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Game Models for Ordering and Channel Decisions of New and Differentiated Remanufactured Products in a Closed-Loop Supply Chain with Sales Efforts

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Abstract: Environmental responsibility and economic benefits have promoted the development of closed-loop supply chains (CLSCs), and shortages and channels are considered to be two important issues in a CLSC. This paper explores the ordering and channel decisions in a CLSC with new and differentiated remanufactured products; considers the price and sales-effort-dependent demands, as well as the proportion of emergency orders determined by emergency order costs and backorder losses; and establishes integrated and decentralized CLSC game models. We introduce a stochastic sales effort, which affects two types of products. The numerical results show that sales effort and the order quantity of new and remanufactured products exhibit concave and convex functions, respectively. The upper limit of sales effort has a greater impact on supply chain decisions. High sales efforts can serve as a means of coordinating dispersed supply chains. Moreover, in different cases, the decisions of an integrated channel are better than those of a decentralized channel. Finally, whether the supply chain adopts an emergency order strategy depends on the relative cost of emergency orders and out-of-stock costs. According to this research, some management insights are also provided.

1. Introduction

In recent years, the quantity of end-of-life products has increased sharply, resulting in environmental pressures and a waste of resources, which has led to the development of closed-loop supply chains (CLSCs). Recycling and remanufacturing through a CLSC not only help reduce environmental pollution but also enable resource recycling and saving and promote a circular economy [1]. Remanufacturing after the recycling of waste tires, electronic products, and automobile engines can effectively reduce the use of raw materials by more than 30% and energy consumption by more than 15%. Apple reduced their carbon emissions per dollar of revenue by 15.4% through remanufacturing; Canon and Xerox saved millions of dollars by recycling and remanufacturing [2]. Both economic and noneconomic factors promote the implementation of recycling and remanufacturing, while economic benefits may be the key factor that attracts enterprises [3]. Therefore, research determining how to achieve higher efficiency and utility in a remanufacturing closed-loop supply chain has high practical value.

In a CLSC, in on addition producing new products, manufacturers carry out remanufacturing using remanufacturable end-of-life products. Some consumers strongly agree that they tend to buy socially responsible and environmentally friendly products [4]. However, most consumers are keptical and take a wait-and-see approach with remanufactured products. Sales efforts, such as advertising, commitment, and promotion, can increase the demand for remanufactured products by changing consumers' degree of cognition regarding remanufactured products and helping consumers identify the environmental value of

remanufactured products [5]. Thus, sales efforts are crucial for increasing the market share of remanufactured products. It is worth noting that the sales efforts for remanufactured products also affect the demand for new products due to the cannibalization effect [1]. Based on the above, one of the innovations of this study is exploreingclosed-loop supply chain decisions where the demand for two types of products is simultaneously influenced by sales efforts for remanufactured products.

Due to demand uncertainty and information asymmetry, a CLSC is more likely to experience shortages. In a CLSC, remanufacturing begins with the recovery of end-of-life products, which is full of uncertainty. Furthermore, consumers' preference for the two types of products and sales efforts for remanufactured products increase the uncertainty of demand. All of the above emphasize the shortage issues in a CLSC and are reasons to avoid profit and nonprofit losses caused by shortages. There is little research on closed-loop supply chains that considers emergency order strategies. Therefore, in this study, we explored the ordering and channel decisions of CLSC members under different supply chain structures, in which the sales effort of remanufactured products affect the market demand of remanufactured and new products. Furthermore, we considered models without and with emergency orders to deal with the shortage problem. In models without emergency orders, backlogged demands are lost, while in models with emergency orders, backlogged demands can be met through these emergency orders but in a certain proportion, which is related to the cost of the emergency order and the cost of the out-of-stock loss.

This study explored a two-echelon CLSC with one manufacturer and one retailer, where the market demands are related to the retail price and sales efforts. Game theoretic models of an integrated CLSC and a decentralized CLSC without and with emergency orders were established to answer the following questions:

What are the optimal order quantities for the two types of products in different channel structures?

What is the influence of sales effort and the costs of emergency order and losses of out-of-stock on channel strategies and profits?

Does an emergency order have a positive impact on order quantity and profit in a CLSC with sales effort?

The remainder of this paper is organized as follows: Section 2 reviews the related literature. In Section 3, the model is described. Section 4 presents and analyzes different CLSC models. Section 5 provides numerical examples to examine the propositions and optimal results, and the impact of sales effort and emergency order strategy are discussed. Section 6 concludes the paper and outlines future research directions. Some implications are also discussed in this section.

2. Literature Review

This paper presents a literature review of four areas: competition between new products and remanufactured products, sales efforts for remanufactured products, emergency order, and centralized and decentralized CLSC structures.

2.1. Competition between New Products and Remanufactured Products

Some studies in the literature assume that in a CLSC, remanufactured and new products are not distinguishable [2,6], while others hold that remanufactured products are different from new products [7,8]. In either case, especially the latter, according to product cannibalization, remanufacturing products leads to reductions in the sales volume, sales revenue, or market share of new products. Considering the competition between remanufactured products and new products, Jian et al. [9] studied how manufacturers and remanufacturers can benefit each other, taking into account patent protection and carbon emissions. Dai et al. [10] analyzed the impact of different CSR ratios on product pricing and profits from the perspective of product differential pricing. Souza [1] found that the remanufacturing decision of products brings about two results: a market expansion effect, in which remanufactured products help to expand market share, and the cannibalization

effect, in which remanufactured products entering the market reduce the sales of new products of the same type or model. Based on the cannibalization effect of remanufactured products, we analyzed the supply chain management of the impact of sales effort on the demand for remanufactured and new products.

2.2. Sales Efforts for Remanufactured Products

In practice, consumers may have different preferences for new products and remanufactured products. For example, some consumers are psychologically biased against remanufactured products and will not buy them no matter how cheap they are [11]. Michaud and Llerena [12] found that consumers tend to develop a friendly attitude towards remanufactured products when they are informed about the significance of remanufactured products for environmental protection. Consequently, implementing sales efforts becomes imperative to promote consumers' perception of remanufactured products. The existing literature confirms the important role of sales efforts in improving consumers' acceptance of remanufactured products [3]. In recent years, Mondal and Giri [4] constructed a two-cycle, green, closed-loop supply chain model in which the market demand depends on selling price, green level, and sales effort and studied the effects of green innovation, sales effort, and waste product recovery rate on CLSCs. Yang et al. [13] discussed how the level of sales effort of retailers affects the economic performance and sustainability of remanufacturing. Sane Zerang et al. [14] studied the effects of sales effort and recovery rate on decision variables, demonstrating the superiority of a manufacturer-led closed-loop supply chain. Taleizadeh et al. [15] explored pricing and reverse channel selection decisions in a CLSC, wherein the demand depended on retail prices and advertising efforts. Khorshidvand et al. [16] developed a new demand function based on price elasticity, cross-sensitivity, product green quality, and advertising level, providing a two-stage approach to modeling and resolving issues within a sustainable CLSC. Ma et al. [17] studied the combined effects of marketing efforts and recycling rates on the profitability of retailers, manufacturers, and CLSCs. Zhang et al. [18] considered that manufacturers pay attention to the fairness of profit distribution when making sales efforts in a CLSC.

The majority of the aforementioned literature assumes that sales effort is a definite variable. In this study, considering the sales effort varies with industry dynamics and time changes, we assumed that it is a random variable following a uniform distribution. The stochastic sales effort also affects the demands for two types of products. Specifically, the demand for remanufactured products increases with the deepening of the sales effort. In contrast, the demand for new products decreases due to the cannibalizing effect.

2.3. Emergency Order

In situations where the inventory is out of stock or in some special circumstances, emergency orders can be used as a supplement to the regular order, which can reduce the losses and costs [19]. Models with emergency orders have been studied extensively [20,21]. Zhang et al. [22] compared two different approaches to deal with excess demands. The first involves excess demand losses and shortage penalties, while the second is that excess demand can be met by emergency orders. Which way is better depends on the comparison between emergency ordering cost and the sales price plus the shortage penalty under the demand loss system. When the possibility of an emergency order exists, it is necessary to consider emergency ordering costs. Poormoaied and Demirci [23] studied an inventory strategy described by a continuous-time Markov chain model, in which emergency orders could be issued based on stock levels prior to the disruption. The results showed that, particularly in instances characterized by high costs of sales losses, long break periods, and a low percentage of supplier availability, providing emergency ordering opportunities in the event of disruption could yield significant cost savings. Notably, emergency orders affect the social and environmental sustainability goals of the supply chain, profits are affected in a CLSC for remanufactured products made from recycled waste products,

and the demand for remanufactured products is not independent of the demand for new products. Regrettably, there is little literature on this aspect.

2.4. CLSC and CLSC Structure

The closed-loop supply chain is of great significance to society, the environment, and the economy. Mondal and Giri [3] stated that the return of waste products helps businesses in many ways, such as preserving resources for future use, reducing environmental hazards, understanding the gap between expected and actual performance, the practical utilization of product characteristics, and building proactive relationships with consumers. Ding and Zhu [24] demonstrated that companies can reduce manufacturing costs by about 40–65% through closed-loop supply chain management. Johari and Hosseini-Motlag [25] contended that establishing a CLSC can reduce the demand for raw materials, minimize negative impacts on the environment, encourage customers to take back used products, create new employment opportunities, and achieve economic, environmental, and social goals. Li et al. [26] showed that the stability of the supply chain system correlates positively with increased recovery rates.

