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Evaluating Information Risk Propagation in Complex Public Opinion Environments Based on the Improved Grey Relational Analysis—Decision Making Trial and Evaluation Laboratory Method

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Abstract: The propagation of information risk in complex public opinion environments not only leads to severe direct reputational losses for companies but also results in significant economic damages. Therefore, during the nascent stage of information risk, identifying potential propagation pathways, determining key dissemination channels, and taking timely measures become crucial. To address this issue, this paper proposes a multi-criteria decision-making method for evaluating information risk propagation in complex public opinion environments. In this method, this paper utilizes probabilistic hesitant fuzzy sets to express the evaluation information, and provide several distance and similarity measurement methods for probabilistic hesitant fuzzy elements. To ensure the rationality of the evaluation indicator weights, this study first applies these distance measurement methods to improve the Grey Relational Analysis—Decision Making Trial and Evaluation Laboratory (GRA-DEMATEL) method for determining the objective weights of evaluation indicators. Next, this paper uses the Delphi method to establish the subjective weights of each evaluation indicator. Finally, by employing a weight synthesis operator, this paper combines the subjective and objective weights to obtain the final indicator weights. Additionally, this paper utilizes the similarity measurement methods for probabilistic hesitant fuzzy elements to improve the combined compromise solution (CoCoSo) method in evaluating and ranking potential information risk propagation pathways. Furthermore, this paper incorporates the "Probability Splitting Algorithm" to handle probabilistic hesitant fuzzy elements, enabling their application in these methodologies. Finally, based on a case study of information risk propagation in the catering industry, we conducted a sensitivity analysis and effectiveness verification of the proposed approach. The results demonstrate the effectiveness of the method and its ability to address real-world issues.

Keywords: complex public opinion environment; information risk propagation; probabilistic hesitant fuzzy sets; CoCoSo method; GRA-DEMATEL method

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Citation: Luo, Z.; Xue, Y.; Su, J. Evaluating Information Risk Propagation in Complex Public Opinion Environments Based on the Improved Grey Relational Analysis—Decision Making Trial and Evaluation Laboratory Method. *Systems* **2023**, *11*, 472. https://doi.org/10.3390/systems11090472

Academic Editor: William T. Scherer

Received: 10 August 2023
Revised: 5 September 2023
Accepted: 12 September 2023
Published: 13 September 2023

1. Introduction

With the rapid development of the internet, the general public now has access to numerous channels to express their opinions and emotions, utilizing features such as clicks, likes, or shares to convey their sentiments and attitudes. Online platforms such as blogs, forum posts, short videos, and more have become avenues for people to share their viewpoints and express their emotions. Public opinion has evolved into an openly expressed collective voice, encompassing opinions, emotions, and attitudes that are no longer confined to individual views. It has emerged as a prominent force shaping discussions and debates, no longer relegated to mere potential influence [1]. However, it is essential to note that the diversity of online public opinion content prevents it from entirely representing a consistent consensus among the masses. Additionally, emotions and attitudes expressed on online platforms are susceptible to manipulation, leading to variations in public opinion. With the rise of "We Media", certain influential online figures with a large number of followers, known as "internet celebrities" or "online influencers," may, at times, intentionally distort hot events and public opinion for personal purposes. This distortion can deviate from the true public sentiment, mislead the public, and create biases [2]. In such a complex public opinion environment, companies are susceptible to negative press and public dissatisfaction, which can have adverse effects on their brand image and reputation. Furthermore, information risk can lead to a decrease in a company's competitiveness, a reduction in market share, and a breakdown in investor confidence, adversely affecting the company's sustainable development. In the complex public opinion environment, information risk propagation involves multiple aspects and dimensions and exhibits sudden characteristics. This implies that when evaluating information risk propagation, we face significant uncertainty and challenges. Therefore, research on the evaluation of information risk propagation in complex environments becomes highly necessary. Through the assessment of information risk propagation, companies can proactively identify potential dissemination pathways before negative public opinion evolves into a major issue. By taking targeted and effective measures, they can prevent significant economic and reputational losses to the organization.

However, when evaluating the propagation of information risks, traditional singular assessment methods fail to encompass all relevant factors. In such a context, a multi-criteria approach becomes a crucial evaluation method. This method takes into account multiple key criteria or indicators in the process of information propagation, thereby enabling a comprehensive analysis and assessment of the likelihood and extent of risk dissemination from various perspectives. By comprehensively considering criteria such as propagation speed, information accuracy, and scope of influence, we can better grasp the essence of information risk propagation and formulate effective risk management strategies. Therefore, we will employ a multi-criteria group decision-making approach to evaluate the pathways of information risk propagation. This method allows us to consider the potential dissemination pathways of information risk from multiple dimensions by integrating the opinions of expert groups, thereby conducting the evaluation. Currently, it has been applied to various evaluation issues, such as evaluations in the pharmaceutical sector [3], military matters [4], energy problems [5], and more. Multi-criteria group decision making is a crucial and emerging decision strategy that addresses decision problems by considering the preferences of multiple experts. However, for each expert, effectively evaluating the subject matter is a challenging task. Moreover, uncertainty always plays a dominant role in any decision process concerning the evaluation of objects [6]. To address this problem, Torra [7] proposed hesitant fuzzy sets. This fuzzy set utilizes a series of numbers ranging from 0 to 1 to describe information in decision making, effectively capturing the uncertainty inherent in the decision-making process. However, the hesitant fuzzy set has also encountered a fatal limitation in practical applications, namely, the potential loss of information [8].

To address this limitation, the concept of probability hesitant fuzzy sets (PHFS) emerged [9]. This type of hesitant fuzzy set presents repeated evaluation information in a probabilistic form, ensuring the integrity of assessment information. Furthermore, Xu and Zhou [9] also developed the score function, deviation function, comparison rules, and basic operations for probability hesitant fuzzy sets. Zhang et al. [10] provided the weighted average operator and weighted geometric operator for probability hesitant fuzzy sets. Subsequently, Jiang and Ma [11] proposed the Frank weighted average operator and weighted geometric operator under probability hesitant fuzzy sets. They applied these operators to a multi-attribute group decision-making method for evaluating the efficiency of listed companies. Fang [8] introduced the hybrid entropy and hybrid cross-entropy measures for probability hesitant fuzzy sets, applying them to multi-attribute decision making. Wang et al. [12] presented the axiom definition of PHF entropy for PHFS and explored the relationship between distance measures and entropy measures. Finally, they developed a probabilistic hesitant fuzzy multi-criteria decision-making (PHF-MCDM) model capable of addressing completely unknown attribute weights and used this method to assess the survival of the Yangtze finless porpoise.

In this study, to address uncertainty and ensure the incorporation of evaluation information from all experts, this paper will utilize probability hesitant fuzzy sets as the representation tool for evaluation data. Additionally, this paper will propose a composite evaluation method to evaluate information risk communication in complex public opinion environments. In this method, this paper employs a combination of the Delphi method and GRA-DEMATEL to determine the weights of each evaluation criterion. Additionally, this paper utilizes an improved CoCoSo approach based on similarity to evaluate the various potential pathways of information propagation. Overall, the contributions of this study can be summarized as follows:

(a) Introduced the "Probability Splitting Algorithm" to handle probability hesitant fuzzy sets and proposed Dice similarity and Jaccard similarity in the context of probability hesitant fuzzy sets. Based on these two similarity measures, two distance measurement methods for probability hesitant fuzzy elements were proposed.
(b) Improved the GRA-DEMATEL method using Dice distance and Jaccard distance in the framework of probability hesitant fuzzy sets to calculate the weights of evaluation criteria. Additionally, a combination of objective and subjective weights was used to derive comprehensive weights for each evaluation criterion.
(c) Improved the standardization process of the traditional CoCoSo method using similarity measures to effectively handle probability hesitant fuzzy information. Presented a novel multi-criteria group decision-making and evaluation method.
(d) Summarized information propagation pathways in complex public opinion environments and identified relevant evaluation criteria, providing a theoretical reference and research basis for future related studies.

The remaining sections of this study are organized as follows: In Section 2, a review of relevant theoretical literature is presented. Section 3 introduces new distance measurement and similarity measurement methods in the context of probability hesitant fuzzy sets and proposes the evaluation method used in this paper. In Section 4, potential information risk propagation pathways and corresponding evaluation criteria are identified through a literature review. Section 5 presents a real-life case study to apply the proposed methods in this paper, and the rationality of the methods is validated through a discussion and analysis. In Section 6, the research conclusions, limitations, and future research directions are provided.

