

Superfluidity and Superconductivity in Neutron Stars

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By compressing matter to densities up to several times the density of atomic nuclei, the catastrophic gravitational collapse of the core of stars with a mass $M \gtrsim 8M_{\odot}$ during supernova explosions and the neutron star left behind (see, e.g., Refs. [1–3]) provide unique opportunities to probe matter under extreme conditions which are inaccessible to terrestrial experiments (see, e.g., Refs. [4,5]). Even though neutron stars are initially very hot with temperatures $\sim 10^{12}$ K, they rapidly cool down to $\sim 10^9$ K within days by releasing neutrinos (see, e.g., Refs. [6–8]). Recalling that the baryon chemical potential inside neutron stars can reach a few gigaelectronvolts, their interior is thus highly degenerate and is expected to undergo various quantum phase transitions, such as superfluidity and superconductivity observed in some terrestrial materials at low-enough temperatures. Despite extensive laboratory research on quantum condensates, little is known about the characteristics of their counterpart in neutron stars (for recent reviews, see, e.g., Refs. [9–11]). This Special Issue deals with some recent advances in our understanding of these phenomena, from both theoretical and observational perspectives.

Only a few months after the publication of the Bardeen–Cooper–Schrieffer (BCS) theory of conventional superconductivity, based on the formation and condensation of electron pairs, its implications for atomic nuclei were discussed by Aage Bohr, Ben Roy Mottelson, and David Pines. They speculated that a similar pairing mechanism between nucleons might explain the energy gap in the excitation spectra of nuclei [12]. They also anticipated that nuclear pairing could explain odd–even mass staggering, and the reduced moments of inertia of nuclei. Neutron superfluidity in neutron stars was first predicted by Arkady Migdal in 1959 [13], and the first calculations were performed a few years later by Vitaly Ginzburg and David Kirzhnits [14]. Proton superconductivity was first studied by Richard Wolf [15]. During these periods, the existence of neutron stars remained speculative. Since the discovery of pulsars, many theoretical calculations based on different many-body methods have been carried out and it was predicted that a neutron star may exhibit various types of superfluid and superconducting phases [11]. In the inner crust and in the outer core neutrons are expected to form 1S_0 Cooper pairs like electrons in conventional superconductors. The properties of this neutron superfluid have been studied by Palkanoglou and Gezerlis [16]. These authors discuss some numerical techniques to improve the extraction of 1S_0 pairing gaps at zero temperature from ab initio calculations (such as those based on quantum Monte Carlo methods) in a box with a finite number of neutrons by imposing twisted boundary conditions. The critical temperature as a function of density has been calculated by Durel and Urban in this issue within the in-medium T-matrix formalism using the effective low-momentum interaction. They find that a small reduction in the density of states, interpreted as a pseudogap, is present in the normal phase at temperatures above the critical temperature. The implications for neutron stars remain to be investigated. These calculations were performed in pure neutron matter. A further complication is that some neutrons are bound inside clusters in the crust and this can affect the superfluid properties. In turn, neutron pairing can influence the internal constitution of the crust, as shown by Shelley and Pastore in this issue. These effects can be studied self-consistently within the Hartree–Fock–Bogoliubov method [17]. These calculations are computationally costly, especially in the deep crust, but can be accurately approximated



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by the extended Thomas–Fermi approach with shell effects added perturbatively via the Strutinsky integral [18] and neutron pairing included in the local density approximation. The outer core of a neutron star is an admixture of neutrons, protons and leptons. Similarly to neutrons, protons in the outer core are expected to pair in the 1S_0 channel. The ground state of these neutron and proton condensates has been studied by Wood and Graber and is presented in this issue within the Ginzburg–Landau approach. They find that the proton superconductor could be in an intermediate state between type I and type II, in which quantised magnetic flux tubes form bundles in a hexagonal lattice separated by flux-free Meissner regions.

A superfluid, as exemplified by helium, exhibits hydrodynamical phenomena not observed in ordinary fluids. One of the most striking manifestations is the spectacular gushing of a superfluid out of its container when subjected to a slight increase in temperature. Such phenomena are caused by the coexistence of two separate dynamical components, both of which carry mass but with distinct velocities: one is a normal viscous fluid, whereas the other has no entropy and flows without resistance [19,20]. A neutron star containing superfluid neutrons and superconducting protons is therefore a complex multi-fluid system, which will be reviewed by Andersson in this issue. These fluids cannot flow completely independently. They are weakly coupled by non-dissipative mutual entrainment effects caused by nuclear interactions [21]. The modeling of the global dynamics of neutron stars thus requires the knowledge of a number of microscopic parameters. In this issue, Allard and Chamel have calculated 1S_0 pairing gaps, chemical potentials and entrainment coefficients in the outer core of a neutron star consistently over the whole range of temperatures and velocities for which superfluidity can exist, in the framework of the nuclear energy-density functional theory. As neutron stars are rotating and magnetized, their interior is threaded by quantised neutron vortices and proton flux tubes [14,22]. The neutron superfluid dynamics in the crust are studied by Gavassino, Antonelli, and Haskell in this issue. They show that the phenomenological equation of motion for a quantized vortex should incorporate an additional transverse force arising from the ambiguity which is present in the specification of which neutrons are “free” and which are confined in clusters. This force is analogous to the Iordanskii force introduced in the context of superfluid helium. To make matters more difficult, a neutron star is so compact that it must eventually be described by Einstein’s theory of general relativity. In this issue, Cheung-Hei Yeung, Lap-Ming Lin, Andersson, and Comer present general-relativistic simulations of superfluid neutron stars within a two-fluid model: a neutron superfluid and a fluid made entirely of charged particles. They show that the moment of inertia, the spin-induced quadrupole moment, and the tidal deformability follow practically the same universal relations as nonsuperfluid neutron stars. This implies that superfluidity does not significantly affect the gravitational-wave signal emitted by binary neutron stars.

Superfluidity may leave an imprint on various astrophysical phenomena. The strongest evidence of this comes from pulsar frequency glitches associated with sudden spin-ups of the neutron star thought to be triggered by the unpinning of quantised superfluid vortices (see, e.g., Refs. [23,24] for recent reviews). In this issue, Montoli, Antonelli, Haskell, and Pizzochero revisit the determination of the glitch activity and its implication for the superfluid regions of neutron stars. The cooling of neutron stars [8] provides another venue for probing superfluidity, as shown by Wei, Burgio, and Schulze. The reason for this lies in the fact that superfluidity modifies both transport properties and neutrino emissivities [25], as reviewed by Manuel and Tolos [26]. The role of low-energy collective excitations, so-called phonons, and their theoretical determination are reviewed by Baldo [27].

The inner core of the most massive neutron stars may not only contain nucleons and leptons but also other particles such as hyperons or even deconfined quarks (see, e.g., Ref. [28] for a review). Some compact stars could be entirely made of strange quarks. Such strange matter is expected to become superconducting at low temperatures (see, e.g., Refs. [29,30] for reviews). Celi, Mariani, Orsaria, and Tonetto [31] have calculated the structure of such stars and they show that their mass and radius are consistent with

constraints inferred from current electromagnetic and gravitational-wave observations. They also present results for various oscillation modes.

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