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A Source Seeking Method for the Implicit Information Field Based on a Balanced Searching Strategy

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Abstract: To address the issue of low efficiency in source seeking within implicit information fields, this paper proposes an autonomous sourcing method based on a balanced search strategy inspired by biological homing behaviors. At the outset of the research, the task of source seeking boiled down to a multi-objective convergence problem. By utilizing feasibility search behaviors as individual samples in evolutionary population, drawing on the principles of evolutionary algorithms, motion searching was integrated with population evolution to guide carriers towards completing source seeking tasks by solving multi-objective problems. Furthermore, the distribution entropy was also considered to measure the searching bias in the process of source seeking. In combination with the requirements of the source seeking process, a new method for balanced searching was designed. Ultimately, through theoretical analysis and simulation verification, we confirmed the effectiveness and rationality of this proposed method.

Keywords: source seeking; biological homing behaviors; implicit information field; balanced searching strategy

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1. Introduction

In near-Earth space, any environmental location can be described by various information features. According to the difference in sensor detection methods, some information can be observed in a large range, which is called an explicit information field. At present, it is also one of the main ways for humans to carry out autonomous positioning, such as using the feature information of environmental images to carry out matching navigation. The other information field has certain observation limitations, and the neighborhood data of adjacent space cannot be obtained during measurement, so it is often necessary to obtain data through field measurement [1,2]. We define the geomagnetic field, gravity field, and odor field as implicit information fields, and this kind of information field is an important navigational information source for organisms in nature.

An explicit information field is often used to carry out navigation and positioning, and a prior database of the information field needs to be obtained in advance. In general, it is difficult to obtain large-scale and high-precision prior databases, which undoubtedly limits the autonomy of mobile carriers. In nature, creatures such as turtles and homing pigeons can use natural navigation information fields to achieve long-distance migration and homing. In this process, it is obviously unimaginable that creatures such as turtles and homing pigeons store complete information field databases in advance. Therefore, this paper used implicit information as navigation clues to carry out source search research, which is a useful supplement to the existing autonomous navigation methods.

From the perspective of characteristic distribution, although the implicit information distribution between adjacent elements has a certain continuity, its gradient direction varies; therefore, it is difficult to use the measured data to deduce the data in the unreachable area. This issue causes difficulties for the development and utilization of implicit information fields, such as geomagnetic navigation [3,4] and odor source finding [5]. In the modern industrial environment, there are similar scenarios, such as in nuclear leakage environments, carrying out the search for a nuclear leakage source [6]; in fingerprint positioning applications, the reverse positioning of the faulty fingerprint source is reversed [7].

At present, for the source seeking problem of implicit information fields, the searching method is mainly based on searching by path planning and searching by data driving.

The searching method based on path planning means that the carrier moves along the way that was planned to search for the target source, and the typical methods are traversal search and Z-shape search [8–10]. This method does not distinguish the types of information sources and achieves certain universality characteristics by the optimization of the path and the expansion of detection capabilities [11–14]. By optimizing the path and expanding the detection ability, the search efficiency can be improved to a certain extent. However, the shortcoming of this method lies in the exponential relationship between the searching time and the searching space, which is difficult to accept in the context of large-scale applications.

The data-driven searching method refers to the research on source seeking methods from the perspective of data acquisition and data utilization [15–17]. For example, the formation cooperative searching method improves the measurement ability of the environmental characteristics of implicit information fields and then can obtain more data. However, it is prone to information redundancy or conflict, which leads to the failure of the source search [18,19]. Probabilistic predictive searching based on historical measured data uses the method of statistical probability to predict the position of the target point and adjusts the predicted results constantly in the subsequent searching, so as to guide the carrier to realize the source searching [20]. However, this method easily falls into the partial minimum.

At present, with the development of technology and our exploration of the environment, knowledge seeking forms and occasions are becoming more and more diverse. However, not all scenarios allow the prediction of structure and internal correlation, and many scenarios are presented as black box solving problems [21–23]. There are some black box problems with solvable properties. For example, underwater organisms can complete thousand-mile migrations by using magnetic field characteristics without any prior database, but it is difficult for humans to achieve navigation tasks without a prior geomagnetic database [24,25]. Dogs can accurately detect odor sources in unfamiliar environments without relying on the misty rain model of odor. Similar posterior detection problems are presented in scenarios of nuclear radiation source search, pseudo-WIFI source search, and alien exploration.

We conducted our research under the following scenarios:

(1) The acquisition of information has a clear field measurement attribute, which meets the requirements of an implicit information field. This means that environmental information about unarrived at locations cannot be obtained in advance.
(2) The source search task does not depend on a prior database, and the source search path cannot be obtained in advance.

This paper proposes a source seeking method based on a balanced searching strategy from the perspective of autonomous search, which was inspired by biological source seeking behavior [26,27]. From the perspective of biological autonomous search, and due to the lack of reference to an a priori database, the source seeking behavior shows the exploration of the environment and the exploitation of the previous information in the implicit information field. The autonomous searching behavior is carried out by taking the characteristic parameters of the information source at the target as the convergence target. In this paper, under the condition of limited perceptual ability, the search behavior in posterior problem solving is explored by taking the source finding of the implicit information field as the object.

The main contributions of this paper are as follows:

(1) Research on navigation and positioning under conditions of limited information detection, drawing inspiration from animal homing behavior;
(2) A balanced search strategy approach was proposed from a search bias perspective to address black box problems, including implicit information;
(3) By conducting theoretical analysis and simulation experiments, the algorithm's convergence was confirmed, and the optimal search bias value was determined. This provides a solid theoretical foundation for future research.

The paper proceeds as follows: Section 2 describes the problem of seeking the source of an implicit information field. Section 3 presents the detailed approach. The algorithm performance analysis and experimental results are shown in Sections 4 and 5, with a discussion in Section 6.

2. Problem Description

In general, the implicit information fields that can be sourced or used for localization have many parameters to be described and can be uniquely characterized by a variety of feature parameters [28], as shown in the following equation:

$$\begin{cases} E = \{e_1, e_2, \cdots, e_n, n \in R\} \\ P(x, y, z) \to E \end{cases} \tag{1}$$

where E stands for the set of implicit information field features, characterized by n parameters $e_1, e_2, \cdots, e_n$, and P stands for any position in space (x, y, z), with a one-to-one mapping relationship between P and E. In the implicit information field, this mapping is ambiguous. As a result, historical information cannot be used to accurately build mapping models.

Here, we note the target information source features as E^T. Then, the problem of source seeking for implicit information fields can be described as the process of the carrier using the measured data combined with autonomous searching to achieve the searching of the target source. Without a loss of generality, the implicit information source finding problem can be reduced to the convergence of various environmental parameters to the target environmental parameters without a prior database [29], as shown below:

$$\min F(E, k) = (f_1(e_1, k), f_2(e_2, k), \cdots, f_n(e_n, k))^T \tag{2}$$

where k represents moment information, and objective function F is k-time and the difference between E^K, the measured environmental characteristics E^T, and the target environmental characteristics.