2. Literature Review

2.1. Complex Public Opinion Environment and Information Risk Propagation

Internet public opinion, also known as online sentiment, is a form of social discourse popular on the internet, where people express different views on societal issues [13]. It represents influential and biased opinions of the public on hot and trending topics in real life. With the rapid development of the internet, the role and impact of internet public opinion have become increasingly prominent. As a result, scholars have started to pay more attention to the study of internet public opinion. Deng et al. [14] used information correlation to classify negative online public sentiment and used this classification to predict the risks associated with negative online public sentiment. Ding [1] studied the influencing factors of internet public opinion and explored the psychological effects of these factors in shaping public sentiment. Peng [2] elaborated on the concept, connotation, causal map, and formation mechanism of misleading public sentiment from the perspective of misleading information. He et al. [15] employed stochastic evolutionary game theory to reveal the impact of the distribution structure of netizen clusters on the evolution of negative online public sentiment. Jin et al. [16] focused on the evolution of netizen emotions and constructed a dynamic model for the polarization of online public sentiment groups, conducting simulation research from three dimensions: emotion awakening, emotion interference, and emotion release. From these studies, it can be observed that existing research on internet public sentiment mainly focuses on exploring certain mechanisms behind public sentiment. However, there is limited research that specifically investigates information risk propagation in complex public opinion environments and evaluates such risk propagation. However, risk management plays an important role in reducing corporate risk [17]. Therefore, this paper will evaluate the evaluation of information risk communication in complex public opinion environments from the perspective of information risk communication.

2.2. Information Risk Evaluation Based on Multi-Criteria Decision-Making Methods

Due to the ability of multi-criteria decision-making (MCDM) methods to evaluate and make decisions about objectives from multiple dimensions, the utilization of such methods for a risk assessment and analysis has garnered attention from scholars in recent years. For instance, Bozanic et al. [18] utilized triangular interval fuzzy numbers to modify the AHP method and apply this approach to risk assessment. Hua et al. [19] combined MCDM with a Failure Mode and Effects Analysis (FMEA) to enhance the performance of FMEA, allowing for more flexible expression of risk information. Jasiński et al. [20] employed MCDM techniques to assess the supply security of raw materials used in automotive manufacturing, based on supply risk evaluation criteria. Under a circular economy context, Yazdani et al. [21] employed a multi-criteria decision-making framework for agricultural product risk management. Su et al. [22] used a multi-criteria group decision-making approach for risk assessment of live e-commerce platforms. In addition, scholars have also applied multi-criteria decision-making methods to various other evaluation problems. For example, Deivanayagampillai et al. [23] employed a multi-criteria decision-making method called Single Value Neutrosophic TODIM to study the causal relationships among obstacles in Industry 5.0 adoption. Saraswat and Digalwar [5] applied an integrated fuzzy multi-criteria decision-making method with Shannon entropy to evaluate energy substitution options. In order to enhance the performance of the healthcare sector, Torkayesh et al. [3] proposed a multi-criteria decision-making method that integrates the best worst method (BWM), Level-Based Weight Assessment (LBWA), and CoCoSo for the evaluation of healthcare departments. Abdel-Basset et al. [24] used a hybrid multi-criteria decision-making method to assess sustainable bioenergy production technologies.

In these multi-criteria evaluation methods, the ranking of alternatives in the CoCoSo method proposed by Yazdani et al. [25] is based on a compromise strategy obtained through the application of the Weighted Sum Model (WSM) and Weighted Product Model (WPM). This compromise strategy serves as the final criterion function for ranking the alternative solutions. When the ratings of alternative solutions in the initial decision matrix are consistent, the combination of compromise strategies produces objective evaluation results. In recent years, this method has been widely applied. Ecer and Pamucar [26] used it for sustainable supplier selection. Deveci et al. [27] applied the method for real-time traffic management in the prioritization of autonomous vehicles. Ghoushchi et al. [28] utilized the method for wind turbine fault mode assessment. Inspired by these studies, this paper will incorporate the probability hesitant fuzzy sets and improve the CoCoSo method to evaluate potential pathways of information risk propagation.

2.3. Weight Determination Methods

When evaluating the propagation of information risks, apart from considering the factors influencing information risk propagation and utilizing multi-criteria decision-making methods to assess these factors, it is also essential to account for the importance of these factors, which is reflected in their respective weights. Different weights assigned to these factors will directly influence the final evaluation outcome. Existing weight determination methods can be classified into three categories: objective weighting methods, subjective weighting methods, and methods that combine both objective and subjective approaches [29]. Objective weighting methods commonly include the entropy weighting method [30], maximum deviation method [31], GRA-DEMATEL method [19], and more. Subjective weighting methods include the AHP method [32], Delphi method [33], and others. The combined objective and subjective weighting methods, as the name implies, combine the approaches of both objective and subjective weighting, such as the BWM-LBWA method [3]. DEMATEL is a classical approach in multi-criteria decision making [34], and Paul et al. [35] used a fuzzy DEMATEL method to evaluate criteria weights. Mao et al. [36] proposed a fuzzy DEMATEL method to deal with the ambiguity of evaluation in the decision-making process and to determine the weights of the evaluation criteria. Gandomi et al. [37] used DEMATEL to determine the inter-relationships between the criteria. The main advantages of a Grey Relational Analysis (GRA) are that it is not limited by sample size and normal distribution of data, and its calculation process is simple and easy to understand [38]. It is often used to enhance certain classical multi-criteria decision-making methods. Silva et al. [39] proposed a multi-criteria decision-making model composed of the "CRITIC (Criteria Importance through Intercriteria Correlation)" method and a Grey Relational Analysis (GRA), aiming to select the best alternatives to include in an investment portfolio. Jagatheswari et al. [40] introduced an improved TOPSIS method based on a Grey Relational Analysis (GRATOPSIS) as a collaborative execution strategy, used to evaluate the trustworthiness provided with each mobile node in a network to ensure Quality of Service (QoS). Zhou et al. [41] constructed a Fuzzy Fermate optimization model based on a Grey Relational Analysis to calculate the weights of criteria in multi-criteria decision making. A Grey Relational Analysis (GRA) can replace subjective pairwise comparisons with grey relational coefficients, addressing the limitations of DEMATEL. In this context, GRA and DEMATEL are combined to leverage their strengths and calculate the weights of risk factors based on objective risk information. This approach ensures a fairer and more effective consideration of the correlations among risk factors. The GRA-DEMATEL approach inherits the capability of the DEMATEL model to identify critical features in the decision-making process, while also harnessing the advantages of GRA to overcome challenges associated with extensive risk factors. This includes the substantial workload and implementation difficulties in pairwise comparisons when dealing with a large number of risk factors, as well as the issues of ensuring consistency in pairwise comparisons and the significant changes in the overall relationship matrix due to minor differences in the direct relationship matrix [42]. Therefore, it is an effective method for calculating objective weights for indicators. With the Delphi method in determining the subjective weights, experts need to be anonymous and go through several rounds of discussion to determine the consistent weights [43]. This makes the determined subjective weights more scientific. Inspired by Torkayesh et al. [3], after combining the advantages of GRA-DEMATEL and the Delphi method, we combine them to determine the weights of evaluation indicators.

3. Methodology

3.1. A Brief Review of Probabilistic Hesitant Fuzzy Sets

In this section, we will provide a brief review of some fundamental concepts of probabilistic hesitant fuzzy sets, laying the theoretical groundwork for subsequent sections.

Definition 1. *Ref. [44] Let Φ be a non-empty finite set. The probabilistic hesitant fuzzy set (PHFS) with respect to Φ is represented as follows:*

$$PHFS = \{\langle \phi, h_\phi(p_\phi) \rangle \phi \in \Phi\}$$

where $h_\phi(p_\phi) = \{\zeta^\alpha | p^\alpha, \alpha = 1,2,3,\ldots,l\}$; it is referred to as an element of the probabilistic hesitant fuzzy set (PHFS), and $h_\phi(p_\phi)$ is termed as the probabilistic hesitant fuzzy element. In the expression $h_\phi(p_\phi)$, l denotes the total number of elements in the PHFS; ζ^α represents the possible membership degree of $\phi \in \Phi$; and p^α indicates the probability of ζ^α occurring, satisfying the normalization condition (i.e., $p^\alpha \in (0,1]$) and $\sum_{\alpha=1}^{l} p^\alpha \leq 1$. In this study, $\sum_{\alpha=1}^{l} p^\alpha = 1$.

