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Cooperation Dynamic through Individualistic Indirect Reciprocity Mechanism in a Multi-Dynamic Model

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Abstract: This research proposes a new variant of Nowak and Sigmund's indirect reciprocity model focused on agents' individualism, which means that an agent strengthens its profile to the extent to which it makes a profit; this is using agent-based modeling. In addition, our model includes environmentally related conditions such as visibility and cooperative demand and internal poses such as obstinacy. The simulation results show that cooperators appear in a more significant proportion with conditions of low reputation visibility and high cooperative demand. Still, severe defectors take advantage of this situation and exceed the cooperators' ratio. Some events show a heterogeneous society only with conditions of high obstinacy and cooperative demand. In general, the simulations show diverse scenarios, including centralized, polarized, and mixed societies. Simulation results show no healthy cooperation in indirect reciprocity due to individualism.

Keywords: computational social science; social systems; agent-based model; indirect reciprocity; individualism; cooperation

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1. Introduction

Why is cooperation between humans so tricky? What is required for cooperation to exist? How can cooperation be established in a world of selfish people (without a central authority)? When should a person cooperate, and when should they be selfish (in continuous interaction)? Cooperation has been identified as a method for personal and entity development paths for towns, cities, and countries, among others. The most crucial aspect of evolution is the ability to generate cooperation in a competitive world [1]. A simple definition of cooperation is that one individual pays a cost for another to receive a benefit. Costs and benefits are measured in terms of reproductive success [2].

The evolution of human cooperation has been explained by five main mechanisms: direct reciprocity, indirect reciprocity, spatial selection, multilevel selection, and relationship selection. These main regularities of interaction are called mechanisms, which are necessary for the evolution of cooperation. However, they are too different for the evolution of cooperation and behaviors that require an evolutionary explanation (such as strong reciprocity, upstream reciprocity, etc.) [2].

Indirect reciprocity operates if there are repeated encounters within a population and third parties observe or know about some of these encounters. Information about these meetings could be spread through communication, affecting the reputation of the participants. Individuals can adopt strategies in which their decisions to donate are based on the recipient's reputation. Its main variants are rewards (why altruism spreads), punishment (why rules disseminate), and deception (why cheating spreads). The cost–benefit calculation for cooperation is not always deliberately or consciously conducted. The donor expects a return from someone, and it is not necessarily the recipient who benefits. Indirect reciprocity develops because interactions are repeated or flow among a society's members and because information about subsequent interactions could be gleaned from observing the reciprocal interactions of others. Indirect reciprocity involves reputation and status; the results of each person in a social group are continually being assessed and reassessed by past and potential interactants based on their interactions with others. Indirect reciprocity presupposes rather sophisticated players and, consequently, is likely to be affected by anticipation, planning, deception, and manipulation [3].

This paper proposes contributions to this context. It proposes an agent-based model to represent the individuals in a virtual world, i.e., the internal and external dynamics based on the indirect reciprocity mechanism by reputation. It is used to analyze cooperative behavior with individualism. This model includes the visibility (of reputation), cooperative demand, and agents' obstinacy. Furthermore, we seek to characterize when cooperation is established according to the different values of simulation parameters.

Another contribution is the agent-based model's methodology application, which was developed in NetLogo and analyzed with Python with three parameters without a social network. We use the hierarchical cluster analysis (HCA) data analysis method to classify the simulation results. The HCA intuitively analyzes the similarity between the histograms (results of simulations) in terms of their means, standard deviations, and skewness. The results obtained were transformed into three-dimensional parameter spaces. The use of data science methods was a helpful tool to determine and visualize the behavioral patterns of simulation results. They were displayed in different clusters, where each cluster had similar behaviors according to the *k*-profile.

For our study, we synthesize various real-life social systems that manifest indirect reciprocity (with individualism), like community networks [4], social networks [5], or knowledge-sharing systems [6]. In these communities, individuals engage in cooperative acts with the expectation of receiving help from others in the future. The groups help strangers with some costs, and helpers' reputations improve. We emphasize [7], as a notable research finding in this sense, which complete social network modeling provides ideas and methodology for the methods of this research with a comprehensive set of variables, parameters, relationships, processes, and results. This research addresses three sentimental evolution dynamics in users and communities; the first can be seen as cooperation in maintaining the social network (or communities).

This article is structured into five sections. The second is a literature review, introducing articles that underpin our research. The third section covers the material and methods, detailing the mechanisms of indirect reciprocity and individualistic indirect reciprocity, along with their experimental design and mathematical formulation. In the fourth section, we present the simulation results and their components. This section systematically displays and quantitatively describes the outcomes of the experiments. The final section comprises discussions, conclusions, and concluding remarks.

2. Related Literature

The following paragraphs are the primary investigations that contextualize this research and constitute the guideline of the indirect reciprocity mechanism model proposed by Nowak and Sigmund [8]. Their model, in essence, records the cooperation, updates the image score, and defines the strategy for the donor and receptor when the interaction of cooperation is set using a criterion (see the corresponding paragraph in the next section).

Nowak and Sigmund [9] made the first theoretical and simulated work based on Alexander's definition of indirect reciprocity. First, they show that individual selection can favor cooperative strategies directed toward recipients who have previously helped others. Later, they analytically showed that discriminating altruism could resist invasion by defectors. However, indiscriminate altruists can invade via random drift and establish a complex dynamical system.

Lotem et al. [10] conducted a realistic experiment of indirect reciprocity mechanisms that allowed individuals to carry D phenotype defectors with a nonheritable phenotype strategy, where k = +7. They concluded that a paradoxical effect whereby phenotype defectors stabilize discriminating altruism could be explained analytically. Another experimental research designed to distinguish between the two proposed mechanisms of indirect reciprocity, discriminator image scoring and standing strategies, was performed by Milinski et al. [11] They studied 23 groups comprising 7 players each and determined that standing strategies demand too much work and a large amount of second- if not third- and fourth-order information about the history of social interactions.

One research on the evolution of indirect reciprocity based on image scoring is the strategy of aiming for "good standing", which was studied by Leimar and Hammerstein [12]. They show that it has superior properties; it could be an evolutionarily stable strategy, and, in some cases, it tends to outperform the image score. Another model with a framework for the evolution of indirect reciprocity via social information is proposed by Mohtashemi and Mui [13]. Its information is selectively retrieved from and propagated through dynamically evolving networks of friends and acquaintances. They analytically show that for indirect reciprocity to be evolutionarily stable, the different probabilities of trusting and helping a reputable individual over a disreputable individual, at a point in time, must exceed the altruistic act's cost-to-benefit ratio.

Another paper that focused on indirect reciprocity with experimental evidence is presented by Seinen and Schram [14]. In their experiments, indirect reciprocity is mainly based on norms regarding how frequently the recipient should have helped others in the past. They show that these norms develop similarly within groups of interacting subjects but distinctly across groups, which leads to the emergence of group norms. From another viewpoint, Yutaka Nakai [15] demonstrates that a fixed tag and reputation cause indirect reciprocity within the group and in-group favoritism. He did this using Nowak and Sigmund's model (replacing *k*-profile) inside the group selection model, conducted evolutionary simulations, and found the emergence of in-group favoritism.

3. Materials and Methods

3.1. Social System

A social system is a system of action. It is made up of the interactions of individuals [16]. Social systems refer to complex structures of interactions and relationships among individuals, groups, organizations, and institutions in a society [17]. These systems are comprised of behavior patterns, norms, values, roles, and other forms of organization that influence people's behavior in a social context. Social systems can be approached from various disciplines, such as sociology, anthropology, social psychology, and political science [18].

Social system is a model of a social organization that possesses a distinctive total unity beyond its component parts, which is distinguished from its environment by a clearly defined boundary, and whose subunits are at least partially interrelated within relatively stable patterns of social order. Social systems exist at all "levels": persons, families, organizations, communities, societies, and cultures [19].

There are various social system types, among which society is the general social system encompassing all other systems. Within the systems included in society, the significance of the so-called functional systems has been noted [20]. Some prominent and generally accepted functional systems in society include political, economic, religious, scientific, legal, educational, healthcare, literary, artistic, media, and sports systems [21].

3.2. Indirect Reciprocity Mechanism and Its Modeling by Nowak-Sigmund

Alexander shows that the essence of moral systems is in patterns of indirect reciprocity [22,23]. He establishes that indirect reciprocity ("I help you and someone else will help me later") based on reputation is what occurs when direct reciprocity ("I help you and you help me") occurs in the presence of an interested audience in cooperative interactions (see Figure 1). In the indirect reciprocity mechanism, the probability of the same two individuals interacting again is low. However, in the direct reciprocity, there are repeated encounters between the same individuals.