The termination conditions of the source seeking task are given from the perspectives of parametric convergence and position convergence, respectively, as shown in the following equation:

$$\begin{cases} \left| e_i^k - e_i^T \right| \to 0,\ i \in n \\ \sqrt{(x_k - x_T)^2 + (y_k - y_T)^2 + (z_k - z_T)^2} \le \gamma \end{cases} \tag{3}$$

where T represents the target location; e_i^T is the characteristic parameter of the i-th target environment; (x_T, y_T, z_T) is the spatial position P_T; and γ is a constant, representing the source precision. The above equation represents the true spatial position of the k-time, the environmental feature e converges to the respective target value, and the carrier reaches or approaches the target point.

By utilizing the measurable properties of adjacent local or global range information features, a corresponding relationship between function F and geographical coordinates can be established in the explicit information field. This enables one to obtain the local or complete form of function F. The identification of sources can be accomplished through either spatial matching or sequential planning.

However, in the realm of implicit information, it is only possible to obtain environmental characteristics of a location, rendering the establishment of function F in Equation (2) unfeasible. In the process of source identification, there exists significant uncertainty at every stage of the search. Overcoming thousands of miles to reach the target point is a formidable challenge akin to that faced by a turtle. It is important to note that the focus of this paper does not lie in determining the specific form of function F, but rather assumes its unknown nature.

These challenges manifest in situations such as olfactory detection, deep-sea navigation, and the autonomous exploration of uncharted territories.

3. A Source Seeking Method Based on a Balanced Searching Strategy

Obviously, it is too difficult to solve the multi-objective convergence problem described in Equation (2) when the specific form of F is unknown. Therefore, in this section, firstly, the characteristics and existence conditions of the solution of the implicit information field that can be found are given. Then, by introducing the search path into the source seeking problem, the relationship between search behavior and parameter variation is established. Finally, according to the principle of information tendency, a search method based on a balanced searching strategy is proposed, and a source seeking algorithm is given by introducing the idea of an evolutionary algorithm.

3.1. Characteristic Analysis of the Source of an Implicit Information Field

In nature, there are a variety of implicit information fields that can be used as navigation information sources for animals, such as odor fields, geomagnetic fields, gravity fields, and other hidden information fields [17,18]. These can, for example, guide animals whose source seeking span is often tens or even thousands of kilometers to complete homing, migration, and other source seeking tasks. In this process, it is obvious that the simple brain structure of animals cannot store a complete "map" beforehand. It is believed that in the process of animal source searching, the implicit information field provides one or more winding paths to guide animals to the source.

In order to accelerate the analysis of the source solution characteristics for the implicit information field, we can assume that the distribution of the environmental feature E in Equation (2) is known, so the source seeking problem can be reduced to a multi-objective convergence problem.

Therefore, the distribution characteristics of the implicit information field source seeking problem solution can be analyzed from the perspective of a Pareto optimal solution with the help of multi-objective optimization theory.

Consider a two-dimensional plane, assuming that the carrier is located at the k-moment $P_k = (x_k, y_k)$. Within adjacent cells, there is a closed region G consisting of Pareto solution sets (see Figure 1).

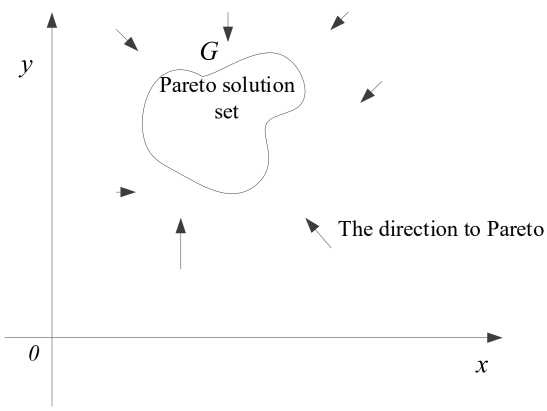

Figure 1. Schematic diagram of static solution distributed in a two-dimensional plane.

In Figure 1, $\exists x^* \in G$ within the region G satisfies $F\langle x^*, t_i \rangle < F(x, t_i)$, that is, the region G represents the closed region composed of the Pareto solution set, and arrow "$\rightarrow$" represents the direction pointing to the Pareto solution set.

The solution of the source seeking problem can be regarded as the process of finding the region G in the search space within a particular time. Generally, when the number of searches is enough that the parameter space can be traversed, then the search path can enter the Pareto solution set region and obtain the source direction at the current moment.

When the time changes from k to $k + 1$, region G also moves, and its changing process is shown in Figure 2.

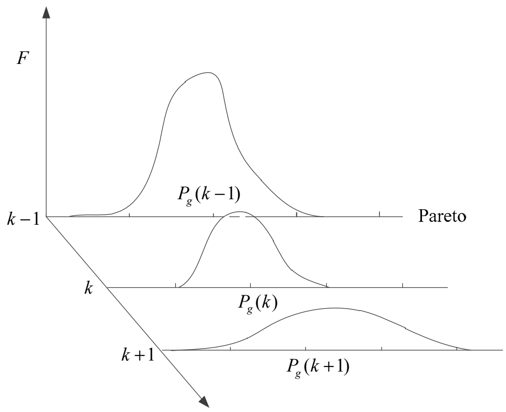

Figure 2. The distribution of solutions at different times.

In Figure 2, $P_g(k)$ shows the optimal solution in time k within region G. It can be seen that the size and position of the optimal solution change as the moment changes. When $k \in [k_1, k_g]$ changes continuously, there exists an optimal solution $P_g(k)$ at every moment, which is enclosed by the optimal solution cluster enclosed by $P_S(k)$ (see Figure 3).

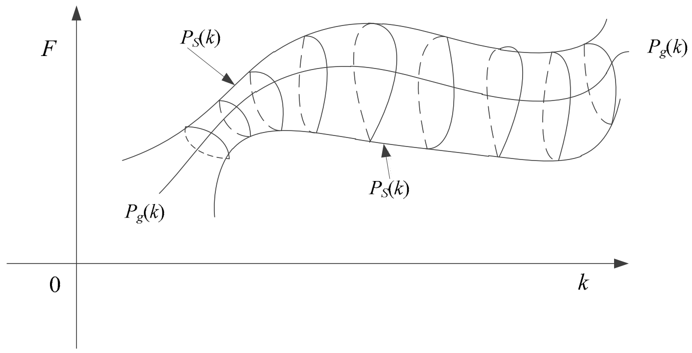

Figure 3. Schematic diagram of the distribution of solutions in continuous time.

In Figure 3, the time-optimal solution cluster for continuous time is presented as a channel shape. By connecting the optimal solutions for continuous time in the channel, a curve is obtained, which is called the solution curve. The solution curve connects the known solution (starting point) and the solution (ending point), and the solution curve can be tracked to solve the multi-objective problem.

The solution curve is the result of the source seeking problem at different times, and tracking the curve can guide the search algorithm to converge to the global optimal solution. In the source finding problem, this curve is the moving trajectory of the carrier in space, connecting the starting point and the end point of navigation, so this curve can also be called the source finding solution curve.