Definition 2. *Ref. [45] Let $h_\phi = \{\zeta^\alpha | p^\alpha, \alpha = 1,2,3,\ldots,l\}$ be a probabilistic hesitant fuzzy element. The scoring function (SF) and deviation degree (DD) are computed as follows:*

$$SF(h_\phi) = \sum_{\alpha=1}^{l} \zeta^\alpha p^\alpha$$

$$DD(h_\phi) = \sum_{\alpha=1}^{l} \left[\zeta^\alpha - SF(h_\phi)\right]^2 p^\alpha$$

According to the scoring function and deviation degree of probabilistic hesitant fuzzy elements, for any two probabilistic hesitant fuzzy elements h^a and h^b, the following relationship exists:

(a) *If $SF(h^a) > SF\left(h^b\right)$, then $h^a \succ h^b$;*

(b) *If $SF(h^a) = SF\left(h^b\right)$, then If $DD(h^a) > DD\left(h^b\right)$, then $h^a \succ h^b$; if $DD(h^a) = DD\left(h^b\right)$, then $h^a \approx h^b$.*

Definition 3. *Refs. [9,46] For any three probabilistic hesitant fuzzy elements h, h^a, and h^b, with the existence of parameter $\updownarrow > 0$ and constant c, the following computation rules hold:*

(1) $(h_\phi)^c = \bigcup_{\alpha=1,2,\ldots,l} \{(1-\zeta^\alpha)|p^\alpha\}$, $(h_\phi)^{\updownarrow} = \bigcup_{a=1,2,3,\ldots,l} \left\{(\zeta^\alpha)^{\updownarrow}|p^\alpha\right\}$

(2) $\updownarrow h_\phi = \bigcup_{a=1,2,3,\ldots,l} \left\{1-(1-\zeta^\alpha)^{\updownarrow}|p^\alpha\right\}$, $h_\phi \ominus \mathrm{c} = \bigcup_{\alpha=1,2,\ldots,l} \{(\zeta^\alpha - c)|p^\alpha\}$

(3) $h^a \oplus h^b = \bigcup_{\alpha_1=1,2,3,\ldots l, \alpha_2=1,2,3,\ldots l} \{(\zeta^{\alpha_1} + \zeta^{\alpha_2} - \zeta^{\alpha_1}\zeta^{\alpha_2})|p^{\alpha_1}p^{\alpha_2}\}$

(4) $h^a \otimes h^b = \bigcup_{\alpha_1=1,2,3,\ldots l, \alpha_2=1,2,3,\ldots l} \{\zeta^{\alpha_1}\zeta^{\alpha_1}|p^{\alpha_1}p^{\alpha_2}\}$

3.2. Some New Similarity and Distance Measures for Probabilistic Hesitant Fuzzy Sets

In this section, we extend the Jaccard similarity [47] and Dice similarity [48] to define Jaccard distance and Dice distance for probabilistic hesitant fuzzy sets (PHFS). The distance formulas are introduced based on the work of Jin et al. [49], who applied these distance measures in the context of interval-valued spherical fuzzy environments to compute distances between interval-valued spherical fuzzy sets. Their research demonstrated that these distance measures can prevent information loss and reduce biased computation results. As a result, we adopt these distance measures to calculate the distance between two probabilistic hesitant fuzzy elements.

In practical decision-making environments, it is indeed common to encounter situations where two probabilistic hesitant fuzzy elements have different lengths. To address this issue, this paper introduces the "Probability Splitting Algorithm" to handle such cases and ensure that the PHFS used in the distance calculation have consistent lengths. The core idea of this algorithm is to find the minimum probability from the probabilistic hesitant fuzzy element and then expand the element with fewer probabilities based on this minimum probability. The specific calculation process is illustrated in Example 1, and the detailed algorithm can be referred to in the works of Fang [8] and Lin et al. [50].

Example 1. *Let there be three probabilistic hesitant fuzzy sets: $h_\phi^1 = \{0.3|0.2, 0.5|0.1, 0.7|0.2, 0.8|0.5\}$, $h_\phi^2 = \{0.5|0.3, 0.7|0.3, 0.8|0.4\}$, and $h_\phi^3 = \{0.6|0.4, 0.8|0.6\}$. By applying the Probability Splitting Algorithm, we can expand these sets as follows:*

$$h_\phi^1 = \{0.3|0.2, 0.5|0.1, 0.7|0.1, 0.7|0.1, 0.8|0.1, 0.8|0.4\}$$

$$h_\phi^2 = \{0.5|0.2, 0.5|0.1, 0.7|0.1, 0.7|0.1, 0.7|0.1, 0.8|0.4\}$$

$$h_\phi^3 = \{0.6|0.2, 0.6|0.1, 0.6|0.1, 0.8|0.1, 0.8|0.1, 0.8|0.4\}$$

Definition 4. *Let there be two probabilistic hesitant fuzzy elements,* $h_\phi^1\left(p_\phi^1\right) = \{\zeta^\alpha|p^\alpha, \alpha = 1,2,3,\ldots,l\}$ *and* $h_\phi^2\left(p_\phi^2\right) = \left\{\zeta^b\middle|p^b, b = 1,2,3,\ldots,l\right\}$. *The Jaccard similarity and distance between them are defined as follows:*

$$S_{Jaccard} = \sum_{i=1}^{L}\left(\frac{\overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}}{\overline{\overline{\zeta_i^a}}^2 + \overline{\overline{\zeta_i^b}}^2 - \overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}}\right)\overline{\overline{p}}^i \tag{1}$$

$$d_{Jaccard}\left(\overline{h_\phi^1}, \overline{h_\phi^2}\right) = 1 - S_{Jaccard} \tag{2}$$

where $\overline{h_\phi^1}$ *and* $\overline{h_\phi^2}$ *represent the probabilistic hesitant fuzzy elements obtained after applying the Probability Splitting Algorithm. The elements* $\overline{\overline{\zeta_i^a}}$ *and* $\overline{\overline{\zeta_i^b}}$ *denote elements within the processed probabilistic hesitant fuzzy sets, and* $\overline{\overline{p}}^i$ *represents the probabilities associated with these elements. L represents the length of the processed probabilistic hesitant fuzzy elements.*

Theorem 1. *The Jaccard distance between probabilistic hesitant fuzzy elements satisfies the following property:*

(1) $0 < d_{Jaccard}\left(h_\phi^1, h_\phi^2\right) \le 1$,
(2) $d_{Jaccard}\left(h_\phi^1, h_\phi^2\right) = d_{Jaccard}\left(h_\phi^2, h_\phi^1\right)$,
(3) *if* $h_\phi^1 = h_\phi^2$, *then* $d_{Jaccard}\left(h_\phi^1, h_\phi^2\right) = 0$.

Proof. (1) For any two probabilistic hesitant fuzzy elements h_ϕ^1 and h_ϕ^2, their internal elements ζ_i^a and ζ_i^b always lie within the interval $[0,1]$. Therefore, according to the Cauchy–Schwarz inequality, we have $\overline{\overline{\zeta_i^a}}^2 + \overline{\overline{\zeta_i^b}}^2 \ge 2\overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}$, $\overline{\overline{\zeta_i^a}}^2 + \overline{\overline{\zeta_i^b}}^2 - \overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}} \ge \overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}$, and $\frac{\overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}}{\overline{\overline{\zeta_i^a}}^2 + \overline{\overline{\zeta_i^b}}^2 - \overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}} \le 1$, then $0 < d_{Jaccard}\left(\overline{h_\phi^1}, \overline{h_\phi^2}\right) \le 1$ and $0 < d_{Jaccard}\left(h_\phi^1, h_\phi^2\right) \le 1$. (2) and (3) clearly hold and the proof is sketchy. □

Definition 5. *Let there be two probabilistic hesitant fuzzy elements,* $h_\phi^1\left(p_\phi^1\right) = \{\zeta^\alpha|p^\alpha, \alpha = 1,2,3,\ldots,l\}$ *and* $h_\phi^2\left(p_\phi^2\right) = \left\{\zeta^b\middle|p^b, b = 1,2,3,\ldots,l\right\}$. *The Dice similarity and distance between them are defined as follows:*

$$S_{Dice} = \sum_{i=1}^{L}\left(\frac{2\overline{\overline{\zeta_i^a}}\,\overline{\overline{\zeta_i^b}}}{\overline{\overline{\zeta_i^a}}^2 + \overline{\overline{\zeta_i^b}}^2}\right)\overline{\overline{p}}^i \tag{3}$$

$$d_{Dice}\left(\overline{h_\phi^1}, \overline{h_\phi^2}\right) = 1 - S_{Dice} \tag{4}$$

where, $\overline{h_\phi^1}$ *and* $\overline{h_\phi^2}$ *represent the probabilistic hesitant fuzzy elements obtained after applying the Probability Splitting Algorithm. The elements* $\overline{\overline{\zeta_i^a}}$ *and* $\overline{\overline{\zeta_i^b}}$ *denote elements within the processed probabilistic hesitant fuzzy sets, and* $\overline{\overline{p}}^i$ *represents the probabilities associated with these elements. L represents the length of the processed probabilistic hesitant fuzzy elements.*

Theorem 2. *The Dice distance between probabilistic hesitant fuzzy elements satisfies the following property:*

(1) $0 \le d_{Dice}\left(h_\phi^1, h_\phi^2\right) \le 1$,
(2) $d_{Dice}\left(h_\phi^1, h_\phi^2\right) = d_{Dice}\left(h_\phi^2, h_\phi^1\right)$,

(3) *if* $h^1_\phi = h^2_\phi$*, then* $d_{Dice}\left(h^1_\phi, h^2_\phi\right) = 0$.

