Electronics **2014**, *3*, 553-563; doi:10.3390/electronics3030553

OPEN ACCESS

electronics

ISSN 2079-9292

www.mdpi.com/journal/electronics

Article

Cognitive Spectrum Sensing with Multiple Primary Users in Rayleigh Fading Channels

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Received: 27 May 2014; in revised form: 3 September 2014 / Accepted: 12 September 2014 / Published: 23 September 2014

Abstract: Accurate detection of white spaces is crucial in cognitive radio networks. Initial investigations show that the accurate detection in a multiple primary users environment is challenging, especially under severe multipath conditions. Among many techniques, recently proposed eigenvalue-based detectors that use random matrix theories to eliminate the need of prior knowledge of the signals proved to be a solid approach. In this work, we study the effect of Rayleigh multipath fading channels on spectrum sensing in a multiple primary user environment for a pre-proposed detector called the spherical detector using the eigenvalue approach. Simulation results show interesting outcomes.

Keywords: cognitive radio (CR); spectrum sensing; spherical detector; Rayleigh fading channels

1. Introduction

The cognitive radio network (CRN) approach is considered as a promising solution for solving the scarcity in radio spectrum resulting from the unprecedented growth in wireless communications. J. Mitola [1] was the first to propose the concept of CRN where a communication device is aware of the radio environment surrounding it and has the ability to adjust its parameters, such as carrier frequency and transmission rate, according to that environment. Since CRN devices are designed to co-exist with licensed primary users, dynamically sensing the radio spectrum in order to decide the presence or absence of the primary users is a very important function of a CRN device. Accurate sensing will minimize the interference to the primary users from the secondary users who use the same frequency band while ensuring high throughput to the secondary users [2]. This is done by lowering the missed detection probability.

In the literature, there are several methods of spectrum sensing (or spectrum detection) based on the type of information, such as frequency, modulation, signal and noise variances, collected about the primary user [3]. However, the most important aspect of spectrum sensing is the primary user signal detection, where a binary hypothesis is tested to declare the presence or the absence of the primary user signal.

Energy detection, matched filter detection and cyclostationary feature detection are the widely investigated sensing methods. Among these, energy detection [4–6] is an easy to implement technique in spectrum sensing, since it measures only the energy of the primary user signal. It does not require information about primary user signal parameters, such as modulation, pulse shape, *etc*. However, noise variance needs to be known for accurate energy detection, and any uncertainty in noise estimation will affect detector performance. Moreover, energy detection schemes are not optimal to detect correlated signals [7]. A generalized likelihood ratio detector (GLR) is proposed in [8] to overcome some or all unknown parameters needed for optimal energy detection, such as noise uncertainty, channel gain and primary user variance. Cyclostationary and matched filter detectors [9,10] are optimal detectors. However, they require prior knowledge of primary signal parameters, which is hard to implement in many practical situations.

Eigenvalue-based detectors [3,7,11–15] have been recently proposed for spectrum sensing purpose. These use the latest random matrix theories to avoid the requirement for *a priori* knowledge of signals. Hence, this technique alleviates the difficulties in estimating unknown parameters that exist in other spectrum sensing methods. There are different eigenvalue-based algorithms based on the type of statistics used for testing the hypothesis. Accordingly, these can be classified as the largest eigenvalue-based detector (LED), the maximum-minimum eigenvalue detector (MMED), the scaled largest eigenvalue-based detectors (SLED), *etc*. Eigenvalue-based detectors outperform conventional detectors especially in multiple primary user scenarios, where multiple antennas (or multiple cognitive users) are collaboratively working to detect the spectrum holes. Typically, in these scenarios, it is hard to get *a priori* knowledge of the noise power, detected signals powers, number of primary user signals, *etc*. In other words, this is considered as blind detection, and the eigenvalue-based detectors are the best choice in such cases.

Previously in the literature, the energy detection hypothesis in multipath fading channels has been investigated for the single primary user scenario, and different closed form expressions for the average detection probability have been derived [5,6,16,17]. Furthermore, cooperative spectrum sensing for single-input-multiple-output (SIMO) and multiple-input-multiple-output (MIMO) techniques have been revisited, and detection parameters have been derived for different multipath fading channels and different combining schemes [18–21]. However, to the best of the authors' knowledge, cooperative spectrum sensing in a multiple primary user scenario considering multipath fading channels using eigenvalue-based detectors has not been reported yet. This is the focus of this paper. In this paper, we investigate the effect of multipath fading channels on cognitive spectrum sensing, specifically considering Rayleigh distributed channels.