3.2. Source Seeking Behavior of Implicit Information Field Based on Motion Path

Due to the limitations of measurement for implicit information field characteristics, one cannot obtain the data of an unreached area or unit. The measurement of environmental characteristics in implicit information fields depends on the motion path. For example, in olfactory navigation, one cannot predict the intensity of odors in neighboring locations. Therefore, it is necessary to introduce the motion path into the problem described in Equation (2).

Considering the moving carrier as a particle in a two-dimensional plane, the kinematic equation of the carrier can be described as follows:

$$\begin{cases} x_{k+1} = x_k + L\cos(\theta) \\ y_{k+1} = y_k + L\sin(\theta) \\ u = (L, \theta) \end{cases} \tag{4}$$

where $P_k = (x_k, y_k)$ represents the position of the carrier at time k, and the motion parameter u consists of step length L and motion direction θ. The position of the carrier at successive moments is recorded, constituting a motion trajectory guiding the carrier to reach the target point.

Equation (4) is introduced into the problem of source seeking for implicit information fields, and then the multi-objective search problem is obtained as shown in Equation (5) below.

$$\begin{cases} \min F(E,k) = (f_1(e_1,k), f_2(e_2,k), \cdots, f_n(e_n,k))^T \\ s.t. \quad G(E,k,u) \le 0 \end{cases} \tag{5}$$

where G is the constraint function of search behaviors, which is composed of environment parameter E, motion parameter u, and time information k. After the introduction of constraint function G, the change in objective function F can be caused by the execution of u, and the connection between objective function F and u is indirectly established, which provides the possibility of solving the subsequent source search problem.

3.3. Balanced Searching Strategy

3.3.1. Exploration and Exploitation

Exploration and exploitation are two basic strategies in the process of search optimization [26,30]. Exploration refers to the searching strategy aimed at obtaining the information of the objective function from the perspective of breadth in the searching process. On the other hand, exploitation refers to the searching strategy based on the function information obtained by exploration, which aims to find the optimal solution at the depth level.

In nature, animals homing in unknown environments through the exploration of the unknown environment and the exploitation of historical information. The source seeking process can be described as follows: in the initial stage, animals may explore the environment through their own movement to obtain the distribution information of features to make up for the lack of environmental cognition; then, they use the accessed information, search for the source path, and respond to changes in the environmental feature distribution and implement search behaviors to maintain an equilibrium between exploration and exploitation in order to obtain maximum profits (here, the benefits include the cognition of the environment and the optimization of the target point of convergence); thus, individuals are guided to arrive at the target point.

Inspired by the homing behaviors of animals, we propose a balanced searching strategy, which focuses on the exploration and exploitation of environmental information and the dynamic balance of information returns and optimization returns, then combines different stages of searching to carry out equilibrium searching. In the early stage of searching, the environment information is explored through random roaming, and the trend movement is gradually formed. In the stage of source searching, the searching bias is dynamically adjusted between exploration and exploitation for the purpose of maximizing revenue.

3.3.2. The Algorithm of Source Seeking

(1) Search behavior design based on evolutionary algorithm

In the following, a source seeking algorithm is proposed based on the balanced searching strategy.

The searching process depends on the movement of the carrier, and its searching behaviors can be characterized by the motion parameter u. Drawing on the idea of an evolutionary algorithm, the evolutionary population is constructed by taking the feasible searching behaviors as the sample of the evolutionary population. Thus, the j-th sample individual can be defined as follows:

$$Q_j = \theta_R = D_\theta \times R, \quad j = 1, \cdots, N_{pop} \tag{6}$$

where $R \in [1, \cdots, 2\pi/D_\theta]$ is a random number; D_θ is the search space compression ratio; and N_{pop} stands for the population size, which is usually set as $N_{pop} > (2\pi/D_\theta)$.

The carrier's search for the target features of the environment depends on its movement in space. Multi-target searching and the movement of the source space have the temporal characteristics shown in Figure 4.

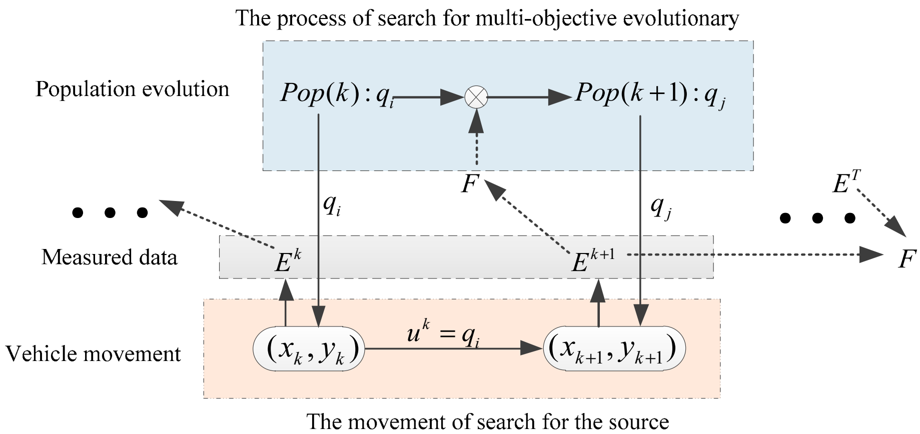

Figure 4. Diagram of the principle of source seeking.

Combined with Figure 4, the source seeking process can be described as follows. At the time k, a certain Q_i^k is selected from the evolutionary population $Pop(k)$ with medium probability as the motion parameter of the carrier, and the carrier obtains the motion displacement of L^k by executing Q_i^k. The E^k and E^{k+1} at positions P_k and P_{k+1} are measured by carrier movement and then substituted into Equation (5) for the multi-objective solution, and the multi-objective function F is calculated. According to the convergence state of the multi-objective function F, the source seeking performance of the executed samples is evaluated, and the breeding or elimination operation is used to increase or decrease the proportion of such samples. Through the mutation operation to improve the population diversity, a new population $Pop(k + 1)$ is obtained, and the next round of the searching process is re-entered. Through repeated iterations, the multi-objective function converges to the minimum, and the source search task is realized.

(2) Search bias measure

The bias of search behaviors can be measured by the diversity of the evolving population. The higher the population diversity, the more scattered the sample distribution in the population, resulting in the greater randomness of the search behaviors, and the search behaviors are biased towards exploration at this moment; the lower the population diversity, the more concentrated the distribution of samples in the population, resulting in less randomness in the search behaviors, which are biased toward development at this time.

Currently, diversity has garnered significant attention in performance research and the development of evolutionary algorithms; however, there is a relative scarcity of quantitative descriptions of diversity. Population diversity is commonly described through the measurement of spatial location differences among sampled individuals across multiple dimensions. However, the search ability and group search bias of the population are not adequately reflected, resulting in instances where diversity is equivalent between a few individuals deviating significantly from the majority and a more dispersed majority. In this case, the low probability of distribution for a few individuals results in a low likelihood of being selected for execution. As such, the search behavior of the population is dominated by aggregating individuals and biased towards exploitation. However, when individuals are more widely dispersed, the distribution probability of each sample becomes closer, and the likelihood of being selected for execution is similar. This results in a bias towards exploratory search behavior. For the above reasons, the concept of distributed entropy is introduced here.