The proof is the same as Theorem 1.

3.3. Weight Determination Method Combining Subjective and Objective Approaches

Weight determination has always been an important research direction in fuzzy evaluation. This paper proposes the following weight determination method based on considering both subjective and objective weights. The method is divided into three stages. In the first stage, evaluators need to assign subjective weights to each evaluation criterion based on their own experiences and risk attitudes. To ensure the scientificity and rationality of the weights, this paper adopts the Delphi method to determine the subjective weights. The second stage is the objective weighting stage. In this stage, this paper will use the GRA-DEMATEL method to determine the weights of each evaluation criterion based on the evaluation data provided by experts. The third stage involves the integration of subjective and objective weights.

(1) Stage 1: Subjective Weight Assignment

In this stage, the evaluators invite experts to use the Delphi method to determine the weights of the evaluation criteria. Let there be m evaluation scenarios ($Eva_i, i = 1,2,3,\ldots,m$), and n evaluation indicators ($Ind_j, j = 1,2,3,\ldots,n$). After the Delphi method evaluation, the following subjective weights are obtained:

$$W^s = \{w_j, j = 1,2,3,\ldots,n\} \tag{5}$$

(2) Stage 2: Objective Weight Assignment

As a relatively new method for weight determination, GRA-DEMATEL combines the advantages of a Grey Relational Analysis, which is not restricted by sample size and normal distribution of data, and overcomes the subjectivity in determining the direct correlation matrix in DEMATEL. Therefore, this paper uses this method to determine the weights of each evaluation indicator. However, since the evaluation data we are dealing with are probabilistic hesitant fuzzy numbers, this study will utilize the distance calculation method proposed in Section 3.3 to improve the grey relational coefficient.

Definition 6. *Let the evaluation matrix based on probabilistic hesitant fuzzy numbers be denoted as P. It can be rewritten as* $P^{Gray} = (P^G_1, P^G_2, P^G_3, \ldots, P^G_n)$*, where* $P^G_j = \left(h^G_{1j}, h^G_{2j}, h^G_{2j}, \ldots, h^G_{mj}\right)^T$ *is a column vector. By reorganizing the evaluation matrix, we obtain n sequences. Let one of these sequences,* $P^G_t = \left(h^G_{1t}, h^G_{2t}, h^G_{2t}, \ldots, h^G_{mt}\right)^T$*, be the behavior characteristic sequence, and the remaining n-1 sequences,* $P^G_1 = \left(h^G_{11}, h^G_{21}, h^G_{21}, \ldots, h^G_{m1}\right)^T$*, ...,* $P^G_{t-1} = \left(h^G_{1(t-1)}, h^G_{2(t-1)}, h^G_{2(t-1)}, \ldots, h^G_{m(t-1)}\right)^T$*,* $P^G_{t+1} = \left(h^G_{1(t+1)}, h^G_{2(t+1)}, h^G_{2(t+1)}, \ldots, h^G_{m(t+1)}\right)^T$*, ...,* $P^G_n = \left(h^G_{1n}, h^G_{2n}, h^G_{2n}, \ldots, h^G_{mn}\right)^T$*, be the related factors. Thus, the direct correlation matrix,* $DRM = \left(\hbar_{tj}\right)_{n\times n}$*, can be calculated using the following formula:*

$$\hbar_{tj} = \frac{1}{m}\sum_{s=1}^{m} \frac{\min\limits_{j,j\neq t}\min\limits_{s}\left\{D\left(P^G_{st}, P^G_{sj}\right)\right\} + c\max\limits_{j,j\neq t}\max\limits_{s}\left\{D\left(P^G_{st}, P^G_{sj}\right)\right\}}{\left\{D\left(P^G_{st}, P^G_{sj}\right)\right\} + c\max\limits_{j,j\neq t}\max\limits_{s}\left\{D\left(P^G_{st}, P^G_{sj}\right)\right\}} \tag{6}$$

where $D\left(P^G_{st}, P^G_{sj}\right)$ *represents the distance calculation formula proposed in this paper. Since the related factor sequences are different when calculating the values of* $\hbar_{ij}$ *and* $\hbar_{ji}$*, we can easily obtain* $DRM = \left(\hbar_{tj}\right)_{n\times n}$*, which is generally a non-symmetric matrix, and c denotes the resolution coefficient, which is usually set as* $c = 0.5$*. Due to the fact that when* $j = t$*, the distance* $D\left(P^G_{st}, P^G_{sj}\right)$ *equals 0, therefore, in the correlation matrix, we set the elements on the diagonal to 0.*

Based on Definition 6, we can easily obtain the direct correlation matrix for the evaluation results. Next, by further processing the direct correlation matrix, we can calculate the objective weights of each evaluation indicator.

Step 1: Normalize the direct correlation matrix to obtain $\overline{DRM} = \left(\overline{\hbar_{tj}}\right)_{n\times n}$.

$$\overline{\hbar_{tj}} = \frac{\hbar_{tj}}{\max\left(\max\left(\sum_t \hbar_{tj}\right), \max\left(\sum_j \hbar_{tj}\right)\right)} \tag{7}$$

Step 2: Calculate the total direct correlation matrix $\overline{TRM} = \left(\overline{T_{tj}}\right)_{n\times n}$.

$$TRM = \lim_{N\to\infty}\left(\overline{DRM} + \overline{DRM}^2 + \cdots + \overline{DRM}^N\right) = \overline{DRM}\left(E - \overline{DRM}\right)^{-1} \tag{8}$$

where E represents the identity matrix.

Step 3: Calculate the sums of each row and column.

$$Row_t = \sum_{j=1}^{n} \hbar_{tj},\ Col_j = \sum_{t=1}^{n} \hbar_{tj} \tag{9}$$

Step 4: Calculate the objective weights of each evaluation criterion based on Formulas (8) and (9).

$$w_j^o = \frac{Row_j + Col_j}{\sum\limits_{j=1}^{n}\left(Row_j + Col_j\right)} \tag{10}$$

(3) Stage 3: Integration of Subjective and Objective Weights

Subjective weighting is based on expert judgments and is highly subjective. Therefore, it may not fully reflect the true weights of each evaluation criterion. On the other hand, objective weighting is calculated based on evaluation data using objective methods. The weights obtained from this method are entirely based on objective data but may overlook some subjective factors. As a result, they may not completely represent the true weights of the evaluation criteria. To address this issue, this paper uses a method that combines subjective and objective approaches to handle the weights. Drawing from the studies of Yazdani et al. [51] and Torkayesh et al. [3], this study proposes the following weight aggregation operators:

$$w_j = \frac{w_i^s w_i^o}{\sum\limits_{i=1}^{n} w_i^s w_i^o} \tag{11}$$

3.4. PHFS-CoCoSo Method Based on Similarity

Building upon the ideas of Simple Additive Weighting (SAW), Weighted Aggregated Sum Product Assessment (WASPAS), and Multiplicative Exponential Weighting (MEW) methods, Yazdani et al. [25] proposed the CoCoSo method and compared it with these three approaches, providing reliable results. However, the traditional CoCoSo method deals with precise numerical information. Although there are some related studies in fuzzy environments, they improve the CoCoSo method by introducing fuzzy operators to process the data [26,27]. In contrast to these studies, here, this paper preprocesses the data using distance measures and directly uses probabilistic hesitant fuzzy numbers for the decision matrix. The specific steps are as follows:

Step 1: Let there be m evaluation schemes and n evaluation criteria. We have the following evaluation matrix based on probabilistic hesitant fuzzy sets.

$$EM = \begin{bmatrix} h_{11}^P & h_{12}^P & \ldots & h_{1n}^P \\ h_{21}^P & h_{22}^P & \ldots & h_{2n}^P \\ . & . & & . \\ . & . & \ldots & . \\ . & . & & . \\ h_{m1}^P & h_{m2}^P & \ldots & h_{mn}^P \end{bmatrix} \quad (12)$$

Step 2: Normalize the evaluation matrix to obtain DEM.

$$DEM = \begin{bmatrix} S_{11} & S_{12} & \ldots & S_{1n} \\ S_{21} & S_{22} & \ldots & S_{2n} \\ . & . & & . \\ . & . & \ldots & . \\ . & . & & . \\ S_{m1} & S_{m2} & \ldots & S_{mn} \end{bmatrix} \quad (13)$$

where $S_{ij}(i = 1, 2, \ldots, m, j = 1, 2, \ldots, n)$ represents the standardized evaluation information after normalization. It is determined with the following equation:

$$S_{ij} = \begin{cases} \frac{S\left(h_{ij}^P, h_j^-\right)}{S\left(h_j^+, h_j^-\right)}; \text{for benefit index} \\ \frac{S\left(h_{ij}^P, h_j^+\right)}{S\left(h_j^+, h_j^-\right)}; \text{for non-benefit index} \end{cases} \quad (14)$$

where $h_j^- = \bigcup_{i=1}^{l} \min \min_{i=1,2,3,\ldots,m} \left(h_{ij}^P\right)$ and $h_j^+ = \bigcup_{i=1}^{l} \max \max_{i=1,2,3,\ldots,m} \left(h_{ij}^P\right)$; $S(\bullet)$ represents the similarity between two probabilistic hesitant fuzzy sets.

