

Article

Hybrid Backstepping-Super Twisting Algorithm for Robust Speed Control of a Three-Phase Induction Motor

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Abstract: This paper proposes a Hybrid Backstepping Super Twisting Algorithm for robust speed control of a three-phase Induction Motor in the presence of load torque uncertainties. First of all, a three-phase squirrel cage Induction Motor is modeled in MATLAB/Simulink. This is then followed by the design of different non-linear controllers, such as sliding mode control (SMC), super twisting SMC, and backstepping control. Furthermore, a novel controller is designed by the synergy of two methods, such as backstepping and super twisting SMC (Back-STC), to obtain the benefits of both techniques and, thereby, improve robustness. The sigmoid function is used with an exact differentiator to minimize the high-speed discontinuities present in the input channel. The efficacy of this novel design and its performance were evidenced in comparison with other methods, carried out by simulations in MATLAB/Simulink. Regression parameters, such as ISE (Integral Square error), IAE (Integral Absolute error) and ITAE (Integral Time Absolute error), were calculated in three different modes of operation: SSM (Start-Stop Mode), NOM (Normal Operation Mode) and DRM (Disturbance Rejection Mode). In the end, the numerical values of the regression parameters were quantitatively analyzed to draw conclusions regarding the tracking performance and robustness of the implemented non-linear control techniques.



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Keywords: backstepping control; super twisting control; sliding mode control; 3-phase induction motor; uncertainties

1. Introduction

Early electricity generation through a three-phase induction machine was deemed a revolutionary advance in the power industry. An induction motor is the most widely utilized electrical machine in the energy industry. Nearly 80% of the energy utilized in industries is produced by three-phase induction motors. A three-phase induction motor has a wide number of applications in areas such as Electric Vehicles, energy saving, and monitoring systems [1–3]. This article centered on the nonlinear speed control of a three-phase induction motor. Even though the linear control techniques are simpler and computationally inexpensive, these control techniques cannot handle disturbance rejection and model uncertainties [4]. Moreover, due to the discontinuous nature of the nonlinearities, the linear approximation becomes an issue. These “hard nonlinearities” comprise of saturation, Coulomb friction, dead-zones hysteresis, and backlash. They are frequently found in control system engineering and their properties cannot be derived from linear procedures. In [5], the performance of a surface mounted permanent magnet synchronous motor (SPMSM) was analyzed by comparing the results of a conventional Proportional Integral (PI) controller with the proposed tracking differentiator–proportional integral and derivative (TD–PID) controller. From the published results, it is evident that

the conventional linear PI controller falls short in terms of peak overshoot, chattering and settling time.

Many nonlinear control techniques have been utilized to achieve optimum speed control of this multivariable machinery. Researchers have introduced different nonlinear control techniques, such as sliding mode control [6], input-output linearization control [7], direct torque control [8], backstepping control [9] and so on to achieve high-performance control for induction motors [10]. The novel idea presented in this paper has significant weight as it combines two different non-linear controllers, namely, backstepping and super twisting algorithm. This combination results in a controller that has the advantages of both techniques for robust and efficient speed tracking.

The rotor's speed can be controlled by the variable supply provided to the stator. Nowadays, most electricity generation is done using a three-phase induction machine. Efficient speed control of such highly nonlinear dynamic machinery is a challenging task. Load uncertainties and additional nonlinear disturbances can further complicate the task of designing a controller [11].

Backstepping control calls for the division of entire systems into subsystems making it easier to derive and compute the desired control input. It is a recursive process extending outwards to consecutive subsystems until the final optimal control is reached. In [12], integral and classical backstepping approaches based on IFOC (Indirect Field Orientation Control) were applied for robust speed control of a squirrel cage three-phase induction motor. The integral approach provided global system stability and increased robustness in the presence of model uncertainties. However, classical Backstepping approaches result in fluctuations in the armature current and a slight steady state error between the actual and desired rotor speed. A simple Backstepping control might not be able to reject the disturbances effectively. Different robust backstepping techniques have been implemented in literature. In [13], a robust adaptive backstepping technique was applied to mobile robotic manipulators. The simulations demonstrated better tracking performance and robustness in comparison with a conventional PID controller. In [14], a sliding mode observer for estimation of flux components and actual speed was utilized in conjunction with a Backstepping controller to improve robustness. The motor was run in speed inversion mode as well. The results depicted no significant changes in the speed, currents, or voltages of the induction motor.

The sliding mode control (SMC) is a robust design technique that is useful for compensating model uncertainties. It provides very effective tracking control. An SMC combined with input-output feedback linearization for two quasi-induction motor drives was presented in [15]. The motors were connected in two configurations: series and parallel. The results in speed start-up and speed reversal modes depicted a small tracking error but the chattering effect was reduced by replacing the signum function with the saturation function. A robust variable step perturb-and-observe sliding mode controller was designed in [16] for a permanent magnet synchronous generator. The results demonstrated an increase in efficiency and enhanced settling time as opposed to a simple variable step perturb-and-observe controller. In [17], SMC in conjunction with a type-2 neuro-fuzzy controller was applied to an induction motor. The speed response depicted satisfactory behavior with small peaks occurring at fast transitions. The results were depicted in terms of the amount of overshoot and learning features. SMC is a very robust technique; however, due to the discontinuous nature of the control input, the system experiences the chattering effect. In [18], a classical SMC was compared to a fuzzy SMC approach for robust speed control of a doubly-fed induction motor. The speed tracking results showed a profound chattering in classical SMC in comparison with a fuzzy SMC.

The chattering effect can usually be avoided by using higher-order SMC approaches [19]. The super twisting algorithm is a sub-branch of the higher order sliding mode (HOSM) control. In [20], a linearized block control, in conjunction with a super twisting algorithm, was applied to a squirrel cage induction motor. This technique provided reduced chattering, along with disturbance rejection, in the presence of variable load torque. A computerized

tuning method for the parameters of the Super Twisting controller technique could further minimize chattering [21] and minimize core losses in a three-phase induction motor [22]. An adaptive Backstepping super twisting SMC was designed and compared with different techniques in [23]. The proposed design showed superior cyclic path tracking and disturbance rejection qualities.

The SMC techniques have been modified and extended, and their effects have been further enhanced over the years to improve the performance of many nonlinear systems. All non-linear controllers have their merits and demerits. For instance, SMC offers satisfactory tracking performance and disturbance rejection; however, it lacks optimum performance due to the chattering effect [24]. The Backstepping controller does not have to deal with the chattering effect; however, the disturbance rejection of load torque is not as effective as in the case of SMC [25]. The super twisting control reduces the chattering effect in the actual speed of the rotor which is dependent on the exact parameter tuning of the controller [21].

Even though multiple non-linear controllers have been designed for the speed control of a three-phase Induction motor, very few have addressed the high-speed discontinuities that emerge, due to the differentiation of approximated step changes in input. Moreover, an extensive performance comparison is due. Through an extensive performance comparison, different non-linear control techniques could be analyzed and the most appropriate technique could be selected for a particular application.

Owing to the above mentioned facts, this article has the following contributions:

- A Novel Backstepping super twisting SMC with exact differentiation and signum approximation (Back-STC-EA) was designed for the robust speed control of a three-phase Induction Motor.
- This controller not only reduces the chattering effect, as opposed to a basic SMC, but also improves the disturbance rejection capability, in comparison with the classical Backstepping controller.
- The exact differentiation and signum approximation reduces the overall effect of high-speed discontinuities present in the desired speed response.
- An extensive performance comparison was carried out between the conventional Backstepping controller, SMC, Back-SMC, Back-STC and the novel Back-STC-EA controller.
- A quantitative and graphical analysis was performed in terms of regression parameters (ISE, IAE, ITAE) and simulation results. This analysis is performed under three different modes of operation: SSM, NOM and DRM.

The rest of this article is organized as follows. The problem is formulated in Section 2. The mathematical model and the uncompensated simulation results for the squirrel cage three-phase induction motor are presented in Section 3. The design methodologies of the nonlinear control techniques, such as Backstepping control, SMC, Back-SMC, Back-STC and Back-STC-EA are proposed in Section 4. The simulation results, numerical comparison and evaluation are presented in Section 5. Section 6 presents conclusions and future work. Appendices are present at the end of the article.

2. Problem Formulation

The proposed problem is stated as follows:

2.1. System Description

The non-linear system considered in this paper can be represented by the following equations:

$$\dot{\varphi} = f(\varphi(t), \omega(t), c(t)) \quad (1)$$

$$\dot{\omega}(t) = g(\varphi(t), \eta(t)) \quad (2)$$

where $\varphi(t)$ are the state variables, $\omega(t)$ is the output, $c(t)$ is the control input and $\eta(t)$ is the disturbance in load. However, the following conditions apply:

- In Start Stop Mode (SSM) and Normal Operation Mode (NOM) disturbance in load $\eta(t)$ is not considered. (i.e., $\eta(t) = 0$).
- In Disturbance rejection mode (DRM), a disturbance in load ($\eta(t)$) is introduced.

The following assumption is made:

- The functions $f(\cdot)$ and $g(\cdot)$ are continuously differentiable, or are made continuously differentiable, by using exact differentiation and signum approximation.

The illustration diagram for the formulated problem is provided in Figure 1.

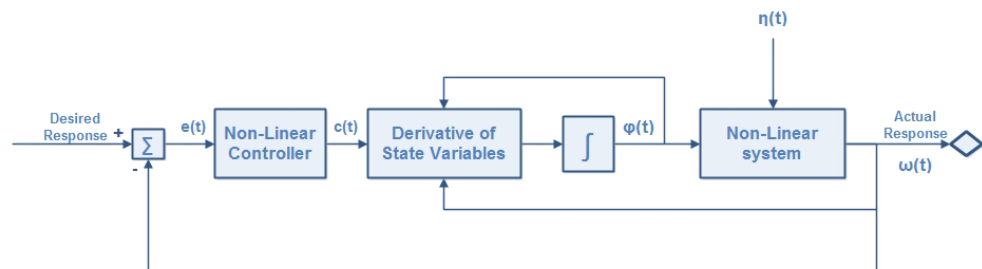


Figure 1. Illustration diagram for Problem Formulation.

2.2. Problem Statement

Design a control input $c(t)$ using different non-linear control techniques for the system (1) and (2), such that a robust and stable output is achieved in the presence of uncertainties. Afterwards, numerically compare and analyze the results of the non-linear control techniques under consideration.