Definition 1. *Distributed entropy: The sample type is class N, and the sample individual can be represented as $C_1, \cdots, C_N$. At some point in the evolution of the group, the proportion of sample individuals in the group is $p_1, \cdots, p_N$ and satisfies $\sum_{i=1}^{N} p_i = 1$; then, the distribution entropy is:*

$$H(p_1, p_2, \cdots, p_N) = -\sum_{i=1}^{N} p_i \ln p_i \tag{7}$$

The distribution entropy is nonnegative, symmetric, and additive in the distribution space, and it is a strictly concave function. When all samples are uniformly distributed, H has a unique maximum value; when one type of individual dominates the population, that is, $p_1 \to 1,\ p_i \to 0(\forall i > 1)$, it is true that:

$$\lim_{p_1 \to 1} H = \lim_{p_1 \to 1} \left[-p_1 \ln p_1 - \sum_{i=2}^{N} p_i \ln p_i \right] = 0 \tag{8}$$

The distribution entropy is the lowest currently.

The distribution entropy quantifies the population diversity and reflects the searching bias of the population. More exactly, the larger the entropy value is, the stronger the global exploration behaviors of the population will be. The smaller the entropy value is, the stronger the local exploitation ability of the population will be.

(3) The strategy of balance

Combined with the analysis of the source seeking path of the implicit information field in Section 3.1, the source seeking path will change with the spatial position. It is not conducive to the tracking of the source path when the distribution entropy of the evolutionary population is too large or too small in the source seeking process. Therefore, we combined the structural characteristics of evolutionary algorithms and the distribution characteristics of source paths to provide a specific algorithm for a balanced searching strategy. We divided the source searching process into three stages (see Figure 5).

In the first stage, the initial stage of source seeking, the searching behaviors are mainly exploration. When the population sample species at time k is greater than 1, the process will enter the source seeking stage.

In the second stage in the process of source seeking, the searching behaviors change dynamically between exploration and exploitation.

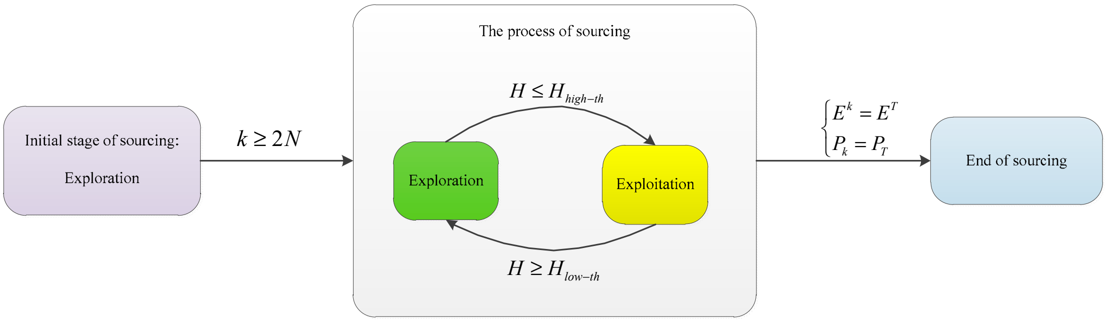

Figure 5. Schematic diagram of decomposition into source searching stages.

When the distribution entropy H is less than or equal to the highest threshold entropy $H_{high-th}$, the searching process is exploitation-oriented to avoid too much random motion leading to the failure of the search. When the distribution entropy H is greater than or equal to the lowest threshold entropy H_{low-th}, the process of source seeking enters the searching process dominated by exploration to avoid the premature population problem caused by low population diversity. Other time, the carrier carries out the search task according to the results of population evolution.

In the third stage, the end of source seeking, when the parameter space and the real space in Equation (3) are reached, the source seeking task can be completed.

So far, we have presented a source seeking method based on a balanced searching strategy, and then combined mathematical analysis and experimental simulation to verify the effectiveness and rationality of the algorithm.

4. Algorithm Performance Analysis

4.1. Analysis for the Convergence of the Algorithm

Using the balanced searching method based on genetic evolution, we could obtain the source path by solving the multi-objective function and guide the carrier to the target source position. The algorithm is still essentially a multi-objective evolutionary algorithm.

For the optimization problem, the global convergence of the algorithm should meet the following two conditions: (1) strictly covered variation distribution, so that any individual $x' \in X$ can be generated by individual $x \in X$ according to a certain mutation probability; (2) in evolutionary algorithms, the population sequence $Pop(0), Pop(1), \cdots, Pop(k), \cdots$ is monotone, meaning that the entire evolutionary process does not degenerate the population due to the loss of the optimal individual.

Condition (1) is easily guaranteed in evolutionary algorithms that include mutation operations.

Considering that the transfer probability from individual i to individual j is r_{ij}, and considering the limiting behaviors in the finite search space $k \to \infty$, there exists $\lim\limits_{k\to\infty} r_{ij}(\infty) = 1$. This means that the balanced searching strategy with genetic variation as the core can ensure that any individual can be generated by another individual.

Proof.

Condition (2) can be proved from the perspective of population learning.

During the searching process, the progeny population consists of two parts. One part is the parent population that was not selected Pop', and the other part is the new population that has been learned through trial and error pop_N.

The source seeking algorithm selects only one sample of the population to reproduce in any one evolution.

When the population reaches a certain size, the performance $h'_p(k)$ of the old population Pop' is equal to the performance $h_p(k-1)$ of the parent population $Pop(k-1)$, so $h'_p(k) = h_p(k-1)$.

The pop_N performance of the progeny population obtained through learning $h_{N-p}(k)$ is superior to $h_p(k-1)$, so $h_{N-p}(k) > h_p(k-1)$.

If sample q_i is selected and the evaluation result is good, the breeding operation is performed.

The next generation population is composed of the progeny population pop_N and the parent population $Pop'(k)$. Then, the performance of the next generation population is:

$$h_p(k) = h_p(k-1) + h_{N-p}(k) > h_p(k-1) \tag{9}$$

If sample q_i is selected and the evaluation result is inferior, and the elimination operation is performed to generate new sample individuals according to the probability. In terms of results, its performance is no lower than that of its parent population.

Therefore, after performing an evolutionary operation, the performance of the progeny population is no worse than that of the parent population, and the population sequence for continuous time $Pop(0), Pop(1), \cdots, Pop(k), \cdots$ is monotonous. □

4.2. Analysis for the Performance of Source Seeking

As for the source seeking algorithm based on a balanced searching strategy, its source searching ability can be clarified from the perspective of obtaining and tracking the solution.

At time *k*, the purpose of the source seeking is to find the closed region *G* composed of the Pareto solution set. Because the carrier searching behaviors are limited, and the mutation operation is used between behaviors, the region has connective reachability.

Therefore, the source seeking issue at time *k* can be optimized.