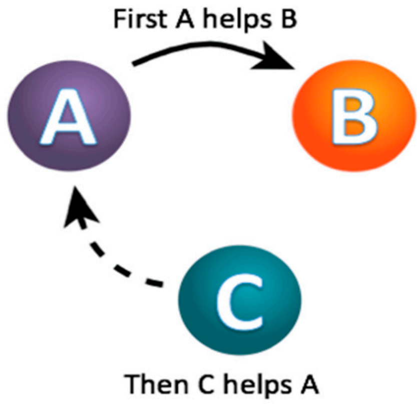

Figure 1. Indirect reciprocity cooperation mechanism [24].

The indirect reciprocity cooperation mechanism has been extensively studied. Investigations of the indirect reciprocity mechanism have analyzed each of its elements with variants. They have determined new types, but the most researched type is the downstream type [25]. It has been studied under various conditions, such as by adding social norms [26]. Additionally, it has been studied using mathematical theories such as game theory, computational methods, social theories, biological theories (see the references of Rand and Nowak, 2013 [24]) and even experimentally (as presented in the Literature Review section).

The model proposed by Nowak and Sigmund [8] has a strategy k, image score s, and pay function p for each agent. The payoff function is the sum benefit received minus the cost of cooperating, the image score is a function that increases or decreases by 1 whether or not someone cooperates with another person (the image score of a recipient does not change), and the strategy k (k-psychological profile or simply k-profile) updated is based on the inheritance or offspring (mutation and selection) evolution. At the beginning of each generation, all players have a pay function and image score value 0; m donor–recipient pairs are chosen randomly, one as a donor, the other as the recipient. The donor cooperates if the image score of the recipient is greater than or equal to the donor's k-profile value. Cooperation means the donor pays a cost, c, and the recipient obtains a benefit, b. There is no payoff in the absence of cooperation.

3.3. Individualistic Indirect Reciprocity Mechanism Multi-Dynamic (IIRMMD) Model

Could altruism emerge in an individualistic population (a social setting where the focus lies on personal goals, independence, and self-reliance, rather than prioritizing the needs, interactions, goals, etc. of the collective group)? Under what conditions or social norms is there altruism in an individualistic society? The concept of individualism has evolved since it was coined; it has had various connotations; and has been studied in sociology, psychological, philosophical, and economic contexts [27]. The Oxford dictionary defines individualism "as the quality of being different from other people and doing things in its way, i.e., the behavior of someone who does things in their way without worrying about what other people think or do".

Taking from the first meaning of individualism, the simplified essential idea from investigations that have studied it [28,29], individualism could be considered a fundamental criterion for deciding or acting by people in its prioritizing criterion for one's benefit. It is a characteristic of society, and it always arises in people unwilling to do something without first achieving an advantage.

The indirect reciprocity mechanism simulation with the k-profile (Nowak and Sigmund's style model with pay function p, image score s, and k-profile) considers that an updated agent's profile is based on the offspring evolution. In the IIRMMD model, we replace the way of updating the agents' k-profile and propose that it will update based on an individualistic criterion (see Figure 2). The rest is almost like Nowak and Sigmund's model. The individualism criterion is somewhat natural to societies, and it is a way of updating an agent's cooperation psychological profile, depending on how their experience goes, which is an internal dynamic for each agent.

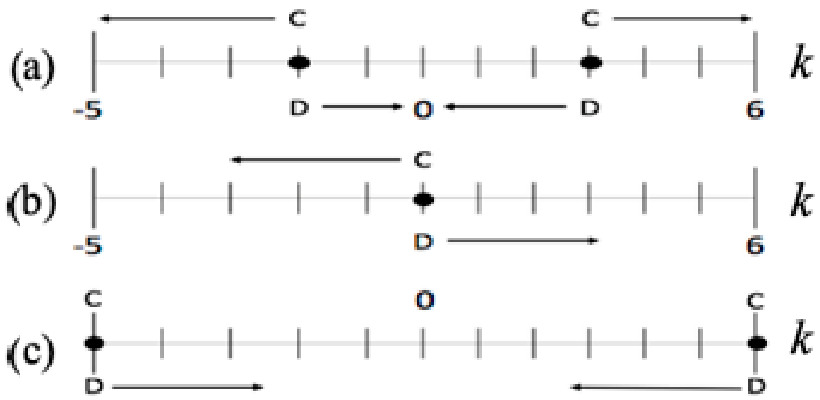

Figure 2. Updated individualistic indirect reciprocity k-profile. When an agent has profile k = –2 and someone cooperates with him, he will update his profile to –3; otherwise, he will move to –1. Likewise, if the agent's profile is $k = 3$ and someone cooperates with him, the agent will reinforce his strategy by updating to $k = 4$; however, if the agent does not receive cooperation, he will move to $k = 2$. The $k = 0$ behavior is similar to a cooperator's; this behavior relates to having an agent's proportion balance in its way of acting.

When the k-profile varies from -1 to 1, the agents are considered as discriminators; when an agent i has $k_i = 0$, or $k_i = -1$, he is a discriminator agent with a tendency to be a cooperator. This is analogous to $k_i = 1$. If $k_i \leq -2$, the agent is called a cooperator; if $k_i \geq 2$, he is considered a defector. An agent is considered as a severe defector if $k_i = 6$ and an unconditional cooperator or altruist if $k_i = -5$. Note that our definition is similar to Nowak and Sigmund's [8].

In the IIRMMD model, we define the individualism criterion as the act in which an agent "strengthens" his k-profile based on their past earnings interactions (if you help me, I have a "good" profile, so I strengthen it; otherwise, I attenuate it changing to the opposite profile). This individualism simulation idea is aligned with Sullivan and Haklay's ABM simulation conception [30]. In Figure 2, each mark represents a potential situation of k-profile for any agent. If the agent is cooperative and someone cooperates with him, he will increase his cooperative profile, moving left; otherwise, he moves to the right. Similarly, if he is a defector and someone cooperates with him, his profile will increase, moving to the right; otherwise, he moves to the left (Figure 2a). When the agent is a discriminator, he becomes a cooperator if someone cooperates with him and defectors if not (Figure 2b). If an agent has profile $k = -5$, he will keep his profile if someone cooperates with him; otherwise, he will move to the right (Figure 2c), similar to $k = 6$.

The IIRMMD model includes environmentally related conditions such as visibility of cooperation and cooperative demand and obstinacy like internal dynamics. These are described below.

Reputation or image score is part of the IIRMMD model, but how do you show (or look at) the reputation of someone with whom you will cooperate? What do you do if it's impossible to see someone's reputation and you still have to cooperate? Or, how accurately do you need to know someone's reputation in order to cooperate effectively? Reputation, as a social scientific concept, refers to the recognition of a social persona or the organizing principle by which a person's (or a group's, organization's, or collectivity's) actions are linked into a common assessment. This process through which we are exposed to the reputations of others through formally sanctioned knowledge is analyzed by collective memory [8]. It is not easy to establish the visibility of agents' reputations in society, as there are different levels of knowledge that can change over time. So, as there are several possibilities or variants of someone's reputation perception, we will assume this knowledge or visibility as a probability parameter in the population, of knowing or seeing the reputation of the agents with whom one interacts.

Overall, to have equity in societies, laws or precepts are implemented explicitly, implicitly, local, global, and others to obtain fairness, impartiality, egalitarianism, or equitableness. The IIRMMD model has the rule to compromise agents' cooperation with the cooperation demand parameter to represent this idea and obtain some equity. This parameter represents part of the second individualism conception [29].

How can the acceptance flexibility of new ideas or impositions be characterized? How can agents' (some) elements be identified to provide a sense of flexibility criteria for changing their ideas? The Oxford Dictionary defines obstinacy as the attitude of somebody who refuses to change their opinions, way of behaving, and so forth; when other people try to persuade them to do something, they engage in behavior that reflects this. In this sense, in the IIRMMD model, agents could update their profile at the first good/lousy experience or after several similar experiences, depending on the obstinacy parameter value. This parameter represents part of the third individualism conception [29].

3.4. IIRMMD Agent-Based Simulation Model with Internal and External Dynamics

The IIRMMD model aims to model and simulate cooperation through the individualistic indirect reciprocity cooperative mechanism with internal and external dynamics in artificial societies. Agents' internal dynamics are characterized by strengthening their profile according to their obstinacy and consecutive positive/negative results (payments). The agents' environment includes the conditions to show the agent's reputation with a probability and the obligation to cooperate parameters.

3.4.1. Basic Assumptions

A population of n agents is established. Each agent *i* has three attributes or variables as in [8]:

- Profile $k_i(t)$ with a range of integer values from -5 to 6.
- Reputation or image score $s_i(t)$ with integer values from -5 to 5.
- Payment function $p_i(t)$ bounded with values from -5 to 5.