Step 3: Calculate the sum of weighted comparability (WC_i) and power-weighted comparability sequences ($PWCS_i$) for each alternative scheme according to the following equation:

$$WC_i = \sum_{j=1}^{n} \left(w_j S_{ij}\right),\ PWCS_i = \sum_{j=1}^{n} \left(w_j\right)^{S_{ij}} \quad (15)$$

Step 4: Based on the arithmetic mean of the sums of scores obtained from the Weighted Sum Method (WSM) and Weighted Product Method (WPM), the relative score compared to the best alternative scheme for WSM and WPM, and the balanced compromise score of WSM and WPM models, three aggregation scores are defined to calculate the relative weights of evaluation schemes. Here, although the value of δ is between 0 and 1, the threshold is typically set at 0.50.

$$\begin{cases} AAS_{ia} = \frac{WC_i + PWCS_i}{\sum_{i=1}^{m} (WC_i + PWCS_i)} \\ AAS_{ib} = \frac{WC_i}{\min_i WC_i} + \frac{PWCS_i}{\min_i PWCS_i} \\ AAS_{ic} = \frac{\delta(WC_i) + (1-\delta)(PWCS_i)}{\delta \max_i WC_i + (1-\delta) \max_i PWCS_i}; 0 \le \delta \le 1 \end{cases} \quad (16)$$

Step 5: Calculate the overall score ($Evaluation_i$) based on the following formula and rank the evaluation schemes according to the overall score.

$$Evaluation_i = (AAS_{ia} \times AAS_{ib} \times AAS_{ic})^{\frac{1}{3}} + \frac{1}{3}(AAS_{ia} + AAS_{ib} + AAS_{ic}) \quad (17)$$

Based on the methods presented in the previous sections, this paper provides the following evaluation process, as shown in Figure 1:

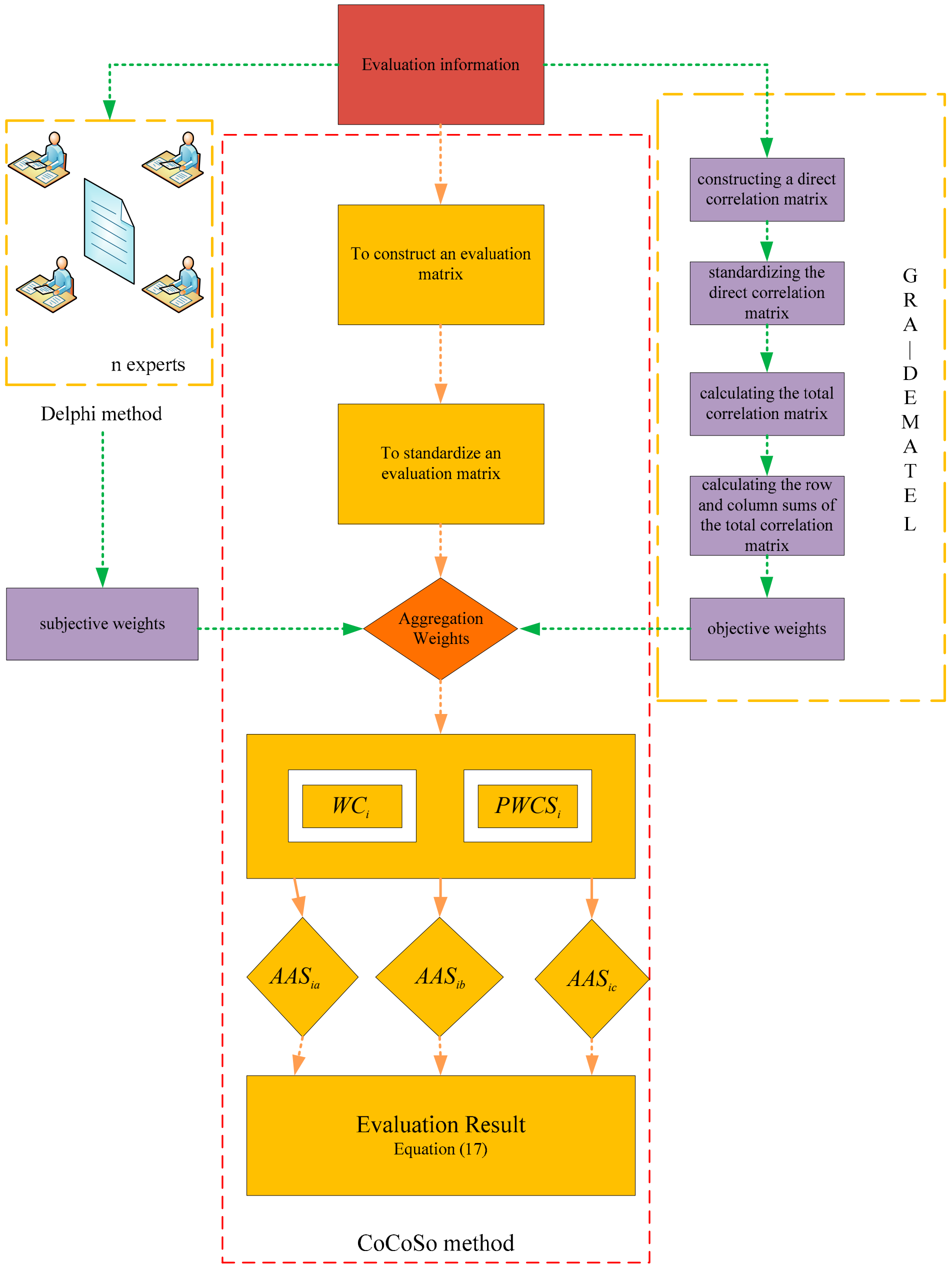

Figure 1. Evaluation process.

4. Evaluation of Information Risk Propagation in Complex Public Opinion Environments

In order to evaluate the information risk propagation in complex public opinion environments, this section will present some potential channels for information risk propagation and construct relevant indicators to assess these propagation channels. The purpose is to assist enterprises in identifying potential propagation channels and determining the most significant ones in a reasonable and timely manner when information risks arise. By using the methods and evaluations presented in this paper, enterprises can promptly discover potential propagation channels and identify the most critical ones. This will enable them to determine response measures promptly and avoid significant losses.

4.1. Information Risk Propagation Channels

Today, the rapid speed and convenience of information dissemination and interaction, coupled with reduced costs, have brought tremendous benefits to human society. However, there is a prevalence of misinformation that spreads widely on various information platforms and media at a low cost and with little effort. Leveraging the astonishing speed and global reach of the internet, misinformation can easily deceive people, manipulate public opinion, and cause significant harm. In this context, this study defines information risk in complex public opinion environments as the potential risks individuals, organizations, or society face in a diverse and rapidly spreading information environment, along with the negative impacts that these risks may bring. Information risk propagation in this paper refers to the dissemination of these risks. By summarizing existing literature, this paper has identified the following potential channels for information risk propagation, as shown in Table 1.

Table 1. Potential Information Risk Propagation Channels.