When the closed region *G* is obtained at time *k*, and the optimal solution *G* contained in this region is wrapped by the Pareto boundary, a meandering channel is formed to reach the global optimal solution.

Under the constraint of a balanced searching strategy, the solution curve can be obtained by the search behaviors. Additionally, the multiple environmental parameters converge to the global optimal solution at time *T*.

Therefore, the source seeking process based on a balanced searching strategy can converge to the target point when one or more source paths exist in the source search space.

5. Experiment

5.1. Simulation Background Field and Source Seeking Parameter Setting

The geomagnetic field is a natural resource of the Earth with abundant characteristic parameters and a natural navigation information source in nature. It is a typical implicit information field because of the complex nonlinear mapping relationship between parameter distribution and geographical location. In this paper, we took the source seeking behaviors of animals using the geomagnetic field, such as migration, homing, and migration, as the prototype to verify the source seeking method.

(1) Parameter setting for carrier movement

The global geomagnetic model WMM2020 was employed to construct the source seeking environment in MATLAB. Considering the accuracy of the actual geomagnetic field sensor, we set the movement step length *L* = 500 m, and each movement step caused a change in the total field strength of about 1~2 nt.

The geomagnetic field is characterized by seven parameters that are perceptible to humans, among which we selected the northward component *Bx*, the eastward component *By*, and the vertical component *Bz*. This parameter combination has been validated in the literature [31] and exhibits favorable searchability.

The characteristic parameters of the geomagnetic environment were selected as $\mathbf{B} = \{B_x, B_y, B_z\}$.

Therefore, we could obtain the form of the normalized G function as follows:

$$G(B,k) = \sum_{i=1}^{3} \left| \frac{B_i^T - B_i^k}{B_i^T - B_i^o} \right| \tag{10}$$

(2) Parameter setting for BSS algorithm

We set the sampling interval as $D_\theta = 30^\circ$, the population size as $N_{pop} = 36$, the propagation operator as $P_b = 2$, and the mutation operator as $P_{mut} = 0.02$.

We chose the intermediate threshold entropy $H_{th} = \frac{H_{high-th} + H_{low-th}}{2}$, and $H \in [\min(H), \max(H)]$, where $\min(H) = 0$, $\max(H) = 2.3$. Here, we set $H_{th} = 1.5$, $H_{high-th} = 1.8$, and $H_{low-th} = 1.2$.

5.2. Comparison of Different Algorithms

Gradient descent algorithm (GDA): Assuming that the mobile carrier can obtain the magnetic field distribution of adjacent elements, the source finding problem described in Equation (5) was transformed into a dynamic multi-objective optimization problem, and the source finding path could be solved using the gradient descent method. This result was denoted as the ideal path.

A timing evolution searching strategy (TES) from the literature [31,32] was employed as a reference algorithm.

Here, we set the target parameter set as (28,126 nT, −3121.3 nT, 54,480 nT).

The above three algorithms were used in the simulation experiment, and the source seeking path was obtained (see Figure 6).

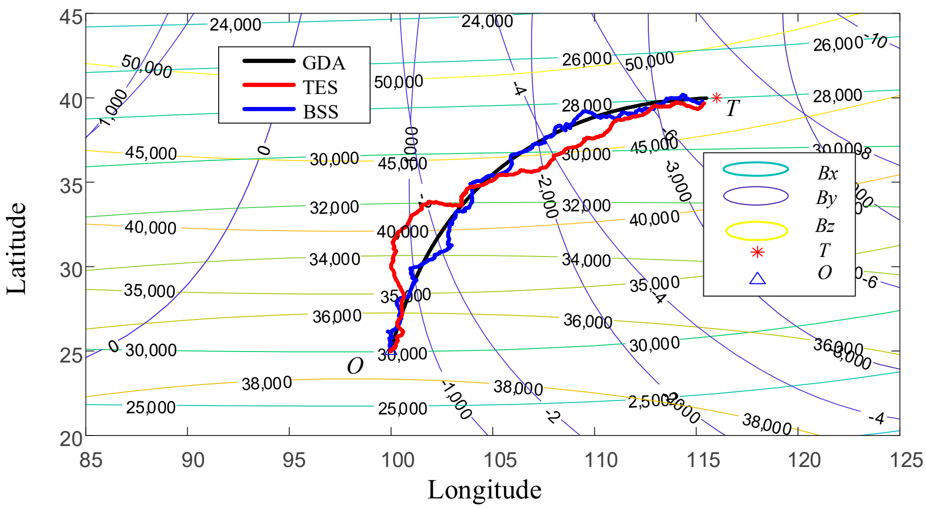

Figure 6. The route of source seeking.

In Figure 6, O is the initial position of the carrier, and T is the position of the source target. The black route is the source seeking route obtained using GDA, which does not exist in the real environment due to the limitation of the sensor perception ability and was only used as an ideal route to test the effectiveness of the algorithm. The red route is the source seeking route obtained by TES. The blue route is the route obtained using the proposed BSS algorithm. It can be seen that all three routes converged to the target point. Compared with the red route, the blue route and the black route were more closely aligned, indicating that the proposed algorithm could better track the solution curve and find the target point.

In the background field, five groups of source searching tasks were randomly set. Furthermore, three algorithms (GDA, TES, and BSS) were used for source seeking. Among them, TES and BSS were used to carry out 1000 source seeking experiments; then, the time taken for source seeking was calculated, as shown in Table 1.

Table 1. Comparison of time taken for source finding by different algorithms.

Algorithm	Source Seeking Task				
	1	**2**	**3**	**4**	**5**
GDA	2787	2266	2406	2490	2413
TES	5828	4868	5036	5168	5110
BSS	5016	4223	4406	4486	4398

As can be seen from Table 1, the source searching time of TES was about 2.0 times that of GDA, while the source seeking time of BSS was about 1.8 times that of GDA. Obviously, the source seeking effect of BSS was better than that of TES.

5.3. Influence of Different Parameters on Algorithm Performance

5.3.1. Analysis of the Influence of H_{th}

Under the condition that ΔH was unchanged, the setting of different behaviors and entropy equilibrium points H_{th} and the influence of sensor noise on source seeking performance were analyzed.

In the simulation, we set ΔH = 0.4; the equilibrium point H_{th} was set from 1.1 to 2.0, and the threshold entropy was set as follows:

$$\begin{cases} H_{low-th} = H_{th} - \Delta H \\ H_{high-th} = H_{th} + \Delta H \end{cases} \quad (11)$$

The other parameters were the same as those in the previous section. Each group of experiments was simulated 1000 times, and we obtained the simulation results shown in Figure 7.