We use the cooperation criterion $k_i < s_j$ to have a balanced cooperation behavior. All possible situations are in Table 1, so k_i could be interpreted as a strategy (a complete set of actions) to cooperate or not with agent *i*.

Table 1. Cooperation criterion $k_i < s_j$, The *x*-axis is k_i, and the *y*-axis is s_j. Furthermore, "*" represents cooperation and "x" represents defection.

s_j													
5	*	*	*	*	*	*	*	*	*	*	x	x	
4	*	*	*	*	*	*	*	*	*	x	x	x	
3	*	*	*	*	*	*	*	*	x	x	x	x	
2	*	*	*	*	*	*	*	x	x	x	x	x	
1	*	*	*	*	*	*	x	x	x	x	x	x	
0	*	*	*	*	*	x	x	x	x	x	x	x	
−1	*	*	*	*	x	x	x	x	x	x	x	x	
−2	*	*	*	x	x	x	x	x	x	x	x	x	
−3	*	*	x	x	x	x	x	x	x	x	x	x	
−4	*	x	x	x	x	x	x	x	x	x	x	x	
−5	x	x	x	x	x	x	x	x	x	x	x	x	
	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	k_i

The IIRMMD's global variables are visibility (V), obstinacy (O), and cooperative demand (CD). These are three simulation model-controllable parameters. Therefore, agents' cooperation not only depends on visual reputation but is also conditioned by cooperative agents' attitude, like personal sensibility obstinacy, and the environment rules as cooperative demand. We briefly describe each one of them.

The visibility parameter (V) has a range of values comprising $\{0,\ 0.1,\ 0.2,\ \cdots,\ 0.9,\ 1\}$. In the cooperative process, a donor agent i will "see" (or know), with probability V, the recipient agent's j reputation s_j. When he sees reputation, the cooperative criterion $k_i < s_j$ applies. Otherwise, he will suppose that the recipient has a "good reputation" and will continue the cooperative process based on the cooperative demand parameter value.

The obstinacy parameter (O) has has a range of values comprising $\{0,\ 1,\ \cdots,\ 6\}$. In the IIRMMD, an agent will strengthen his profile when he obtains profits (after several times); otherwise, he will update to the opposite side (cooperator/defector). The obstinacy parameter regulates this behavior. When the obstinacy parameter $O = a$, an agent will strengthen (or update) its profile only after a consecutive times that he was the recipient of cooperation (or he was not). An obstinacy parameter of $O = 0$ means that agents will strengthen their profile at each interaction.

The cooperative demand parameter (CD) has a range of integer values comprising $\{0,\ 1,\ \cdots,\ 6\}$. This parameter forces the agents to cooperate. When the cooperative demand parameter has the value of $CD = V$, cooperation is forced for donor agents with $k_i < V$, even though they cannot see the recipient's reputation.

3.4.2. General Description

The simulation model is represented by a population of $n = 100$ agents, and players have an initial reputation and payout function with values s = 0 and p = 0, respectively. In each experiment, the initial values of visibility (V), obstinacy (O), and cooperative demand (ED) are established. Each experiment consists of 1000 time steps. The time scale t is discrete.

In each time step, a generation is created. The population is composed of n agents. Each agent i has a profile $k_i(t)$, a reputation $s_i(t)$, and a payment function $p_i(t)$. For each iteration t, m pairs of agents are randomly selected for the interactions. Within each couple, one agent is randomly chosen to be a possible "donor" (denoted by i), and the other is a "recipient" (denoted by j). With probability V, donor i knows the reputation s_j of agent j. He cooperates based on the cooperative criterion, $k_i < s_j$; if this inequation does not hold, the cooperative demand criterion is applied. If agent i is a donor and agent j is a recipient, the following dynamical system of equations defines an actualization of the agent's attribute values in time $t + 1$, for agents i and j.

$$k_i(t+1) = k_i(t)$$

$$s_i(t+1) = s_i(t) + \delta_c$$

$$p_i(t+1) = p_i(t) - \mu_c c$$

$$k_j(t+1) = k_j(t) + \eta_c$$

$$s_j(t+1) = s_j(t)$$

$$p_j(t+1) = p_j(t) + \mu_c b$$

where

$$\delta_c = \begin{cases} 1 \ \ if\ i\ cooperate \\ -1\ if\ i\ no\ cooperate \end{cases}$$

$$\mu_c = \begin{cases} 1\ if\ i\ cooperate \\ 0\ if\ i\ no\ cooperate \end{cases}$$

$$\eta_c = \begin{cases} -1\ if\ \ i\ (no)\ cooperate\ and\ -4 \le k_j(t) \le 0\ \left(k_j(t) > 0\right) \\ \ 0\ if\ \ i\ cooperate\ and\ k_j(t) = -5\ or\ k_j(t) = 6. \\ \ \ 1\ if\ \ i\ (no)\ cooperate\ and\ 0 < k_j(t) \le 5\ \left(k_j(t) \le 0\right) \end{cases}$$

When donor i cooperates, his reputation score s_i increases by one unit; and if not, s_i decreases by one. When cooperation occurs, the donor pays a cost, c; and the recipient obtains a benefit, b. There is no reward in the absence of cooperation.

Note that

$$\eta_c = \begin{cases} \delta_c \cdot sgn(k_j) & if\ k_i(t) \neq -5,\ 6 \\ 0 & if\ k_i(t) = -5\ or\ 6 \end{cases}$$

where

$$sgn(k_j) = \begin{cases} 1\ if\ 0 \leq k_j(t) \leq 5 \\ -1\ if - 4 \leq k_j(t) \leq 0 \end{cases}$$

Observe that η_c is the update value for the individualism criterion.

The pseudo-code process and flowchart diagram simulation are shown in Appendix A.

3.5. Experimental Design

The research method consists of two parts. The first is the application of agent-based models (ABMs) from [30]'s perspective and the methodology of [31], which are applied in the NetLogo simulator [32] (https://ccl.northwestern.edu/netlogo/, accessed on 20 August 2023). The second part comprises the simulation results' analysis, which are performed with the Python programming language; a point group analysis method called hierarchical cluster analysis (HCA) was used to see the connections between the objects inside the cluster and dendrograms to look at the agglomerations [33].

The following initial conditions are common to all experiments. First, a population of $n = 100$ agents is established. Later, the benefit (b) and cost (c) parameters are set to $b = 1$ and $c = 0.1$, respectively (to avoid negative profits, we add 0.1 at the beginning of each interaction). Finally, the reputation value s of each agent and its payment p function value are set equal to 0. The k-profile values are randomly scattered following a uniform distribution (a histogram represents this).

For each experiment, the parameters' values V, O, and CD are fixed on their range, respectively. Every experiment consists of 1000 iterations (ticks or time steps).

Randomly, m pairs of agents are selected; one agent is chosen as a donor, and the other is selected as a recipient (some players may never be chosen). Once one pair of agents has interacted, their profiles, payments, and reputations are updated asynchronously, as follows: The recipient strengthens its k-profile based on whether he received cooperation (or not) and the obstinacy parameter condition, and adds b to his payments (if he received cooperation); thus, his reputation is unchanged. The donor does not change his k strategy, increases his reputation by 1 in case of having been cooperative, or subtracts 1 if he refused to cooperate, and subtracts c only if he cooperates (c is the cost of cooperating).

After one thousand iterations, the data $\overline{p}$, $\overline{s}$ graphs, and k-values are collected. This amount of iterations stabilizes $\overline{p}$ and $\overline{s}$, which ensures there will not be (statistical) variations in the last k-histogram.

Note that each agent has an average of 2 m/n interactions, either as a donor or recipient. The probability that a player meets the same player again at the r-th time is $\left(1 - \frac{125}{4826}\right)^{r-2} \frac{125}{4826}$, with $n = 100$, $m = 125$ y $r \geq 2$ (so indirect reciprocity holds).

Considering the values of the three primary parameters (visibility, obstinacy, and cooperative demand), we have 539 possible combinations of experiments (without counting the randomness).

4. Simulation Results

We will focus on the final frequency distribution profiles. Average payment and reputation behavior are considered to validate simulations' stable state (for the nonce). Each simulation generated values for the strategy, reputation, and payouts for every iteration of time t. The final simulation results of the k-profile are structured in a histogram with 12 values distributed ranging from -5 to 6. We use three of their graphical characteristics to classify these histograms: the mean, standard deviation, and skewness. We call this group of parameters the characteristic space. The characteristic space is a new database. The first coordinate corresponds to the mean value, the second contains the standard deviation, and the third one is the asymmetry value.

