Blind Channel Estimation for FBMC/OQAM Systems Based on Subspace Approach

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Abstract: The conventional channel estimation schemes for filter bank multicarrier with offset quadrature amplitude modulation (FBMC/OQAM) systems are mainly based on preamble methods. However, the utilization of preamble for channel estimation decreases the system’s spectrum efficiency. In this paper, we propose a modified subspace blind channel estimation method for FBMC/OQAM systems. The proposed method distinguishes itself from previously preamble based methods by utilizing spatial diversity technique to introduce data redundancy for blind channel estimation, which leads to high spectral utilization. Thus, the proposed method can provide significant root mean square error (RMSE) performance improvement compared to conventional preamble based methods at high SNRs. Simulation results verify the validity of the proposed method in FBMC/OQAM systems.

Keywords: FBMC/OQAM; blind channel estimation; subspace method; preamble; RMSE

1. Introduction

Over the past decades, multicarrier communication techniques have been widely adopted in many communication systems for high data rate transmission. Orthogonal frequency division multiplexing (OFDM) [1–3] is certainly one of the most famous and accepted multicarrier technologies among the mainly wireless communication systems. This is because of its immunity to multipath fading and simplicity of channel estimation and suitability for multiple input multiple output (MIMO) systems. However, it suffers from some inherent drawbacks, such as high spectral band leakage, sensitivity to carrier frequency offset, and cyclic prefix (CP) overhead. It should be noted that the use of CP in OFDM systems could decrease the spectrum efficiency.

Due to the abovementioned drawbacks in OFDM systems, the filter bank multicarrier with offset quadrature amplitude modulation (FBMC/OQAM) system has recently drawn increasing attention from many researchers [4–8]. FBMC/OQAM well utilizes time frequency localization (TFL) property pulse shaping via an IFFT/FFT-based filter bank, and staggered OQAM symbols, real symbols at twice the symbol rate of FBMC/QAM, are loaded on the subcarriers. Regardless of the higher complexity compared to OFDM, FBMC/OQAM can provide remarkably reduced out of band emissions, robustness against carrier frequency offset, and better spectral efficiency as CP is not required. In fact, FBMC/OQAM has its root in the pioneering studies of Chang [9] and Saltzberg [10] who introduced multicarrier techniques (including OFDM technique) over two decades ago. However, unlike OFDM that transmits complex-valued symbols at a given symbol rate, FBMC/OQAM transmits real-valued symbols at twice this symbol rate. As FBMC/OQAM only holds the orthogonality in the real field, the received symbols are contaminated with an intrinsic imaginary interference came from
neighboring subcarriers and symbols. The intrinsic interference complicates signal processing tasks, such as channel estimation. Hence, channel estimation methods developed for CP/OFDM cannot be directly applied in FBMC/OQAM systems since the subcarrier functions are only orthogonal in the real field.

Much effort has been devoted to solve the channel estimation problem for FBMC/OQAM systems. A common method is based on the preamble approach. Interference approximation method (IAM) [11] and interference cancellation method (ICM) [12] are the two classical preamble based methods, which achieve the effect of channel estimation by avoiding the inherent interference or constructively utilizing the interference. Inspired by IAM and ICM, the authors [13] propose a novel preamble structure (NPS) for channel estimation in FBMC/OQAM systems. Simulation results demonstrate that the novel preamble structure can obtain better performance than traditional methods. Recently, channel estimation based on compressive sensing (CS) is a hot research topic. However, most of the studies [14–16] focus on channel estimation based on CS in OFDM systems. Few of them have studied the CS based channel estimation method for FBMC/OQAM systems. A preamble based channel estimation method by utilizing CS approach for FBMC/OQAM systems has been first proposed in [17]. The approach is using orthogonal matching pursuit (OMP) greedy CS algorithm to reconstruct channel impulse response, where the sparsity level of the channel is provided as a priori information. Then, the authors [18] propose a sparse adaptive channel estimation method based on CS for FBMC/OQAM systems. It has been verified that a CS based approach can obtain significant channel estimation performance improvement compared with the traditional preamble based approach. However, in a preamble based scheme, the preamble should be protected from the subsequent data transmission and the previous frame by inserting null symbols, which causes longer preamble. Periodic transmission of training sequence decreases bandwidth utilization. Besides, some semi-blind channel estimation methods [19,20] have been proposed by utilizing the preamble and inherent interference in the FBMC/OQAM signals. This approach also requires the usage of preamble for channel estimation. In [21], the authors propose linearly precoded isotropic orthogonal transform algorithm (IOTA) based multicarrier systems to achieve blind channel estimation by utilizing the structure of auto-correlation and cross-correlation matrices introduced by precoding. The simulations demonstrate the validity of the proposed method. An algorithm for the blind identification of time-dispersive channels in pulse shaping OFDM/OQAM systems has been proposed in [22], and the approach exploits cyclostationarity induced by the use of overlapping pulse shaping filters and uses second-order statistics only. Simulations demonstrate the performance of the algorithm. A drawback of the algorithm is that it requires a lot of averaging to arrive at good estimates.