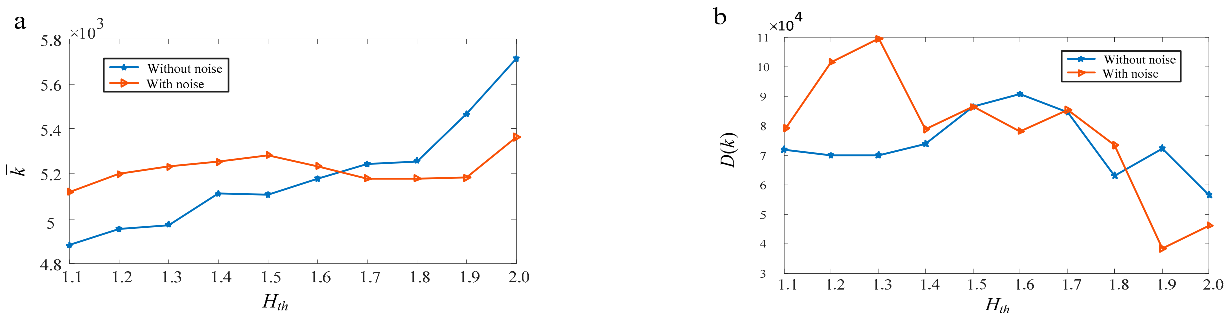

Figure 7. Influence of H_{th} on source seeking performance. (**a**) The average source seeking time; (**b**) the variance in the source seeking time.

In Figure 7, the blue line segment represents the data obtained without noise, and the red line represents the data obtained when there was noise interference. Figure 7a shows the average of the source seeking time, and Figure 7b shows the variance in the source seeking time.

It can be seen that a change in H_{th} had a significant impact on the performance of the algorithm. When H_{th} was small, the algorithm was biased towards exploitation; when this parameter was large, the algorithm was biased towards exploration. Overall, when the algorithm was biased towards exploitation, the time consumption was significantly lower than that when the algorithm was biased towards exploration. In combination with the variance statistics, it can be seen that the time taken for source seeking was shorter and the performance of source seeking was better in the interval $H_{th} \in [1.3, 1.6]$. When $H_{th} \geq 1.6$, the source seeking effect was better than that in the environment without noise.

5.3.2. Analysis of the Influence of ΔH

We set H_{th} = 1.4. According to Equation (10), the threshold entropy ΔH under different conditions could be obtained. The other parameters were the same as those in the previous section. Each group of experiments was simulated 1000 times, and the simulation results are presented in Figure 8.

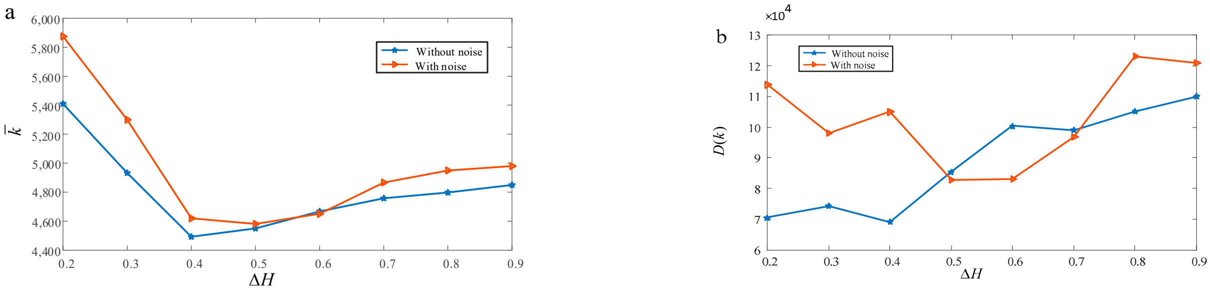

Figure 8. Influence of ΔH on source seeking performance. (**a**) The average source seeking time; (**b**) the variance in the source seeking time.

In Figure 8, the blue line segment represents the data obtained without noise, and the red line represents the data obtained when there was noise interference. Figure 8a shows the average of the source seeking time, and Figure 8b shows the variance in the source seeking time.

It can be seen that a change in ΔH had a significant impact on the performance of the algorithm. When the value of ΔH was small, the algorithm search bias was more binding; when the value of ΔH is large, the weaker the constraint on the search bias of the algorithm. In an implicit information field, the behaviors of source seeking need to explore the environment. However, when the constraint is strong, the algorithm cannot easily explore the surroundings. Therefore, the duration of and variance in source seeking were large. Then, with the continuous increase in ΔH, the constraint of the threshold entropy on the algorithm bias was weakened, the source searching process depended on the search bias regulation of the evolutionary algorithm itself, and the time taken for source seeking tended to be constant. On the whole, the source searching performance was better when $\Delta H \in [0.4, 0.7]$.

5.3.3. Analysis of the Influence of Search Bias Migration Speed

After a thorough discussion, we investigated the impact of the search bias direction and binding force on the source detection performance. Moving forward, we will delve into the influence of the search bias migration rate on the source finding efficacy. In the BSS searching algorithm-based search process, the population migration rate could be represented by the change in distribution entropy H, which was reflected in the increase in distribution probability for new optimal solutions through breeding operations, leading to population migration towards new modes. If the migration rate was too high, the algorithm could become trapped in local convergence, while if it was too low, tracking the solution curve over time became challenging. However, appropriate migration rates could facilitate the rapid movement of populations towards new modes. Among them, the migration rate of the population was determined by the propagating operator P_b.

In order to assess the algorithm's performance, it was imperative to compare its results in a noise-free environment, a noisy environment, and under local extreme conditions. Using digital simulation methods, we selected multiple values for the propagation operator $P_b \in [1, 25]$ and conducted 1000 simulation experiments. We then compared and analyzed the source finding performance in different environments. Figure 9 shows the simulation results.

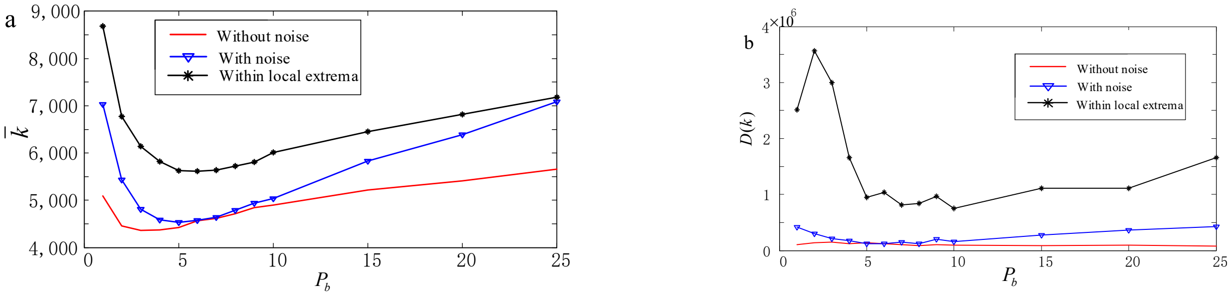

Figure 9. Influence of P_b on source seeking performance. (**a**) The average source seeking time; (**b**) the variance in the source seeking time.

The statistical graph of the source search time is presented in Figure 9, where the red line represents the simulation results under ideal conditions, the blue line represents the simulation results with measurement noise, and the black line represents the simulation results with local extreme values.