We use HCA to group the characteristic space's points by similarity and generate a dendrogram with the Ward method [33]. This analysis generates 12 clusters when cutting at a height of 2 (which means that clusters' centroids are at a distance less than 2 in the characteristic space). The dendrogram calculated with HCA intuitively represented the similarity between the histograms (simulations) in terms of their means, standard deviations, and skewness.

We visualize how the different groupings are distributed using the parameters' space in a three-dimensional view. The axes comprise visibility, obstinacy, and cooperative demand. Thus, we finally visualize 12 clusters in a 3D space and attach representative histograms to each of them.

4.1. Basic Case: Parameters Values Set to Zero

Figure 3 shows selected insights into the temporal evolution of the experiment for $V = 0$, $O = 0$, and $CD = 0$. Figure 3a shows the initial conditions of the population ($t = 0$), a heterogeneous society. Figure 3b shows the temporal state at $t = 25$. Figure 3c displays the historical state for $t = 100$. Figure 3d $t = 1000$ shows the final state of the simulation. In this case, we observe that society is distributed with discriminators' dominance, around 30%, and the rest is distributed almost equally between cooperators and defectors; note that there are no unconditional altruists.

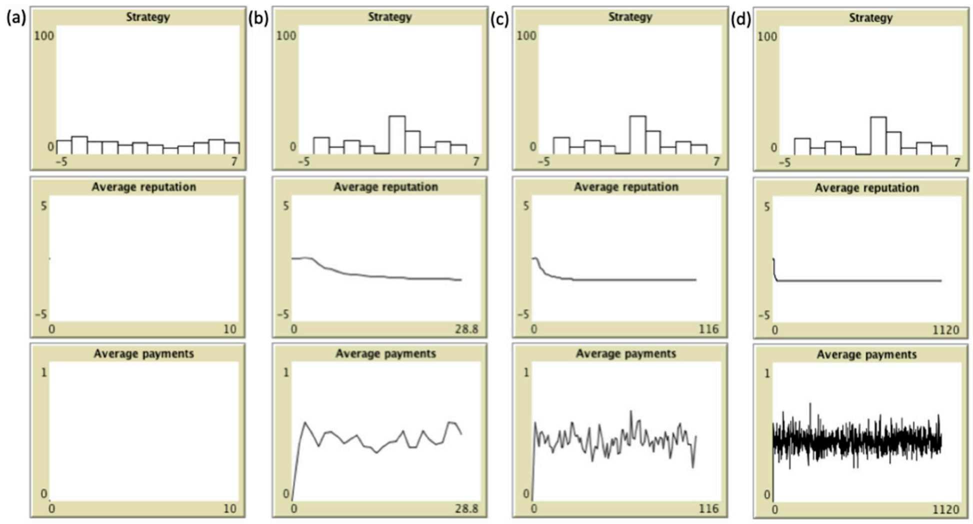

Figure 3. Temporal evolution (**a**) $t = 0$, (**b**) $t = 25$, (**c**) $t = 100$, and (**d**) $t = 1000$. Only these values of t are shown because, after 100, the graphs present almost identical behavior. Histograms show the agents proportions of *k*-profile values. The other two monitors are the average of reputation and payments showing the temporal evolution. The software places the bar up to the right of each number in the histograms; it shows the lower and upper limits of histogram values. Also, it displays slightly more on the *x*-axis, and the parameter mark is how far the curve goes for the average reputation and payment graphs.

4.2. Clusterization and Numerical Analysis Results

The following figures represent 12 clusters derived from HCA (see Figures 4–15). Each one has two representative histograms with similar features but some skewness differences. A numerical description of each cluster clarifies its properties.

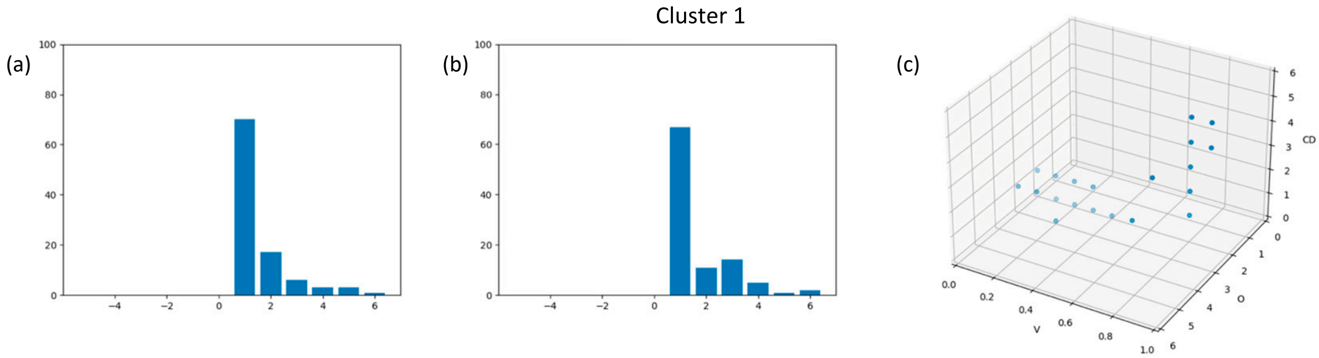

Figure 4. Cluster 1: (20 histograms) (**a**) or (**b**) This cluster has a significant proportion of agents at bin 1, 75% of whole agents. The rest are in bins 2 to 6. (**c**) In the parameter space 3D view, the parameter that displays the grouping for this cluster is obstinacy (O) since they are clustered in values 2 and 3. There are two groups in the visibility parameter (V); the first is in the lower part of the grouping of points, which has values ranging from 0 to 0.4 and the second at 0.6. The cooperation demand parameter (CD) shows two groups, one with 0 and the other where the points are agglomerated between 3 and 5.

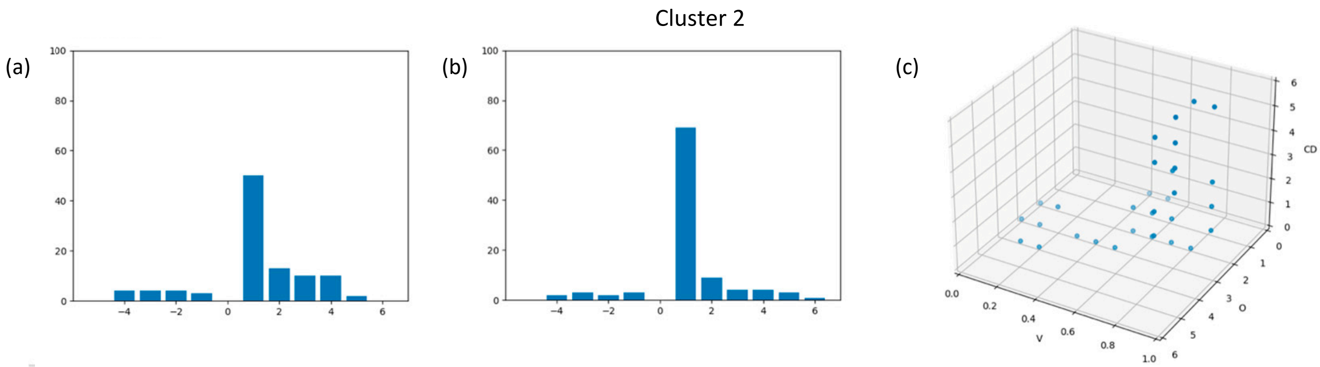

Figure 5. Cluster 2: (32 histograms) (**a**) or (**b**) The histograms representing this cluster are characterized by three groups of proportions. The first proportion, with no more than 20%, is in the bins from −4 to −1. The second, with 50% to 70%, is in bin 1, and the third one ranges from bins 2 to 6; this one has between 20% and 40% of the agents. (**c**) In the parameter space, obstinacy is the most significant characteristic, which takes values 2 and 3. Two groups can be identified in the visibility parameter, one with values ranging from 0 to 0.4 and the other with values between 0.5 and 0.9. The cooperation demand mostly has two groups, one with a value of 0 and another where values range from 2 to 6.