Channels	Mark	Content	References
Social media platforms	Ev_1	Social media platforms such as Facebook, Twitter, and Weibo have become the main channels for information dissemination. Users can spread information through various means, including posting messages, sharing links, and commenting, which may include various types of content, such as real, false, or misleading information.	Wu and Yang [52] Liu and Ma [53] Li et al. [54]
News media	Ev_2	News agencies and media play a crucial role in information dissemination. They spread information through reporting, interviews, commentary, and other means. However, at times, media can also engage in inaccurate reporting, sensationalism, or biased coverage, leading to risks in information propagation.	Bi and Tian [55]
Online forums and blogs	Ev_3	Online forums and blogs provide platforms for discussion and communication, but they also come with information dissemination risks. Users can spread information through posting threads, comments, and replies, including false information, malicious attacks, and hate speech, among other things.	Wu and Yang [52]
Traditional media	Ev_4	Traditional media, including television, radio, newspapers, and magazines, still play a significant role in information dissemination. These media channels deliver information to the public through news reports, opinion articles, television programs, and more.	Di and Mingchen [56] Djerf-Pierre and Shehata [57]
Short video platforms	Ev_5	Short video platforms have become increasingly popular in recent years, providing a quick and engaging way for users to share and consume information. These platforms, such as TikTok and Instagram Reels, allow users to create and share short videos, making them a powerful tool for information dissemination. However, similar to other social media platforms, there is a risk of spreading misinformation, false content, and potentially harmful messages through short video content.	Li et al. [58]

4.2. Construction of Evaluation Indicators

In the current era of advanced internet technology, information risks often spread and disseminate through various potential channels. Therefore, it is of great importance to evaluate these potential dissemination pathways and identify the main routes of information propagation. This assessment directly impacts how decision makers can take effective measures to address these information risks and avoid significant losses. In this context, this paper considers the characteristics of information risk propagation in a complex public opinion environment, such as the lack of barriers to dissemination, micro-content, micro-formats, real-time nature, interactivity, and vitality. Additionally, we take into account the networked distribution patterns of internet users. Based on these factors and referencing previous scholarly research, this paper proposes five indicators for evaluating information propagation pathways. The specific details are presented in Table 2.

Table 2. Evaluation Indicators for Information Risk Propagation Pathways.

Indicators	Mark	Content	References
Trustworthiness and reliability	Ind_1	When evaluating information dissemination pathways, the primary criteria to consider are their trustworthiness and reliability. The dissemination pathways should be based on credible sources, authoritative institutions, or reliable platforms to ensure the authenticity and accuracy of the information.	Hajli et al. [59]; Song et al. [60]
Dissemination speed and breadth	Ind_2	In the complex public opinion environment, information dissemination tends to be rapid and extensive. When evaluating dissemination pathways, it is important to consider their speed of propagation and the breadth of their influence. Some pathways may have the ability to spread rapidly, while others may have a more significant impact within specific social circles or target audiences.	Glenski et al. [61] Tian et al. [62]
Information filtering mechanisms	Ind_3	When evaluating communication channels, it is essential to consider their information filtering mechanisms. Some channels may have robust information screening and quality control mechanisms, which can reduce the dissemination of false, misleading, or harmful information. On the other hand, other channels may be relatively loose, making it easier for low-quality and inaccurate information to spread.	Denizci Guillet et al. [63]
Participation and interactivity	Ind_4	In complex public opinion environments, it is important to consider the level of participation and interactivity when evaluating communication channels. Some channels may actively encourage user engagement, interaction, and information sharing, thereby promoting information dissemination. On the other hand, other channels may be more passive, with information dissemination primarily controlled by media or organizations.	Woo-Young [64] Chen and Zhao [65]
Social impact and public opinion orientation	Ind_5	When evaluating communication pathways, it is essential to consider their social impact and public opinion orientation. Some pathways may have a significant influence on public opinion, social emotions, and attitudes, which in turn can affect the public's decision making and behavior. Assessing the public opinion orientation of communication pathways helps understand their impact on society.	You et al. [66] Feng [67]

5. Case Study

In China, there is a well-known restaurant chain brand with over 900 outlets. This brand is highly favored by consumers for providing delicious food and a pleasant dining experience. It places great emphasis on food safety and hygiene standards, establishing a positive reputation within the industry. The brand strictly controls the procurement, storage, and processing of ingredients to ensure consumers enjoy fresh, safe, and hygienic cuisine. Additionally, the brand values employee training and management to ensure staff possess good hygiene awareness and professional skills, delivering high-quality catering services. Over the years, the brand's unique cuisine and professional dining experience have continuously attracted new customers, building a large base of loyal consumers.

However, recently, the brand faced a public relations crisis due to a food safety incident. On 1 January 2023, a consumer posted a video on social media exposing food safety issues observed during their visit to one of the brand's outlets. The video showed poor hygiene conditions in the food preparation area, with improper food storage and the presence of cockroaches. The consumer's video quickly gained widespread attention on social media platforms, being widely shared and commented on. Other consumers also began sharing their negative experiences and concerns about food safety issues at the brand's outlets. Soon, the company's management became aware of the situation and started collecting relevant information. According to available data, the consumer's video was viewed and shared over 20,000 times on social media. Numerous bloggers wrote more than 60 papers about the brand's food safety issues. The total number of posts and comments related to the brand's food safety problem on social media exceeded 10,000.

In order to prevent the situation from escalating further, the management of the company decided to form an expert group to analyze the incident and determine appropriate response measures. Now, based on the evaluation criteria from Section 4.2, the potential dissemination pathways mentioned in Section 4.1 are assessed to facilitate the formulation of response strategies. Due to the urgency and uncertainty of the situation, the expert group's evaluation information will be presented using probability hesitant fuzzy sets. The specific evaluation information is shown in Table 3.

Table 3. Evaluation Information for the Catering Company Regarding the Emergency Incident.

	Ind_1	Ind_2	Ind_3	Ind_4	Ind_5
Ev_1	{0.8 | 0.2, 0.9 | 0.6, 0.95 | 0.2}	{0.7 | 0.1, 0.8 | 0.4, 0.9 | 0.4, 0.95 | 0.1}	{0.9 | 0.6, 0.95 | 0.4}	{0.85 | 0.2, 0.9 | 0.4, 0.95 | 0.4}	{0.8 | 0.1, 0.9 | 0.6, 0.95 | 0.3}
Ev_2	{0.9 | 0.7, 0.95 | 0.3}	{0.6 | 0.3, 0.7 | 0.4, 0.8 | 0.2, 0.9 | 0.1}	{0.8 | 0.4, 0.85 | 0.4, 0.9 | 0.2}	{ 0.8 | 0.1, 0.9 | 0.5, 0.95 | 0.4}	{0.8 | 0.4, 0.9 | 0.4, 0.95 | 0.2}
Ev_3	{0.8 | 0.2, 0.9 | 0.8}	{0.5 | 0.2, 0.6 | 0.4, 0.7 | 0.4}	{0.9 | 0.5, 0.95 | 0.5}	{ 0.8 | 0.2, 0.9 | 0.6, 0.95 | 0.2}	{0.7 | 0.1, 0.8 | 0.2, 0.9 | 0.5, 0.95 | 0.2}
Ev_4	{0.8 | 0.1, 0.9 | 0.7, 0.95 | 0.2}	{0.5 | 0.3, 0.6 | 0.4, 0.65 | 0.3}	{0.7 | 0.2, 0.8 | 0.4, 0.85 | 0.4}	{0.6 | 0.3, 0.7 | 0.5, 0.8 | 0.2}	{0.7 | 0.2, 0.8 | 0.4, 0.9 | 0.4}
Ev_5	{0.9 | 0.9, 0.95 | 0.1}	{0.85 | 0.6, 0.9 | 0.4 }	{0.9 | 0.2, 0.95 | 0.8}	{0.9 | 0.4, 0.95 | 0.6}	{0.85 | 0.1, 0.9 | 0.4, 0.95 | 0.5}

Stage 1: After conducting four rounds of anonymous evaluation using the Delphi method, the final subjective weights for the five evaluation indicators are determined as follows:

$$W^s = [0.15, 0.3, 0.1, 0.25, 0.2] \quad (18)$$

Stage 2: Use GRA-DEMATEL to determine the weights of individual evaluation indicators.

Step 2.1: Based on Definition 6, the evaluation information is rewritten, and the direct correlation matrix is obtained according to Formula (6) as shown in (18).

$$DRM = \begin{pmatrix} 0. & 0.5312 & 0.6528 & 0.697 & 0.73 \\ 0.5312 & 0. & 0.5322 & 0.5422 & 0.5469 \\ 0.6493 & 0.6847 & 0. & 0.7098 & 0.7397 \\ 0.8021 & 0.716 & 0.7222 & 0. & 0.7363 \\ 0.6745 & 0.6806 & 0.7217 & 0.7396 & 0. \end{pmatrix} \tag{19}$$

Step 2.2: Calculate the normalized direct correlation matrix using Formula (7) as shown in (19).

$$\overline{DRM} = \begin{pmatrix} 0. & 0.2033 & 0.2483 & 0.2592 & 0.2652 \\ 0.1999 & 0. & 0.2024 & 0.2017 & 0.1987 \\ 0.2333 & 0.246 & 0. & 0.255 & 0.2657 \\ 0.2695 & 0.2405 & 0.2426 & 0. & 0.2474 \\ 0.2395 & 0.2417 & 0.2562 & 0.2626 & 0. \end{pmatrix} \tag{20}$$

Step 2.3: Calculate the total correlation matrix using Formula (8) as shown in (20).

$$TRM = \begin{pmatrix} 4.29802 & 4.42396 & 4.52222 & 4.63519 & 4.63238 \\ 3.80804 & 3.60521 & 3.83116 & 3.92036 & 3.91278 \\ 4.54996 & 4.51496 & 4.38644 & 4.69699 & 4.69698 \\ 4.5747 & 4.51195 & 4.58309 & 4.49532 & 4.687 \\ 4.55629 & 4.51419 & 4.59259 & 4.70412 & 4.48935 \end{pmatrix} \tag{21}$$

Step 2.4: Calculate the sums of rows and columns, and the objective weights of each evaluation indicator using Formulas (9) and (10). The results are shown in Table 4.