3. Mathematical Model

A mathematical model, when consciously selected, can reduce the amount of work and produce more accurate results. A mathematical model of a three-phase nonlinear induction motor was selected and implemented in MATLAB/Simulink [26–28]. For simplification of the model, park transformation was used. The park transformation rotated the abc reference frame to a dq (direct-quadrature) reference frame. The park transformation is utilized very often in MATLAB/Simulink with three-phase induction motors, due to the perfect alignment of the rotor flux with the d-axis, which implies that the q-axis component of the rotor flux can be taken as zero. These reference voltages are further utilized to compute the flux linkages, which, in turn, compute the rotor and stator current. These currents are then used to derive the final equations for speed and electromagnetic torque [27]. The overall block diagram is presented in Figure 2. The final equations for the flux linkage variables are given as follows (Symbols are given in Abbreviations):

$$\frac{dF_{sq}}{dt} = \omega_b \left[V_{sq} - \frac{\omega_e F_{sd}}{\omega_b} + \frac{R_s}{X_{ls}} \left\{ \frac{F_{rq} X_m}{X_{lr}} + F_{sq} \left(\frac{X_m}{X_{ls}} - 1 \right) \right\} \right] \quad (3)$$

$$\frac{dF_{sd}}{dt} = \omega_b \left[V_{sq} + \frac{\omega_e F_{sq}}{\omega_b} + \frac{R_s}{X_{ls}} \left\{ \frac{F_{rd} X_m}{X_{lr}} + F_{sd} \left(\frac{X_m}{X_{ls}} - 1 \right) \right\} \right] \quad (4)$$

$$\frac{dF_{rq}}{dt} = \omega_b \left[\frac{\omega_e - \omega_r}{\omega_b} F_{rd} + \frac{R_r}{X_{lr}} \left\{ \frac{F_{sq} X_m}{X_{ls}} + F_{rq} \left(\frac{X_m}{X_{lr}} - 1 \right) \right\} \right] \quad (5)$$

$$\frac{dF_{rd}}{dt} = \omega_b \left[\frac{\omega_e - \omega_r}{\omega_b} F_{rq} + \frac{R_r}{X_{lr}} \left\{ \frac{F_{sd} X_m}{X_{ls}} + F_{rd} \left(\frac{X_m}{X_{lr}} - 1 \right) \right\} \right] \quad (6)$$

The final equations of the speed and electromagnetic torque are as follows:

$$\frac{d\omega_r}{dt} = k_1 [T_e - T_L] \quad (7)$$

$$T_e = k_2 [F_{sd} i_{sq} - F_{sq} i_{sd}] \quad (8)$$

where $k_1 = \frac{P}{2J}$, and $k_2 = \frac{3p}{4\omega_b}$. These equations may be further utilized to achieve the optimum control of the induction motor. The field distribution variables include the supply voltage variables, and the stator and rotor currents, which can be used as the control inputs in the design of nonlinear control systems. In (7), the actual speed of the rotor is dependent on the load torque. Hence, when the load varies, the rotor speed varies as well. The effect of uncertainties present in load torque is compensated by automatically adjusting the electromagnetic torque.

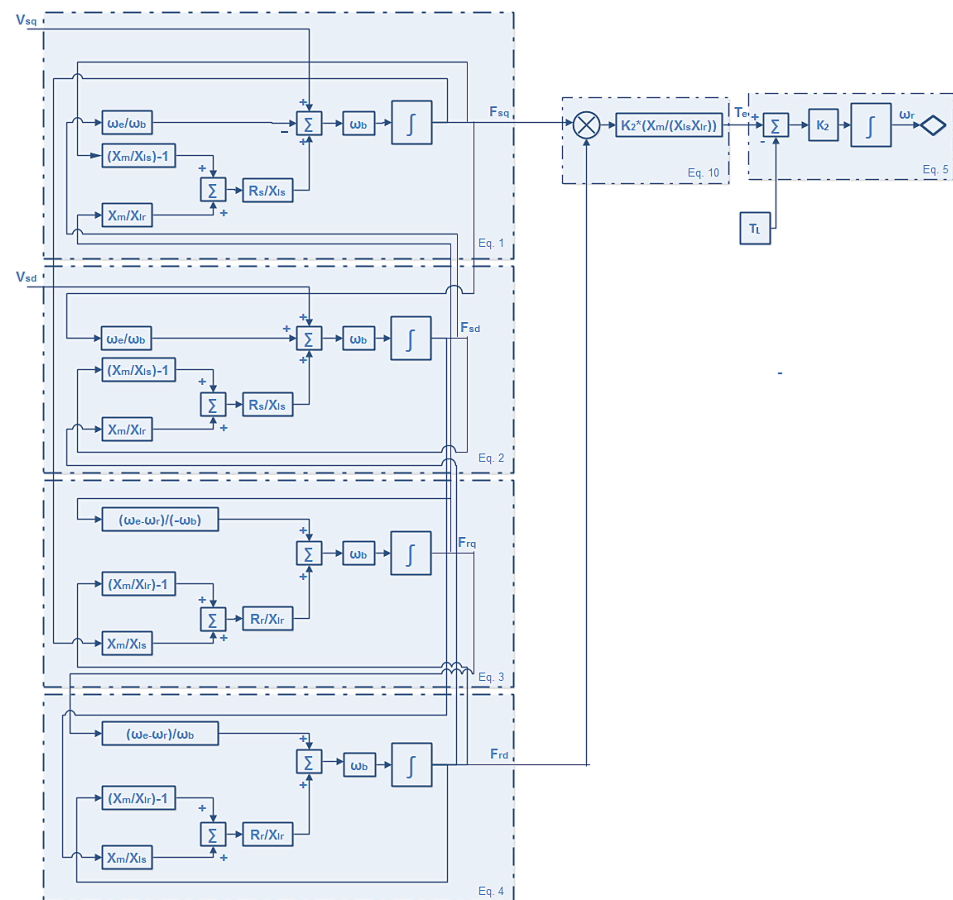


Figure 2. Internal dynamics of 3-phase induction motor model.

3.1. Simulation Results and Findings of Uncompensated 3-Phase Induction Motor

3.1.1. Motor under Test

A 1.1 KW, 220 V, 50 Hz, 4 poles Squirrel cage induction motor has been selected as a plant. The parameters are given in Table 1.

Table 1. Parameters of the induction motor under test.

Induction Motor Parameters	Symbol	Numerical Value	Unit
Stator Resistance	R_s	0.19	Ω
Rotor Resistance	R_r	0.39	Ω
Leakage Stator Inductance	L_{ls}	0.21×10^{-3}	H
Leakage Rotor Inductance	L_{lr}	0.60×10^{-3}	H
Magnetizing Inductance	L_m	4×10^{-3}	H
Nominal Current	I_m	10	Amps
Rotor's Inertia	J	0.0226	kg m ²
Base Speed	ω_e	314.159	rad/s

3.1.2. Simulation Study

The d–q model of the 3-phase induction motor was implemented using Simulink. The obtained actual rotor speed is illustrated in Figure 3. The frequency of supplied voltage to the stator was 50 Hz. The base speed of the motor was produced by the Rotational Magnetic Field. In Figure 3, the actual rotor-speed is plotted with the base-speed (reference speed) for the 3-phase induction motor. It can be seen that the actual speed tracked the base speed. However, as soon as the load torque of 10 Nm was applied at 0.875 s, the rotor's speed dropped, which showed that the machine was not invariant to load disturbance. There were undesired oscillations as well at the start. To solve these issues related to robustness and speed tracking, a number of different nonlinear controllers were designed, as described in the next Section.

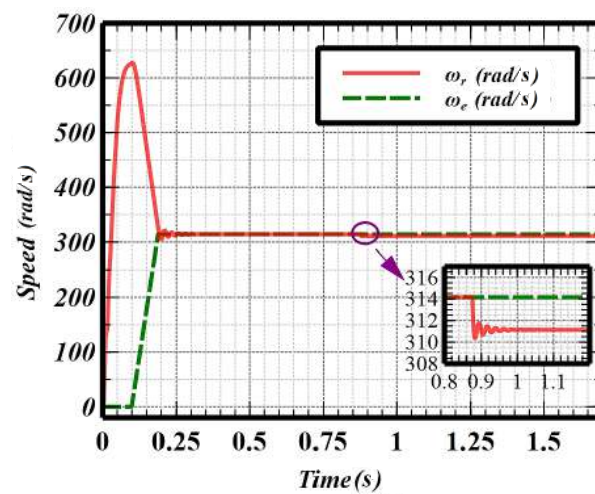


Figure 3. Base Speed (ω_e) and actual rotor speed (ω_r).

4. Design and Simulation of Nonlinear Controllers

4.1. Backstepping Controller

The backstepping control technique is widely utilized for achieving the control of numerous nonlinear systems [29]. It has applications in robotics, military and biomedical engineering services [30]. Referring to (3) to (6) it can be seen that the flux variables are taken as the state variables. The overall block diagram of a backstepping controller with a 3-phase induction motor is presented in Figure 4.

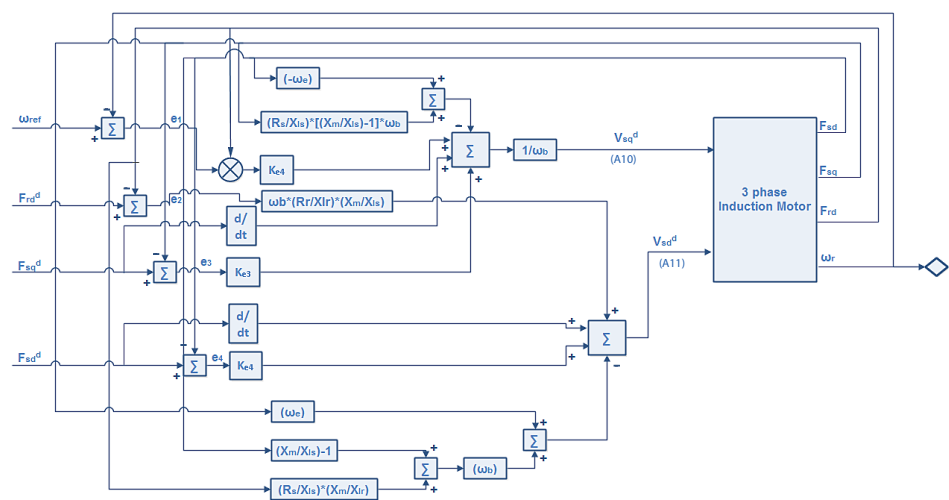


Figure 4. Internal Dynamics of backstepping control for 3-phase induction motor model.