In this paper, we investigate blind channel estimation for FBMC/OQAM systems by exploiting subspace method [23]. The proposed blind channel estimation method avoids using preamble sequences, which allows more data for transmission and can acquire high spectral efficiency. The main contributions of this paper are listed as follows:

1. To the best of our knowledge, blind channel estimation based on the modified subspace approach for FBMC/OQAM systems has not yet been studied in the literatures. In this paper, modified subspace method is exploited for FBMC/OQAM systems. This method does not require the utilization of preamble but utilizes receiver diversity technique to introduce data redundancy for blind channel estimation.

2. Amplitude estimation of channel tips is used to evaluate the estimation accuracy. Root mean square error (RMSE) is provided to evaluate the estimation performance over multipath fading channels. Conventional preamble based channel estimation methods are utilized as a benchmark for simulation comparisons.

3. The accuracy of the analytical results is verified by numerical simulations under different conditions. The different conditions include different numbers of receiver diversity, different modulation modes, and different numbers of FBMC/OQAM receiving symbols. Simulation
results show that the proposed blind channel estimation method has a better performance than the conventional preamble based scheme at higher SNRs.

The purpose of this paper is to propose a blind channel estimation method based on subspace approach for FBMC/OQAM systems. We would like to convince the reader with the potential of the proposed method as an efficient performance channel estimator.

The remainder of the paper is organized as follows. In Section 2, the transmission system model of FBMC/OQAM is introduced and the classical preamble based channel estimation method is reviewed. Based on modified subspace approach, a blind channel estimation method by using spatial diversity technique to introduce data redundancy is proposed in Section 3. Simulation results are presented in Section 4. Finally, Section 5 gives the concluding remarks.

2. FBMC/OQAM System

2.1. Transmission Model

The baseband transmitted signal in FBMC/OQAM systems can be expressed as

\[ x(t) = \sum_{m=0}^{N-1} \sum_{n} a_{m,n} g_{m,n}(t) \]  

where \( N \) is the number of subcarriers, \( a_{m,n} \) denotes the real valued OQAM symbols, and \( g_{m,n}(t) \) is the pulse shaping basis function, which is derived from the time-frequency prototype function \( g(t) \) in the equation

\[ g_{m,n}(t) = g(t - n\tau_0)e^{j2\pi mf_0 t}e^{j\phi_{m,n}} \]  

where the subscript \((.)\) \( m,n \) is the \((m,n)\)-th frequency time (FT) point, it denotes the \( m \)-th subcarrier and the \( n \)-th symbol time instant. \( T_0 \) is the OFDM symbol duration time, and \( \tau_0 \) is the OQAM symbol time offset between the real and imaginary parts. \( F_0 \) is the subcarrier spacing, with \( F_0 = 1/T_0 = 1/2\tau_0 \). In addition, \( e^{j\phi_{m,n}} \) is determined by the additional phase term \( \phi_{m,n} = \frac{\pi}{2}(m + n) \).

To guarantee distortion-free data recovery, subcarrier pulse shaping function \( g_{m,n}(t) \), which is defined in (2), should satisfy the orthogonality in the real field as

\[ \Re\{ \langle g_{m,n} | g_{p,q} \rangle \} = \Re\{ \sum_{t} g_{m,n}(t)g_{p,q}^*(t) \} = \delta_{m,p}\delta_{n,q} \]  

where \( \delta_{ij} \) denotes the Kronecker delta, and is defined as \( \delta_{m,p} = 1 \) if \( m = p \) and \( \delta_{m,p} = 0 \) if \( m \neq p \). However, even with perfect time and frequency synchronization in a distortion-free channel, some purely imaginary interference still exists. Thus, we define the interference weights as

\[ \langle g_{m,n} | g_{p,q} \rangle = -j\langle g_{m,n} | g_{p,q} \rangle \]  

where \( \langle g_{m,n} | g_{p,q} \rangle \) denotes a purely real term for \((m,n) \neq (p,q)\).