Many researchers have examined integrated and decentralized game structures in CLSC from various perspectives, for example, recycling decisions [27], pricing decisions [3], and ordering strategies [28]. Maiti and Giri [29] explored the effect of channel power structures on the pricing strategy, collection rate, quality, and profits, showing that integrated policies are always better. In this study, we focused on ordering decisions. He et al. [30] investigated three possible channel structures for a manufacturer with government subsidies. Solaleh et al. [31] studied the behavior of a two-stage decentralized channel within a CLSC. Zhou et al. [32] investigated the channel leadership and performance in a three-echelon CLSC.

2.5. Game Theory in CLSC

Game theory (GT) has been widely used to solve the operational problem of CLSCs [33]. Shekarian [34] conducted a review and analysis of articles focusing on CLSC models grounded in GT, revealing that almost all pricing and coordination problems have been established using GT. In addition, Lyu et al. [35] explored the cooperation problem among manufacturers by establishing a mathematical model of a decentralized closed-loop supply chain and applying game theory methods. Lee [36] established six different Stackelberg game models to identify the optimal choice for the market leadership in a CLSC. Liu et al. [37] constructed a differential game model to analyze the impact of goodwill on decision-making processes within closed-loop supply chains.

Table 1 presents a comparison between this study and the relevant literature.

Table 1. A comparison of this study with previous studies.

Authors (Year)	New and Remanufactured Products	Sales Efforts	Emergency Order	CLSC
Modak et al. [2]	No difference	×	×	√
Gu et al. [6]	No difference	×	×	√
Jian et al. [9]	Different	×	×	√
Dai et al. [10]	Different	×	×	√
Taleizadeh et al. [15]	No difference	A fixed value	×	√
Behrooz et al. [16]	Different	A fixed value	×	√
Poormoaiedand Demirci [19]	-	×	√	×
Zhang et al. [22]	-	×	√	×
This paper	√	A random variable	√	√

Notes: × represents that the study did not include the corresponding point, while √ represents that it did include it.

3. Problem Definition

We conside a CLSC composed of one retailer and one hybrid manufacturer that provides remanufactured products using end-of-life products and manufactures new products from raw materials. We assumed that all the recycled products can be remanufactured. The manufacturer provides two types of products with unit wholesale prices, denoted as w_n, w_r ($w_n > w_r$), to the retailer. The retailer is tasked with selling both types of products at different prices p_n, p_r ($p_n > p_r$), based on which the remanufactured products are clearly different from the new product [1,2]. In order to enhance the perception of remanufactured products and positively influence the market demand, the retailer pays for a certain amount of sales activities, such as advertising, promotion, and service commitment. Let s denote the level of a composite index of sales effort input. The retailer's investment in the level of sales effort s is assumed to follow a convex function, defined as $\frac{1}{2}gs^2$ [3]. The sales effort s may vary across industries and consumer groups and evolve over time. Therefore, we assume s to be a random variable following a uniform distribution in the range of [c, d] ($0 = < c < d$, where c is the lowest limit, and d is the upper limit. If the sales effort surpasses d, the cost paid may be greater than the benefit gained, leading to no economic gain). The effect of the sales effort on demand is changing the marketing share of the two types of products. In addition, both demands are functions of individual prices. The demand function for each product type is expressed as:

$$D_n = a\left(1 - b\left(1 - e^{-s\theta}\right) - \alpha\right) - \xi p_n$$

$$D_r = a\left(b\left(1 - e^{-s\theta}\right) + \alpha\right) - \xi p_r$$

where index n stands for the new product, index r refers to the remanufactured product, a represents the initial market potential, and ξ is the sensitivity of demand to price. $b\left(1 - e^{-s\theta}\right)$ symbolizes the increased demand share of remanufactured product from the sales effort. θ and b ($0 < b < 1$) are the influence coefficients of the sales effort on demands. α is a positive parameter measuring the initial quota of the demand of the remanufactured product when the sales effort is 0.

In manufacturing, the unit production cost of a new product, denoted as c_n, is more costly than the unit remanufacturing cost of a used product, denoted as c_r, i.e., $c_n > c_r$. In retailing, the retailer decides how much to order for both types of products according to market demand, defined as Q_n and Q_r. If the quantity ordered exceeds the demand, the remainder is disposed of as a salvage value, recorded as V_n, V_r ($V_n > V_r$). If the order quantity is lower than the demand, the unmet demand is lost in the case that emergency orders are not allowed, resulting in out-of-stock losses, denoted as B_n and B_r, ($B_n > B_r$), respectively. When emergency orders are allowed, a proportion of such orders shall be determined according to the relative relationship between the emergency order cost and the loss incurred due to shortages. Much of the literature posits that the cost of emergency order is borne by retailers [3]. However, it is more possibly borne by manufacturers. Therefore, we analyzed the impact of the emergency order cost borne by the manufacturer. Costs associated with emergency orders are recorded as k_n, k_r ($k_n < k_r$), respectively. The reason for the high cost of emergency orders for the remanufactured product lies in the uncertainty of the recovery process. Table 2 summarizes the major notations in our following model development.

Table 2. Model parameters and decision variables.

Model Parameters	
a	Initial market potential
ξ	Price elasticity coefficient of demand

Table 2. *Cont.*

Model Parameters	
θ/b	Influence coefficients of sales effort on demand
α	Initial quota of remanufactured product
s	Sales effort level for remanufactured product
c	Lower bound on sales effort
d	Upper bound on sales effort
g	Sales effort investment coefficient
p_n, p_r	Unit selling prices of new and remanufactured products
c_n, c_r	Unit costs of production or remanufacturing of new and remanufactured products
B_n, B_r	Unit shortage costs of new and remanufactured products
V_n, V_r	Unit salvage values of new and remanufactured products
k_n, k_r	Unit extra costs of the emergency order
Decision variables:	
w_n, w_r	Unit wholesale prices of new and remanufactured products
Q_n, Q_r	Order quantities of new and remanufactured products

4. Model Formulation and Solution

In this section, the CLSC and its members' profit functions are formulated, and optimal decisions regarding Q_n, Q_r, w_n, and w_r in each scenario are derived. The concavity of profit functions is proven, and closed-form relationships for decision variables Q_n and Q_r are extracted in both decentralized and integrated scenarios under both situations without and with emergency orders.

4.1. Models under Different Channel Power Structures without Emergency Order
4.1.1. Integrated Scenario

In the integrated scenario, in order to maximize the benefit of the CLSC, the members of the CLSC make decisions under the promotion of the central decision maker. The profit function of the integrated scenario model is

$$\pi_I^N(Q_n, Q_r) = p_n(D_n \wedge Q_n) + V_n(Q_n - D_n)^+ - B_n(D_n - Q_n)^+ - c_n Q_n$$
$$+ p_r(D_r \wedge Q_r) + V_r(Q_r - D_r)^+ - B_r(D_r - Q_r)^+ - c_r Q_r - \frac{1}{2}gs^2$$

where $x \wedge y = min\{x, y\}$ and $(x - y)^+ = max\{x - y, 0\}$

According to the properties of the demand function, the expected profit function is described as follows:

$$E_I^N(\Pi(Q_n, Q_r)) = p_n(\int_{z_n}^{d} D_n f(s)ds + \int_{c}^{z_n} Q_n f(s)ds) + V_n\int_{z_n}^{d}(Q_n - D_n)f(s)ds - B_n\int_{c}^{z_n}(D_n - Q_n)f(s)ds - c_n Q_n$$
$$+ p_r(\int_{c}^{z_r} D_r f(s)ds + \int_{z_r}^{d} Q_r f(s)ds) + V_r\int_{c}^{z_r}(Q_r - D_r)f(s)ds - B_r\int_{z_r}^{d}(D_r - Q_r)f(s)ds - c_r Q_r - \frac{1}{8}g(d+c)^2$$

where $z_n = \dfrac{ln[\frac{ab}{-a+ab+a\alpha+\xi p_n+Q_n}]}{\theta}$; $z_r = \dfrac{ln[\frac{ab}{ab+a\alpha-\xi p_r-Q_r}]}{\theta}$

Proposition 1. *Under the integrated scenario without emergency orders, the expected profit function $E_I^N(\prod(Q_n, Q_r))$ is concave with respect to Q_n and Q_r; hence, there is a unique optimal combination of decision variables $(Q^{NI*}{}_n, Q^{NI*}{}_r)$ that maximizes $E_I^N(\prod(Q_n, Q_r))$, and the optimal production quantities of the two types of products are*

$$Q^{NI*}{}_n = a + ab\left(-1 + e^{-\frac{\theta(cB_n + (-c+d)c_n + cp_n - dV_n)}{B_n + p_n - V_n}}\right) - a\alpha - \xi p_n \tag{1}$$

$$Q^{NI*}{}_r = a\left(b - be^{-\frac{\theta(dB_r + (c-d)c_r + dp_r - cV_r)}{B_r + p_r - V_r}} + \alpha\right) - \xi p_r \tag{2}$$