In this study, we consider multiple primary users and eigenvalue-based spectrum sensing using a spherical test detector, extending the previous work [22] that just considered the AWGN channel. The closed-form expression derived for the detection probability in [22] is used as a performance metric to investigate the impact of Rayleigh multipath fading on multiple primary user spectrum sensing in this work.

The rest of the paper is organised as follows. In Section 2, we describe the spectrum sensing system model with multiple primary users. Section 3 describes finding the average detection probability in Rayleigh fading channels. Section 4 outlines simulation results, and Section 5 concludes the paper.

2. System Description

Let us assume that K secondary users collaboratively sense the presence of P primary users, where N samples are received. Hence, in the signal detection algorithm, two hypotheses, H_0 and H_1, are defined for the absence (null) and the presence (alternative) of the primary user signal, respectively, as follows:

$$H_0 : y\,[n] = w\,[n] \tag{1}$$

$$H_1 : y\,[n] = h\,s\,[n] + w\,[n]\,. \tag{2}$$

Here, y, w, h and s denote the discrete received samples, noise, channel gain coefficient and transmitted primary signal, respectively. Under H_0, as in (1), no signal is detected, and the received samples are nothing but the additive complex Gaussian noise; hence, the absence of the primary user is declared. While under H_1, as in (2), the received samples are signal samples in addition to the noise; hence, the presence of the primary user is declared. Consequently, the received observation sample vector is:

$$\mathbf{y} = \mathbf{H}\,\mathbf{s} + \mathbf{w}. \tag{3}$$

Here, $\mathbf{y} \in \mathbb{C}^{KP\times N}$ is a dimensional complex vector, which represents the receiving data vector to sense P primary users. The $K\times 1$ vector $\mathbf{w}$ represents the complex Gaussian noise vector with i.i.d. distribution $N \sim (0, \sigma_w^2)$ having zero mean and complex noise power σ_w^2, respectively. The $K \times P$ dimension matrix $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_p]$ represents the channel gain coefficients. For instance, $\mathbf{h}_1$ represents the vector of all of the channels between the first primary user and all secondary users (K sensors); similarly, for all $\mathbf{h}_P$ entries, as shown in Figure 1. We can write the $\mathbf{H}$ matrix as follows:

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2}, \cdots & h_{1,P} \\ \vdots & \ddots & \vdots \\ h_{K,1} & h_{K,2}, \ldots. & h_{K,P} \end{bmatrix}. \tag{4}$$

The P primary user signals are represented by a $P \times 1$ vector $\mathbf{s} = [s_1,\ s_1, \ldots, s_P]^T$, where the primary users signals samples are i.i.d. with complex Gaussian distribution with zero mean and σ_s^2 variance, *i.e.*, $\mathbf{s} \sim N\left(0, \sigma_s^2\right)$ and uncorrelated with noise. The $\mathbf{H}$ matrix is assumed constant during the sensing period; hence, it is deterministic. According to (3) and (4), the received observations are represented by a $K \times N$ matrix $\mathbf{Y}$ as:

$$\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_1, \ldots, \mathbf{y}_N]. \tag{5}$$

In fact, each vector of $\mathbf{y}_N$ in (5) represents one sample of all detected primary signals received by all cognitive sensors (equivalently by multiple antennas installed in one cognitive sensor) as follows:

$$\begin{aligned}\mathbf{y}_{K,1} = &\begin{bmatrix} h_{1,1} & h_{1,2}, \cdots & h_{1,P} \\ \vdots & \ddots & \vdots \\ h_{K,1} & h_{K,2}, \cdots & h_{K,P} \end{bmatrix} \\ &\times [s_1, s_2, \ldots, s_P]^T + [w_1, w_2, \ldots, w_K]^T.\end{aligned} \tag{6}$$