Table 4. Sums of Rows and Columns, and Objective Weights of Evaluation Indicators.

The i-th Row/Column/Indicator	1	2	3	4	5
Sums of Rows	22.51176	19.07755	22.84532	22.85205	22.85653
Sums of Columns	21.78701	21.57026	21.9155	22.45196	22.41849
Objective Weights of Evaluation Indicators	0.2011	0.18452	0.20319	0.20566	0.20553

Stage 3: Calculate the comprehensive weights.

According to the subjective weights obtained using the Delphi method in Stage 1 and the objective weights obtained using GRA-DEMATEL in Stage 2, the comprehensive weights are calculated using Formula (11). The results are shown in Table 5.

Table 5. Comprehensive Weights for Each Evaluation Indicator.

Indicator	Ind_1	Ind_2	Ind_3	Ind_4	Ind_5
The Subjective Weights	0.15	0.3	0.1	0.25	0.2
The Objective Weights	0.2011	0.18452	0.20319	0.20566	0.20553
The Comprehensive Weights	0.15207	0.27907	0.10244	0.2592	0.20722

Stage 4: Rank potential information risk propagation pathways using the similarity-based PHFS-CoCoSo method.

Using the similarity-based PHFS-CoCoSo method, the normalized evaluation matrix under the probability hesitant fuzzy set is obtained as shown in Equation (21). Based on Equations (15)–(17), the evaluation results for WC_i, $PWCS_i$, AAS_{ia}, AAS_{ib}, and AAS_{ic}, as well as the comprehensive evaluation results $Evaluation_i$, are calculated and presented in Table 6.

Table 6. The results of WC_i, $PWCS_i$, AAS_{ia}, AAS_{ib}, AAS_{ic}, and $Evaluation_i$.

	Ev_1	Ev_2	Ev_3	Ev_4	Ev_5
WC_i	1.02684	1.05124	1.06292	1.08874	1.01894
$PWCS_i$	0.96407	0.93401	0.92345	0.88915	0.97476
AAS_{ia}	0.20041	0.19984	0.19995	0.19910	0.20069
AAS_{ib}	2.09200	2.08215	2.08174	2.06850	2.09628
AAS_{ic}	0.96482	0.96208	0.96262	0.95851	0.96618
$Evaluation_i$	1.22058	1.21480	1.21500	1.20695	1.22321
Ranking	2	4	3	5	1

Based on the results from Table 6, we can observe that the short video platform is the primary information risk communication channel for the restaurant company in this public opinion incident. Next in line are social media platforms, online forums and blogs, news media, and traditional media. It is not surprising that the short video platform emerged as the main communication channel for this event. Currently, China has a large number of short video users. Compared to other platforms, short video platforms have lower entry requirements for users, as they only need to upload videos taken on their mobile phones. Additionally, videos spread quickly, making the short video platform the most significant channel for this event's propagation. The restaurant company needs to take timely measures against these potential communication channels' ranking to prevent the event from becoming a hot topic and causing irreparable damage to the company's reputation.

6. Discussion

6.1. Sensitivity Analysis

The proposed method in this paper involves two parameters, namely coefficient c and δ. Among them, coefficient c will affect the objective weights of individual evaluation indicators, thereby influencing the evaluation results, while coefficient δ directly affects the final evaluation results. Therefore, in this section, we will explore the impact of different values of c and δ on the evaluation results. Regarding parameter c, as its value changes from 0 to 1, the weights of each evaluation indicator undergo minor fluctuations. However, the ranking of all indicators' weights remains unchanged, indicating that the weight determination method in this paper exhibits good stability when considering both subjective and objective factors. The results are shown in Figure 2. From Figure 3, it can be observed that due to the small changes in weights, the variations in the potential dissemination pathways are also minimal. Among all potential dissemination pathways, only pathways Eva_2 and Eva_3 exhibit slight changes in their rankings. The reason for this variation is that the difference in evaluation data between pathways Eva_2 and Eva_3 is small, leading to minor changes in weights and subsequent alterations in the rankings.

Parameter δ influences the final evaluation results by affecting the value of Q in Formula (16). From Figure 4a, it can be observed that as δ varies, the change trend is significant. When $\delta < 6$, the rankings among different potential dissemination pathways remain unchanged despite the variation in δ; however, when $\delta \geq 6$, the rankings among potential dissemination pathways change. From Figure 4b, it can be seen that the variations among potential dissemination pathways also follow the same trend as δ, with $\delta = 6$ as the dividing line. This is because the value of AAS_{ic} directly affects the evaluation results; hence, AAS_{ic} and the evaluation results exhibit the same change trend.

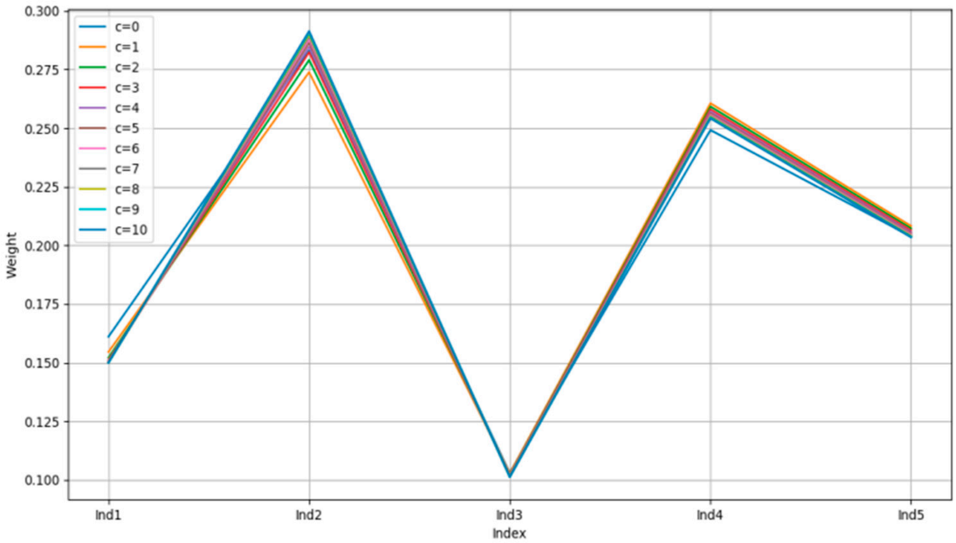

Figure 2. Weights at different resolutions.

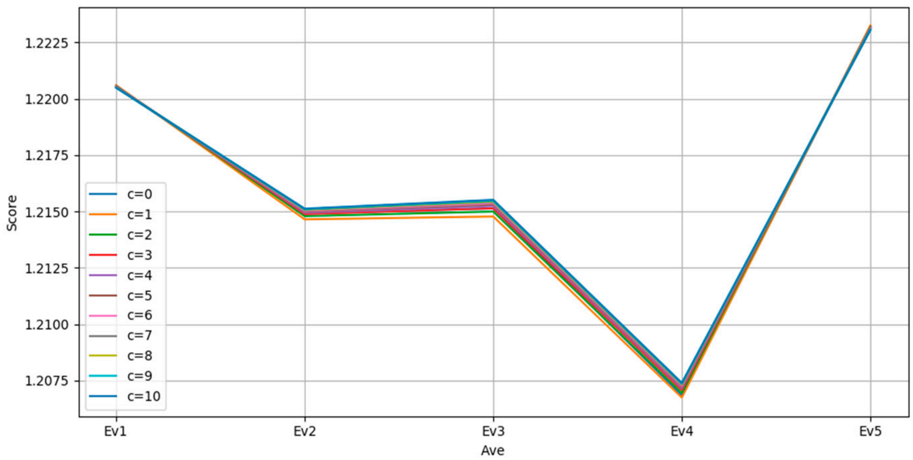

Figure 3. Ranking of scenarios under different weights.