Proof. To prove concavity, the Hessian matrix of $E_I^N(\prod(Q_n, Q_r))$ is calculated

$$\begin{bmatrix} \frac{B_n+p_n-V_n}{(c-d)\theta(a(-1+b+\alpha)+\xi p_n+Q_n)} & 0 \\ 0 & -\frac{B_r+p_r-V_r}{(c-d)\theta(-a(b+\alpha)+\xi p_r+Q_r)} \end{bmatrix}$$

It is obvious that $(a(-1+b+\alpha)+\xi p_n+Q_n) = abe^{-\theta s} > 0$ and $(-a(b+\alpha)+\xi p_r+Q_r) = -abe^{-\theta s} < 0$. Since all parameters are positive, the first principal minor $\frac{B_n+p_n-V_n}{(c-d)\theta(a(-1+b)+\xi p_n+Q_{(n)})}$ is negative, and the second principal minor $\frac{B_n+p_n-V_n}{(c-d)\theta(a(-1+b+\alpha)+\xi p_n+Q_n)} *$ $\frac{B_r+p_r-V_r}{(c-d)\theta(-a(b+\alpha)+\xi p_r+Q_r)}$ is positive. Thus, the Hessian matrix is negative definite, and $E_I^N(\prod(Q_n, Q_r))$ is concave for all decision variables. Solving $\frac{\partial E_I^N(\prod(Q_n, Q_r))}{\partial Q_n} = 0$ and $\frac{\partial E_I^N(\prod(Q_n, Q_r))}{\partial Q_r} = 0$, Equations (1) and (2) are obtained. $\square$

Corollary 1. *In the integrated model, the optimal order quantities for new products Q^{NI*}_n are convex and the optimal order quantities for remanufactured products Q^{NI*}_r are concave with respect to the lower and upper bound of sales efforts c and d.*

Proof. Since $\frac{\partial^2 Q^{NI*}_n}{\partial c^2} = \frac{abe^{-\frac{\theta(c(B_n+p_n)-dV_n+(-c+d)c_n)}{B_n+p_n-V_n}}\theta^2(B_n+p_n-c_n)^2}{(B_n+p_n-V_n)^2} > 0; \frac{\partial^2 Q^{NI*}_n}{\partial d^2} = \frac{abe^{-\frac{\theta(cB_n+cp_n-dV_n+(-c+d)c_n)}{B_n+p_n-V_n}}\theta^2(-V_n+c_n)^2}{(B_n+p_n-V_n)^2} > 0; \frac{\partial^2 Q^{NI*}_r}{\partial c^2} = -\frac{abe^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)c_r)}{B_r+p_r-V_r}}\theta^2(V_r-c_r)^2}{(B_r+p_r-V_r)^2} < 0; \frac{\partial^2 Q^{NI*}_r}{\partial d^2} = -\frac{abe^{-\frac{\theta(dB_r+(c-d)c_r+dp_r-cV_r)}{B_r+p_r-V_r}}\theta^2(B_r-c_r+p_r)^2}{(B_r+p_r-V_r)^2} < 0$ then Q^{NI*}_n is convex, and Q^{NI*}_r is concave in c and d. $\square$

4.1.2. Decentralized Scenario

In the decentralized CLSC, each channel member makes decisions independently to maximize their individual profits. The manufacturer acts as a Stackelberg leader and determines the wholesale price. The retailer, guided by its own profit function, determines order quantities based on the manufacturer's decisions. Backward induction is adopted in pursuit of equilibrium decisions.

The retailer's profit in a general form can be expressed as

$$\pi_R^N(Q_n, Q_r) = p_n(D_n \wedge Q_n) + V_n(Q_n - D_n)^+ - B_n(D_n - Q_n)^+ - w_n Q_n$$
$$+p_r(D_r \wedge Q_r) + V_r(Q_r - D_r)^+ - w_r Q_r - B_r(D_r - Q_r)^+ - \frac{1}{2}gs^2$$

Similarly, the expected profit function of the retailer is described as follows:

$$E_R^N(\Pi(Q_n, Q_r)) = p_n(\int_{z_n}^d D_n f(s)ds + \int_c^{z_n} Q_n f(s)ds) + V_n \int_{z_n}^d (Q_n - D_n)f(s)ds - B_n \int_c^{z_n} (D_n - Q_n)f(s)ds - w_n Q_n$$
$$+p_r(\int_c^{z_r} D_r f(s)ds + \int_{z_r}^d Q_r f(s)ds) + V_r \int_c^{z_r} (Q_r - D_r))f(s)ds - B_r \int_{z_r}^d (D_r - Q_r)f(s)ds - w_r Q_r - \frac{1}{8}g(d+c)^2$$

Proposition 2. *Under the decentralized scenario without emergency orders, the expected profit function $E_R^N(\prod(Q_n, Q_r))$ is concave with respect to Q_n and Q_r; hence, there is a unique optimal combination of decision variables (Q^{NR*}_n, Q^{NR*}_r) that maximizes $E_R^N(\prod(Q_n, Q_r))$, and the optimal production quantities of the two products are*

$$Q^{NR*}_n = a + ab\left(-1 + e^{-\frac{\theta(cB_n+(-c+d)w_n+cp_n-dV_n)}{B_n+p_n-V_n}}\right) - a\alpha - \xi p_n \tag{3}$$

$$Q^{NR*}_r = a\left(b - be^{-\frac{\theta(dB_r+(c-d)w_r+dp_r-cV_r)}{B_r+p_r-V_r}} + \alpha\right) - \xi p_r \tag{4}$$

Proof. To prove concavity, the Hessian matrix of $E_R^N(\prod(Q_n, Q_r))$ is calculated as follows:

$$\begin{bmatrix} \frac{B_n+p_n-V_n}{(c-d)\theta(a(-1+b+\alpha)+\xi p_n+Q_n)} & 0 \\ 0 & -\frac{B_r+p_r-V_r}{(c-d)\theta(-a(b+\alpha)+\xi p_r+Q_r)} \end{bmatrix}$$

The proof process is the same as for Proposition **1**. $\square$

Corollary 2. *In the decentralized model, the optimal order quantities $Q^{NR*}{}_n$ are convex and $Q^{NR*}{}_r$ is concave with respect to the lower and upper bound of sales efforts c and d.*

Proof. Since $\frac{\partial^2 Q^{NI*}{}_n}{\partial c^2} = \frac{abe^{-\frac{\theta(c(B_n+p_n)-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}}\theta^2(B_n+p_n-w_n)^2}{(B_n+p_n-V_n)^2} > 0; \frac{\partial^2 Q^{NI*}{}_n}{\partial d^2} = \frac{abe^{-\frac{\theta(cB_n+cp_n-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}}\theta^2(-V_n+w_n)^2}{(B_n+p_n-V_n)^2} > 0; \frac{\partial^2 Q^{NI*}{}_r}{\partial c^2} = -\frac{abe^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)w_r)}{B_r+p_r-V_r}}\theta^2(V_r-w_r)^2}{(B_r+p_r-V_r)^2} < 0; \frac{\partial^2 Q^{NI*}{}_r}{\partial d^2} = -\frac{abe^{-\frac{\theta(dB_r+(c-d)w_r+dp_r-cV_r)}{B_r+p_r-V_r}}\theta^2(B_r-w_r+p_r)^2}{(B_r+p_r-V_r)^2} < 0.$ Then, the optimal order quantity $Q^{NR*}{}_n$ is convex and $Q^{NR*}{}_r$ is concave in c and d. $\square$

The manufacturer's profit function is formulated as

$$E_M^N(\Pi(w_n, w_r)) = w_n Q_n - c_n Q_n + p_r Q_r - c_r Q_r$$

Proposition 3. *Under the decentralized scenario without emergency orders, the expected profit function $E_M^N(\prod(w_n, w_r))$ is concave with respect to w_n and w_r if $2(B_n + p_n - V_n) - (d - c)\theta(-c_n + w_n) > 0$.*

Proof. To prove concavity, the Hessian matrix of $E_M^N(\prod(w_n, w_r))$ is calculated as follows:

$$\begin{bmatrix} \frac{ab(c-d)e^{-\frac{\theta(c(B_n+p_n)-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}}\theta(X_n+(c-d)\theta(-c_n+w_n))}{(B_n+p_n-V_n)^2} & 0 \\ 0 & \frac{ab(c-d)e^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)w_r)}{B_r+p_r-V_r}}\theta(X_r+(c-d)\theta(c_r-w_r))}{(B_r+p_r-V_r)^2} \end{bmatrix}$$

where $X_i = 2B_i + 2p_i - 2V_i (i = n, r)$.