For instance: if we have $K-$sensors, then the first received sample $(N = 1)$ of the observations vector equals the sum of all primary users signals filtered by the channel plus the complex Gaussian additive noise as:

$$\boldsymbol{y}_{K,1} = \begin{bmatrix} h_{1,1}s_1 + h_{1,2}s_2 + \; .... + h_{1,P}s_P + w_1 \\ \vdots \\ h_{K,1}s_1 + h_{K,2}s_2 + \; .... + h_{K,P}s_P + w_1 \end{bmatrix}. \tag{7}$$

Hence, the received observations data by K multiple antennas (installed in a single sensor or K sensors working cooperatively or any such combination forms) is a $\mathbf{Y}$ matrix with $K \times N$ dimension, where $K \leq N$:

$$\mathbf{Y} = \begin{bmatrix} \boldsymbol{y}_{1,1}, & \boldsymbol{y}_{1,2}, \; .... & \boldsymbol{y}_{1,N} \\ \vdots & \ddots & \vdots \\ \boldsymbol{y}_{K,1}, & \boldsymbol{y}_{K,2}, \; .... & \boldsymbol{y}_{K,N} \end{bmatrix}. \tag{8}$$

Consequently, $\mathbf{Z}$ denotes the $K \times K$ test statistics of the observations data covariance matrix:

$$\mathbf{Z} = \mathbf{Y}\,\mathbf{Y}^H. \tag{9}$$

Here, $(.)^{\,H}$ denotes the Hermitian complex conjugate. In general, the covariance matrix $\mathbf{Z}$ follows the Wishart distribution $\mathbf{Z} \sim W_K\,(N, \Sigma)$ with K dimension and N degrees of freedom. Σ is a population covariance matrix that is the expectation of the observations covariance matrix averaged by the number of the received samples as follows:

$$\Sigma = E[\mathbf{Y}\,\mathbf{Y}^H]/N. \tag{10}$$

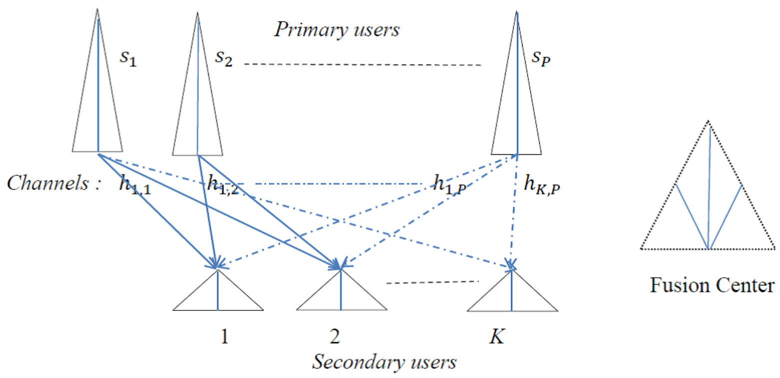
Next, the spherical test statistics $T_{ST} = \frac{|\mathbf{Z}|}{\left(\frac{1}{K}\,tr(\mathbf{Z})\right)^K}$ is compared against pre-calculated threshold λ for a specified false alarm probability in order to decide the presence or absence of the primary users as follows:

$$Z = \begin{cases} \Sigma = \sigma_w^2\,\mathbf{I}_K & \textit{if } H_0 \\ \Sigma = \sigma_w^2\,\mathbf{I}_K + \sum_{i=1}^P A_i\,h_i h_i^H & \textit{if } H_1. \end{cases} \tag{11}$$

Under the H_0 null hypothesis, where no primary user is detected, the covariance data matrix $\mathbf{Z}$ follows an uncorrelated complex central Wishart distribution [22] with i.i.d. distribution $N \sim (0, \sigma_w^2 \, \mathbf{I}_K)$ having zero mean and population covariance matrix $\Sigma = \sigma_w^2 \, \mathbf{I}_K$. While under the H_1 hypothesis, a primary user signal is detected and its presence is declared. Consequently, the covariance data matrix $\mathbf{Z}$ follows a correlated non-central complex Wishart distribution with the population covariance matrix $\Sigma = \sigma_w^2 \, \mathbf{I}_K + \sum_{i=1}^{P} A_i \, h_i h_i^H$. The i-th primary user transmission power A_i is $A_i = E[s_i s_i^H]$, and its corresponding signal-to-noise ratio (SNR) across K sensors is:

$$\gamma_i = A_i ||h_i||^2 / \sigma_w^2. \tag{12}$$

Figure 1. Cooperative multiple primary users spectrum sensing.