The received based signal after passing through the channel can be expressed as

\[ r(t) = \sum_{m=0}^{N-1} \sum_{n} a_{m,n} g_{m,n}(t)H_{m,n}(t) + \eta(t) \]  

with

\[ H_{m,n}(t) = \int_{0}^{\tau_{\text{max}}} h(t,\tau)e^{-2j\pi mf_0 \tau}d\tau \]  

where \( H_{m,n}(t) \) denotes the complex response of the fading channel at instant \( t \), with \( h(t,\tau) \) the impulse response of the channel at time \( t \) and delay \( \tau \), and \( \eta(t) \) is the additive noise. For simplicity, it is assumed that each subcarrier channel is a flat fading channel. It means that during the duration of the prototype,
the channel is constant. After that, we can get \( H_{m,n}(t) = H_{m,n} \). The IFFT/FFT implementation diagram of the FBMC/OQAM system [24] is shown in Figure 1.

\[
\begin{align*}
j^\ast (m+2n) & \quad \text{Re} \quad \text{IFFT} \quad \text{Raised Cosine Filter Banks} \quad g(n) \quad \text{PS} \\
\end{align*}
\]

\[
\begin{align*}
j^\ast (m+2n+1) & \quad \text{Im} \quad \text{IFFT} \quad \text{Raised Cosine Filter Banks} \quad g(n+\frac{N}{2}) \quad \text{P/S} \\
\end{align*}
\]

\[
\begin{align*}
\hat{a}_{m,n} & \quad \text{Re} \quad \text{equalization} \quad \text{FFT} \\
\end{align*}
\]

\[
\begin{align*}
\hat{a}_{m,n} & \quad \text{Im} \quad \text{equalization} \quad \text{FFT} \\
\end{align*}
\]

\[
\begin{align*}
H(t) & \quad \text{channel} \\
\end{align*}
\]

\[
\begin{align*}
x(t) & \quad r(t) \quad \text{S/P} \\
\end{align*}
\]

Figure 1. The IFFT/FFT implementation diagram of the FBMC/OQAM system.

2.2. Preamble Channel Estimation Analysis

Noise is omitted for simplicity, the demodulation output symbol at the frequency and time (FT) point \((p, q)\) is

\[
y_{p,q} = H_{p,q} a_{p,q} + j \sum_{(m_0,n_0) \neq (0,0)} a_{p+m_0,q+n_0} H_{p+m_0,q+n_0} \langle y, p_{p,q} \rangle \\
\tag{7}
\]

where \( \langle , \rangle \) represents inner product. At the receiver, the zero-forcing equalized signal can be expressed as

\[
y_{p,q} = a_{p,q} + I_{p,q} \\
\tag{8}
\]

where \( I_{p,q} = j \sum_{(m_0,n_0) \neq (0,0)} a_{p+m_0,q+n_0} \frac{H_{p+m_0,q+n_0}}{H_{p,q}} \langle y, p_{p,q} \rangle \). The estimated value of the output symbol is

\[
\hat{a}_{p,q} = \Re \left\{ \frac{y_{p,q}}{H_{p,q}} \right\} = a_{p,q} - \sum_{(m_0,n_0) \neq (0,0)} a_{p+m_0,q+n_0} \Im \left\{ \frac{H_{p+m_0,q+n_0}}{H_{p,q}} \right\} \langle y, p_{p,q} \rangle \\
= a_{p,q} + \Re \{ I_{p,q} \} \\
\tag{9}
\]

with \( \Re \{ I_{p,q} \} \) denotes the inter-symbol interference of the real part, it should be noted that the interference makes it difficult to obtain accurate estimates of symbols. In the analysis, \( \hat{a}_{p,q} \) can be obtained by an approximately estimate.