The first principal minor is negative, and the second principal minor is positive if

$$2(B_n + p_n - V_n) - (d - c)\theta(-c_n + w_n) > 0 \tag{5}$$

By holding Condition (5), the Hessian matrix is negative definite and $E_M^N(\prod(w_n, w_r))$ is concave in both w_n and w_r. Hence, there is a unique optimal combination of decision variables $(w^{N*}{}_n, w^{N*}{}_r)$ that maximizes $E_M^N(\prod(w_n, w_r))$. To obtain the optimal values of the decision variables, $\frac{\partial E_M^N(\prod(w_n, w_r))}{\partial w_n} = 0$ and $\frac{\partial E_M^N(\prod(w_n, w_r))}{\partial w_r} = 0$ need to be solved, that is,

$$a + ab\left(-1 + e^{-\frac{\theta(cB_n+cp_n-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}}\right) - a\alpha - \xi p_n - \frac{ab(-c+d)e^{-\frac{\theta(cB_n+cp_n-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}}\theta(-c_n+w_n)}{B_n+p_n-V_n} = 0 \tag{6}$$

$$a\left(b - be^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)w_r)}{B_r+p_r-V_r}} + \alpha\right) - \xi p_r + \frac{ab(c-d)e^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)w_r)}{B_r+p_r-V_r}}\theta(-c_r+w_r)}{B_r+p_r-V_r} = 0 \tag{7}$$

As Equations (6) and (7) are transcendental equations, a numerical method needs to be used. We discuss the results using numerical examples in Section 5. $\square$

Corollary 3. *The order quantity of the two types of products in the centralized channel is higher than that in the decentralized channel:* $Q^{NI*}_n > Q^{NR*}_n; Q^{NI*}_r > Q^{NR*}_r.$

Proof. $\quad Q^{NI*}_n - Q^{NR*}_n = ab\left(e^{-\frac{\theta(cB_n+(-c+d)c_n+cp_n-dV_n)}{B_n+p_n-V_n}} - e^{-\frac{\theta(c(B_n+p_n)-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}}\right) > 0;$

$Q^{NI*}_r - Q^{NR*}_r = ab\left(-e^{-\frac{\theta(dB_r+(c-d)c_r+dp_r-cV_r)}{B_r+p_r-V_r}} + e^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)w_r)}{B_r+p_r-V_r}}\right) > 0.\ \square$

4.2. Models under Different Channel Power Structures with Emergency Orders

In scenarios involving emergency orders, not all the unmet demands can be addressed by emergency ordering. When the cost of emergency ordering is high, emergency ordering is uneconomical. If the loss incurred due to stockouts is significant, the company permits emergency ordering. Therefore, we introduce the concept of the ratio between the emergency order costs and the backorder losses to determine the proportion of emergency orders, denoted as β_i:

$$\beta_i = 1 - \frac{k_i}{B_i}\ (k_i < B_i \quad i = n,r)$$

4.2.1. Integrated Scenario

When emergency ordering is allowed, a part of the demand exceeding the regular order quantity is satisfied by the emergency order. The CLSC obtains additional revenue from the emergency order, concurrently reducing backorder losses. The profit function of the integrated scenario model is

$$\pi^Y_I(Q_n, Q_r) = p_n(D_n \wedge Q_n) + V_n(Q_n - D_n)^+ + \beta_n(p_n - k_n)(D_n - Q_n)^+ - B_n(1-\beta_n)(D_n - Q_n)^+ - c_n(Q_n + \beta_n(D_n - Q_n)^+)$$
$$+ p_r(D_r \wedge Q_r) + V_r(Q_r - D_r)^+ + (p_r - k_r)\beta_r(D_r - Q_r)^+ - B_r(1-\beta_r)(D_r - Q_r)^+ - c_r(Q_r + \beta_r(D_r - Q_r)^+) - \tfrac{1}{2}gs^2$$

The expected profit function is described as follows:

$$E^Y_I(\Pi(Q_n,Q_r)) = p_n(\int_{z_n}^d D_nf(s)ds + \int_c^{z_n} Q_nf(s)ds) + V_n\int_c^{z_n}(Q_n - D_n)f(s)ds + (p_n - k_n)\int_{z_n}^{z_n}\beta_n(D_n - Q_n)f(s)ds - B_n\int_c^{z_n}(1-\beta_r)(D_n - Q_n)f(s)ds - c_n(Q_n + \int_c^{z_n}\beta_n(D_n - Q_n)f(s)ds)$$
$$+ p_r(\int_c^{z_r} D_rf(s)ds + \int_{z_r}^d Q_rf(s)ds) + V_r\int_c^{z_r}(Q_r - D_r)f(s)ds + (p_r - k_r)\int_{z_r}^d\beta_r(D_r - Q_r)f(s)ds - B_r\int_{z_r}^d(1-\beta_r)(D_r - Q_r)f(s)ds - c_r(Q_r + \int_{z_r}^d\beta_r(D_r - Q_r)f(s)ds) - \tfrac{1}{8}g(d+c)^2$$

Proposition 4. *Under the integrated scenario with emergency orders, the expected profit function* $E^Y_I(\prod(Q_n,Q_r))$ *is concave with respect to* Q_n *and* Q_r *if* $k_i(B_i + p_i) - B_iV_i > 0$ *; hence, there is a unique optimal combination of decision variables* (Q^{YI*}_n, Q^{YI*}_r) *that maximizes* $E^Y_I(\prod(Q_n,Q_r))$*, and the optimal production quantities of the two types of the products are*

$$Q^{YI*}_n = abe^{-\frac{\theta(ck_np_n + B_n(ck_n + (-c+d)c_n - dV_n))}{k_n(B_n+p_n)-B_nV_n}} - a(-1+b+\alpha) - \xi p_n \tag{8}$$

$$Q^{YI*}_r = a\left(b - be^{-\frac{\theta(dk_rp_r + B_r(dk_r + (c-d)c_r - cV_r))}{k_r(B_r+p_r)-B_rV_r}} + \alpha\right) - \xi p_r \tag{9}$$

Proof. To prove concavity, the Hessian matrix of $E^Y_I(\prod(Q_n,Q_r))$ calculated as follows:

$$\begin{bmatrix} \frac{k_n(B_n+p_n)-B_nV_n}{(c-d)\theta B_n(a(-1+b+\alpha)+\xi p_n+Q_n)} & 0 \\ 0 & -\frac{k_r(B_r+p_r)-B_rV_r}{(c-d)\theta p_r(-a(b+\alpha)+\xi p_r+Q_r)} \end{bmatrix}$$

By holding the condition $k_i(B_i + p_i) - B_iV_i > 0$, the first principal minor $\frac{k_n(B_n+p_n)-B_nV_n}{(c-d)\theta B_n(a(-1+b+\alpha)+\xi p_n+Q_n)}$ is negative. The second principal minor $\frac{k_n(B_n+p_n)-B_nV_n}{(c-d)\theta B_n(a(-1+b+\alpha)+\xi p_n+Q_n)}$ $*\left(-\frac{k_r(B_r+p_r)-B_rV_r}{(c-d)\theta p_r(-a(b+\alpha)+\xi p_r+Q_r)}\right)$ is positive. Thus, the Hessian matrix is negative definite and $E^Y_I(\prod(Q_n,Q_r))$ is concave for all decision variables. $\frac{\partial E^Y_I(\prod(Q_n,Q_r))}{\partial Q_n} = 0$ and $\frac{\partial E^Y_I(\prod(Q_n,Q_r))}{\partial Q_r} = 0$ result in Equations (8) and ((9). $\square$

Corollary 4. *In the integrated model with emergency orders, the optimal order quantities $Q^{YI*}{}_n$ are convex and $Q^{YI*}{}_r$ is concave with respect to the lower and upper bound of sales efforts c and d.*

Proof. Since $\dfrac{\partial^2 Q^{YI*}{}_n}{\partial c^2} = \dfrac{abe^{-\frac{\theta(ck_n p_n + B_n(ck_n + (-c+d)c_n - dV_n))}{k_n(B_n + p_n) - B_n V_n}}(\theta k_n(B_n + p_n) - \theta B_n c_n)^2}{(k_n(B_n + p_n) - B_n V_n)^2} > 0;$

$\dfrac{\partial^2 Q^{YI*}{}_n}{\partial d^2} = \dfrac{abe^{-\frac{\theta(ck_n p_n + B_n(ck_n + (-c+d)c_n - dV_n))}{k_n(B_n + p_n) - B_n V_n}}\theta^2 B_n^2 (c_n - V_n)^2}{(k_n(B_n + p_n) - B_n V_n)^2} > 0; \dfrac{\partial^2 Q^{YI*}{}_r}{\partial c^2} =$

$-\dfrac{abe^{-\frac{\theta(dk_r p_r + B_r(dk_r + (c-d)c_r - cV_r))}{k_r(B_r + p_r) - B_r V_r}}\theta^2 B_r^2 (c_r - V_r)^2}{(k_r(B_r + p_r) - B_r V_r)^2} < 0; \dfrac{\partial^2 Q^{YI*}{}_r}{\partial d^2} =$

$-\dfrac{abe^{-\frac{\theta(dk_r p_r + B_r(dk_r + (c-d)c_r - cV_r))}{k_r(B_r + p_r) - B_r V_r}}(\theta k_r(B_r + p_r) - \theta B_r c_r)^2}{(k_r(B_r + p_r) - B_r V_r)^2} < 0,$ then $Q^{YI*}{}_n$ is convex and $Q^{YI*}{}_r$ is concave in c and d. $\square$

4.2.2. Decentralized Scenario

In the decentralized scenario, the retailer issues an emergency order when it is out of stock, and the manufacturer makes supplies for the emergency order and pays the emergency order cost. Remanufacturing takes recycled products as materials, and the recycling process is usually more uncertain, resulting in higher reaction costs compared to new products.

