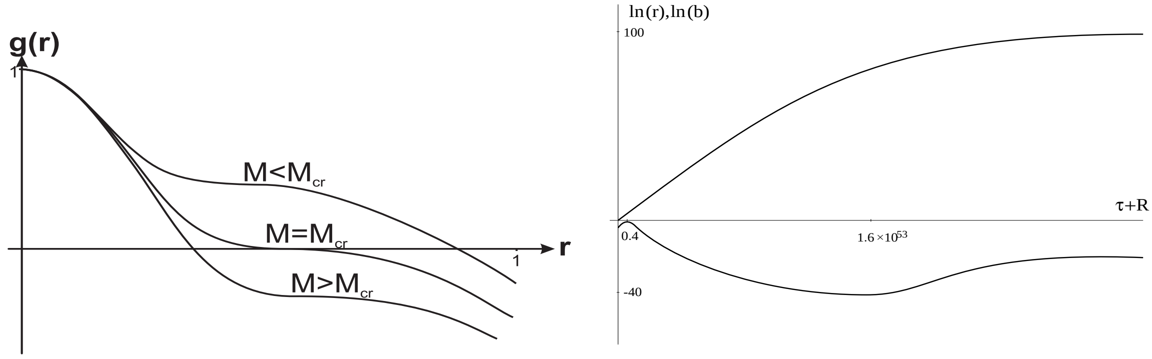

Figure 7. Evolution towards triple-horizon space-time (**left**) and evolution of scale factors (**right**).

The triple-horizon space-time is distinguished by quantum dynamics of the cosmological horizon as the only thermodynamically stable final product of its evaporation. Evaporation stops with the finite non-zero value of the cosmological constant given by the finite value of q_{cr}, so that the space-time symmetry acts as the symmetry which provides, via the Holographic Principle, the mechanism of relaxing of the cosmological constant to a certain tightly fixed non-zero value.

It is well known that in the frame of FLRW cosmology it is impossible to describe the initial inflation and today accelerated expansion via a single self-consistent theoretical scheme. It is possible in the frame of the Lemaître cosmology with anisotropic perfect fluid. For the case of the source terms specified by (2), regular cosmological Lemaître class models are asymptotically de Sitter at the early and late time, and can describe evolution from an inflationary beginning to the late time acceleration, guided by dynamical vacuum dark energy closely related to space-time symmetry [26,30].

The static spherically symmetric metric (1) can always be transformed to the Lemaître form ([26] and references therein) with the line element

$$ds^2 = d\tau^2 - b^2(R,\tau)dR^2 - r^2(R,\tau)d\Omega^2 \tag{29}$$

where $b^2(R,\tau) = (\partial r/\partial R)^2$ for the case when each 3-hypersurface $\tau = const$ is flat [58], which guarantees fulfilment of the spatial flatness condition $\Omega = 1$ required by observations.

One-horizon space-time have the same global structure as that for the de Sitter space-time (3-d curve on the right in Figure 3), for spatially flat models with $\Omega = 1$ evolution starts with the non-singular non-simultaneous de Sitter bang at $(\tau + R) \to -\infty$ from the regular time-like surface $r(R,\tau) = 0$. It is followed by the anisotropic stage at which the expansion in the transversal direction with $\partial_\tau r > 0$ is accompanied by shrinking in the radial direction where $\partial_R |g_{RR}| < 0$ until $dg(r)/dr < 0$ [26,30].

The evolution of two scale factors in the course of the evolution is shown in Figure 7 (right) [29] (the upper curve for $lnr(\tau, R)$, the lower for $lnb(\tau, R)$) plotted with the density profile (4) and the model parameters given above the Figure 7 [29]. As we see, at the first inflationary stage and the present stage of accelerated expansion the behavior of two scale factors is similar (curves run parallel and differ only by constant) and corresponds to the flat de Sitter scale factor. The maximum in the scale factor $b(\tau, R)$ at $\tau + R \simeq 0,4$ corresponds to the maximum of the transversal pressure (the variable $(\tau + R)$ is normalized to the GUT time $t_{GUT} = r_\Lambda/c \simeq 0.6 \times 10^{-35}$ s) [29].

4. Summary and Discussion

Algebraic classification of stress-energy tensors implies the possibility to introduce in general setting dynamical vacuum dark energy which is directly related (via the Einstein equations) to space-time symmetry and can be evolving and clustering. In the spherically symmetric case it is specified by $T_t^t = T_r^r$ $(p_r = -\rho)$. Regular solutions to the Einstein equations are described by the metrics of the Kerr-Schild class, $ds^2 = g(r)dt^2 - g(r)^{-1}dr^2 - r^2 d\Omega^2$, and have obligatory de Sitter centers provided that stress-energy tensors satisfy the weak energy condition. Dependently on a mapping (reference frame), they describe regular cosmological models with time-evolving and spatially inhomogeneous vacuum dark energy, and regular compact objects with de Sitter vacuum interiors: regular black holes, their remnants and self-gravitating vacuum solitons, generically related to vacuum dark energy.