Define a neighborhood \( \Omega_{\Delta m, \Delta n} \) for a given FT point \((p, q)\), \((\Delta m, \Delta n) \neq (0,0)\)

\[
\Omega_{\Delta m, \Delta n} = \{ (m_0, n_0), |m_0| \leq \Delta m, |n_0| \leq \Delta n, |H_{p+m_0,q+n_0} - H_{p,q}| \}
\tag{10}
\]
With a good time frequency pulse filter, a consensus is that the interference is only influenced by the first-order neighborhood $\Omega_{1,1}$ of a given FT point $(p, q)$, where $\Omega_{1,1} = \{(m, n), |m| \leq 1, |n| \leq 1, (m, n) \neq (0, 0)\}$. Then, we can rewrite (8) to

$$\frac{y_{p,q}}{H_{p,q}} = a_{p,q} + j \sum_{(m_0,n_0) \in \Omega_{1,1}} a_{p+m_0,q+n_0} \langle g \rangle_{p+m_0,q+n_0} + n_0$$

For good time and frequency localized pulses $g$, the interference comes from FT points outside a neighborhood $\Omega_{1,1}$ of $(p, q)$ is negligible. As an example with the IOTA filter function [11], when $(m_0, n_0) \notin \Omega_{1,1}$, we have $|\langle g \rangle_{p+m_0,q+n_0} + n_0| < 0.04$, and

$$\frac{\sum_{(m_0,n_0) \notin \Omega_{1,1}} |\langle g \rangle_{p+m_0,q+n_0} + n_0|^2}{\sum_{(m_0,n_0) \in \Omega_{1,1}} |\langle g \rangle_{p+m_0,q+n_0} + n_0|^2} \approx 0.02$$

Therefore, we also have

$$\left| \sum_{(m_0,n_0) \notin \Omega_{1,1}} a_{p+m_0,q+n_0} \frac{H_{p+m_0,q+n_0}}{H_{p,q}} \langle g \rangle_{p+m_0,q+n_0} + n_0 \right| < \left| \sum_{(m_0,n_0) \in \Omega_{1,1}} a_{p+m_0,q+n_0} \langle g \rangle_{p+m_0,q+n_0} + n_0 \right|$$

Then, (11) can be approximated as

$$\frac{y_{p,q}}{H_{p,q}} \approx a_{p,q} + j \sum_{(m_0,n_0) \in \Omega_{1,1}} a_{p+m_0,q+n_0} \langle g \rangle_{p+m_0,q+n_0} + n_0$$

Let us denote $a_{p,q}^{(i)} = \sum_{(m_0,n_0) \in \Omega_{1,1}} a_{p+m_0,q+n_0} \langle g \rangle_{p+m_0,q+n_0}$, where $a_{p,q}^{(i)}$ is a pure real value. Taking the real part in (14), we can get the estimated value

$$\hat{a}_{p,q} = \Re \left\{ \frac{y_{p,q}}{H_{p,q}} \right\} \approx a_{p,q}$$

Consequently, the output symbol at the FT point $(p, q)$ can be represented as

$$y_{p,q} \approx H_{p,q}(a_{p,q} + ja_{p,q}^{(i)})$$

Assuming that the symbol on the receiver FT point $(p, q)$ is a prior known preamble symbol, with noise taken into consideration, the preamble based channel estimation [11] is

$$\hat{H}_{p,q} = \frac{y_{p,q}}{a_{p,q} + ja_{p,q}^{(i)}} \approx H_{p,q} + \frac{\eta}{a_{p,q} + ja_{p,q}^{(i)}}$$

where $\eta$ is the noise term in the output of demodulation. It should be noted that the larger the power of $a_{p,q} + ja_{p,q}^{(i)}$, the better the estimation will be. This observation underlies the classical preamble based method (e.g., IAM), and these approaches focus on increasing the power of interference to estimate the channel.
3. Proposed Blind Channel Estimation Method

In OFDM systems, the traditional subspace channel estimation methods use CP to introduce data redundancy. However, it cannot be directly used in FBMC/OQAM systems as CP is not required. The subspace method should be modified, so that it can be applied to FBMC/OQAM systems for improving the estimation performance.