2.1. Detection Parameters

Typically, the spherical test statistics detector is derived under the generalized likelihood ratio test (GLRT) [23]. Detection parameters are obtained through the beta approximation function as follows [22].

False alarm probability:

$$P_F(\lambda) \cong \frac{B_\lambda(\alpha_0, \beta_0)}{B(\alpha_0, \beta_0)}. \tag{13}$$

Detection probability:

$$P_D(\lambda) \cong \frac{B_\lambda(\alpha_1, \beta_1)}{B(\alpha_1, \beta_1)}. \tag{14}$$

Here, α_0, β_0, α_1 and β_1 are calculated as in [22].

3. Spectrum Sensing in Rayleigh Multipath Fading Channel

In fact, the probability parameters introduced in the previous section are derived for the AWGN channel, since constant channel gain coefficients $h_{K,P}$ are assumed. Therefore, the channel model for $\mathbf{H}$ may not need to be specified [22]. Consequently, when channel characteristics are not specified, it is considered an ideal channel [24], which means no multipath fading was considered. It is worthwhile to mention that assuming constant channel coefficients would be realistic if:

(1) the symbol time duration of the detected signal were smaller than the channel coherence time;
(2) the sensing time were short enough, *i.e.*, shorter than the coherence time of the channel; and
(3) there were no relative motion between the primary and secondary users; otherwise, it would be unrealistic.

Actually, this assumption does not reflect the real situation of the spectrum sensing process; hence fading is unavoidable, and it has a great impact on the spectrum sensing process, as we will show in the next section. Consequently the fading effect has to be included in evaluating the detection probability.

A few important issues are considered in our study: maximizing the dynamic spectrum sensing accuracy, maximizing secondary user throughput and reducing interference to the primary user. Typically, the throughput can be maximized by minimizing the false alarm probability P_F, since this indicates falsely that a vacant band (hole) of the spectrum is occupied. As a result, secondary user throughput is decreased, since not all of the vacant bands are exploited.

On the other hand, protecting the licensed primary user's transmission from any interference with the secondary users transmission is achieved by reducing the missed detection probability P_m; since the P_m mistakenly indicates that there is no active primary user, when actually there is. Consequently, a vacant band is mistakenly declared, and the cognitive user may start transmission. Minimizing this harmful interference is done by maximizing the detection probability, which will also reduce the missed detection probability, since $P_m = 1 - P_D$.

In this work, we consider Rayleigh multipath fading, which is more commonly experienced in realistic wireless networks. In Rayleigh fading channels, the instantaneous SNR has the following probability distribution function (pdf) [16,25]:

$$f_\gamma(\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}. \tag{15}$$

Here, γ and $\overline{\gamma}$ represent instantaneous and average SNR, respectively. We need to consider only P_D, the detection probability in averaging, since it is a function of SNR (see (14)). Hence, average P_D in the Rayleigh fading channel gives:

$$\overline{P}_{DRay} = \int_0^\infty P_D(\lambda, \gamma)\, f(\gamma)\, \mathrm{d}\gamma. \tag{16}$$

Substituting (14) and (15) in (16) results in,

$$\overline{P}_{DRay} = \int_0^\infty \frac{B_\lambda(\alpha_1, \beta_1)}{B(\alpha_1, \beta_1)} \left(\frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}\right) \mathrm{d}\gamma. \tag{17}$$

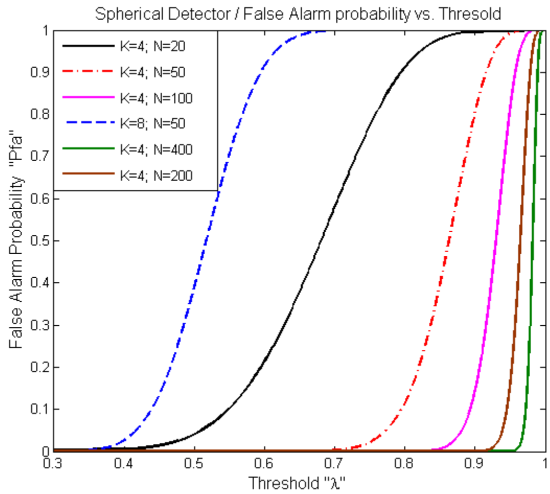
Here, $B(.)$ and $B_\lambda(.)$ denote beta and incomplete beta functions, respectively.