Figure 9a shows the average time spent. In all three environments, the source search time decreased first and then increased with the increase in P_b. Figure 9b presents the statistical outcomes of time variance $D(k)$, indicating that the statistical results and trends of both ideal and noisy environments were highly similar. The red and blue lines in Figure 9b represent $D(k) < 0.5 \times 10^6$. In the local extreme environment, the time variance showed a large change. In the range of $P_b \leq 5$, the time variance first increased and then decreased, and the maximum value exceeded 3.5×10^6. In the $P_b > 5$ range, the time variance $D(k)$ was around 1×10^6. If the $P_b \in [1, 4)$ selection was too limited, the migration speed of the population slowed down, resulting in poor information transmission and increasing the time cost of source searching; with the increase in $P_b \in [4, 8)$, the speed of population migration was accelerated, enabling the timely tracking of the solution curve by the population. At this stage, satisfactory results were observed in terms of source search time and consistency. When $P_b \in [8, 25]$ continued to increase, rapid population migration resulted in search behaviors that were overly sensitive to environmental changes. Even minor alterations could significantly impact the population and prolong the time required to locate resources. The time spent on source searching was too long at this point, which in turn reduced the variance in time consumption.

To enhance the efficiency of the source detection algorithm and integrate its performance across all three environments, it is recommended to opt for propagating operator $P_b \in [4, 8]$.

5.3.4. Analysis of the Influence of D_θ

The sampling interval D_θ is the interval at which the feasible search direction θ is sampled, and its value directly impacts the compression of the search space as well as the variety and quantity of search behaviors. The appropriate selection of sampling interval D_θ has a significant impact on the source search performance. This section explores how the sampling interval affects the source search behavior.

From a theoretical perspective, the greater the range of selection for D_θ, the lower the amount of information that can be acquired through search behavior. Meanwhile, the value of D_θ determines the number of species in the population sample. A smaller D_θ leads to a larger variety of search behaviors and an extended duration for exploring all accessible spaces, which is not conducive to effective source searching behavior. However, if the D_θ selection is excessively large, the amount of information obtained from a single-step movement will be significantly less than that contained in the unexplored space. Therefore, it was imperative to investigate the impact of sampling intervals on sourcing behavior.

There exists a causal relationship between the sampling interval D_θ and the species M of population samples, which can be described as $MD_\theta = 2\pi$. Simultaneously, the probability of population sample selection is subject to alteration in response to variations in population size, N_{pop}. Therefore, we bifurcated the population and sample sizes into two categories to conduct distinct analyses of their effects on navigation performance.

(1) Impact of D_θ on source localization performance in the presence of constant

Figure 10 presents the statistical data of 1000 simulation results in three different environments under varying values of D_θ. Specifically, Figure 10a displays the mean source search time $\overline{k}$, while Figure 10b illustrates the variance in source search time $D(k)$.

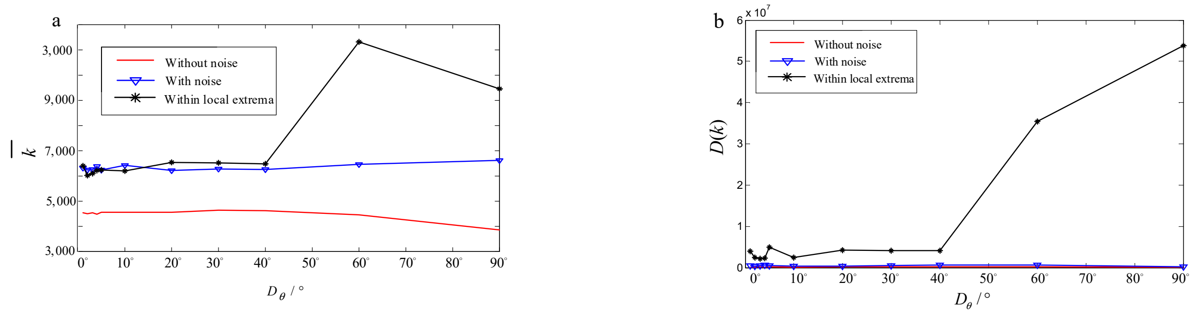

Figure 10. Influence of D_θ on source seeking performance in the presence of constant N_{pop} on source seeking performance in the presence of constant. (**a**) The average source seeking time; (**b**) the variance in source seeking time.

As depicted in Figure 10, for a given population size, distinct D_θ values exhibited varying effects on source detection performance across the three environments.

Compared to a noise-free environment, the time required for source searching in noisy and local minimum environments increased overall from a time-consuming perspective. (1) In a noisy environment, the increase in D_θ did not significantly decrease $\overline{k}$, and the time spent on source finding remained relatively stable. In terms of time variance statistics, the increase in D_θ did not result in significant changes to $\overline{k}$'s time variance, and overall $\overline{k}$ exhibited a relatively stable trend. (2) In the local extreme environment, when between $D_\theta \in [1°, 40°]$, the average time $\overline{k}$ consumed in this environment was close to the $\overline{k}$ in the noise environment. However, significant increases were observed in $D_\theta > 40°$, $\overline{k}$ and $D(k)$ during the period under study. In the context of time variance statistics, $D(k)$ exhibited a significantly higher value in the local extreme value environment compared to the other two environments. Furthermore, there was a significant increase in $D(k)$ following $D_\theta > 40°$.

(2) Influence of D_θ on source search performance with equal sample size

To further investigate the impact of the sampling interval on the algorithm, a statistical analysis was conducted on the source detection time and time variance across different population sizes. $N_{pop} = 4 \times 360^{\circ} / D_\theta$ was established with an initial population size of four samples. As D_θ increased, the population size N_{pop} decreased. The sampling interval fell within the $D_\theta \in [1°, 60°]$ range, and 1000 independent experiments were conducted at multiple value positions to determine the mean time $\overline{k}$ and time variance $D(k)$. The results are presented in Figure 11 below.

As depicted in Figure 11a, the average time consumption $\overline{k}$ exhibited a decreasing trend with the increase in D_θ in both the environment with noise and that without noise. Upon reaching $D_\theta > 30^{\circ}$, $\overline{k}$ converged towards the respective stable values.

As depicted in Figure 11b, after the occurrence of $D_\theta > 40^{\circ}$, the time variance $D(k)$ in the local extreme value environment experienced a sharp increase, leading to a deterioration in the consistency of the sourcing behavior. The performance of the BSS navigation algorithm remained consistent across all three environments.