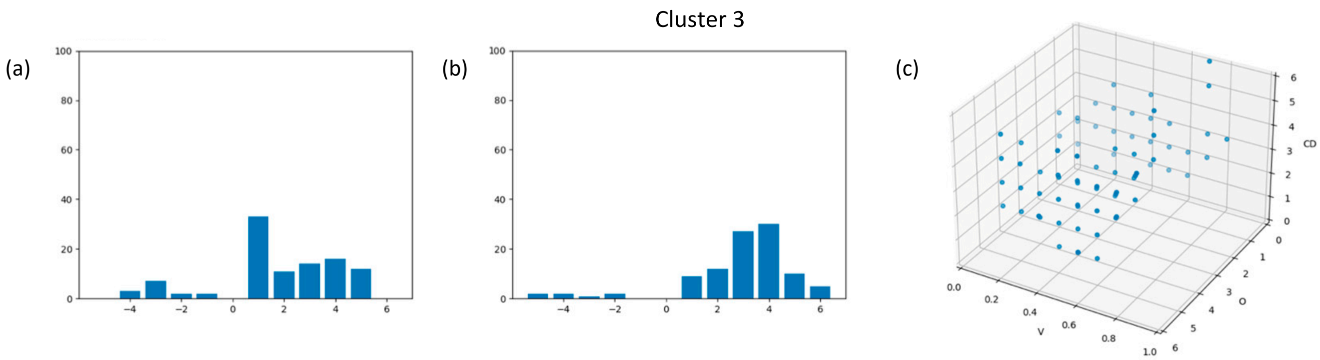

Figure 6. Cluster 3: (74 histograms) (**a**) or (**b**) The histograms representing this cluster have two parts; one part is in bins from −4 to −1 and one which is more representative and has values in bins ranging from 1 to 5. (**c**) There are two groups in which obstinacy is the most relevant parameter in the parameter space. Points in the first group are located from 2 to 4 values, and the second group is

at a value of 0. The parameter of visibility's values ranges from 0 to 0.6, and the cooperation demand parameter is distributed from 0 to 4.

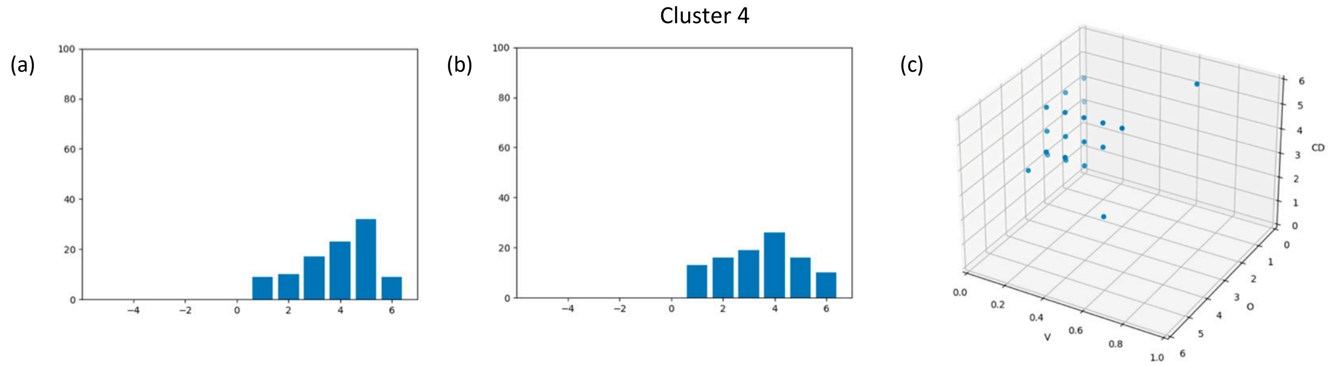

Figure 7. Cluster 4: (20 histograms) (**a**) or (**b**) The histograms' proportions of this cluster are in bins from 1 to 6. (**c**) Obstinacy is the most characteristic parameter, with most values being 2, visibility has values from 0 to 0.2, and cooperation demand has values 3 and 4.

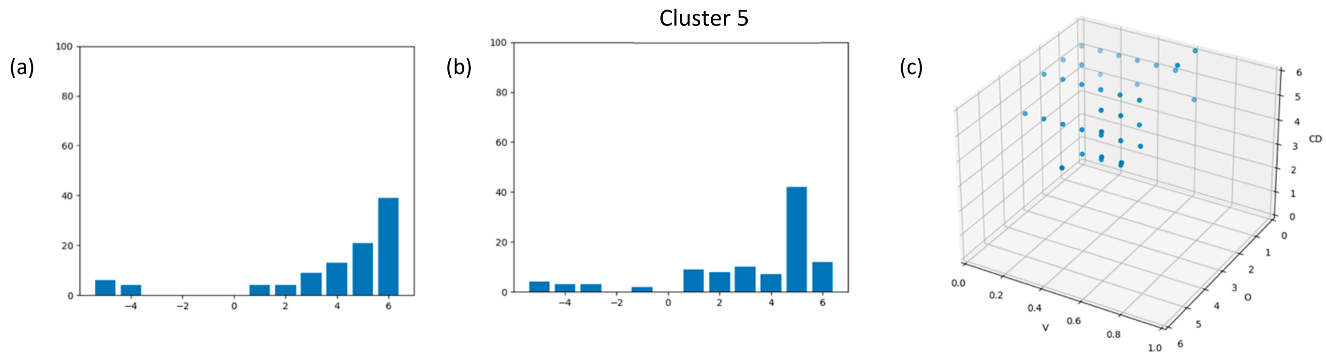

Figure 8. Cluster 5: (36 histograms) (**a**) or (**b**) The histograms representing this cluster have proportions skewed to the right; this cluster has two frequency groups, one with bin values of −5 and −4 and the other with bin values from 1 to 6. (**c**) In the characteristics space, visibility and obstinacy values are the most representative; the visibility values' range is from 0 to 0.5, while obstinacy has two parts, the first one has values of 3 to 4, and the second has a value of 0. The cooperation demand parameter has values ranging from 4 to 6.

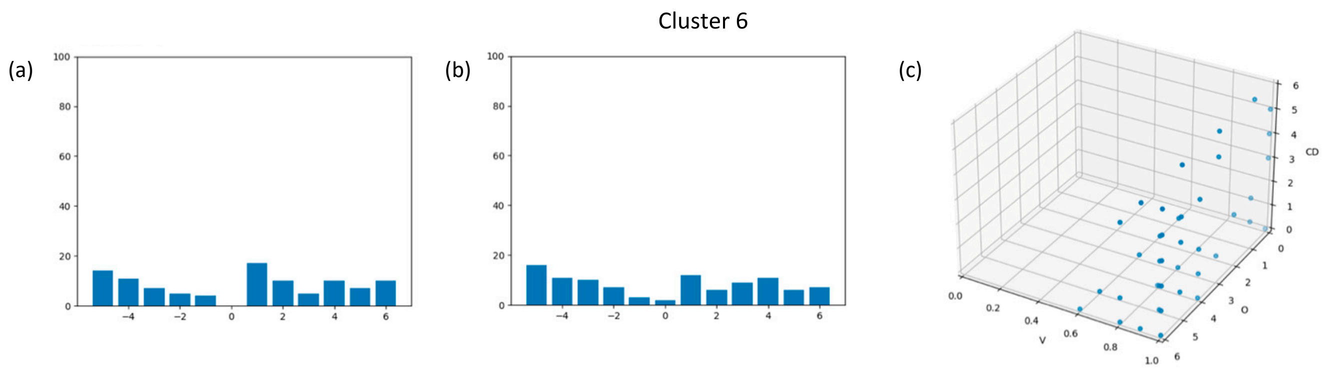

Figure 9. Cluster 6: (40 histograms) (**a**) or (**b**) There is a heterogeneity proportion of k values in this cluster, ranging from −5 to 6. (**c**) In the parameter space, the most representative attributes are in the visibility and obstinacy parameters; the visibility parameter axis has values between 0.8 and 1. Obstinacy values are mostly between 5 and 6.

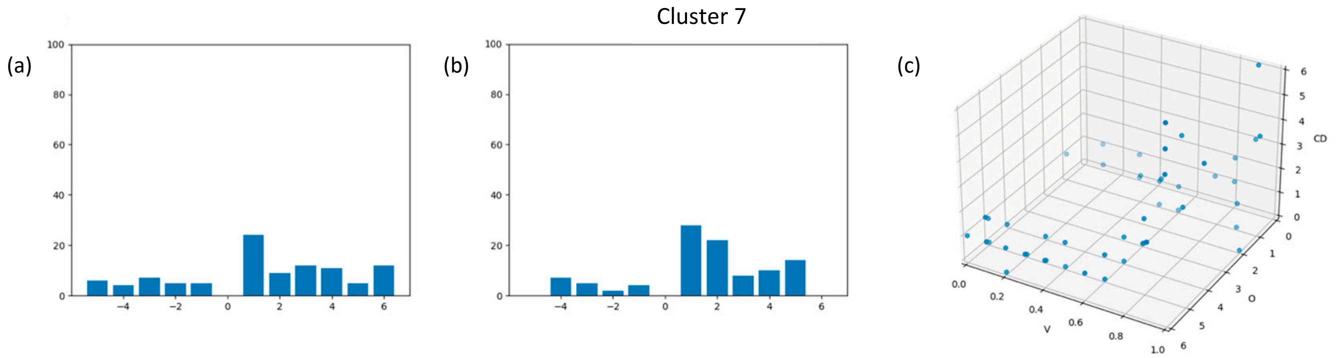

Figure 10. Cluster 7: (50 histograms) (**a**) or (**b**) This cluster has frequency histograms represented with values ranging from −5 to −1 and from 1 to 6. (**c**) In the parameter space, two groupings can be identified. The first group has an obstinacy parameter ranging from 4 to 5; the cooperation demand value is 0; visibility ranges from 0 to 0.6. In the second group, the obstinacy value is 0, cooperation demand is between 0 and 2, and visibility is mostly between 0 and 0.8.