For ease of computation, Equation (8) is rewritten in terms of only the flux variables. This is done by substituting stator current variables in the electromagnetic torque equation, resulting as follows [27]:

$$i_{sq} = F_{sq} \left[\frac{X_{ls} - X_m}{X_{ls}^2} \right] - \frac{X_m}{X_{ls}X_{lr}} F_{rq} \quad (9)$$

$$i_{sd} = F_{sd} \left[\frac{X_{ls} - X_m}{X_{ls}^2} \right] - \frac{X_m}{X_{ls}X_{lr}} F_{rd} \quad (10)$$

where X_{ls} and X_m are stator leakage and magnetizing reactance. The above equations ensure that current variables are now only dependent on the flux linkage variables. Substituting Equations (9) and (10) into (8) yields the following equation of the electromagnetic flux:

$$T_e = k_2 \left(\frac{X_m}{X_{ls}X_{lr}} \right) [F_{rd}F_{sq} - F_{sd}F_{rq}] \quad (11)$$

The park transformation ensures that the rotor flux is only directed along the d-axis, hence flux across the quadrature axis can be assumed to be zero ($F_{rq} = 0$). Substituting the value of F_{rq} into (11), the electric torque could be obtained according to the following simplified expression:

$$T_e \approx k_2 \left(\frac{X_m}{X_{ls}X_{lr}} \right) F_{rd}F_{sq} \quad (12)$$

The value of electromagnetic torque in (12) is substituted into the equation of the rotor's speed in (7), which yields the following:

$$\frac{d\omega_r}{dt} = k_4 F_{rd}F_{sq} - k_1 T_L \quad (13)$$

where $k_4 = \frac{p}{2j} \frac{3p}{4\omega_b} \left(\frac{X_m}{X_{ls}X_{lr}} \right)$.

For designing the backstepping control, the mathematical model of the induction motor is divided into two subsystems.

Subsystem 1: This subsystem consists of the state space equations of the rotor's speed and the rotor's flux in the d-axis direction. The model of the rotor's speed and the rotor's flux dynamics can be written as follows:

$$\frac{d\omega_r}{dt} = k_4 F_{rd}F_{sq} - k_1 T_L \quad (14)$$

$$\frac{dF_{rd}}{dt} = \omega_b \left[\frac{R_r}{X_{lr}} \left\{ \frac{F_{sd}X_m}{X_{lr}} + F_{rd} \left(\frac{X_m}{X_{lr}} - 1 \right) \right\} \right] \quad (15)$$

The stator's fluxes (F_{sq} , F_{sd}) are taken as the intermediate control inputs, which are designed using composite Lyapunov stability criteria. These control inputs are fed to subsystem 2.

Subsystem 2: This subsystem consists of the state space equation of the stator's fluxes in the direction of the dq-axis.

$$\frac{dF_{sq}}{dt} = \omega_b \left[V_{sq} - \frac{\omega_e F_{sd}}{\omega_b} + \frac{R_s}{X_{ls}} \left\{ F_{sq} \left(\frac{X_m}{X_{ls}} - 1 \right) \right\} \right] \quad (16)$$

$$\frac{dF_{sd}}{dt} = \omega_b \left[V_{sd} + \frac{\omega_e F_{sq}}{\omega_b} + \frac{R_s}{X_{ls}} \left\{ F_{sd} \left(\frac{X_m}{X_{ls}} - 1 \right) \right\} \right] \quad (17)$$

In this subsystem, the supply voltages (V_{sq} , V_{sd}) are taken as the final control inputs, which are designed using the combined Lyapunov functions for both subsystems.

Step 1: To control the speed of an induction motor, the speed tracking error should be zero which implies that the rotor's speed follows the reference speed exactly. The following error signals are generated for subsystem 1:

$$e_1(t) = \omega_{\text{ref}} - \omega_r \quad (18)$$

$$e_2(t) = F_{rd}^d - F_{rd} \quad (19)$$

where ω_{ref} and F_{rd}^d are the desired values of speed and flux linkage across the d-axis, respectively. The following Lyapunov stability function is defined to derive the expressions for the intermediate control inputs (F_{sq} and F_{sd}):

$$V_{12} = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 \quad (20)$$

The proof of Lyapunov stability criteria and the derivation of the intermediate control inputs are presented in Appendix A. The final intermediate control inputs are as follows:

$$F_{sq}^d = \frac{k_{e1}e_1 + \frac{d\omega_{\text{ref}}}{dt} + k_1 T_L}{k_4 F_{rd}} \quad (21)$$

$$F_{sd}^d = \frac{k_{e2}e_2 + \frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd}}{\omega_b \frac{X_m}{X_{ls}} \frac{R_r}{X_{lr}}} \quad (22)$$

Step 2: Subsystem 2 takes the control inputs designed by subsystem 1 and, then, using a combined Lyapunov function including all the errors, it designs the final control inputs (V_{sq} and V_{sd}). The following error signals are generated for subsystem 2:

$$e_3 = F_{sq}^d - F_{sq} \quad (23)$$

$$e_4 = F_{sd}^d - F_{sd} \quad (24)$$

To derive the final control inputs (V_{sq} and V_{sd}), we will substitute the intermediate control inputs F_{sq}^d and F_{sd}^d into (23) and (24), respectively:

$$e_3 = \frac{k_{e1}e_1 + \frac{d\omega_{\text{ref}}}{dt} + k_1 T_L}{k_4 F_{rd}} - F_{sq} \quad (25)$$

$$e_4 = \frac{k_{e2}e_2 + \frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd}}{\omega_b e_f} - F_{sd} \quad (26)$$

The Lyapunov stability function for the entire system is as follows:

$$V = \frac{1}{2} [e_1^2 + e_2^2 + e_3^2 + e_4^2] \quad (27)$$

The proof of Lyapunov stability criteria for the entire system and the derivation of the final control inputs are presented in Appendix B. The final control inputs that satisfy the Lyapunov stability criteria are as follows:

$$V_{sq}^d = \frac{k_{e3}e_3 + \frac{dF_{sq}^d}{dt} - \left(-\omega_e F_{sd} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sq} \right] \right) + k_4 F_{rd} e_1}{\omega_b} \quad (28)$$

$$V_{sd}^d = \frac{k_{e4}e_4 + \frac{dF_{sd}^d}{dt} - \left(\omega_e F_{sq} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sd} \right] \right) + \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} e_2}{\omega_b} \quad (29)$$

4.2. Design of Sliding Mode Controller for an Induction Motor

The sliding mode control is a robust design technique that is useful in compensating for model uncertainties and provides very effective tracking control. The sliding mode control starts with the design of a sliding surface [31]. The sliding surface is designed in a way that depicts that the actual parameter tracks the reference value and the system is stable. The basic principle behind sliding mode theory is to design a control algorithm that forces the system to stay on a sliding surface [32]. Sliding mode control has two phases: (i) reaching phase (ii) sliding phase. The model considered in this design has the viscous co-efficient of friction denoted by F_c . The overall block diagram of a sliding mode controller with a 3-phase induction motor is presented in Figure 5.

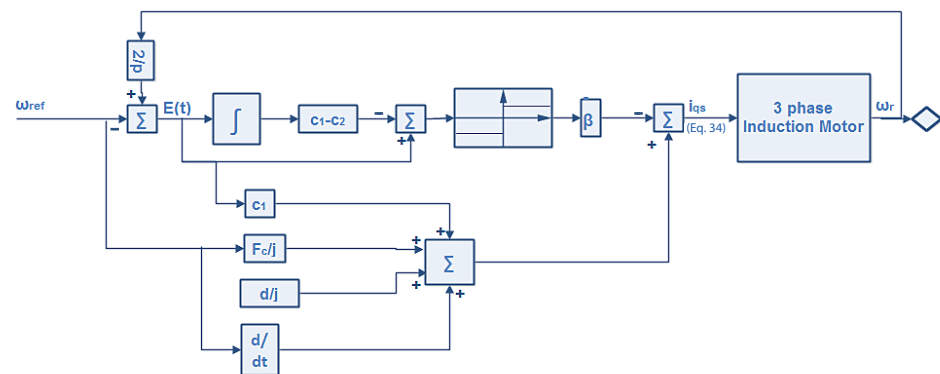


Figure 5. Internal Dynamics of SMC for 3-phase induction motor.

For the design of SMC for an induction motor, the electrical speed is first converted to mechanical speed as follows:

$$\omega_m = 2 \frac{\omega_r}{p} \quad (30)$$

The mechanical equation of the induction motor then becomes:

$$\frac{d\omega_m}{dt} = -\frac{F_c}{j} \omega_m - \frac{T_L}{j} + \frac{T_e}{j} \quad (31)$$

Note that (31) is very similar to (7), except for the additional nonlinearity introduced through the viscous coefficient of friction. For field-oriented control, (31) becomes:

$$\frac{d\omega_m}{dt} = -\frac{F_c}{j} \omega_m - \frac{T_L}{j} + C i_{qs}, \text{ with } C = \frac{3pF_{sd}}{4\omega_b j} \quad (32)$$

The T_L (i.e., Load torque) has an uncertain behavior which is compensated by the sliding mode control (SMC) law. The tracking error for speed is given as:

$$E(t) = \omega_m - \omega_{ref} \quad (33)$$

The derivative of the error signal: (33) is:

$$\frac{dE(t)}{dt} = -\frac{F_c}{j} E(t) + u(t) + d(t) \quad (34)$$

where $u(t)$ is the control law and $d(t)$ denotes the uncertainty due to the load torque. The sliding surface is defined as:

$$S(t) = E(t) - (c_1 - c_2) \int E(\tau) d\tau \quad (35)$$

The two subsystems mentioned in (14)–(17) are considered here. The proof of Lyapunov stability criteria and the derivation of the final control inputs are presented in Appendix C. The Equations (41) and (42) are the final control inputs, which are supplied to the induction motor model to achieve optimum control.

$$V_{sq}^d = \frac{-\varphi_1 - q_1 \text{sign}(z_2) - q_2 z_2}{\varphi_2} \quad (41)$$

$$V_{sd}^d = \frac{-\varphi_3 - q_2 \text{sign}(z_4) - q_4 z_4}{\varphi_4} \quad (42)$$

4.4. Design of the Backstepping Super Twisting Sliding Mode Control (STSMC) Law for an Induction Motor

A traditional SMC has many distinct features but it also has a limitation in terms of the chattering effect. Chattering refers to oscillations with finite amplitude and frequency. Higher order sliding mode provides an additional advantage in terms of eliminating or reducing the chattering phenomena. It also has all the characteristics of a traditional SMC. Hence, in order to eliminate the effect of chattering, a super twisting algorithm was cascaded with a backstepping controller [33]. A super twisting algorithm consists of two parts: (i) a discontinuous function of sliding surface variable (ii) a continuous function of sliding surface variable. The overall block diagram of a Backstepping Super Twisting controller with a 3-phase induction motor is presented in Figure 7.