4. Simulation Results

In this section, we will compare the receiver operating characteristic (ROC) graphs for both AWGN and Rayleigh fading channels using the detection probability closed-form [22] as an evaluation metric under different detection scenarios in order to show the impact of fading on detection probability. Without loss of generality, all tested noise and primary signals are assumed to be i.i.d. and have complex Gaussian distribution with zero means and unity variances. We always aim to maximize P_D

and minimize P_F with some trade-off between them (IEEE 802.22 regulations require $P_F \leq 0.1$). Figure 2 shows the threshold as a function of false alarms for $K \times N$ values: (4,20), (4,50), (4,100), (4,200), (4,400) and (8,50).

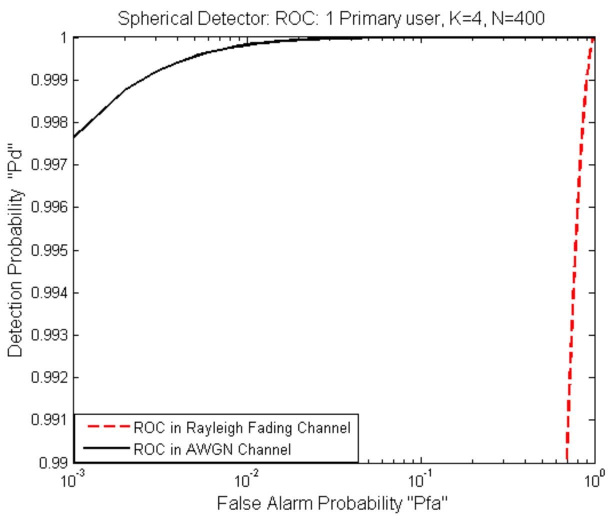
Figure 2. Specified false alarm as a function of the threshold for different K, N.



It is easy to show that for a constant number of secondary users and a specified false alarm value, as the number of samples increases, the corresponding threshold value increases, too. Meanwhile, for a constant number of samples, the corresponding threshold value decreases as the number of secondary users increases for the specified false alarm value.

The integral in (17) has no analytical solution, since the incomplete beta function (some references refer to it as a regularized beta function) appears in the integral to have no analytical solution. MATLAB R2014a (Version 8.3, Massachusetts, New York, USA, 6 Mar 2014) is employed for Rayleigh channel simulation in order to investigate the detection probability ($\overline{P}_{D\,Ray}$) behaviour under such a channel. We have used the same assumed values of the sensing parameters as in [22] for easy comparison with the effect of the multipath fading.

Figure 3. Single primary user with SNR = −3 dB, $K = 4$, $N = 400$, where there is clearly fading, degraded P_D.



Spherical detector is a multiple primary users spectrum sensing technique; however, it could be used to sense single primary users by setting $P = 1$ in (11) for H_1. Assuming a single primary user ($P = 1$) with SNR = -3 dB, $K = 4$ and $N = 400$, Figure 3 shows the fading impact on detection where P_D is reduced.

Similarly for multiple primary users, the detection probability decreases due to the multipath fading effect, as shown in Figure 4, where three primary users are considered ($P = 3$). Three signals with $SNR_1 = -1$, $SNR_2 = -3$, $SNR_3 = -10$, all with $K = 4$ and $N = 200$, are also considered.

Figure 4. Three primary users signals $SNR_1 = -1$, $SNR_2 = -3$, $SNR_3 = -10$, all with $K = 4, N = 200$.