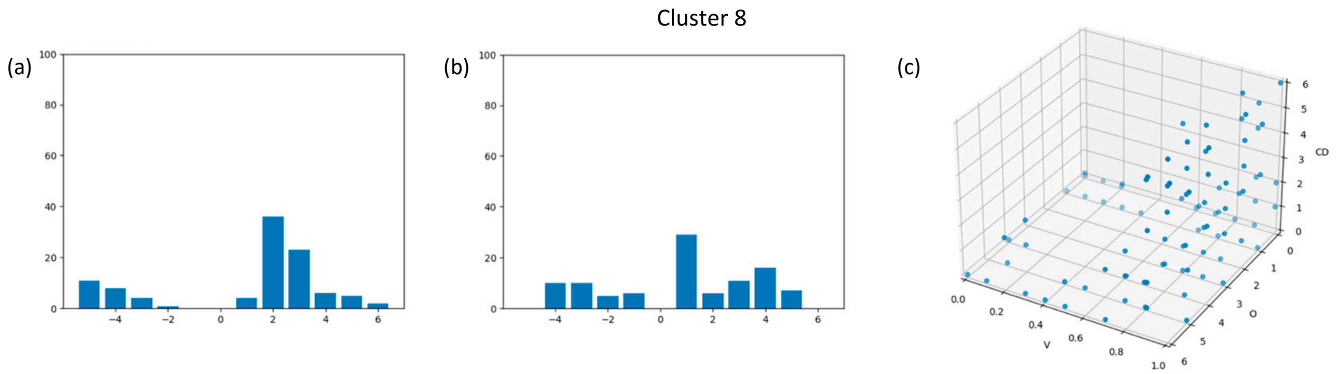

Figure 11. Cluster 8: (96 histograms) (**a**) or (**b**) The histograms representing this cluster have two proportions. The left part is in the bins from −5 to −2 and the right between 1 to 6, which have more profusion. (c) Visibility is the most relevant parameter in the parameter space, with values ranging from 0.8 to 1. Obstinacy and cooperation demand have all value types, but no outstanding groupings exist.

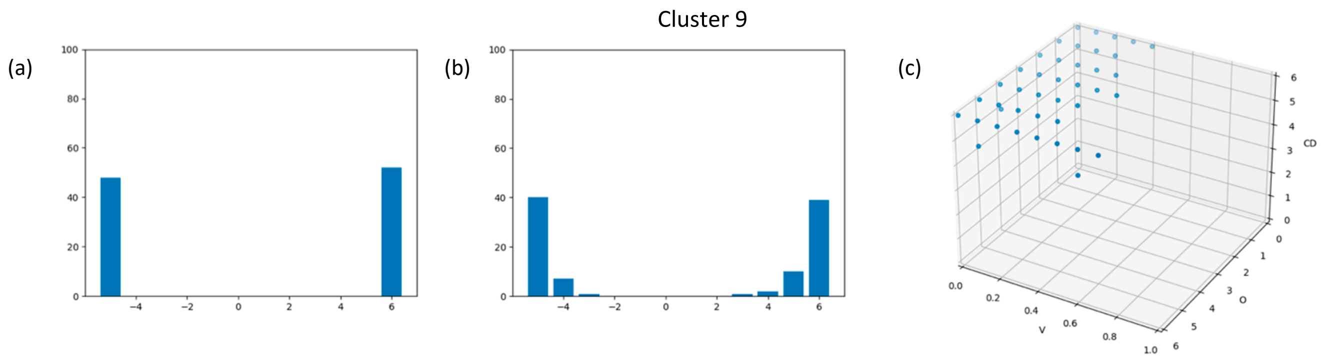

Figure 12. Cluster 9: (41 histograms) (**a**) or (**b**) This cluster is represented by histograms with frequencies at the extremes; it is polarization. The frequencies are in the bins −5 and −4 as well as 5 and 6, almost in the same proportion. (**c**) In the 3D representation parameter space, the cooperation demand value is 6. Low visibility is the second parameter that is projected, with values of 0 and 0.4.

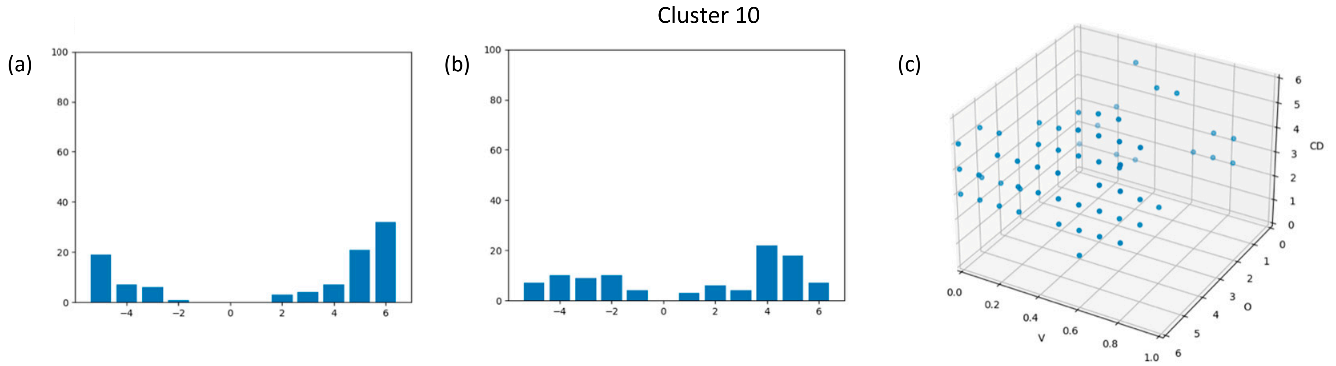

Figure 13. Cluster 10: (60 histograms) (**a**) or (**b**) This cluster is represented with histograms where there are frequencies on both sides, with a greater quantity on the right side and a tendency to the extremes, bins −5 and 6. (**c**) In the parameter space, obstinacy has two groups: the first one with a value of 1 and the other with values of 5 and 6. The cooperation demand parameter has values between 3 and 5, and the visibility ranges from 0 to 0.8.

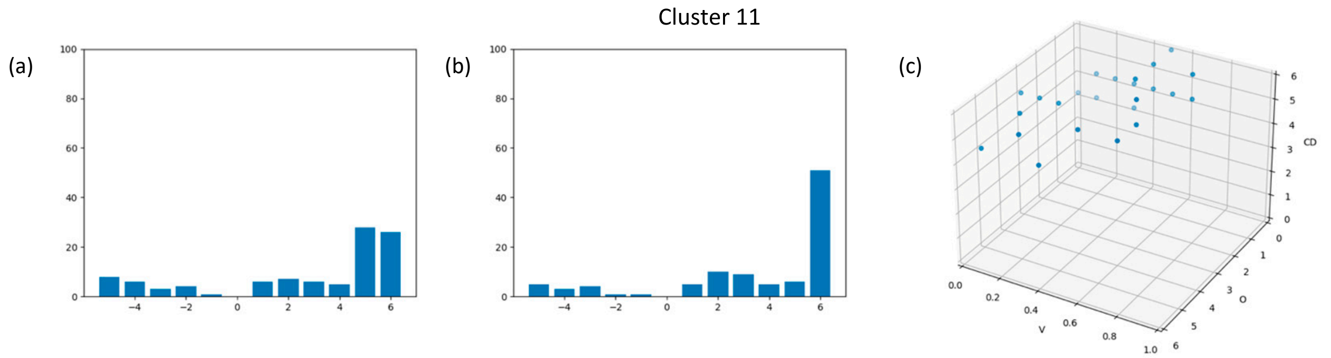

Figure 14. Cluster 11: (24 histograms) In this cluster, (**a**) or (**b**) histograms have a greater proportion on the extreme right side, bins 5 and 6, around 50%, and the rest are distributed in the other bins, except for 0. (**c**) The obstinacy parameter's values identify two groups in the parameter space. The first group has a value of 1, and the second group's values range from 3 to 5. The cooperation demand's values range from 4 to 6 and visibility ranges from 0 to 0.6.