Consider a FBMC/OQAM system with total number of $N$ subcarriers. The $k$-th block of the transmitted symbols (frequency-domain) is

$$s(k) = [s_0(k), s_1(k), \ldots, s_{N-1}(k)]^T$$

(18)

After the frequency-domain block modulation implemented by inverse fast Fourier transform (IFFT), the time-domain FBMC/OQAM block $x(k)$ is

$$x(k) = [x_0(k), x_1(k), \ldots, x_{N-1}(k)]^T = W_N s(k)$$

(19)

where $W_N$ is the $N \times N$ dimensional inverse discrete Fourier transform (IDFT) matrix, define $w = e^{2\pi j/N}$, $W_N$ is given by

$$W_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w^1 & \cdots & w^{(N-1)} \\ 1 & w^2 & \cdots & w^{2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & w^{(N-1)} & \cdots & w^{(N-1)(N-1)} \end{bmatrix}$$

(20)

As CP is not required in FBMC/OQAM systems, the spatial receiver diversity technique can be used to introduce new data redundancy. Define $M$ as the redundancy factor, it means that there exists number of $M$ receiving antennas at the receiver (single input multiple output). The received signal after passing through the $m$-th channel is defined as $r^m(k) = [r_0^m(k), r_1^m(k), \ldots, r_{N-1}^m(k)]^T$, and $\eta^m(k) = [\eta_0^m(k), \eta_1^m(k), \ldots, \eta_{N-1}^m(k)]^T$ denotes the additive white gauss noise (AWGN) in the channel. The channel impulse response at the $m$-th antenna can be denoted as $h^m = [h_0^m, h_1^m, \ldots, h_L^m]^T$, with $L$ the channel order. In order to avoid the effect of inter-block interference on channel estimation, only $N - L$ data symbols are selected for the $k$-th transmitted FBMC/OQAM block, the received block without inter-block interference can be rewritten as

$$r^m(k) = \begin{bmatrix} h_0^m \cdots \ h_L^m \\ h_0^m \cdots \ h_L^m \\ \vdots \\ h_0^m \cdots \ h_L^m \end{bmatrix} \begin{bmatrix} x_0(k) \\ x_1(k) \\ \vdots \\ x_{N-1}(k) \end{bmatrix} + \begin{bmatrix} \eta_0^m(k) \\ \eta_1^m(k) \\ \vdots \\ \eta_{N-1}^m(k) \end{bmatrix}$$

(21)

where $H^m$ is the $(N - L) \times N$ channel matrix and the redefined $\eta^m(k)$ is the $(N - L) \times 1$ dimension AWGN vector.

Define the matrix

$$H = \begin{bmatrix} H^0 \\ H^1 \\ \vdots \\ H^{M-1} \end{bmatrix}$$

(22)
\[ A = HW_N \]  

where \( H \) and \( A \) are both \( M(N-L) \times N \) matrix. The received signal from \( M \) receive antennas yields the following equation

\[
r(k) = \begin{bmatrix} 
H^0 \\
\vdots \\
H^{M-1} 
\end{bmatrix} W_N s(k) + \eta(k) \\
= As(k) + \eta(k) 
\]

By collecting \( N_s \) consecutive FBMC/OQAM blocks for channel estimation, the final FBMC/OQAM received signal model can be written as

\[
Y = [r(1), \ldots, r(N_s)] \\
= A[s(1), \ldots, s(N_s)] + N_n \\
= X + N_n 
\]

where \( X = A[s(1), \ldots, s(N_s)] \) is signal matrix, both \( Y \) and \( X \) are \( M(N-L) \times N_s \) matrix. \( N_n \) is the noise matrix. The singular value decomposition (SVD) of \( X \) is given as

\[
X = \begin{bmatrix} 
U_s & U_n 
\end{bmatrix} \begin{bmatrix} 
\Sigma_s \\
0 
\end{bmatrix} \begin{bmatrix} 
V_s^H \\
V_n^H 
\end{bmatrix} 
\]

where \( \begin{bmatrix} U_s & U_n \end{bmatrix} \) is a \( M(N-L) \times M(N-L) \) matrix. The matrix \( U_s \) constitutes the signal subspace, and the matrix \( U_n \) constitutes the noise subspace. \( U_s \) is corresponding to the diagonal matrix \( \Sigma_s = \text{diag} (\lambda_1, \lambda_2, \ldots, \lambda_N) \). In fact, the estimate value of \( U_n \) can be obtained by the singular value decomposition (SVD) of \( Y \).