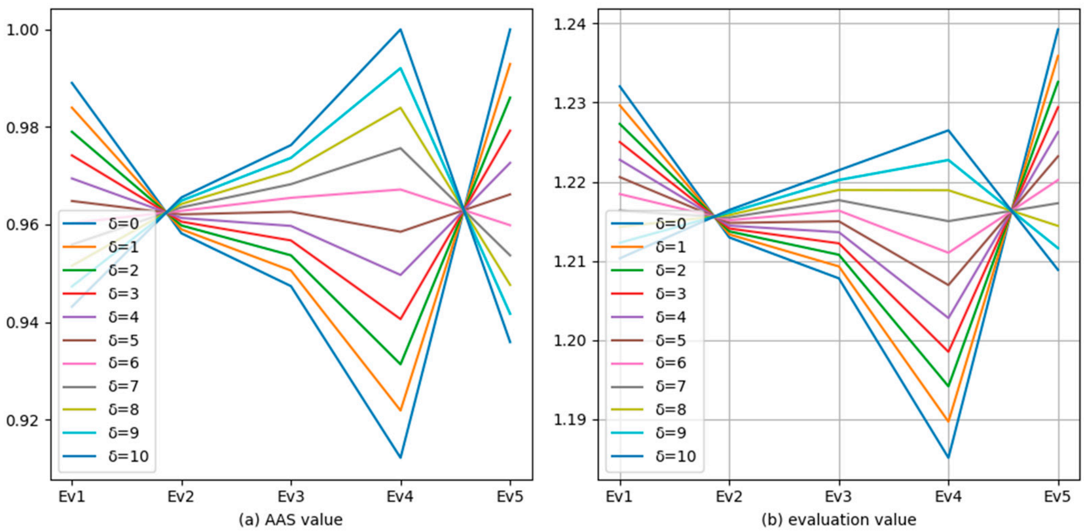

Figure 4. Evaluation results under different values of parameter δ.

6.2. Effectiveness Analysis

In this section, we will verify the effectiveness of the proposed evaluation method for information risk propagation pathways in complex public opinion environments from the following three aspects: First, the improved CoCoSo method with similarity measurement is used to standardize the probabilistic hesitant fuzzy evaluation information. Therefore, when applying different similarity measurement methods to this approach with the same parameters, c and δ, the evaluation results should not have significant deviations. Although different similarity measurement methods may have slight differences in emphasis for the same dataset, the resulting deviations should not be substantial, meaning that the evaluation results will not fluctuate significantly. Second, for a decision matrix, changing the evaluation value of one scheme should not alter the ranking of other schemes. Therefore, this paper will verify the effectiveness of the method by changing the evaluation results of potential propagation pathways in the evaluation matrix. Third, when decomposing the same evaluation problem into multiple sub-problems, using the same method to evaluate these sub-problems should yield consistent evaluation results with the original evaluation.

Based on the evaluation data in Section 5, this paper demonstrates the effectiveness of the proposed method from the three aspects mentioned above. For the first aspect, this study uses Jaccard similarity and Dice similarity as the similarity measurement methods in Formula (14). This paper can observe that although the similarity values and evaluation results are different, the final ranking of potential propagation pathways remains consistent. This does not affect the overall ranking of the potential propagation pathways. For the second aspect, this paper replaces the evaluation values of Ev_1 in the original evaluation matrix with 0.1 while keeping the probabilities unchanged. The results are presented in Table 7. From the results in Table 7, we can see that changing the evaluation information of one option does not influence the ranking of other potential propagation pathways. Although the rankings of Ev_2 and Ev_3 changed after the replacement, this is because Ev_2 and Ev_3 have very similar evaluation results in the original evaluations. Therefore, after replacing the evaluation result of Ev_1, the weights undergo a slight change, leading to a minor shift in their positions. For the third aspect, this paper divided the original evaluation matrix into two sub-evaluation matrices consisting of Ev_1, Ev_3, Ev_5 and Ev_2, Ev_4. This paper also split the subjective weights into $W_1^s = [0.15, 0.1, 0.2]$ and $W_2^s = [0.3, 0.25]$, then recalculated to obtain $W_1^{ss} = [0.33, 0.22, 0.45]$ and $W_2^{ss} = [0.55, 0.45]$. As shown in Figure 5, the final evaluation results are $Ev_5 \succ Ev_1 \succ Ev_3 \succ Ev_2 \succ Ev_4$,$Ev_5 \succ Ev_1 \succ Ev_3$ and $Ev_2 \succ Ev_4$. The evaluation results are consistent with the original decision matrix's ranking. In conclusion, it can be demonstrated that the evaluation method proposed in this paper is effective.

Table 7. Evaluation Results Before and After Data Change.

	Ev_1	Ev_2	Ev_3	Ev_4	Ev_5
Original Evaluation Results	1.22058	1.21480	1.21500	1.20695	1.22321
Original Rankings	2	4	3	5	1
Modified Evaluation Results	2.84894	54.43592	51.99502	43.26278	62.10789
Modified Ranking	5	2	3	4	1

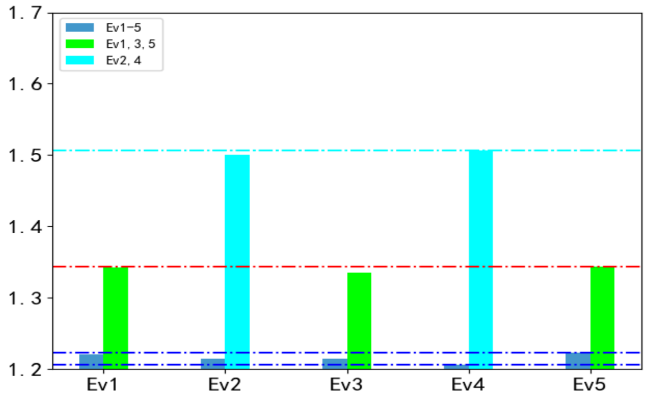

Figure 5. Comparison between sub-schemes and original scheme.

7. Conclusions

The widespread and rapid development of the internet has led to an unprecedented speed of information dissemination. Through various online platforms and social media, information can spread rapidly, with low cost and convenience. However, this has also resulted in information overload and the dissemination of false information. Misleading content, rumors, and inaccurate information can spread instantly, impacting the public and giving rise to the formation and spread of public opinion. In this context, it becomes crucial to evaluate and respond to the dissemination of information risks. Based on the preceding analysis, this study identified five potential pathways for information risk dissemination and determined five evaluation indicators to assess these potential pathways. Considering the need for timely and targeted response measures during information risk events, which are characterized by uncertainty and urgency, this study proposed a probabilistic hesitant fuzzy evaluation model for information risk dissemination. In this evaluation model, new distance and similarity measurement methods for probabilistic hesitant fuzzy elements were introduced. These distance measurement methods were used to improve the GRA-DEMATEL method and determine the objective weights of the evaluation indicators. The similarity measurement method was utilized to enhance the CoCoSo method and evaluate and rank the various potential information risk dissemination pathways. In computing these distances and similarities, the varying lengths of different probabilistic hesitant fuzzy elements were taken into account, and the "Probability Splitting Algorithm" was introduced to handle probabilistic hesitant fuzzy elements. Additionally, considering that the objective weight determination method only relies on known evaluation data to determine the weights of each evaluation indicator, lacking the subjective judgment of evaluation experts, the Delphi method was incorporated to determine the subjective weights of each evaluation indicator. These subjective weights and objective weights were then combined using a weight aggregation operator to obtain the comprehensive weights. Finally, the effectiveness and reliability of the proposed method were further confirmed through a sensitivity analysis and validity analysis. Overall, the results demonstrate that the proposed evaluation method is reliable and effective in assessing information risk dissemination pathways in a complex public opinion environment.

This study involved extensive research on information risk dissemination pathways in complex environments and provided some potential information risk dissemination pathways and evaluation indicators; we also proposed a probabilistic hesitant fuzzy evaluation model based on an improved CoCoSo method and a combination of subjective and objective weights. In these methods, we also extended some new distance and similarity measurement methods for probabilistic hesitant fuzzy elements. However, the identified potential information risk dissemination pathways in this study tend to focus on macro-level dissemination pathways, and the determined evaluation indicators are not comprehensive and specific enough. Therefore, in future research, we will further refine these potential information risk dissemination pathways and incorporate more literature to provide more comprehensive multidimensional and hierarchical evaluation indicators. Additionally, Equation (11) also has some limitations, as evidenced with the fact that after aggregation, certain aggregated values exceed the maximum value participating in the aggregation. While this is a recognized and commonly used aggregation method, in future research, we aim to further refine the approach for aggregating subjective and objective weights.

Author Contributions: Conceptualization, Z.L. and J.S.; methodology, Y.X.; software, J.S. and Z.L.; formal analysis, Z.L.; resources, J.S.; writing—original draft preparation, Z.L. and Y.X.; writing—review and editing, J.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work is supported by the Science and Technology Research Program of Chongqing Municipal Education Commission (Grant No. KJQN202101617).

Data Availability Statement: The data used to support the findings of this study are included within the article.

Conflicts of Interest: The authors declare no conflict of interest.

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