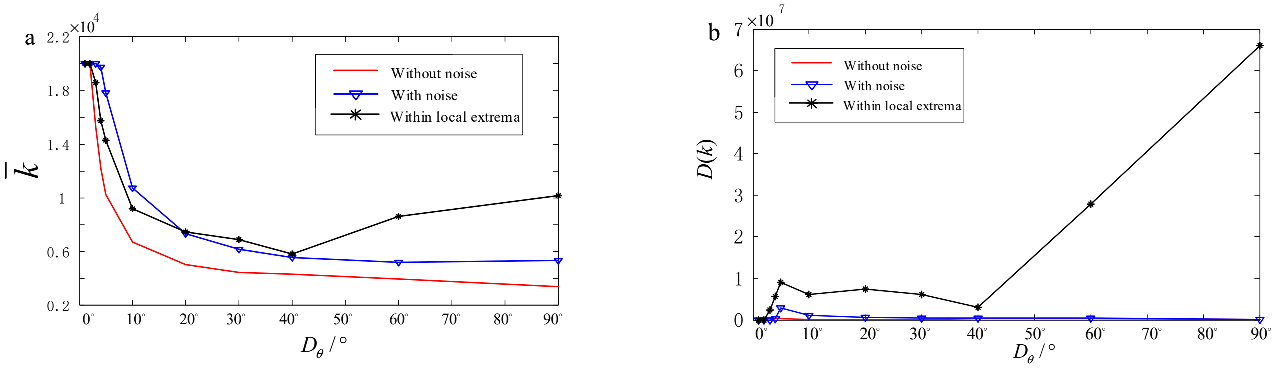

Figure 11. Influence of D_θ on source seeking performance under the same initial sample proportion. (**a**) The average source seeking time; (**b**) the variance in source seeking time.

Based on the above analysis, it can be seen that:

(1) Under the circumstance of an equal population size, altering the sampling interval did not significantly impact the performance of the source detection algorithm.

(2) With equal sample sizes, changes in the sampling intervals significantly impacted the performance of the source finding algorithms. Within a certain range, increasing the sampling interval could greatly reduce the navigation time while diminishing the effect of $D_\theta \in [30°, 90°]$.

Combined with the analysis of the two sets of comparative experiments, it is recommended that $D_\theta \in [30°, 60°]$ be adopted as the sampling interval and $D_\theta = 30°$ be utilized in subsequent studies.

5.3.5. Analysis of the Influence of N_{pop}

From a theoretical perspective, the algorithm's search inertia increases as the population size N_{pop} grows larger, resulting in reduced sensitivity to environmental changes. Conversely, a smaller N_{pop} leads to lower search inertia and greater sensitivity to environmental factors. Therefore, this section will employ numerical simulations to analyze how the population size influenced the source searching behavior.

In a noise-free environment, the impact of different population sizes on the source search performance was simulated and analyzed by measuring the noise levels and extreme local conditions. The statistical results are presented in Figure 12 below.

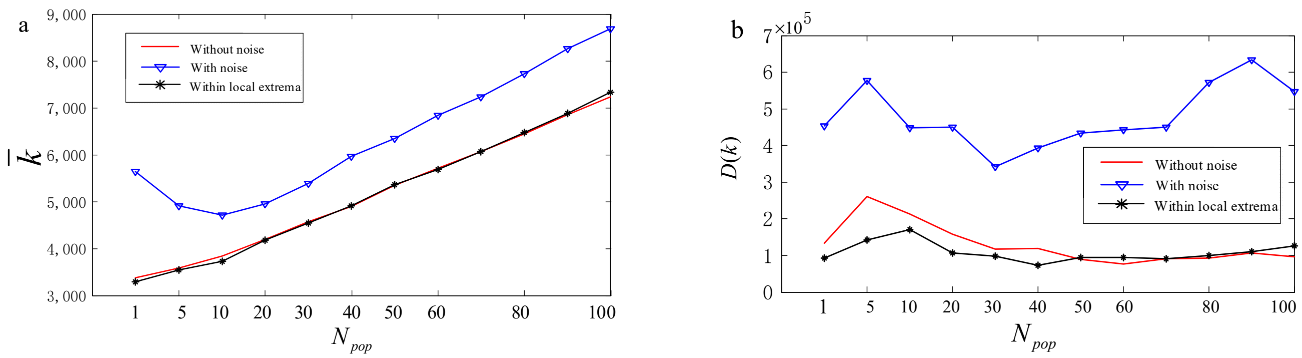

Figure 12. Influence of population size on sourcing performance. (**a**) The average source seeking time; (**b**) the variance in source seeking time.

As depicted in Figure 12, the algorithm exhibited similar source search time characteristics in both noiseless and locally extreme environments under identical sampling intervals, with an increase in population size leading to a corresponding increase in source search time. In a noisy environment, the noise resistance performance was weaker with smaller population sizes, particularly for population size $N_{pop} = 1$, where the time spent in the noisy environment was nearly 2.6 times longer than in the environment without noise. When the population size fell within the range of $N_{pop} \in [20, 70]$, the variance in sourcing time across the three environments exhibited a high degree of similarity.

Furthermore, the algorithm's ability to suppress noise was evaluated and compared in both a noisy environment and a noise-free environment in terms of source localization time. The simulation results are presented in Figure 13 below.

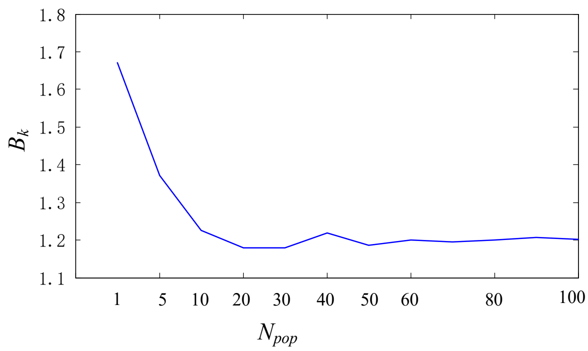

Figure 13. Influence of population size on the noise resistance of the algorithm.

In Figure 13, the vertical axis is the ratio B_k of the mean time $\overline{k}$ in the environment with noise to the mean time $\overline{k}$ in the environment without noise. The closer the ratio approached to 1, the less susceptible the source finding algorithm was to population-size-induced noise.

As depicted in Figure 13, $N_{pop} = 20$ served as the conspicuous demarcation point, and the mean time consumption decreased rapidly with an increase in population size within the $N_{pop} = 20$ range. In contrast, the average time consumption remained relatively stable at approximately 1.2 within the $N_{pop} \geq 20$ interval.

In general, the size of the population had a certain impact on the source finding performance. Specifically, when the population size reached a certain level, the time required for source finding was positively proportional to the population size, while the noise resistance performance improved. Therefore, it is recommended to choose between $N_{pop} \in [20, 70]$.

6. Conclusions

This paper studied an implicit information field source seeking method from the angle of search behavior bias. A balanced searching strategy was proposed, which introduced the search path into the solution of the source seeking problem, constructed an evolutionary population with feasible search behaviors as the individual, introduced the metric of the search bias of distribution entropy, and designed a balanced searching strategy combining evolutionary optimization and the source seeking process. Through the performance analysis and simulation experiments, the validity and rationality of the method were verified. In this research, insufficient attention was given to implicit information fields beyond magnetic fields. Therefore, hidden information fields such as odor and gravity will be compared in order to identify their sources. Additionally, the algorithm still exhibited mutual coupling among certain parameters, necessitating further analysis and research into the impact of additional parameter settings on the algorithm's source finding performance.

Author Contributions: Conceptualization and writing (original draft preparation), K.L.; methodology and software, K.L.; validation and formal analysis, Y.B.; investigation and writing (review and editing), J.L.; Q.Z. analyzed the experimental data and results, realized data visualization, and wrote the first draft of the paper. All authors have read and agreed to the published version of the manuscript.

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