The autocorrelation matrix of the received signal is

\[
R_r = E\{r(k)r^H(k)\} = AR_r A^H + \sigma^2 I 
\]

where \( \sigma^2 \) is the variance of AWGN, and \( I \) is the identity matrix. In practice, for the convenience of calculation, the estimated value of the autocorrelation matrix \( \hat{R}_r \) is used to replace \( R_r \), and \( \hat{R}_r \) is given by the following equation

\[
\hat{R}_r = \frac{1}{N_s} \sum_{i=1}^{N_s-1} r(i)r(i)^H 
\]

The singular value decomposition (SVD) of \( \hat{R}_r \) can be used to acquire the singular vectors of the noise subspace \( U_n \).

The orthogonality property between signal subspace and noise subspace asserts

\[
U_n(i)^H A = 0, \quad i = 1, \ldots, M(N-L) - N 
\]

where \( U_n \) denotes the set of singular vectors of the noise subspace, and \( U_n(i) \) is the \( i \)-th column of \( U_n \).

Note that \( R_r \) is obtained from the estimated value \( \hat{R}_r \), the singular vector of the noise subspace cannot satisfy the formula (29), the channel can be estimated by the least square method

\[
\hat{h} = \arg\min_{i=1}^{M(N-L)-N} \sum_{i=1}^{M(N-L)-N} \|\hat{U}_n(i)^H A\|^2 
\]
with $U_n(i) = [u_i(0), \ldots, u_i(M(N-L)-1)]^T$. $1 \times (N-L)$ sub-matrix $U^m(m = 0, \ldots, M-1)$ is generated from $U_n(i)$, and $U^m = [u_i(m(N-L)) \cdots u_i((m+1)(N-L)-1)]$. Then, $U^m$ can generate $U_i^m$ with dimensions $(L+1) \times N$. With

$$U_i^m = \begin{bmatrix} U^m & 0 & \cdots & 0 \\ 0 & U^m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & U^m \end{bmatrix}$$

(31)

By defining matrix $U_i = [(U_i^0)^T, \ldots, (U_i^{M-1})^T]^T$, we can get the following equation

$$U_n(i)^T H = h^T U_i$$

(32)

By formulas (23) and (32), we can get

$$\|U_n(i)^T A\|^2 = U_n(i)^H W_N W_N^H H^H U_n(i) = h^H (U_i)^* U_i^H h^*$$

(33)

where $h^*$ is the conjugate of $h$. Define $\kappa = [U_1, \ldots, U_{M(N-L)-L}]^*$, the minimum value of $h$ can be converted to the minimum value of $h^*$, and the channel estimation value is determined by

$$\hat{h}^* = \arg\min (h^*)H \hat{\kappa} \hat{\kappa}^H h^*$$

(34)

where $\hat{\kappa}$ is the estimate of $\kappa$, and $\hat{h}^*$ is the eigenvector corresponding to the minimum eigenvalue of matrix $\hat{\kappa} \hat{\kappa}^H$. Taking the conjugate operation to $\hat{h}^*$, we can obtain the final estimated channel value $\hat{h}$.

4. Simulation Results

In this section, simulations are carried out to evaluate the performance of the proposed blind estimation method with comparison to other three preamble-based least square (LS) methods for FBMC/OQAM. The performance of the proposed method is analyzed under different receiver diversities, different number of receiving symbols, and different modulations. Estimation of channel taps is used to evaluate the estimation accuracy, root mean square error (RMSE) is employed to evaluate the estimation error over multipath fading channels.

$$\text{RMSE} = \frac{1}{\|h\|} \sqrt{\frac{1}{DM(L+1)} \sum_{i=1}^{D} \|\tilde{h}_i - h\|^2}$$