Space-time with two vacuum scales can have at most three horizons. Regular cosmological models with vacuum dark energy belong to the Lemaître class cosmologies with anisotropic perfect fluid. Space-time symmetry provides the mechanism of relaxation of a cosmological constant to a certain non-zero value via restoration of space-time symmetry to the de Sitter group.

Generic features of thermodynamics of horizons are obtained in the frame of the Padmanbhan approach, appropriate for a multi-horizon space with a non-zero pressure and based on the canonical ensemble of the Kerr-Schild metrics at the constant temperature of a horizon determined by the periodicity of the Euclidean time in the Euclidean continuation of the Einstein action.

Among the one-horizon models with the same global structure as for the de Sitter space-time and evolution of vacuum dark energy from the first inflation to the present accelerated expansion, the Holographic Principle singles out the triple-horizon space-time as the only thermodynamically stable product of quantum evaporation of the cosmological horizon, which uniquely defines the basic physical parameters including the final non-zero value of the vacuum dark energy density.

For all regular black holes described by the Kerr-Schild metrics generated by any source with $T_t^t = T_r^r$, quantum evaporation of horizons involves a second order phase transition followed by quantum cooling and resulting in a thermodynamically stable double-horizon remnant with zero temperature and positive specific heat capacity.

Regular compact black holes, their remnants and self-gravitating vacuum solitons G-lumps can be responsible for observational effects typically related to a dark matter [24,70] and serve as the source of information on the scale of inhomogeneity of the early Universe [71].

Black hole remnants are considered as a reliable source of dark matter for more than three decades [72–74]. Regular primordial black holes (RPBH), their remnants and gravitational solitons G-lumps with de Sitter vacuum interiors can arise during first and second, $E_{infl} = E_{QCD}$ [75] inflationary stages in a quantum collapse of primordial fluctuations. Probability of tunnelling in a collapse is given by [76]

$$D \geq \exp\left[-4\left(\frac{M}{M_{Pl}}\right)^{3/4}\left(\frac{E_{Pl}}{E_{infl}}\right)\right] \quad for \quad \frac{M}{M_{Pl}} \geq \left(\frac{E_{infl}}{E_{Pl}}\right)^4 \left(\frac{E_{Pl}}{E_{int}}\right)^8 \tag{30}$$

where E_{infl} is the scale of the inflationary vacuum. At the first inflation, $E_{infl} = E_{GUT}$, regular objects with the interior de Sitter vacuum $E_{int} = E_{GUT}$ can arise with masses $M > 10^{11}$ g. For $E_{int} = E_{Pl}$ any mass is possible, as well as for regular objects arising at the second inflationary stage.

RPBHs, their remnants and G-lumps can capture available charged particles and form graviatoms ([76] and references therein)—gravitationally bound ($\alpha_G = GMm/\hbar c$) quantum systems with captured particles. Conditions of the existence of graviatoms [76] constrain the masses of captured particles by $m > 10^9$ GeV for $E_{int} = E_{GUT}$. This can be (i) heavy particles captured at the reheating stage after the first inflation and (ii) leptoquarks survived in galactic haloes [77,78].

Observational signatures of graviatoms as heavy dark matter candidates provide a source of information on a vacuum scale in the epoch when they were formed [71]. Most promising is the oscillatory electromagnetic radiation of graviatom whose characteristic frequency depends on the scale E_{int} of the interior de Sitter vacuum [76]. For the density profile (4), $\hbar\omega = 0.678\hbar c/r_{deS} = 0.678 \times 10^{11}$ GeV $(E_{int}/E_{GUT})^2$. Detection of cosmic photons is possible up to $10^{11.5}$ GeV [79].

Mass of objects with interior de Sitter vacuum is generically related to breaking of space-time symmetry from the de Sitter group in their origins [20,80]. In the Standard Model de Sitter vacuum appears as the false vacuum of the Higgs field responsible for particle masses (the Higgs field is involved in mass generation in its false vacuum state $p = -\rho$). In graviatoms with the GUT scale interiors, where baryon and lepton numbers are not conserved, a remnant component of graviatom may induce proton decay, which could in principle serve as an additional observational signature of graviatom for heavy dark matter searches at the IceCUBE experiment. If the relevant cross section is determined by the geometrical size of nucleon ($\sigma_i \sim 10^{-26}$ cm^2) one could expect in the matter of a 1 km^3 detector, like IceCUBE, up to 300 events per year [71].

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