The retailer's profit in a general form can be expressed as

$$\pi^Y R(Q_n, Q_r) = p_n(D_n \wedge Q_n) + V_n(Q_n - D_n)^+ + p_n \beta_n(D_n - Q_n)^+ - B_n(1 - \beta_n)(D_n - Q_n)^+ - w_n(Q_n + \beta_n(D_n - Q_n)^+)$$
$$+ p_r(D_r \wedge Q_r) + V_r(Q_r - D_r)^+ + p_r \beta_r(D_r - Q_r)^+ w_r(Q_r + \beta_r(D_r - Q_r)^+) - B_r(1 - \beta_r)(D_r - Q_r)^+ - \tfrac{1}{2} g s^2$$

The expected profit function of the retailer is described as follows:

$$E_R^Y(\Pi(Q_n, Q_r)) = p_n(\int_{z_n}^{d} D_n f(s)ds + \int_{c}^{z_n} Q_n f(s)ds) + V_n \int_{z_n}^{d}(Q_n - D_n)f(s)ds + p_n \int_{c}^{z_n}\beta_n(D_n - Q_n)f(s)ds - B_n \int_{c}^{z_n}(1 - \beta_n)(D_n - Q_n)f(s)ds - w_n(Q_n + \int_{c}^{z_n}\beta_n(D_n - Q_n)f(s)ds)$$
$$+ p_r(\int_{c}^{z_r} D_r f(s)ds + \int_{c}^{d} Q_r f(s)ds) + V_r \int_{c}^{z_r}(Q_r - D_r))f(s)ds + p_r \int_{z_r}^{d}\beta_r(D_r - Q_r)f(s)ds - B_r \int_{z_r}^{d}(1 - \beta_r)(D_r - Q_r)f(s)ds - w_r(Q_r \int_{z_r}^{d}\beta_r(D_r - Q_r)f(s)ds) - \tfrac{1}{8}g(d + c)^2$$

Proposition 5. *Under the decentralized scenario with emergency orders, the expected profit function $E_R^Y(\prod(Q_n, Q_r))$ for the retailer is concave with respect to Q_n and Q_r if $k_i(B_i + p_i) - B_i V_i > 0$; hence, there is a unique optimal combination of decision variables $(Q^{YR*}{}_n, Q^{YR*}{}_r)$ that maximizes $E_R^Y(\prod(Q_n, Q_r))$, and the optimal production quantities of the two types of products are*

$$Q^{YR*}{}_n = abe^{-\frac{\theta(ck_n p_n + B_n(ck_n + (-c+d)w_n - dV_n))}{k_n(B_n + p_n) - B_n V_n}} - a(-1 + b + \alpha) - \xi p_n \tag{10}$$

$$Q^{YR*}{}_r = a\left(b - be^{-\frac{\theta(dk_r p_r + B_r(dk_r + (c-d)w_r - cV_r))}{k_r(B_r + p_r) - B_r V_r}} + \alpha\right) - \xi p_r \tag{11}$$

Proof. To prove concavity, the Hessian matrix of $E_R^Y(\prod(Q_n, Q_r))$ calculated as follows:

$$\begin{bmatrix} \dfrac{k_n(B_n + p_n) - B_n V_n}{(c-d)\theta B_n(a(-1+b+\alpha) + \xi p_n + Q_n)} & 0 \\ 0 & -\dfrac{k_r(B_r + p_r) - B_r V_r}{(c-d)\theta p_r(-a(b+\alpha) + \xi p_r + Q_r)} \end{bmatrix}$$

The proof process is the same as for Proposition 4. $\square$

Corollary 5. *In the integrated model with emergency orders, the optimal order quantities $Q^{YR*}{}_n$ are convex and $Q^{YR*}{}_r$ is concave with respect to the lower and upper bounds of sales efforts c and d.*

Proof. Since $\frac{\partial^2 Q^{YI*}{}_n}{\partial c^2} = \frac{abe^{-\frac{\theta(ck_n p_n + B_n(ck_n+(-c+d)w_n - dV_n))}{k_n(B_n+p_n)-B_nV_n}}(\theta k_n(B_n+p_n)-\theta B_n w_n)^2}{(k_n(B_n+p_n)-B_nV_n)^2} > 0; \frac{\partial^2 Q^{YI*}{}_n}{\partial d^2} =$

$\frac{abe^{-\frac{\theta(ck_n p_n + B_n(ck_n+(-c+d)w_n - dV_n))}{k_n(B_n+p_n)-B_nV_n}}\theta^2 B_n^2 (w_n - V_n)^2}{(k_n(B_n+p_n)-B_nV_n)^2} > 0; \frac{\partial^2 Q^{YI*}{}_r}{\partial c^2} =$

$-\frac{abe^{-\frac{\theta(dk_r p_r + B_r(dk_r+(c-d)w_r - cV_r))}{k_r(B_r+p_r)-B_rV_r}}\theta^2 B_r^2 (w_r - V_r)^2}{(k_r(B_r+p_r)-B_rV_r)^2} < 0; \frac{\partial^2 Q^{YI*}{}_r}{\partial d^2} =$

$-\frac{abe^{-\frac{\theta(dk_r p_r + B_r(dk_r+(c-d)w_r - cV_r))}{k_r(B_r+p_r)-B_rV_r}}(\theta k_r(B_r+p_r)-\theta B_r w_r)^2}{(k_r(B_r+p_r)-B_rV_r)^2} < 0,$ then the optimal order quantity $Q^{YI*}{}_n$

is convex and $Q^{YI*}{}_r$ is concave in c and d. $\square$

The manufacturer's profit function is formulated as

$$E_M^Y(\Pi(w_n, w_r)) = (w_n - c_n)Q_n + (w_n - c_n - k_n)\int_c^{z_n} \beta_n(D_n - Q_n)f(s)ds$$

$$+(w_r - c_r)Q_r + (w_r - c_r - k_r)\int_{z_r}^{d} \beta_r(D_r - Q_r)f(s)ds$$

Proposition 6. *Under the decentralized scenario without emergency orders, the expected manufacturer profit function* $E_M^Y(\prod(w_n, w_r))$ *is concave with respect to* w_n *and* w_r *if*

$$2k_n p_n + 2B_n k_n - 2B_n V_n + B_n(c - d)\theta(-c_n + w_n) > 0 \tag{12}$$

Proof. To prove concavity, the Hessian matrix of $E_M^Y(\prod(w_n, w_r))$ calculated as follows:

$$\begin{bmatrix} \frac{ab(c-d)e^{-\frac{\theta(ck_n p_n + B_n(ck_n - dV_n + (-c+d)w_n))}{k_n(B_n+p_n)-B_nV_n}}\theta B_n(Y_n + B_n(c-d)\theta(-c_n+w_n))}{(k_n(B_n+p_n)-B_nV_n)^2} & 0 \\ 0 & \frac{ab(c-d)e^{-\frac{\theta(dk_r p_r + B_r(dk_r - cV_r + (c-d)w_r))}{k_r(B_r+p_r)-B_rV_r}}\theta B_r(Y_i + B_r(c-d)\theta(c_r-w_r))}{(k_r(B_r+p_r)-B_rV_r)^2} \end{bmatrix}$$

where $Y_i = 2k_i p_i + 2B_i k_i - 2B_i V_i$ $(i = n, r)$

By holding Condition (12), the first principal minor is negative, and the second principal minor is positive. Therefore, Hessian matrix is negative definite. To obtain the optimal values of the decision variables, $\frac{\partial E_M^Y(\prod(w_n, w_r))}{\partial w_n} = 0$ and $\frac{\partial E_M^Y(\prod(w_n, w_r))}{\partial w_r} = 0$ need to be solved, that is,

$$abe^{-\frac{\theta(ck_n p_n + B_n(ck_n - dV_n + (-c+d)w_n))}{k_n(B_n+p_n)-B_nV_n}} - a(-1 + b + \alpha) - \xi p_n -$$
$$\frac{ab(-c+d)e^{-\frac{\theta(ck_n p_n + B_n(ck_n - dV_n + (-c+d)w_n))}{k_n(B_n+p_n)-B_nV_n}}\theta B_n(-k_n - c_n + w_n)}{k_n(B_n+p_n)-B_nV_n} = 0 \tag{13}$$

$$a(b - be^{-\frac{\theta(dk_r p_r + B_r(dk_r - cV_r + (c-d)w_r))}{k_r(B_r+p_r)-B_rV_r}} + \alpha) - \xi p_r + \frac{ab(c-d)e^{-\frac{\theta(dk_r p_r + B_r(dk_r - cV_r + (c-d)w_r))}{k_r(B_r+p_r)-B_rV_r}}\theta B_r(-k_r - c_r + w_r)}{k_r(B_r+p_r)-B_rV_r} = 0 \tag{14}$$

As Equations (13) and (14) are transcendental equations, a numerical method needs to be used. We discuss the results using numerical examples in Section 5. $\square$

Corollary 6. *The order quantity of the two types of products in the centralized channel is higher than that in the decentralized channel with emergency ordering:* $Q^{YI*}{}_n > Q^{YR*}{}_n; Q^{YI*}{}_r > Q^{YR*}{}_r.$

Proof. $Q^{YI*}{}_n - Q^{YR*}{}_n = ab\left(e^{-\frac{\theta(ck_n p_n + B_n((-c+d)c_n + ck_n - dV_n))}{k_n(B_n+p_n)-B_nV_n}} - e^{-\frac{\theta(ck_n p_n + B_n(ck_n - dV_n + (-c+d)w_n))}{k_n(B_n+p_n)-B_nV_n}}\right) >$