The control law was designed by adding up the effects of a switching control and the equivalent control of the system. The switching control was implemented by (43) and (44):

$$U_{swc} = -r_1 \sqrt{|S|} \text{sign}(S) + v \quad (43)$$

$$\frac{dv}{dt} = -k_2 \text{sign}(S) \quad (44)$$

The final control law was as follows:

$$U_{control} = U_{swc} + U_{eq} \quad (45)$$

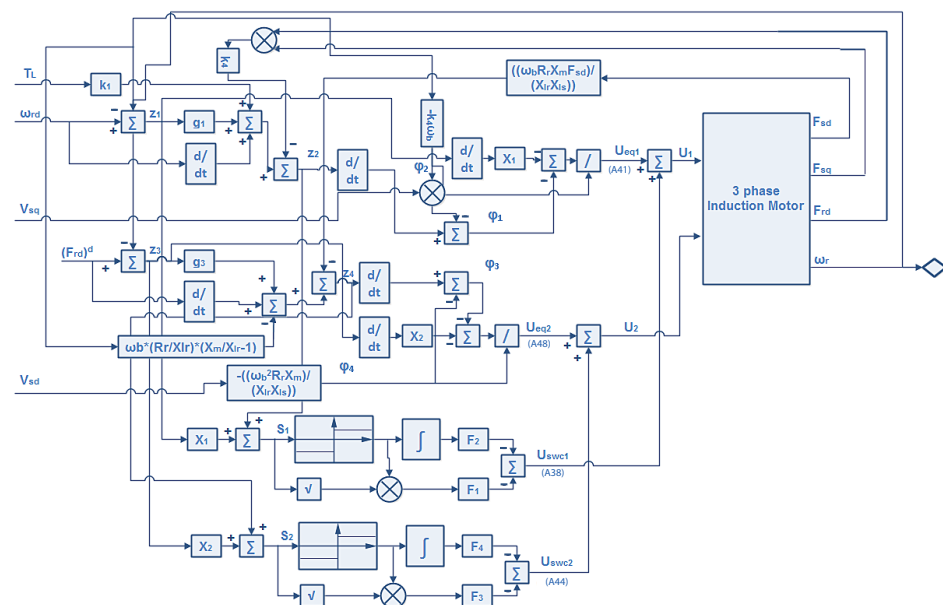


Figure 7. Block diagram of the proposed Backstepping Super Twisting algorithm for an induction motor.

For induction motors, the backstepping controller has already been defined in (A53) and (A55). The control law of HOSM (Super twisting control) for the first subsystem is as follows:

$$U_1 = U_{swc1} + U_{eq1} \quad (46)$$

To derive the switching control law, a sliding surface is defined as:

$$S_1 = z_2 + X_1 z_1 \quad (47)$$

where z_1 and z_2 are given in (A17) and (A24).

Using the super twisting algorithm, the switching control is based on (43) and (44)

$$U_{swc1} = -F_1 \sqrt{|S_1|} \text{sign}(S_1) - F_2 \int \text{sign}(S_1) ds_1 \quad (48)$$

where F_1 and F_2 are positive constants.

To derive the equivalent control input for subsystem 1, the derivative of the sliding surface was computed first:

$$\frac{dS_1}{dt} = \frac{dz_2}{dt} + X_1 \frac{dz_1}{dt} \quad (49)$$

In super twisting algorithm, the derivative of the sliding mode controller is equal to zero ($\dot{s} = 0$). Hence, the equivalent control was designed as follows:

$$\varphi_1 + \varphi_2 V_{sq} + X_1 \frac{dz_1}{dt} = 0 \quad (50)$$

$$U_{eq1} = V_{sq}^d = \frac{-\varphi_1 - \frac{X_1 dz_1}{dt}}{\varphi_2} \quad (51)$$

The control law of HOSM for the second subsystem is as follows:

$$U_2 = U_{swc2} + U_{eq2} \quad (52)$$

To derive the switching control law for this subsystem, we define a sliding surface

$$S_2 = z_4 + X_2 z_3 \quad (53)$$

where z_3 and z_4 have already been defined in (A36) and (A43). Using the super twisting algorithm, the switching control is defined using (43) and (44):

$$U_{swc2} = -F_3 \sqrt{|S_2|} \text{sign}(S_2) - F_4 \int \text{sign}(S_2) ds_2 \quad (54)$$

To derive the equivalent control for the subsystem 2, the derivative of the sliding surface is derived first:

$$\frac{dS_2}{dt} = \frac{dz_4}{dt} + X_2 \frac{dz_3}{dt} \quad (55)$$

The equivalent control was designed as follows:

$$\varphi_3 + \varphi_4 V_{sd} + X_2 \frac{dz_3}{dt} = 0 \quad (56)$$

$$U_{eq2} = V_{sd}^d = \frac{-\varphi_3 - \frac{X_2 dz_3}{dt}}{\varphi_4} \quad (57)$$

The Equations (48), (50), (54) and (57) represent the final control inputs of the Backstepping Super twisting control law.

Back-STC with Exact Differentiation and Signum Approximation (Back-STC + ea)

The control law defined in (48), (50), (54) and (57) contain derivatives of the reference speed and flux values. Differentiating a step change in an input results in a discontinuity, which leads to an error in the final output. To overcome this problem, an exact differentiator [34] and an approximation of sigmoid function [35] were designed to minimize the effect of discontinuity and to obtain better results. The exact differentiator that was used here was as follows:

$$z_0 = v_0 \quad (58)$$

$$v_0 = -\sigma_0 |z_0 - f(t)|^{\left(\frac{n}{n+1}\right)} \text{sign}(z_0 - f(t)) + z_1, \text{ and } \dot{z}_1 = v_1 \quad (59)$$

$$v_1 = -\sigma_1 |z_1 - v_0|^{\left(\frac{n-1}{n}\right)} \text{sign}(z_1 - v_0) + z_2, \text{ and } \dot{z}_2 = v_2, \dots, \quad (60)$$

$$v_{(n-1)} = -\sigma_{n-1} |z_{n-1} - v_{n-2}|^{\frac{1}{2}} \text{sign}(z_{n-1} - v_{n-2}) + z_n \quad (61)$$

$$\dot{z}_n = -\sigma_n \text{sign}(z_n - v_{n-1}) \quad (62)$$

The sigmoid function is approximated as follows:

$$S_{app} = -K_{app} \text{sat}\left(\frac{s}{\varnothing}\right) \quad (63)$$

where s is the sliding surface for the particular system; $\varnothing$ is the scaling factor for the approximation and K_{app} is a positive constant.

5. Numerical Evaluation & Comparison

A comparison study was conducted in terms of Minimization Criteria: Integral Square Error (ISE), Integral Absolute Error (IAE) and Integral Time Absolute Error (ITAE). The induction motor operated under three different modes: Start & stop mode (SSM), Normal Operation Mode (NOM) and Disturbance Rejection Mode (DRM). ISE, IAE and ITAE were statistical parameters used to evaluate the performance of the design system. Furthermore, a graphical analysis was performed to analyze the results more critically.

5.1. Start–Stop Mode (SSM)

In this mode, the speed tracking efficiency of the motor under an approximated step reference signal was examined. An approximated step signal was utilized to avoid very large discontinuities. The reference signal ran the motor between 0.1–2 s. This SSM reference speed signal was applied with all the control techniques implemented in Section 4. The results are plotted in Figure 8. Moreover, the statistical parameters were computed for each nonlinear controller as shown in Table 2.

According to the values of evaluation parameters (ITAE, ISE & IAE) stated in Table 2, SMC could be considered the most promising design in comparison with the rest of the controllers, owing to the fact that it had the least values for the statistical parameters (ITAE: 0.0019 rad/s, IAE: 0.0018 rad/s and ISE: 0.000001 rad/s). However, SMC experienced chattering effects, as depicted in Figure 8b, which made it unreliable from a hardware perspective. On the contrary, Back–SMC had the highest values for all the statistical parameters (ITAE: 0.5893 rad/s, IAE: 0.2987 rad/s and ISE: 2.3060) and could be considered to have the most unsatisfactory behavior in comparison with the other designs. Backstepping cont. (ITAE: 0.0061 rad/s, IAE: 0.0071 rad/s and ISE: 0.0004 rad/s) and Back–STC (ITAE: 0.0018 rad/s, IAE: 0.0020 rad/s, and ISE: 0.00009) also provided satisfactory speed tracking capabilities, considering the small values of the regression parameters. Back–STC–EA also had small values for all three parameters (ITAE: 0.0476 rad/s, IAE: 0.0411 rad/s and ISE: 0.0008 rad/s). However, it exhibits a minimal chattering effect at around 0.1 s; although later it smoothed out as depicted in Figure 8e.

It is worth noting that in Figure 8c, Back–SMC had abrupt transitions at $t = 1.007$ and 2.025 s. These anomalies were the primary cause of the increased value for the ISE parameter.

The magnitudes of these peaks were 0.9544 rad/s and 12.3 rad/s, respectively. This further confirmed the unsatisfactory speed tracking behavior of the designed Back-SMC.