(35)

where $D$ denotes the number of simulation runs, and $\tilde{h}_i$ denotes the $i$-th simulation channel estimation value, and $M$ denotes the number of receive antennas. The influence of spatial correlation correction between receiving antennas is not considered. We take modulation as 4QAM/16QAM, the number of subcarrier is $N = 16, 64$. The square root raised cosine filter is adopted in FBMC/OQAM, the roll off factor of the filter equals to one, and the length of filter is $L_h = 4T$. In Table 1, we show the detailed values of simulation parameters.
Table 1. Simulation parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of subcarriers $N$</td>
<td>16, 64</td>
</tr>
<tr>
<td>Modulation</td>
<td>4QAM, 16QAM</td>
</tr>
<tr>
<td>The number of receive antennas $M$</td>
<td>2, 4</td>
</tr>
<tr>
<td>Channel order $L$</td>
<td>3, 4</td>
</tr>
<tr>
<td>FBMC/OQAM symbol blocks $N_s$</td>
<td>200</td>
</tr>
</tbody>
</table>

We first evaluate the effectiveness of the subspace channel estimation method in AWGN channel. Four random channels ($M = 4$) are simulated, the channel coefficients in the literature [25] are selected as the simulation channel coefficients, and the channel coefficients are listed below. The channel order is $L = 4$, with 4QAM modulation, $N = 64$ and $N_s = 200$.

$$
h^{(1)} = \begin{bmatrix} -0.049 + j0.359 \\ 0.482 - j0.569 \\ -0.556 + j0.587 \\ 1 \\ -0.171 + j0.061 \end{bmatrix}, \quad h^{(2)} = \begin{bmatrix} 0.443 - j0.0364 \\ 1 \\ 0.921 - j0.194 \\ 0.189 - j0.208 \\ -0.087 - j0.054 \end{bmatrix}$$

$$
h^{(3)} = \begin{bmatrix} -0.221 - j0.322 \\ -0.199 + j0.918 \\ 1 \\ -0.284 - j0.524 \\ 0.136 - j0.19 \end{bmatrix}, \quad h^{(4)} = \begin{bmatrix} 0.417 + j0.030 \\ 1 \\ 0.873 + j0.145 \\ 0.285 + j0.309 \\ -0.049 + j0.161 \end{bmatrix}$$

Figures 2 and 3 present the amplitude estimation of the real and imaginary part of channel taps based on subspace method, respectively. It can be found that the proposed method yields good channel amplitude estimation both in the real and imaginary parts of the channel. The channel taps can be estimated accurately in FBMC/OQAM systems by utilizing the proposed blind estimation method based on subspace approach. We can determine that blind estimation method based on subspace approach is effective for channel estimation in FBMC/OQAM systems.

Then, RMSE performance of the preamble-based LS methods and the subspace method for channel estimation in multipath fading channels is compared. The four different channel coefficients are given below. The channel order is $L = 3$.

$$
h^{(1)} = \begin{bmatrix} 1 \\ 0.54 \\ 0.4 \\ 0.28 \end{bmatrix}, \quad h^{(2)} = \begin{bmatrix} 1 \\ 0.35 \\ 0.32 \\ 0.2 \end{bmatrix}, \quad h^{(3)} = \begin{bmatrix} 1 \\ 0.38 \\ 0.25 \\ 0.12 \end{bmatrix}, \quad h^{(4)} = \begin{bmatrix} 1 \\ 0.4 \\ 0.15 \\ 0.08 \end{bmatrix}$$

Figure 4 shows the RMSE performance comparisons of preamble-based LS methods and subspace method in four-path fading channels. The three preamble-based LS methods in the simulation are IAM, ICM, and NPS, which have been introduced in Section 1. It is verified that NPS can provide the best RMSE performance of the three preamble-based LS channel estimation methods in FBMC/OQAM systems. Subspace blind method partly outperforms other three methods. When SNR is in the range of 0–5 dB or more than 20 dB, subspace blind method has a lower RMSE than other three methods. When SNR is in the range of 10 dB–20 dB, conventional preamble-based LS methods can provide slightly better performance than blind method. With the increase of SNR, the RMSE of blind method is reduced and the accuracy of estimation is improved. Besides, when SNR is in the range of 5 dB–40 dB, LS methods maintain at high and stable RMSE values, especially the IAM method. The preamble-based LS methods have poor estimation performance and low accuracy at high SNRs.
When a SNR of 40 dB is considered, the RMSE of the blind method is less than $10^{-3}$, and its estimation performance is significantly better than LS methods. With the increasing of SNR, the blind method of channel estimation improves. The preamble channel estimation methods achieve the best performance in the SNR range of 5 to 15. Then, the preamble methods gradually reach an error floor. With the SNR increases, the inherent interference can lead to the performance deterioration of the preamble method. The blind estimation method has more robust in resisting the inherent interference. It is worth noting that although the blind method outperforms the other methods at higher SNRs, and does not require additional bandwidth, there are still some drawbacks, such as poor flexibility and a high computation load. Therefore, it is not suitable for real-time systems. With the improvement of the electronic technology, it is believed that the proposed blind method will see greater application in the future.