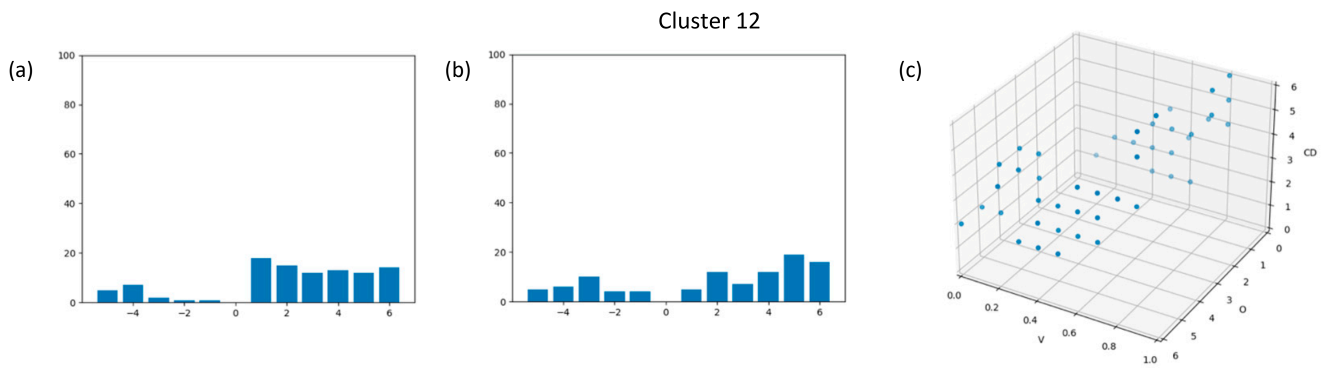

Figure 15. Cluster 12: (46 histograms) (**a**) or (**b**) The histograms representing this cluster have a higher proportion on the right side. The frequencies are distributed around bins 1 to 6 and a few on the other side. (**c**) In the parameter space, two groupings determined by the values of obstinacy are distinguished: the first group with values between 0 to 1 and the other with values ranging from 4 to 6. The visibility has values ranging from 0 to 0.8 and the cooperation demand mostly has values ranging from 2 or 4.

4.3. Particular Patterns and Leading Relations

In general, we can say that each one of the clusters has two or more geometrically defined groups (mainly due to proximity or geometric shape). There are no predominant cooperator societies; as soon as some appear, defectors appear in a greater proportion. Also, defectors or discriminator societies are established or are dominant. There are more discriminators and defectors when the obstinacy parameter value is smaller, and the simulations take more time to stabilize.

The clusters with more than 30% of cooperators are clusters 6, 9, and 10; this is a characteristic showing these clusters share a high obstinacy. The clusters where cooperators disappear are 1 and 4; in them, obstinacy with a value of 2 or 3 is one of the special conditions, in addition to the fact that visibility is low and the cooperation demand is intermediate.

In the clusters with more discriminators are clusters 1, 2, and 3, the pattern comprises low visibility and cooperation demand, with intermediate obstinacy, and values between 2 and 4, although we can also find proportions of around 30% in clusters 7 and 8. In clusters 1 and 2, as soon as the demand or visibility increases, cooperators and defectors emerge (clusters 10 and 3). If we decrease the values of visibility or obstinacy in them, we move to cluster 3, in which the community of defectors increases significantly. However, if the visibility increases slightly, we move to cluster 8, where the defectors predominate.

A society of defectors is established when conditions of visibility and obstinacy are low to intermediate and the cooperation demand is from 2 and above (clusters 3, 4, and 5). However, some cases have high obstinacy and medium cooperation demand. There are always defectors in all the simulation results, but clusters 1, 2, and 8 have a smaller proportion, around 35%.

There is an almost heterogeneous society in clusters 6 and 7; this is when there is high obstinacy and visibility, although there are some exceptional cases with low or intermediate visibility (from 0 to 0.6), and the cooperation demand parameter is low. Furthermore, if we decrease the visibility parameter of cluster 6, it approaches cluster 7, in which the population of defectors increases, with large populations of discriminators. When the obstinacy decreases or the cooperation demand increases, it moves to cluster 8, which defectors dominate.

Cluster 9 has a polarized community; the cooperation demand is high, and visibility is low, forcing cooperation between agents, so individualism generates polarization. Also, there is polarization for cases with an intermediate cooperation demand, visibility until 0.8, and obstinacy almost with a value of 6. If, in this case, the cooperation demand parameter is decreased, it moves to cluster 10, in which the proportions of extremists (defectors or altruists) gradually decrease towards the center in both communities.

Suppose that we consider the initial conditions as the environment and evaluate the special situations when they take the minimum or maximum values. In a neutral environment (all parameters are 0: $V = 0$, $CD = 0$, and $O = 0$), the extremist cooperators and severe defectors disappear, the discriminators' society predominates with around 30%, and the rest is distributed almost equally between the defectors and cooperators. When the parameters take the extreme values, $V = 0$, $CD = 6$, and $O = 6$, a society polarization is established at the extreme values, $k = -5$ and $k = 6$. Other extreme environment parameters values comprise $V = 1$, $CD = 6$, and $O = 6$; in this case, a heterogeneous society is established, but, at the same time, the reputation is low or has negative values, which means that there is no cooperation.

5. Discussion and Conclusions

The simulation results did not show scenarios in which most agents are cooperators, regardless of the parameters' values. Due to the simulation results in the IIRMMD model with heterogeneous initial conditions, it is impossible to have a society of cooperators.

There are discriminator societies (Figures 4 and 5), and when they decrease, the defectors and cooperators grow up simultaneously.

Defector societies (Figures 6–8) are established with almost no cooperators with low conditions parameters. Other defector societies have some cooperators (Figures 10, 14, and 15) and severe defectors.

Some simulation cases establish heterogeneous societies (Figure 9). However, a defector society can be found when there is little knowledge of the agent's reputation and obstinacy.

There is a society that has a better balance of cooperators and defectors proportion (Figure 11); this is the most prominent society (or the giant cluster).

There are simulations with only unconditional cooperators and severe defectors (Figures 12 and 13). In all these cases, when the cooperators appear to grow up severe defectors, it make us think they are in a certain symbiosis.

Simulation results show no conditions for cooperation being established (most of the population is cooperative). However, some conditions cause the emergence of cooperation (cooperators arise). It seems to be beneficial that no discriminators exist, but they are necessary because severe defectors increase if they do not exist.

In the introduction, four general questions about cooperation were settled. These were partially answered from the IIRMMD model's approach. For the first question, the fact that each agent strengthens its k-profile by individualism implies that being a defector generates better profits or fewer losses in any scenario, so the cooperation is complex. For particular parameter values, a few cooperators emerge, and predominant populations of defectors arise. Similarly, a (partial) answer to the second question—the existence of cooperation—is never obtained if cooperation is understood to exist when a majority percentage of the agents is cooperative (i.e., at least 60%). Nevertheless, if the existence of cooperation is considered as the existence of at least 30% of cooperators, then 35% of simulation experiments fulfill it. For question three, with the model's initial conditions (heterogeneous society), a society of cooperators can never be established (over 60%) and much less so if the agents are required to cooperate since defectors always appear in a greater proportion. In order to establish cooperation in a selfish society, it may be necessary to change the initial conditions in relation to the agents' proportions or the type of mechanism. As a partial answer to question four, according to individualism, a cooperator ceases cooperating when his obstinacy threshold of continuous non-cooperation is exceeded, and a defector is more selfish if his obstinacy threshold is surpassed; this is when he acquires many cooperations that exceed his threshold.

A partial answer to the first question of Section 3.3 is that, under special conditions, altruistic societies or groups arise. However, this is not a good situation since it causes severe defectors and a higher percentage. The second question has a similar answer.

Unlike the other models in the literature, this model considers internal attributes such as obstinacy; external characteristics such as cooperative demand; and the characteristic that has been used in other investigations—the probability of seeing image score. This model has two new parameters in the experiments and coincides with the statement of Nowak and Sigmund.

Our work differs from the article of Nowak and Sigmund [8] because we do not consider three agent types of proportions with various forms of payments and p probability criteria to know if an agent is a G-individual. It also differs from the work of Lotem et al. [10] since they are interested in identifying the effect of a person with a $D = +7$ phenotype, fixed for all simulation generations, which we do not do. Leimar and Hammerstein [12] analyze standing strategies or good standing strategies to compare them to image scoring, which also differs from our work.