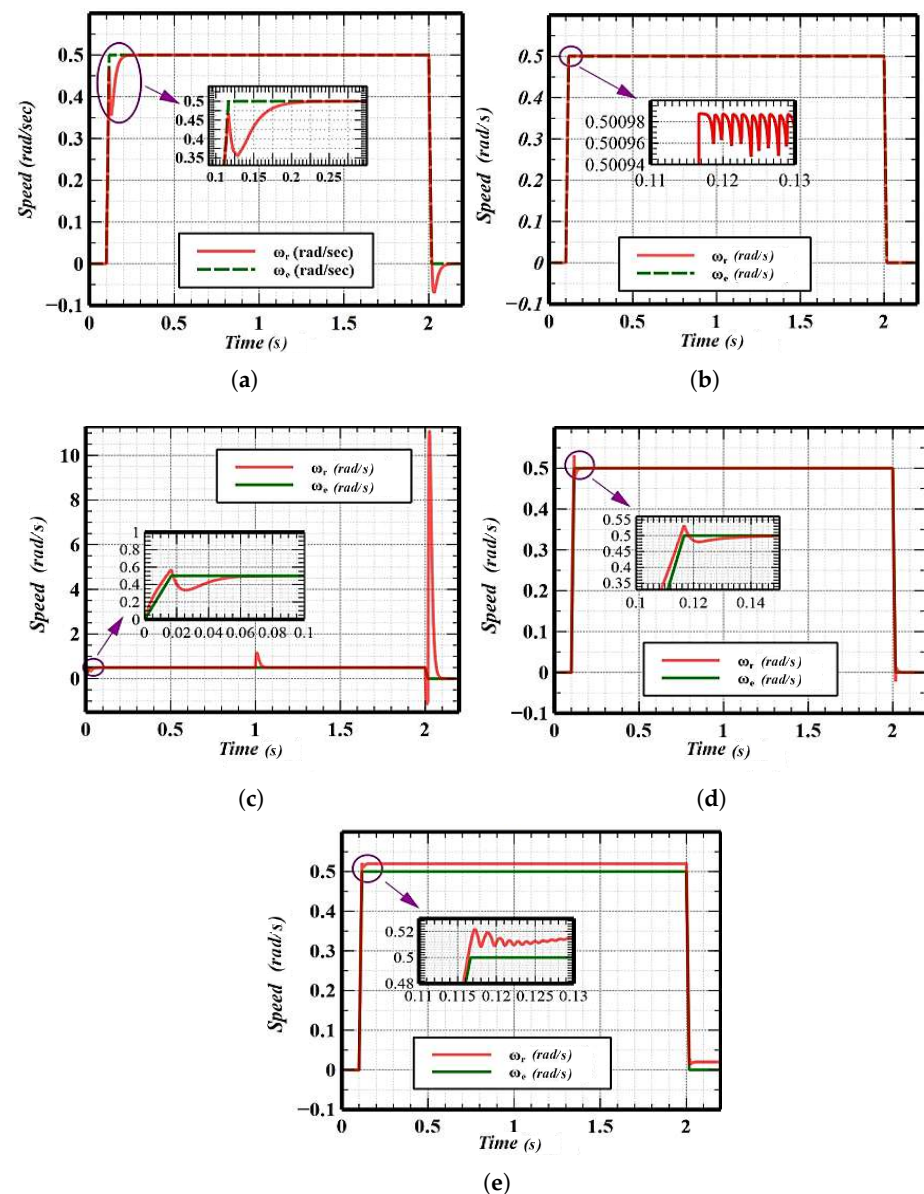


Figure 8. Output speed signals for SSM. (a) Backstepping control. (b) SMC, (c) Back-SMC, (d) Back-STC, (e) Back-STC-EA.

Table 2. Numerical evaluation-start-stop mode.

Control Technique	ITAE (rad/s)	IAE (rad/s)	ISE (rad/s)
Backstepping cont.	0.0061	0.0071	0.0004
SMC	0.0019	0.0018	0.000001
Back-SMC	0.5893	0.2987	2.3060
Back-STC	0.0018	0.0020	0.00009
Back-STC-EA	0.0476	0.0411	0.0008

5.2. Normal Operation Mode (NOM)

The second mode that was used to evaluate the performance of the controllers applied to the nonlinear induction motor model was the NOM. In this mode, a regulated reference speed signal with a maximum tolerance of 1.5% was applied to the respective controllers in order to examine their behavior in response to a varying speed signal. In normal mode, the transition period from one speed to another was longer. Hence, the chances of

discontinuous behavior were smaller than those from the SSM. For a clearer representation of results from NOM, the actual and desired rotor speeds for each control technique were separately plotted in Figure 9. The results of evaluation parameters for NOM are presented in Table 3.

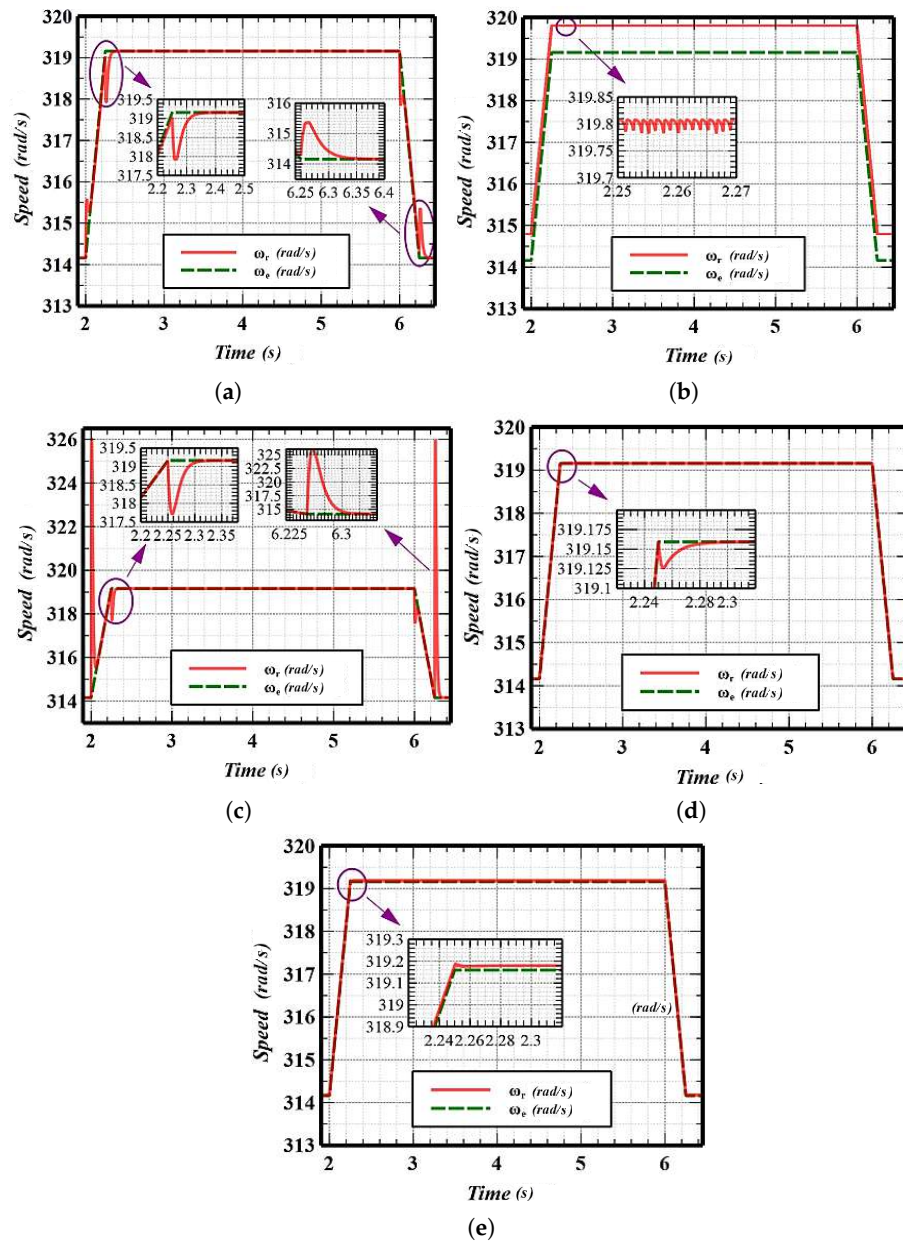


Figure 9. Output speed signals for NOM: (a) Backstepping control, (b) SMC, (c) Back-SMC, (d) Back-STC, (e) Back-STC-EA.

Table 3. Numerical Evaluation—NOM.

Control Technique	ITAE (rad/s)	IAE (rad/s)	ISE (rad/s)
Backstepping cont.	2.315	1.751	24.93
SMC	13.66	4.171	2.682
Back-SMC	2.552	2.848	147.1
Back-STC	0.0073	0.1011	0.2468
Back-STC-EA	0.4187	0.1484	0.152

The values of ITAE, IAE were the highest for SMC, when compared to the other designs (i.e., 13.66 rad/s and 4.171 rad/s, respectively). Moreover, in Figure 9b, it can be seen that the SMC experienced chattering effects and a steady state error of 0.64 rad/s.

Hence, SMC could be regarded as the most unsatisfactory design for the speed control of a 3-phase induction motor, when compared to the other controllers in NOM. Back-SMC had the highest value of ISE (i.e., 147.1) and it can be seen from Figure 9c that abrupt peaks were present at $t = 2$ and 6.25 s. The magnitudes of these peaks were 325.8 rad/s and 325.7 rad/s, respectively. These abrupt transitions were undesirable from a hardware perspective as they could cause damage to the motor under test. Backstepping cont. also had a considerably high ISE (24.93 rad/s). Figure 9a shows that Backstepping cont. also had abrupt transitions at $t = 2, 2.225, 6.02, 6.27$ s. The magnitudes of these abrupt transitions were 319.45, 315.5, 315.7 and 318.2 rad/s respectively. However, the magnitudes of these transitions were lower than for Back-SMC. Both Back-STC and Back-STC-EA had minimum values for ITAE, ISE, and IAE, as compared to the other three controllers, and depicted good speed tracking characteristics as shown in the Figure 9d,e. From Figure 9d, it can be seen that a slight dip was evident at 2.24 s. While in Figure 9e, this dip no longer existed. Hence, it could be concluded that the use of exact differentiation and signum approximation improved the Back-STC algorithm to some extent.

5.3. Normal Operation Mode (NOM) with Bounded Matched Disturbance

To further assess the robustness and efficiency of the non-linear controllers discussed above, a bounded matched disturbance [36] was introduced in the closed loop system for NOM. The bounded disturbance is shown in Figure 10.

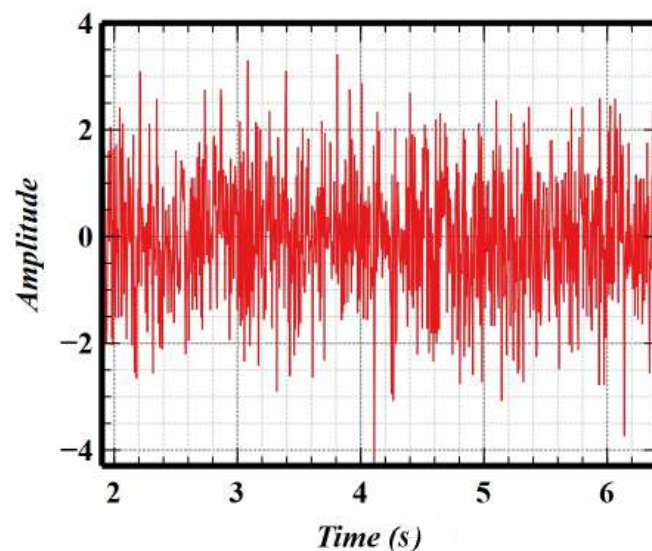


Figure 10. Bounded matched disturbance in NOM.