$0; Q^{NI*}{}_r - Q^{NR*}{}_r = ab\left(-e^{-\frac{\theta(dk_r p_r + B_r((c-d)c_r + dk_r - cV_r))}{k_r(B_r+p_r)-B_rV_r}} + e^{-\frac{\theta(dk_r p_r + B_r(dk_r - cV_r + (c-d)w_r))}{k_r(B_r+p_r)-B_rV_r}}\right) > 0. \square$

Corollary 7. *Under the two channel structures, the difference between product order quantities with and without emergency orders depends on the difference in their respective out-of-stock losses and the cost of emergency orders.*

Proof. $\quad Q^{NR}{}_n - Q^{YR}{}_n = ab\left(\mathrm{e}^{-\frac{\theta(c(B_n+p_n)-dV_n+(-c+d)w_n)}{B_n+p_n-V_n}} - \mathrm{e}^{-\frac{\theta(ck_n p_n+B_n(ck_n-dV_n+(-c+d)w_n))}{k_n(B_n+p_n)-B_n V_n}}\right);$

$Q^{NR}{}_r - Q^{YR}{}_r = ab\left(-\mathrm{e}^{-\frac{\theta(dB_r+dp_r-cV_r+(c-d)w_r)}{B_r+p_r-V_r}} + \mathrm{e}^{-\frac{\theta(dk_r p_r+B_r(dk_r-cV_r+(c-d)w_r))}{k_r(B_r+p_r)-B_r V_r}}\right).$ This is similar under the integrated structure. □

5. Analysis of Equilibrium Results

5.1. Numerical Examples Datasets

In this section, numerical examples are used to analyze the decision results for different models. Due to the difficulty of obtaining market data for remanufacturing closed-loop supply chains, the datasets employed in this study were sourced from reference [5] for data analysis. The datasets adhered to all assumptions and are shown in Table 3. Mathematica 12 (Champagne, IL, USA) was used to solve the models.

Table 3. Numerical examples' datasets.

Parameter	p_n	p_r	B_n	B_r	V_n	V_r	c_n	c_r	k_n
Value	8	6	3	2	1.5	1	4	1.5	1.75
Parameter	a	α	θ	c	d	b	g	ξ	k_r
Value	50	0.015	1.7	0	5	0.25	0.005	2	1.85

5.2. Results of Order Quantities and Profits

For the given datasets, the corresponding decision variables and members' profits under integrated and decentralized models across different scenarios are shown in Table 4.

Table 4. Decision variables and profits in different scenarios.

	Integrated CLSC			Decentralized CLSC			
	Q^*_n	Q^*_r	$E(\pi_I)^*$	Q^*_n	Q^*_r	$E(\pi_R)^*$	$E(\pi_M)^*$
Without emergency order	22.0849	1.2453	78.338	20.9307	0.999	26.7704	50.0612
With emergency order	20.9159	1.2451	84.7575	20.8106	0.996	32.1299	49.4988

According to Table 4, the order quantities of both product types in the decentralized scenario are lower than those in the integrated scenario. Moreover, under the emergency ordering strategy, the order quantities in both the integrated and decentralized scenarios decrease, while the total profit of the CLSC increases. As expected, the total channel profits in the integrated scenario are higher than those in the decentralized scenario, proving that the performance in the integrated scenario for a CLSC is better [4]. This conclusion is independent of the presence or absence of an emergency ordering strategy. Under the emergency order case, emergency orders do not increase profits for both the manufacturer and the retailer. Instead, it makes the retailer more profitable and the manufacturer less profitable [5], and the gap between the profits of the manufacturer and the retailer decreases. The manufacturer's revenue decreases because it bears the emergency ordering cost.

5.3. Impact of Sales Effort

In this part, the influence of sales efforts under different channel structures on the decision variables and profits is presented and analyzed. The sales effort s is defined as a random variable subject to a uniform distribution, where *c* and *d* are the lower and upper

bounds, respectively. Profits and order quantities vary with different combinations of c and d, as shown in Figures 1–11.
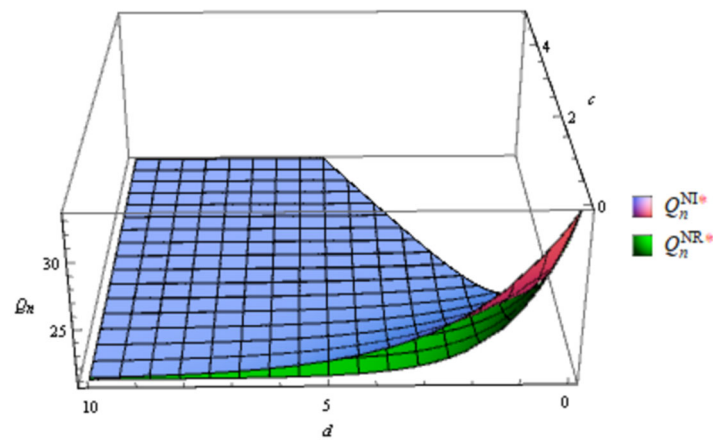
Figure 1. Order quantities of new products in integrated and decentralized scenarios without emergency orders.
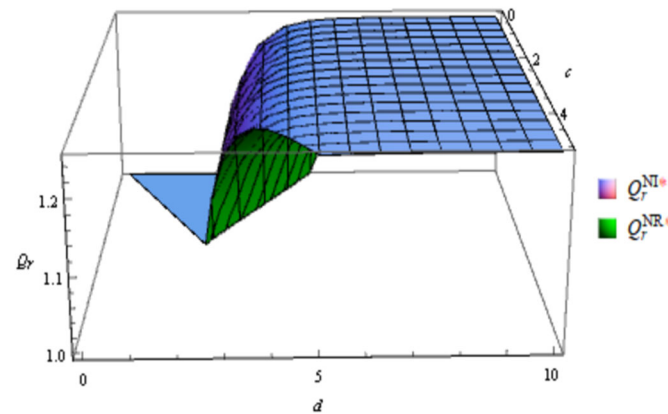

Figure 2. Order quantities of remanufactured product in integrated and decentralized scenarios without emergency orders.
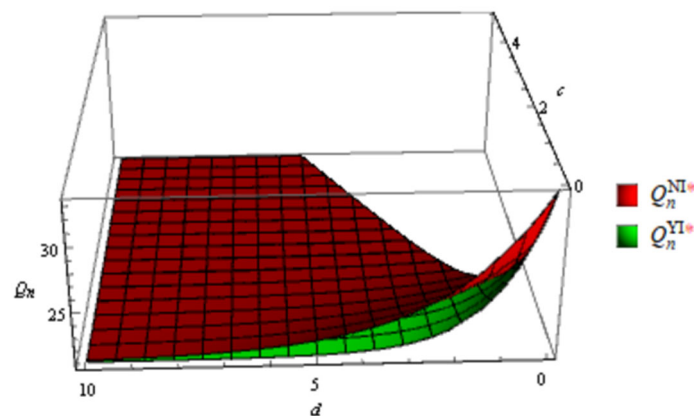

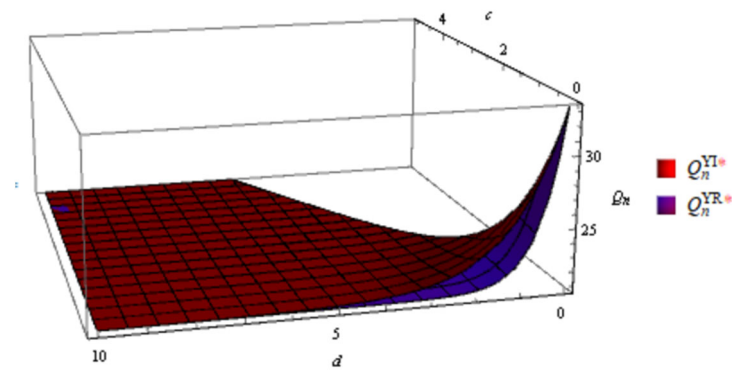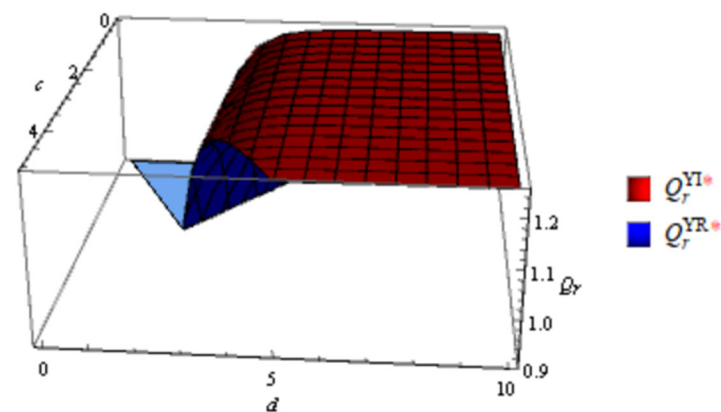**Figure 3.** Order quantities of new product without and with emergency orders in the integrated scenario.