Additionally, our work is quite different from the work of Mohtashemi and Mui [13] since they analyzed the evolution of indirect reciprocity through social information. They determine this evolution with simulations and analytical calculations.

Since our model promotes individualism and includes social parameters, our results are not like previous papers. However, in a certain sense, we agree with Milinski et al. [11] since the cooperative demand parameter is similar to a social norm.

Although our work is different from the work of Seinen and Schram [14] due to the experimental evidence, we obtain information that the "groups of social norms" concepts have similarities regarding the variation of the cooperative demand parameter of our model.

Concluding Remarks

This work proposes a new computer simulation application in a social systems class with internal and external dynamics (based on the ABM) called the IIRMMD model, which is based on the indirect reciprocity mechanism with reputation to analyze cooperative behavior with individualism. This model includes the visibility (of reputation), cooperative demand, and agents' obstinacy.

The use of data science methods was a helpful tool to determine and visualize the behavioral patterns of multiple simulation results. They were efficiently displayed in different clusters, where each cluster had similar behaviors according to the k-profile.

The results summarize that individualism in the indirect reciprocity mechanism generates unhealthy cooperation; whenever there are cooperators, defectors who exploit the situation emerge, gaining benefits without contributing to cooperation. Additionally, with the emergence of discriminatory groups, a balance is created in cooperation, and the abuse stemming from defector groups is reduced. Another conclusion is that heterogeneity reflects social fragmentation (in the sense of cooperation) and occurs when agents know that there is an elevated level of reputation.

The applicability of a simulation model of indirect reciprocity with individualism in real-life social systems is significant and reveals a significant amount of information. This approach models the dynamics of interactions where indirect reciprocity plays a fundamental role, from virtual environments such as social networks to work settings and communities. This model demonstrates its relevance by representing how interactions among individuals impact the formation of bonds, trust, decision making, and collaboration. Its versatility makes it a valuable tool for understanding and predicting collective behavior in a wide range of social contexts, offering an essential perspective for addressing complex social dynamics.

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Data Availability Statement: The simulation model, databases, and Python notebook are available at https://github.com/mmgs-abm/dataset-sim-res-exp-iirmc.

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Appendix A. Pseudo-Code Process and Flowchart

The following MCRII simulation algorithm contains all the parts and processes described above. In summary, it describes what and how p, s, and k values are updated for the agents' population in each iteration of time, starting from parameters' initial conditions V, O, and EC.

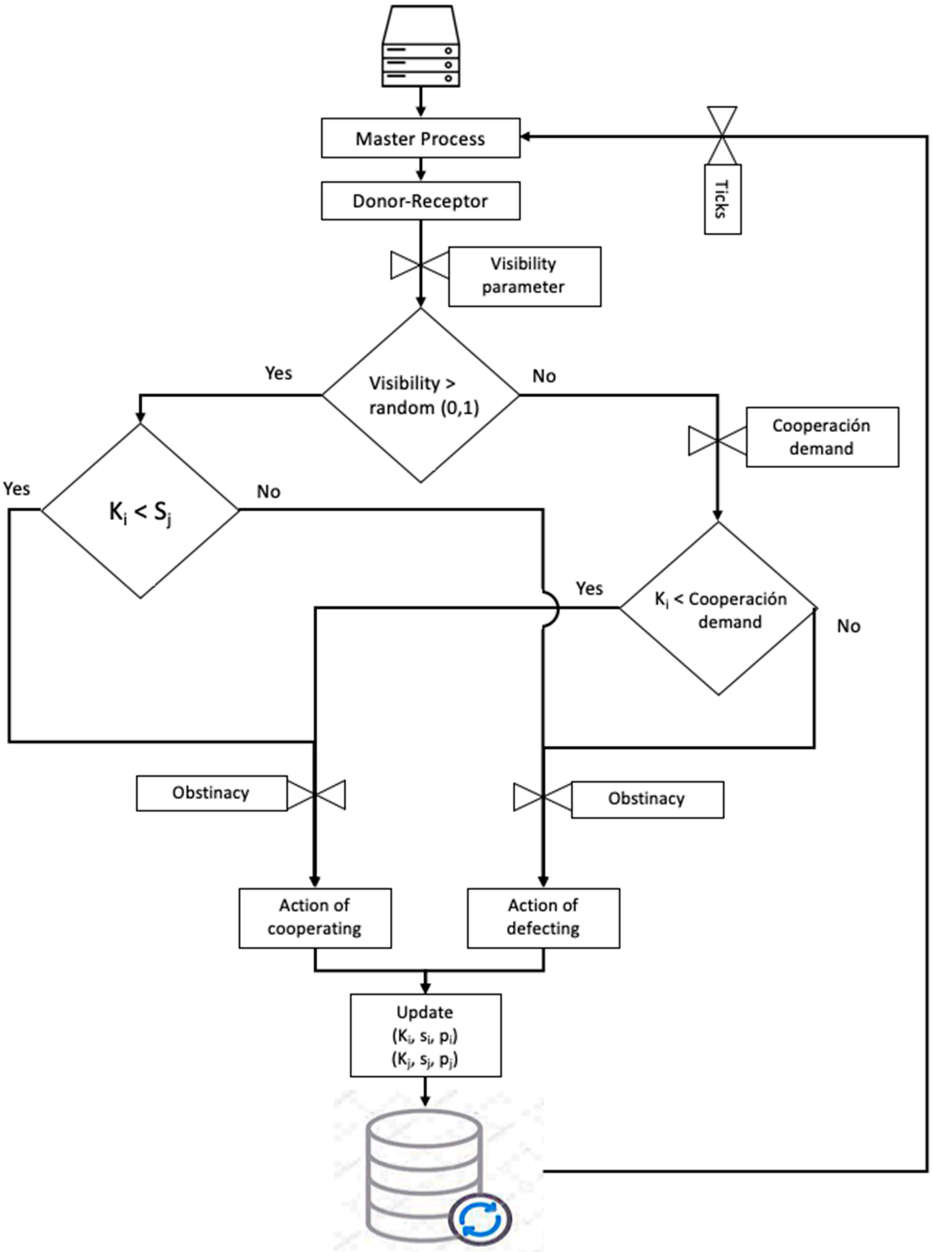
Pseudo-code of the evolutionary dynamics of the simulation model

```
1   Initialize: t = 1000, N = 100, k = {5−, −4, . . . 5, 6}, s = 0, p = 0, O = (0, 6), v = (0, 1), CD = (0, 6), c = 0.1, b = 1
2       for each round do
3           round t
4           repeat select link until (average degree × n)/2
5               randomly donor/recipient
6               if visibility > random (0, 1) then
7                   if k_i < s_j, then
8                       cooperative action for donor and recipient
9                       update obstinacy
10                  else
11                      action of defecting for donor and recipient
12                      update obstinacy
13                  end if
14              else
15                  if k_i < cooperation demand, then
16                      cooperative action for donor and recipient
17                      update obstinacy
18                  else
19                      action of defecting for donor and recipient
20                      update obstinacy
21                  end if
22              end if
23          update (k, s, p)
24      return (k, s, p)
25      end for
26      output (k, s, p)
27  End
```

Appendix A.1. Pseudo-Code Description

The process begins with initializing variables and establishing the parameters of the system's initial state (line 1). The master loop starts for each round (line 2). The first round starts with a value of t, which ranges from 1 to 1000 (line 3). The loop (line 4) includes the repeated action, creating links randomly between agents (as many as the average connected degree link parameter multiplied by n agents and divided by two; this is performed to obtain an average of 2.5 interactions per agent; the average degree parameter is controlled in the master process). Later, the 'donor' and 'recipient' selection is randomly realized (line 5). In line 6, the first conditional cooperation process is applied; if the visibility parameter is greater than some random number between 0 and 1, then the cooperation criterion is applied (line 7). If the conditions cooperation criterion is satisfied, cooperation will occur, the donor pays a cost c, and the recipient obtains a benefit b (line 8). After this, variables s and p are updated (according to the rules); individualism and obstinacy will be considered for updated variable k (line 9). If the cooperation criterion is not satisfied, the above values will be updated in another way, following the defined rules (lines 10–13). When the visibility criterion condition from line 6 is not satisfied, then the process will proceed to line 15, where k_i will be compared with the cooperation demand parameter; if the inequality holds, then the variables will be updated (lines 16–17); otherwise, they will be updated with defection rules (line 18–20). After the conditional process, the cycle updates k, s, and p (line 23) per iteration. The cycle concludes with updated values being returned (line 24) for the following iterations. Finally, the values of all agents' variables are updated (line 26).

Appendix A.2. MCRII's Simulation Flowchart



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