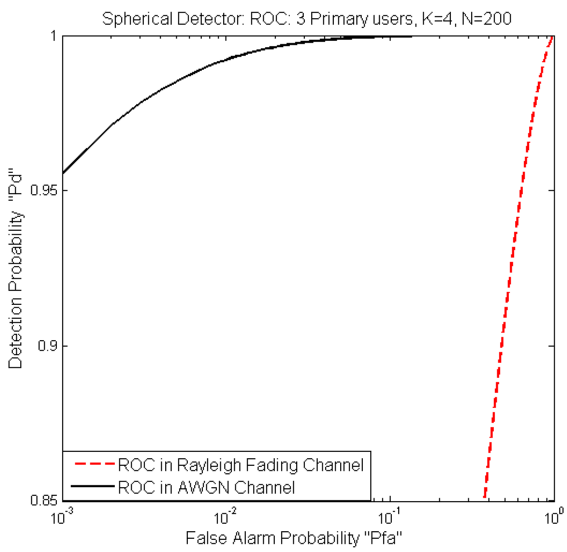
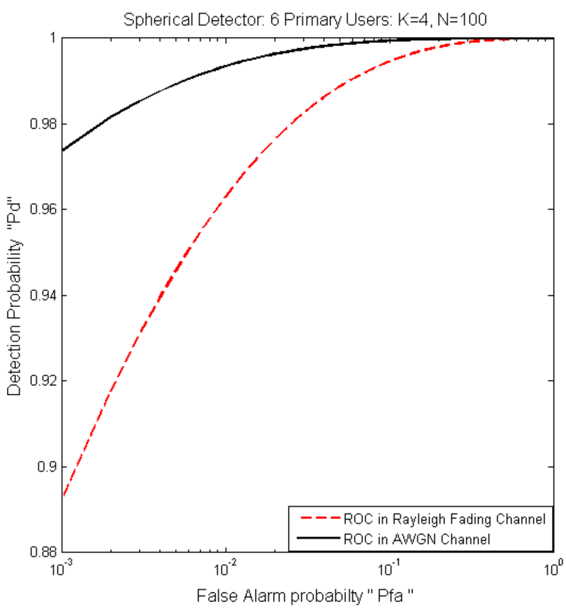


Figure 5 shows six primary users ($P = 6$) simultaneously transmitting six signals with $SNR_1 = 0$, $SNR_2 = -1$, $SNR_3 = -3$, $SNR_4 = -8$, $SNR_5 = -10$ and $SNR_6 = -22$, all with $K = 4$, $N = 100$.

Figure 5. Six primary users with $SNR_1 = 0$, $SNR_2 = -1$, $SNR_3 = -3$, $SNR_4 = -8$, $SNR_5 = -10$, $SNR_6 = -22$, all with $K = 4, N = 100$.



All previous figures clearly show the effect of multipath fading on spectrum sensing, which sharply lowers the detection probability. However, this situation is not necessarily always true. In fact, multipath fading may improve the detection accuracy depending on its type. For example, fading can help improve detection in a frequency selective fading channel when diversity combining schemes are considered. In addition, multipath signals could add constructively or destructively, depending on the phase of the reflected or scattered signal replicas, as in Nakagami-m multipath fading channels [18,26], which will result in further interesting sensing behaviours.

5. Conclusions

A spherical test detector in the Rayleigh multipath fading channel is considered in this paper to detect multiuser primary user signals. Simulation results show the impact of multipath fading on detection probability for both single and multiple primary user scenarios. Interestingly, the drop in the detection probability and corresponding false alarm probability are not the same for the three investigated scenarios. In fact, we observe that as the number of primary users increases, there is less impact due to multipath fading on the detection probability. This can be attributed to a stronger power received when the number of primary users is increased. However, we believe that this phenomenon still needs more investigation and analysis to confirm this behaviour, which will be a part of our future work. The distribution of the observations' covariance matrix in the Rayleigh fading channel needs to be investigated, too, in order to decide whether it is still Wishart or not. Furthermore, our future work will include solving the integral of the average detection probability numerically. Extending the investigation to different channel fading distribution, such as Nakagami-m, is also under consideration. Further, we plan to consider employing a few different types of diversity combining schemes, such as maximal ratio combining.

Acknowledgements

The authors would like to acknowledge NSERC for funding this project.

Author Contributions

Salam Al-Juboori introduced the problem formulation, system modeling, derived the analytical expressions of the system parameters, implemented simulation, produced corresponding figures, wrote the paper, analyzed the results and gave the conclusion. Sattar Hussain participated in introducing the problem format, system modelling, deriving the analytical expressions of the average detection probability and the average false alarm probability. Xavier Fernando is the supervisor of this work. He has been guiding the authors throughout the work.

Conflicts of Interest

The authors declare no conflict of interest.

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