The responses of all the control techniques in the presence of bounded matched disturbance are presented in Figure 11. From Figure 11a, it is evident that the Backstepping controller was not able to fully reject the disturbance, owing to the variations in the actual rotors' speed curve. However, the actual speed of the rest of the controllers, namely SMC (Figure 11b), Back-SMC (Figure 11c), Back-STC (Figure 11d) and Back-STC-EA (Figure 11e) seemed to have rejected the disturbance and were presenting robust results. This could also be seen by comparing the results in Figure 11 with the results given in Figure 9. Comparing Figures 9b and 11b, it is evident that the SMC behaved the same and rejected the bounded matched disturbance; however, the chattering persisted. The speed tracking behavior of the rest of the 3 controllers, namely Back-SMC, Back-STC and Back-STC-EA remained the same, both in disturbance and without disturbance modes. Hence, it could be concluded that these controllers rejected the bounded matched disturbance.

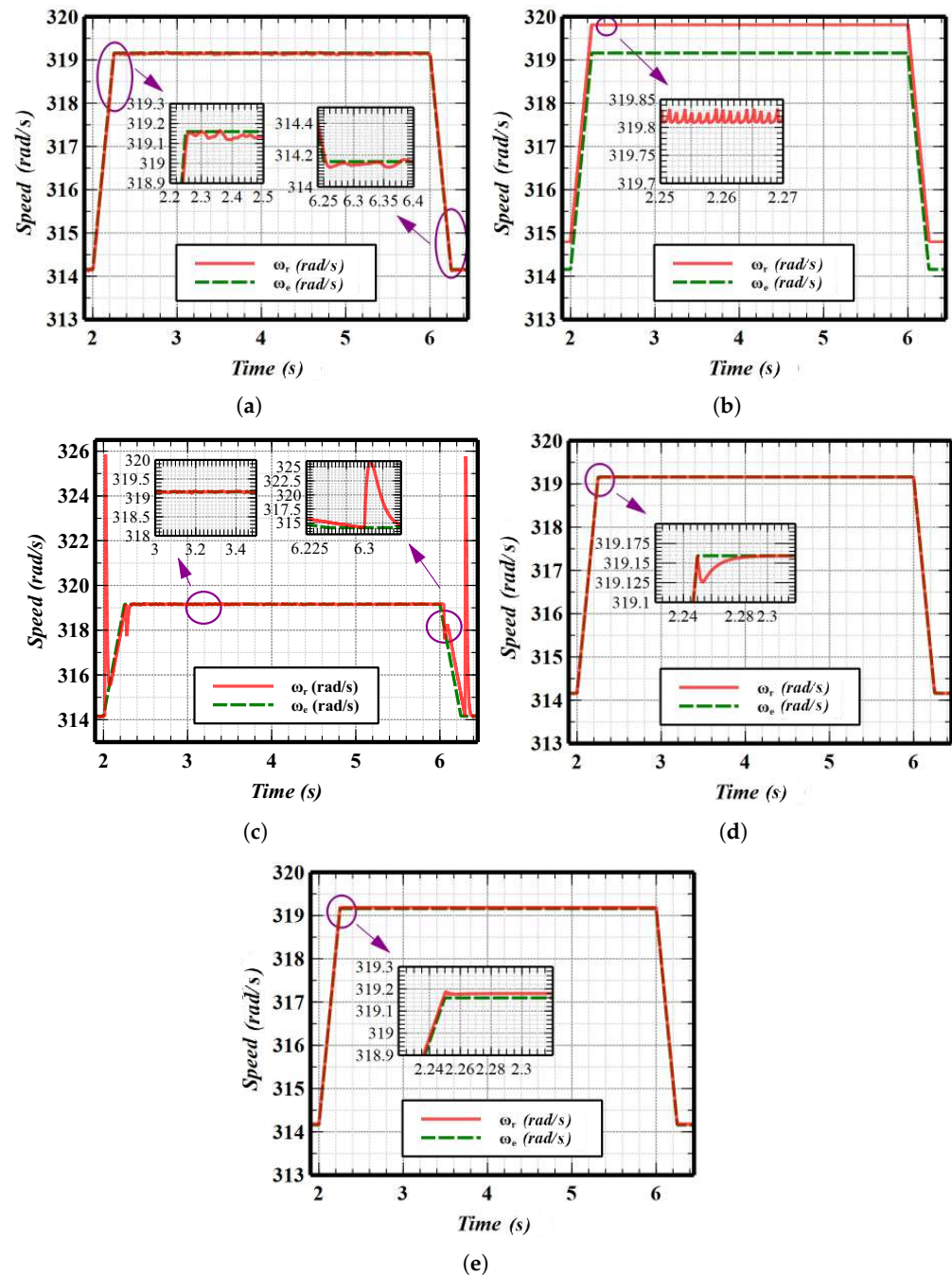


Figure 11. Output speed signals for NOM with bounded matched disturbance. (a) Backstepping control. (b) SMC (c) Back-SMC (d) Back-STC (e) Back-STC-EA.

5.4. Disturbance Rejection Mode (DRM)

The third mode is the most important as it concerns itself with the disturbance rejection in load torque of the 3-phase Induction motor. The parametric uncertainty introduced in the load torque should be compensated by a robust controller. In this mode, uncertainty was introduced in the load torque while the motor was running, as shown in Figure 12. The load torque had a sudden abrupt change of 10 Nm at 2 s. The values of the minimizing regression criteria (ITAE, IAE and ISE) are evaluated and presented in Table 4. The actual speeds of all the implemented nonlinear control techniques are plotted in comparison with the desired speed in Figure 13.

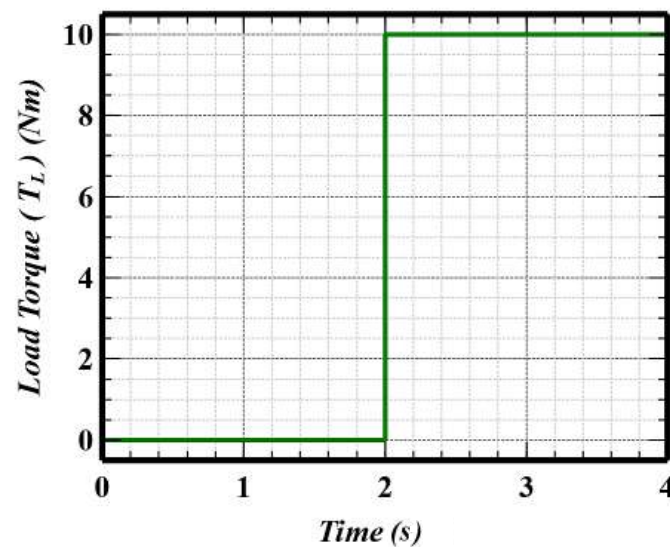


Figure 12. Uncertainty in Load Torque (Nm).

The values of ITAE, IAE and ISE were the highest for Backstepping cont. (i.e., 7.217, 4.156 and 8.417 rad/s, respectively). Moreover, it can be seen in Figure 13a, the backstepping cont. did not reject the disturbance as the rotor speed had a steady state error of 1 rad/s after the disturbance in load torque was introduced at $t = 2$ s. Hence, the designed Backstepping cont. did not achieve robust conditions in its performance.

The SMC also had high values for the three regression parameters (i.e., ITAE: 5.065 rad/s, IAE: 2.353 rad/s and ISE: 1.457 rad/s). This could be attributed to the steady state error of 0.64 rad/s between the rotor's desired and actual speeds. However, it can be seen in Figure 13b that the SMC rejected the disturbance introduced in load torque at $t = 2$ s. Hence, by inspection, the designed SMC approached robust characteristics.

Back-SMC had the lowest values for the evaluating parameters (i.e., ITAE: 0.0424 rad/s, IAE: 0.0514 rad/s and ISE: 0.0437 rad/s). It can be seen in Figure 13c that the Back-SMC rejected the disturbance introduced due to the load torque and came back to the reference speed after a slight peak with an % overshoot of 0.42. Hence, the designed Back-SMC was considered robust.

The values of evaluating parameters for both Back-STC and Back-STC-EA were considerably lower, as shown in Table 4. It can be seen in Figure 13d,e that both these controllers rejected the disturbance in load torque and returned to their pre-disturbed rotor speeds. Hence, both of these designed controllers showed robustness in the presence of load torque uncertainties introduced in the 3-phase Induction Motor operation.

A somewhat similar study was conducted in [33]. A disturbance observer-based Back-SMC with super twisting sliding mode observer was implemented for speed control of an Induction motor. An uncertainty in load torque was introduced to monitor the robustness of the proposed design. For the desired speed of 200 rpm and an uncertainty of 2 Nm a speed fluctuation of -80 rpm was observed. However, in our proposed design (Back-STC-EA), for the desired speed of 314.1593 rpm and uncertainty of 10 Nm, a speed fluctuation of -0.1 rpm was observed, as shown in Figure 13e. This further confirmed the robustness and efficiency of the proposed controller.

Though the work is limited to the MATLAB/Simulink environment, we plan to realize the proposed control strategy in a real 3-phase induction motor in the near future to provide a more rigorous understanding of the motor speed control.