According to Figures 1–8, the order quantity of new products generally has a convex function relationship with both c and d, and the order quantity of remanufactured products is a concave function with sales efforts c and d. This result is consistent with Corollaries 1, 2, 4, and 5. Moreover, when d approaches 0, meaning that retailers do not make sales efforts

for the sale of remanufactured products, the order quantity of remanufactured products is significantly lower. However, when sales efforts begin to take effect, the impact is greater in the early stages. And, when d is large enough, the gap in channel order quantities gradually decreases.
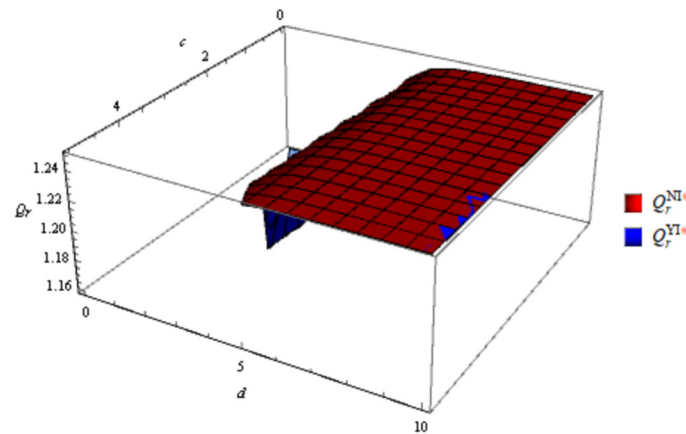
Figure 4. Order quantities of remanufactured products without and with emergency orders in the integrated scenario.
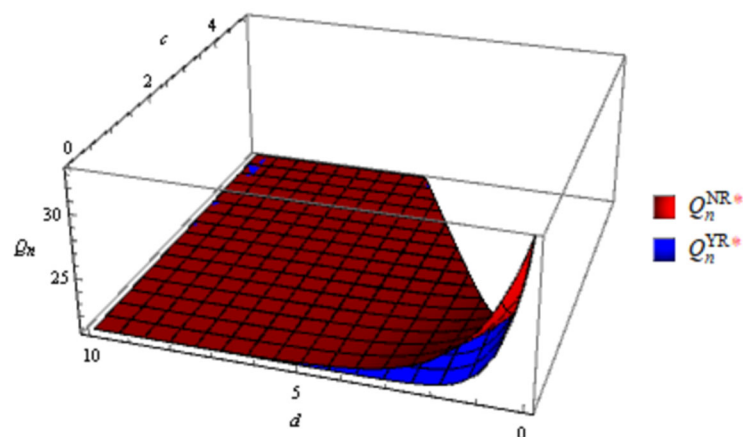
Figure 5. Order quantities of new products without and with emergency orders in the decentralized scenario.
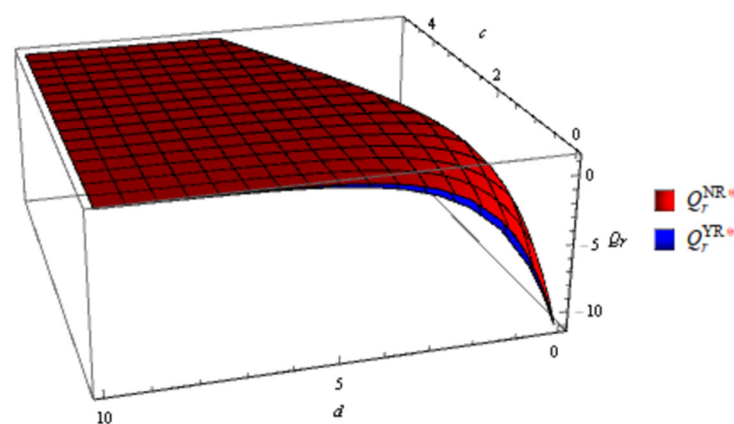
Figure 6. Order quantities of remanufactured products without and with emergency orders in the decentralized scenario.

Figure 7. Order quantities of new products in integrated and decentralized scenarios with emergency orders.

Figure 8. Order quantities of remanufactured products in integrated and decentralized scenarios with emergency orders.
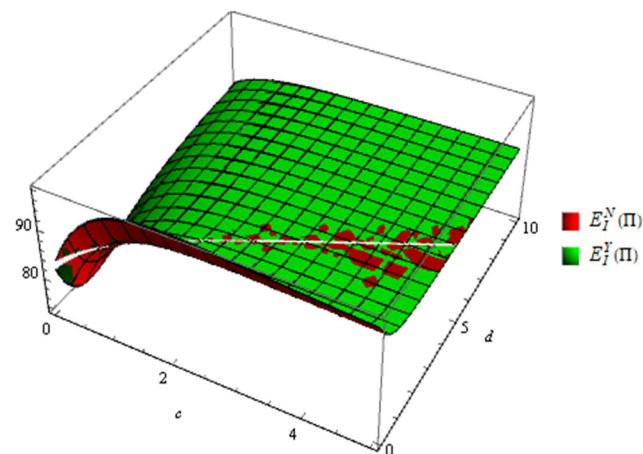
Figure 9. Profits of the CLSC without and with emergency orders in the integrated scenario.

From the channel structure aspect, consistent with Corollaries 3 and 6, it is illustrated that the order quantities are generally higher in the integrated scenario than in the decentralized scenario both with and without emergency orders [4]. Moreover, the order quantity of remanufactured product is not sensitive to the channel structure under different levels of sales efforts. The possible reason for this is that the market share of remanufactured products is relatively small. In addition, high sales efforts reduce channel differences in order quantity.
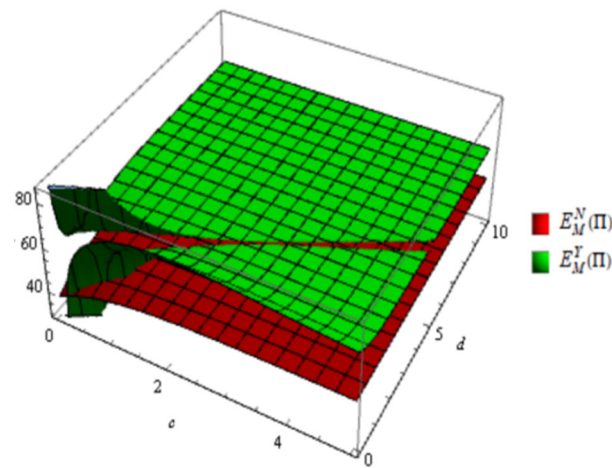
Figure 10. Profits of the manufacturer without and with emergency orders in the decentralized scenario.
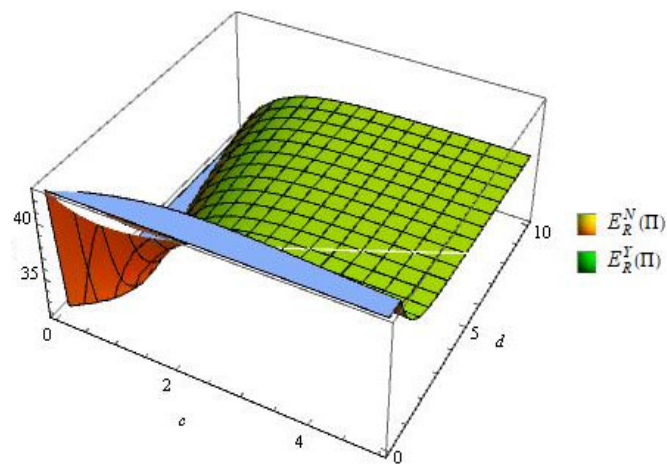
Figure 11. Profits of the retailer without and with emergency orders in the decentralized scenario.

Comparing the order quantities of two types of products with and without emergency orders, it was found that the order quantities with emergency orders are lower than those without emergency orders. This conclusion is in line with the actual situation [22].

As shown in Figure 9, under the integrated structure, in most cases, for different sales efforts, the profits of the CLSC with the emergency order strategy are higher than those without emergency orders, which is consistent with Table 4. However, in some cases, the opposite is true. Additionally, with the improvement in sales efforts, the overall profit of the CLSC also improves. Figures 10 and 11 indicate that in a decentralized CLSC, sales efforts have a greater impact on retailer profits than on manufacturers and are beneficial to retailers. Another observation is that sales efforts have limitations in increasing profits for manufacturers or retailers. When sales efforts reach a certain level, both tend to stabilize.

Additionally, Figure 10 shows that, except in a few cases, emergency orders also bring more profits to the manufacturer. Figure 11 illustrates that the retailer always benefit from emergency orders. Comparing Figure 10 with Figure 11, it can be found that the manufacturer obtains a larger part of the profit of the CLSC. It indicates that the manufacturer has more power in the CLSC.