![Real part of Channel](image1.png)

**Figure 2.** Amplitude estimation of the real part of channel taps.

![Imag Part of Channel](image2.png)

**Figure 3.** Amplitude estimation of the imaginary part of channel taps.
When the number of subcarriers increases, more sub-vectors are generated to compute the correlation with the increase of modulation order, the RMSE performance of the blind method will deteriorate. The channel estimation accuracy of blind method under 16QAM mapping is lower than that of the 4QAM mapping. This phenomenon may due to the existence of the intrinsic interference between subcarriers in FBMC/OQAM systems. The intensity of inherent interference is influenced by the modulation order. When the modulation order increases, the influence of interference is greater, and the accuracy of channel estimation is affected. In Figure 5, the curve with 4QAM and \( N = 16 \) provides a significant SNR gain compared to other three curves at the same MSE level. Reducing the number of subcarriers can get up to about 15 dB SNR gain, when \( RMSE = 10^{-2} \). 4QAM modulation curve can obtain about 11 dB SNR gain compared to 16QAM modulation curve. From Figure 6, the RMSE performance is improved by increasing the number of receive antennas. When \( RMSE = 10^{-3} \) and in the case of the same conditions, the curve with \( M = 4 \) gives performance that is approximately 10 dB better than the curve with \( M = 2 \).

![Figure 4](image1.png)

**Figure 4.** RMSE performance comparisons of preamble-based LS methods and subspace method, with 4QAM, \( N = 64 \), \( M = 4 \), and \( N_s = 200 \).

Figures 5 and 6 depict the RMSE performance of subspace blind method for different modulation and different number of \( N \) with \( M = 2 \) and \( M = 4 \), respectively. From the comparison of the two figures, apparently, the greater the number of receive antennas is adopted, the less the RMSE is induced. When the number of subcarriers increases, more sub-vectors are generated to compute the correlation matrix in (27). Hence, increasing the number of subcarriers degrades the RMSE performance. Besides, with the increase of modulation order, the RMSE performance of the blind method will deteriorate. The channel estimation accuracy of blind method under 16QAM mapping is lower than that of the 4QAM mapping.

![Figure 5](image2.png)

**Figure 5.** RMSE performance of subspace blind method for different modulation and different number of \( N \), with \( M = 2, N_s = 200 \).
Figure 6. RMSE performance of subspace blind method for different modulation and different number of symbols, with $M = 4, N_s = 200$.

Figure 7 shows the RMSE performance of blind method with different number of FBMC/OQAM symbols. 4QAM mapping is adopted. The number of subcarriers is $N = 16$. The number of receiver antennas is $M = 4$. We can find that with different number of symbols, the blind channel estimation method produces almost the same RMSE except in the vicinity of the high bound. When $N_s = 100$, it has the worst RMSE performance. When the number of FBMC/OQAM symbols increases to a certain extent, the RMSE performance has slightly improvement. This indicates that the selection of the symbols $N_s = 200$ is very robust.

Figure 7. RMSE performance of subspace blind method for different number of FBMC/OQAM symbols, with 4QAM, $N = 16, M = 4$.

5. Conclusions

In this paper, a blind channel estimation based on modified subspace approach was proposed in FBMC/OQAM systems. Compared with the traditional preamble based channel estimation method, the proposed method does not require the usage of preamble but employ receiver diversity to introduce data redundancy for blind estimation. First, the effectiveness of the proposed method is evaluated in AWGN channel. Then, the proposed method is compared with other three conventional preamble-based LS methods under a multipath fading channel. The RMSE performance of the proposed method is simulated and analyzed with different conditions. Simulation results and analysis
show that the blind channel estimation based on subspace approach is an efficient method for channel estimation in FBMC/OQAM systems, and it can provide significantly better performance than LS methods at high SNRs. In the future, we intend to further study the channel estimation methods for MIMO-FBMC/OQAM systems.

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