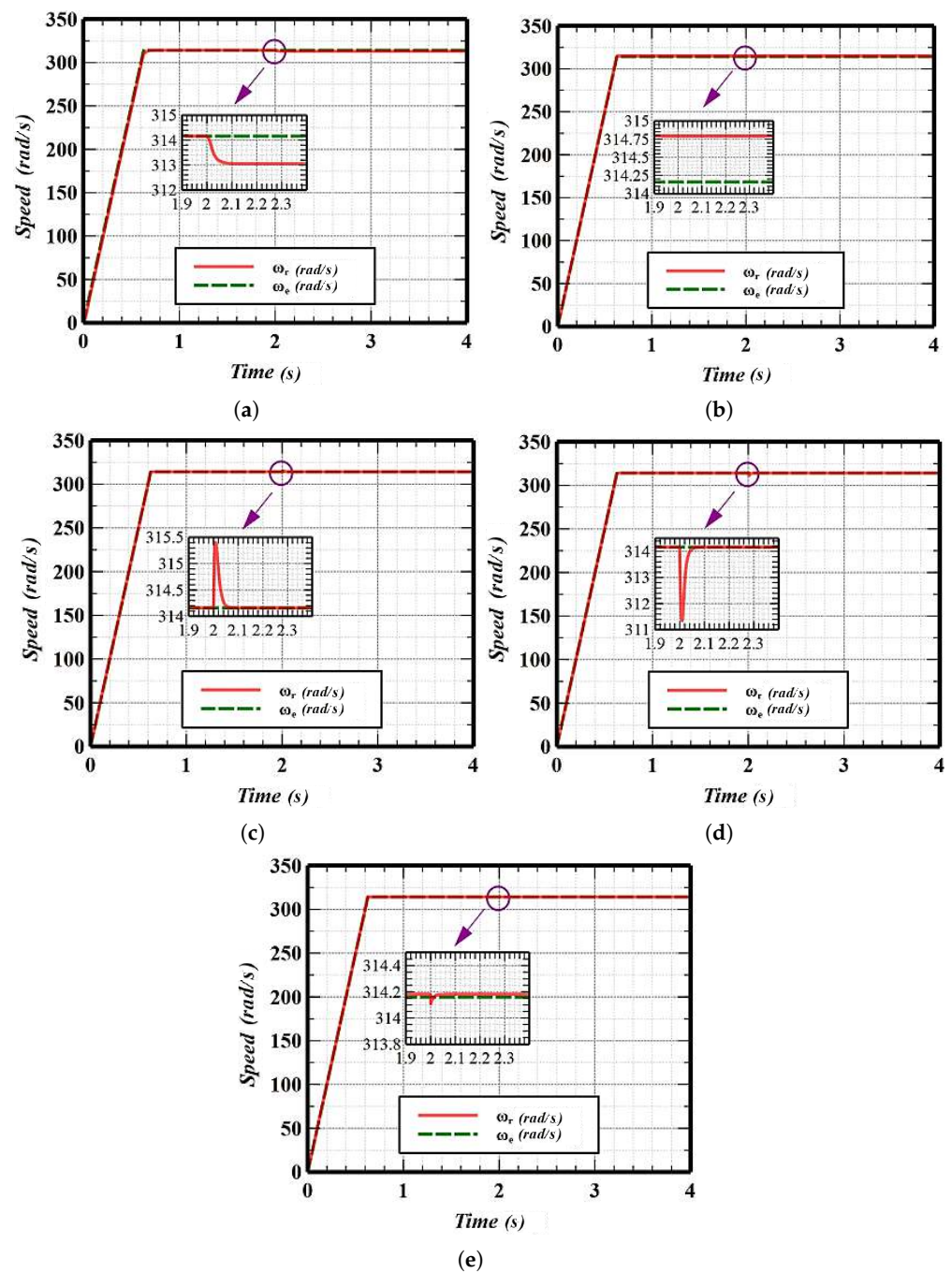


Figure 13. Output speed signals for DRM: (a) Backstepping Control, (b) SMC, (c) Back-SMC, (d) Back-STC, (e) Back-STC-EA.

Table 4. Numerical Evaluation—DRM.

Control Technique	ITAE (rad/s)	IAE (rad/s)	ISE (rad/s)
Backstepping cont.	7.217	4.156	8.417
SMC	5.065	2.353	1.457
Back-SMC	0.0424	0.0514	0.0437
Back-STC	0.1149	0.0888	0.1365
Back-STC-EA	0.1581	0.0790	0.0015

6. Conclusions and Future Work

The Induction motors are used extensively in domestic, as well as in industrial, applications. The objective of this research was to evaluate and compare the performance of dif-

ferent control methods when applied to a three-phase induction motors. Load disturbance rejection is the main concern in speed control applications. Firstly, the mathematical model of a three-phase induction motor was implemented in MATLAB 9.1 R2016b/Simulink software, Islamabad, Pakistan to simulate its behavior. Secondly, different control techniques were applied to study the performance of a three-phase induction motor in different scenarios, such as SSM, NOM and DRM. In SSM, Backstepping cont. The synergy of backstepping and the super twisting sliding mode control technique gave the best results when compared with other methods, owing to having the lowest values of statistical parameters. NOM, Back-STC and Back-STC-EA were the controllers with minimum ITAE. The robust qualities of the speed tracking were thoroughly investigated. Future possibilities include the hardware implementation of the designed controllers with an actual three-phase induction motor. These results could then be compared to the simulation study to further explore the topic.

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Abbreviations

The following nomenclature was used in this manuscript:

U_{sa}, U_{sb}, U_{sc}	Stator Supply voltages in <i>abc</i> frame of reference (V)
ω_e	Base Speed (rad/s)
U_{sd}^s, U_{sq}^s	Stator voltages in d-q stationary frame of reference (V)
V_{sq}, V_{sd}	Stator voltages in d-q dynamic frame of reference (V)
R_s, R_r	Stator & Rotor Resistances (Ω)
i_{sq}, i_{sd}	Stator currents in d-q dynamic frame of reference (A)
Φ_{sd}, Φ_{sq}	Stator Flux Linkages in d-q dynamic frame of reference (Wb-t)
Φ_{sd}^s, Φ_{sq}^s	Stator Flux Linkages in d-q Stationary frame of reference (Wb-t)
i_{rd}, i_{rq}	Rotor currents in d-q dynamic frame of reference (A)
ω_r	Rotor's Actual Speed (rad/s)
Φ_{rd}, Φ_{rq}	Rotor Flux Linkages in d-q dynamic frame of reference (Wb-t)
F_{sd}, F_{sq}	Stator Flux Variables (Wb)
F_{rd}, F_{rq}	Rotor Flux Variables (Wb)
ω_b	Base Frequency (Hz)
L_{ls}, L_{lr}	Stator & rotor's Leakage inductances (H)
L_m	Magnetizing Inductance (H)
Φ_{md}, Φ_{mq}	Magnetizing flux linkage in d-q dynamic frame of reference (Wb-t)
X_{ls}, X_{lr}	Stator & rotor's Leakage reactances (Ω)
X_m	Magnetizing Reactance (Ω)
T_L	Load Torque (Nm)
T_e	Electromagnetic Torque (Nm)
f	Electrical Frequency
$e_1(t), e_2(t), e_3(t), e_4(t)$	Error Signals for Backstepping controller
$k_{e1}, k_{e2}, k_{e3}, k_{e4}$	Tuning parameters for Backstepping Controller
V_{12}, V	Lyapunov Functions for Backstepping Controller

F_{sd}^d, F_{sq}^d	Virtual Control Inputs for backstepping controller (Wb)
V_{sd}^d, V_{sq}^d	Final Control inputs for backstepping Controller (V)
F_c	Viscous Coefficient of Friction
$E(t)$	Error signal for sliding mode controller
$u(t)$	Control law for Sliding mode controller
$d(t)$	Disturbance in load torque (Nm)
c_1, c_2, β	Tuning parameters for sliding mode controller
$V(S)$	Lyapunov Function for Sliding Mode Controller
z_1, z_2, z_3, z_4	Error signals for backstepping sliding mode controller
$V(z_1), V(z_2), V(z_3), V(z_4)$	Lyapunov Functions for backstepping sliding mode controller
g_1, g_2, g_3, g_4	Tuning Parameters for backstepping sliding mode controller
q_1, q_2, q_3, q_4	Tuning Parameters for backstepping sliding mode controller
r_1, K_2	Tuning Parameters for backstepping Super Twisting controller
U_{eq}	Equivalent Control for Super twisting Controller
U_{swc}	Switching Control for Super twisting Controller
S_1, S_2, S_3	Sliding Surfaces backstepping Super Twisting controller
$f(t)$	Real time noisy signal
$z_0, \dot{z}_1, \dots, \dot{z}_n$	Exact differentiators
$\sigma_0, \sigma_1, \dots, \sigma_n$	Tuning parameters
S_{app}	Signum Approximation
k_{app}	Approximation constant
$\emptyset$	Scaling Factor
N	Number of Samples
$e(i)$	Error for regression analysis

Appendix A. Derivation of Intermediate Control Inputs for the Backstepping Controller

Taking the derivative of the Lyapunov function in (20):

$$\frac{dV_{12}}{dt} = e_1 \frac{de_1}{dt} + e_2 \frac{de_2}{dt} \quad (A1)$$

where

$$\frac{de_1}{dt} = \frac{d\omega_{ref}}{dt} - k_4 F_{rd} F_{sq} + k_1 T_L \quad (A2)$$

$$\frac{de_2}{dr} = \frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} \quad (A3)$$

For the system to be stable, the Lyapunov function should be negative definite. To make the derivative of the Lyapunov function negative definite, the term $\frac{de_1}{dt}$ and $\frac{de_2}{dt}$ are made equal to $k_{e1}e_1$ and $k_{e2}e_2$, respectively:

$$\frac{d\omega_{ref}}{dt} - k_4 F_{rd} F_{sq} + k_1 T_L = k_{e1}e_1 \quad (A4)$$

$$\frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} = k_{e2}e_2 \quad (A5)$$

Rearranging these equations, we get the control inputs for subsystem 1 as,

$$F_{sd}^d = \frac{k_{e1}e_1 + \frac{d\omega_{ref}}{dt} + k_1 T_L}{k_4 F_{rd}} \quad (A6)$$

$$F_{sd}^d = \frac{k_{e2}e_2 + \frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd}}{\omega_b \frac{X_m}{X_{ls}} \frac{R_r}{X_{lr}}} \quad (A7)$$

These equations constitute the intermediate control required for obtaining the desired final control input for subsystem 2. Substituting (A4) and (A5) into the derivative of the Lyapunov function, we get:

$$\frac{dV_{12}}{dt} = e_1[-k_{e1}e_1] + e_2[-k_{e2}e_2] \quad (A8)$$

$$\frac{dV_{12}}{dt} = -k_{e1}e_1^2 - k_{e2}e_2^2 \text{ (negative definite if } k_{e1} \text{ and } k_{e1} > 0) \quad (A9)$$

Appendix B. Derivation of Final Control Inputs for the Backstepping Controller

Rearranging (25) and (26), we get

$$k_4 F_{rd} e_3 - k_{e1} e_1 = \frac{d\omega_{ref}}{dt} + k_1 T_L - k_4 F_{rd} F_{sq} = \frac{de_1}{dt} \quad (A10)$$

$$\omega_b e f e_4 - k_{e2} e_2 = \frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{rd} - \omega_b \frac{X_m}{X_{ls}} \frac{R_r}{X_{lr}} F_{sd} = \frac{de_2}{dt} \quad (A11)$$