5.4. Impact of Emergency Order Costs and Out-of-Stock Loss

Figure 12 shows that as kn increases, Q_n also increases, while Q_n decreases with the increase in B_n. Furthermore, from the perspective of the proportion of emergency orders, when k_n is small, and B_n is relatively large, indicating that the proportion of emergency

ordering is higher, the impact of the ratio k_n/B_n on Q_n is almost nonexistent. Moreover, the order quantity of new products in the high-ratio case is higher than that in the low-ratio case. What causes this phenomenon is that the retailer increases the order quantity at the time of the original order when emergency orders are costly.
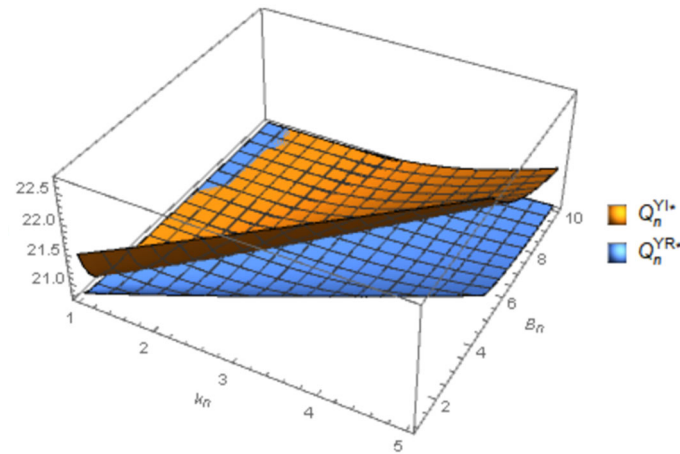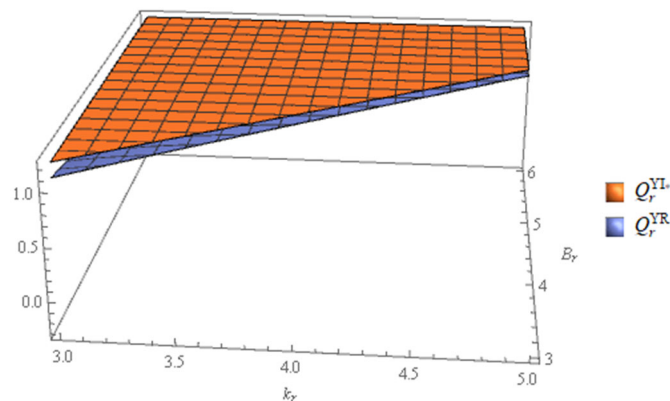

Figure 12. Order quantities of new products in the integrated and decentralized scenario with emergency orders.

According to Figure 13, the results regarding the impact of emergency orders on the order quantity of the remanufactured product are a bit different from those of new products. Specifically, increases in both k_r and B_r result in an increase in Q_r. Additionally, compared to integrated structures, in a decentralized supply chain structure, the variation in the order quantity of remanufactured products is greater. In addition, it can be inferred that when k_r/B_r decreases, indicating an increase in the proportion of emergency orders, the order quantity for remanufactured products decreases.

Figure 13. Order quantities of remanufactured products in the integrated and decentralized scenario with emergency orders.

According to Figures 14–16, the emergency ordering costs and out-of-stock costs of new and remanufactured products affect the profits of the manufacturer, the retailer, and the CLSC. When k_i is rather small, the profits of the CLSC and members are not sensitive to the k_i/B_i ratio. However, in the case of the implementation of emergency ordering, the emergency ordering of the new product makes the manufacturer's profits decrease as k_n increases (see Figure 15a). When B_n is relatively small, the impact of the k_n ratio on the profit of the CLSC depends on k_n, causing the lowest profit. In addition, the profit of the CLSC decreases with the k_n increasing (see Figure 14a). For the retailer, if the loss from stock shortage B_n is small, emergency ordering is not necessary because it causes a large

reduction in profits, as shown in Figure 16a. Moreover, as k_r increases, the profits of both the CLSC and the retailer decrease (see Figures 14b and 16b).
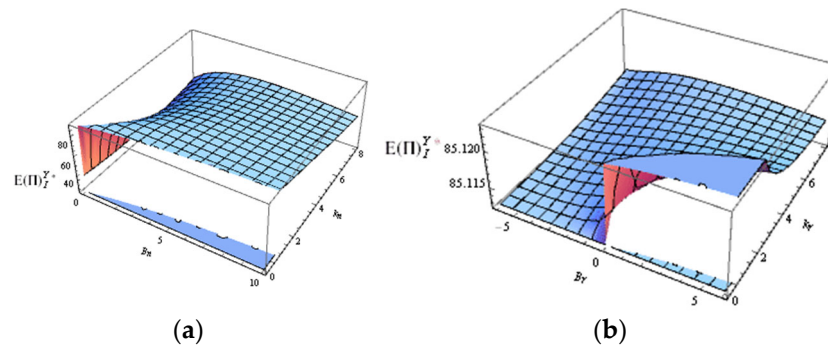

Figure 14. Profits of the CLSC in the integrated scenario with emergency orders. (**a**) The effect of k_n/B_n on the profit of the CLSC (**b**)The effect of k_r/B_r on the profit of the CLSC.
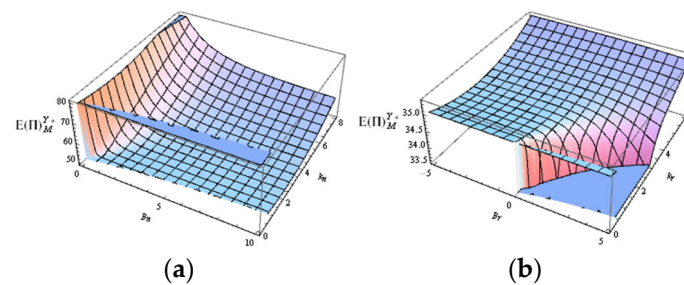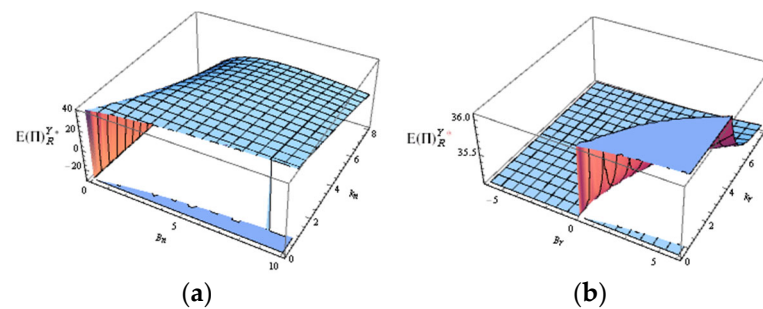

Figure 15. Profits of the manufacturer in the decentralized scenario with emergency orders, (**a**) The effect of k_n/B_n on the profit of the manufacturer, (**b**) The effect of k_r/B_r on the profit of the manufacturer.

Figure 16. Profits of the retailer in the decentralized scenario with emergency orders, (**a**) The effect of k_n/B_n on the profit of the retailer, (**b**) The effect of k_r/B_r on the profit of the retailer.

6. Conclusions

Environmental awareness, resource conservation, and economic benefits have promoted the development of closed-loop supply chains, and remanufactured products are gradually appearing in the market with new products. Relative to new products, the acceptance of remanufactured products in consumers' perception is more influenced by sales effort, especially in the initial stage. Sales efforts also lead to variations in the market share of new products. While there have been some studies on sales effort in closed supply chains, most of them have been based on the assumption of a linear relationship between sales effort and demand. In this study, a CLSC with one manufacturer and one retailer was considered, in which the sales effort, treated as a random variable, impacts the market shares of two types of products. The optimal ordering decisions and wholesale prices were explored by establishing game theory models for both centralized and decentralized

CLSCs without and with emergency ordering. All models were evaluated using numerical experiments. Managerial insights from the proposed model can be summarized as follows:

1. In general, the order quantity of new products usually follows a convex function the sales effort, while the order quantity of remanufactured products exhibits a concave function with sales effort. The sales effort has a stronger impact on the order quantity of new products compared to remanufactured products, wherein the upper bound of sales effort has a greater impact on the order quantities of both types of products rather than the lower bound.

2. In most cases, emergency orders increase the profit of the CLSC and its members. In addition, the order quantity and profit in the CLSC are both affected by the relative ratio between the cost of emergency ordering and the cost of out-of-stock losses rather than solely one of them.

3. The profits and order quantities of the integrated closed-loop supply chain are higher than those of the decentralized scenario, which indicates that the integrated supply chain performs better than the decentralized one, regardless of the implementation of an emergency ordering strategy.

4. A certain degree of sales effort narrows the difference in ordering quantities between decentralized and integrated channels.

The management recommendations that can be derived from the research results are as follows:

5. Retailers need to establish appropriate sales effort limits to avoid excessive resource investment leading to diminishing marginal return and focus on the overall revenue. Additionally, it is necessary for managers to take measures to ensure effective responses to the sales challenges posed by new and remanufactured products and to ensure that sales efforts are maximally translated into an increase in order volume.

6. For the impact of emergency orders on the CLSC, a cost–benefit analysis can be conducted for each product or product category to determine the economic benefits of emergency orders in different situations. Moreover, retailers and manufacturers can establish closer cooperative relationships to better respond to emergency situations. This may involve sharing information, collaborating to solve problems, and ensuring the reliability and stability of the CLSC.

7. A closed-loop supply chain can take the sales effort strategy for remanufactured products as a means to coordinate a decentralized supply chain, ensuring that sales efforts generate more consistent order volumes across various channels, thereby improving overall sales performance and customer satisfaction. For example, sales strategies and train sales teams can be integrated to better achieve synergies between channels.

In this study, it was assumed that the manufacturer directly remanufactures the recycled products. This assumption can be relaxed and the behavior of CLSCs including the recycling process can be explored in the future. Also, the assumption in this study that sales efforts are uniformly distributed can be further extended to explore the impact of different consumer perceptions and sales efforts under different market conditions on closed-loop supply chain decision making. Moreover, multi-echelon CLSC points to interesting future research. The proportion of emergency orders related to the out-of-stock quantity is also a direction that can be studied. Additionally, examining CLSC management with other forms of backorder compensation strategies could provide valuable insights. There are many aspects of closed-loop supply chain management that need to be explored.

Conflicts of Interest: The authors declare no conflicts of interest.

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