Substituting the values of error and their derivatives in the derivative of the Lyapunov function (27), we get:

$$\frac{dV}{dt} = e_1[k_4 F_{rd} e_3 - k_{e1} e_1] + e_2 \left[\omega_b \frac{X_m}{X_{ls}} \frac{R_r}{X_{lr}} e_4 - k_{e2} e_2 \right] \quad (A12)$$

$$+ e_3 \left[\frac{dF_{sq}^d}{dt} - \omega_b V_{sq} - \left(-\omega_e F_{sd} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sq} \right] \right) \right] \quad (A13)$$

$$+ e_4 \left[\frac{dF_{sd}^d}{dt} - \omega_b V_{sd} + \left(-\omega_e F_{sq} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sd} \right] \right) \right] \quad (A14)$$

The final control inputs that satisfy the Lyapunov stability criteria are as follows:

$$V_{sq}^d = \frac{k_{e3} e_3 + \frac{dF_{sq}^d}{dt} - \left(-\omega_e F_{sd} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sq} \right] \right) + k_4 F_{rd} e_1}{\omega_b} \quad (A15)$$

$$V_{sd}^d = \frac{k_{e4} e_4 + \frac{dF_{sd}^d}{dt} - \left(\omega_e F_{sq} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sd} \right] \right) + \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} e_2}{\omega_b} \quad (A16)$$

Substituting these into the derivative of the Lyapunov function, we get:

$$\frac{dV}{dt} = -k_{e1} e_1^2 - k_{e2} e_2^2 - k_{e3} e_3^2 - k_{e4} e_4^2 < 0$$

(negative definite if k_{e1} , k_{e2} , k_{e3} , and $k_{e4} > 0$)

Hence, the condition is proved.

Appendix C. Derivation of Final Control Inputs for Backstepping Sliding Mode Controller

Step 1:

$$z_1 = \omega_{rd} - \omega_r \quad (A17)$$

Taking the derivative of the error signal and substituting the value of the derivative of the rotor's speed, we get:

$$\frac{dz_1}{dt} = \frac{d\omega_{rd}}{dt} - K_4 F_{rd} F_{sq} + k_1 T_L \quad (A18)$$

Considering the following Lyapunov candidate function and its derivative:

$$V(z_1) = \frac{1}{2}z_1^2 \quad (\text{A19})$$

$$\frac{dV(z_1)}{dt} = z_1 \frac{dz_1}{dt} \quad (\text{A20})$$

$$\frac{dV(z_1)}{dt} = z_1 \left(\frac{d\omega_{rd}}{dt} - K_4 F_{rd} F_{sq} + k_1 T_L \right) \quad (\text{A21})$$

For the system to be stable the Lyapunov function should be negative definite. It is obtained by substituting the right-hand side of (A21) (excluding z_1) equal to $g_1 z_1$.

$$(k_4 F_{rd} F_{sq})_d = g_1 z_1 + \frac{d\omega_{rd}}{dt} + k_1 T_L \quad (\text{A22})$$

$$\frac{dV(z_1)}{dt} = -g_1 z_1^2 \quad (\text{A23})$$

Note that (A23) is negative definite if $g_1 > 0$.

Step 2:

Defining the error for the stator flux in the q-axis direction, we get:

$$z_2 = (k_4 F_{rd} F_{sq})_d - (k_4 F_{rd} F_{sq}) \quad (\text{A24})$$

Taking the derivative of (A24) and substituting from the Lyapunov condition (A23) yields:

$$\frac{dz_2}{dt} = \left[g_1 \left(\frac{d\omega_{rd}}{dt} - k_4 F_{rd} F_{sq} + k_1 T_L \right) + \frac{d}{dt} \left(\frac{d\omega_{rd}}{dt} \right) + \frac{dk_1 T_L}{dt} \right] \quad (\text{A25})$$

$$- k_4 \left[F_{rd} \left(\omega_b V_{sq} - \omega_e F_{sd} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sq} \right] \right) \right] \quad (\text{A26})$$

$$+ \left(\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} + \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} \right) F_{sq} \quad (\text{A27})$$

To avoid cumbersome calculations, the following arrangements are made:

$$\frac{dz_2}{dt} = \varphi_1 + \varphi_2 V_{sq} \quad (\text{A28})$$

where

$$\varphi_1 = \frac{g_1 d\omega_{rd}}{dt} - g_1 k_4 F_{rd} F_{sq} + g_1 k_1 T_L + \frac{d}{dt} \left(\frac{d\omega_{rd}}{dt} \right) + \frac{dk_1 T_L}{dt} - k_4 F_{rd} \left(-\omega_e F_{sd} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{ls}} - 1 \right) F_{sq} \right] \right) \quad (\text{A29})$$

$$- k_4 \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} F_{sq} - k_4 \left(\omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} \right) F_{sq}$$

$$\varphi_2 = -k_4 F_{rd} \omega_b \quad (\text{A30})$$

The Lyapunov candidate function considered in this case is:

$$V(z_2) = \frac{1}{2}z_2^2 \quad (\text{A31})$$

Taking the derivative of the Lyapunov function, it is obtained that:

$$\frac{dV(z_2)}{dt} = z_2 \frac{dz_2}{dt} \quad (\text{A32})$$

By substituting (A28), one gets

$$\frac{dV_{z2}}{dt} = z_2(\varphi_1 + \varphi_2 V_{sq}) \quad (\text{A33})$$

For the system to be stable, the Lyapunov function should be negative definite. It is obtained by substituting the R.H.S of (A33), (excluding z_2) equal to $g_2 z_2$. Then, the resulting expressions are:

$$V_{sq} = \frac{-\varphi_1 - g_2 z_2}{\varphi_2} \quad (\text{A34})$$

and,

$$\frac{dV(z_2)}{dt} = -g_2 z_2^2, \text{ (It is negative definite if } g_2 > 0 \text{)} \quad (\text{A35})$$

Step 3: Now considering subsystem 2, the error signal for the rotor flux in d-axis direction is defined as follows:

$$z_3 = F_{rd}^d - F_{rd} \quad (\text{A36})$$

Taking the derivative and substituting the value of $\dot{F}_{rd}$, we obtain:

$$\frac{dz_3}{dt} = \frac{dF_{rd}^d}{dt} - \left[\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} + \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} \right] \quad (\text{A37})$$

The following Lyapunov function is being considered:

$$V(z_3) = \frac{1}{2} z_3^2 \quad (\text{A38})$$

Taking the derivative of the Lyapunov function and substituting the value of $\dot{z}_3$, it is obtained that:

$$\frac{dV(z_3)}{dt} = z_3 \left[\frac{dF_{rd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} \right] \quad (\text{A39})$$

To make it negative definite, it was selected:

$$\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} = g_3 z_3 - \omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} + \frac{dF_{rd}^d}{dt} \quad (\text{A40})$$

Rearranging and substituting (A40) into (A39), it is obtained:

$$\frac{dV_{z3}}{dt} = -g_3 z_3^2 < 0 \text{ (which is negative definite if } g_3 > 0 \text{)} \quad (\text{A41})$$

Step 4: The following error signal is selected for the stator flux in the d-axis direction:

$$z_4 = \left(\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} \right)_d - \left(\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} \right) \quad (\text{A42})$$

Substituting the value of $\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd}$ from (A41) into (A43), we get:

$$z_4 = \left(\frac{dF_{rd}^d}{dt} - \left(\omega_b \frac{R_r}{X_{lr}} \left(\frac{X_m}{X_{lr}} - 1 \right) F_{rd} \right) + g_3 z_3 \right) - \left(\omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} F_{sd} \right) \quad (\text{A43})$$

Taking the derivative of (A43) and simplifying it, we get:

$$\frac{dz_4}{dt} = \varphi_3 + \varphi_4 V_{sd} \quad (\text{A44})$$

where

$$\varphi_3 = \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} \frac{dF_{sd}^d}{dt} - \omega_b \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} \omega_e F_{sq} + \omega_b \left[\frac{R_s}{X_{ls}} \left(\frac{X_m}{X_{lr}} F_{rd} + \left(\frac{X_m}{X_{ls}} - 1 \right) \right) F_{sd} \right] \quad (\text{A45})$$

$$\varphi_4 = -\omega_b^2 \frac{R_r}{X_{lr}} \frac{X_m}{X_{ls}} \quad (\text{A46})$$

A candidate Lyapunov function is selected as follows:

$$V(z_4) = \frac{1}{2} z_4^2 \quad (\text{A47})$$

Then, taking the derivative of the Lyapunov function (A47):

$$\frac{dV(z_4)}{dt} = z_4 \frac{dz_4}{dt} \quad (\text{A48})$$

Substituting (A44) in the derivative of the Lyapunov function, we get:

$$\frac{dV(z_4)}{dt} = z_4 [\varphi_3 + \varphi_4 V_{sd}] \quad (\text{A49})$$

To ensure the definite negative nature of the Lyapunov function, the following value of V_{sd} is selected:

$$V_{sd} = \frac{-\varphi_3 - g_4 z_4}{\varphi_4} \quad (\text{A50})$$

Substituting (A50) in (A49), it is obtained:

$$\frac{dV(z_4)}{dt} = -g_4 z_4^2, \quad \text{which is negative definite if } g_4 > 0 \quad (\text{A51})$$

The SMC is incorporated as follows: consider z_2 and z_4 as the sliding surfaces. Then, the derivatives of these sliding surfaces should be equal to zero, i.e., ($\dot{z}_2 = 0, \dot{z}_4 = 0$). The following adjustments are made to achieve the final control law:

$$\frac{dz_2}{dt} = \varphi_1 + \varphi_2 V_{sq} = -q_1 \text{sign}(z_2) - q_2 z_2 = 0 \quad (\text{A52})$$

$$V_{sd}^d = \frac{-\varphi_1 - \text{sign}(z_2) - q_2 z_2}{\varphi_2} \quad (\text{A53})$$

$$\frac{dz_4}{dt} = \varphi_3 + \varphi_4 V_{sd} = -q_3 \text{sign}(z_4) - q_4 z_4 = 0 \quad (\text{A54})$$

$$V_{sd}^d = \frac{-\varphi_3 - q_3 \text{sign}(z_4) - q_4 z_4}{\varphi_4} \quad (\text{A55})$$

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