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A Novel Approach for Group Decision-Making from Intuitionistic Fuzzy Preference Relations and Intuitionistic Multiplicative Preference Relations

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Abstract: During the decision-making process, evaluation information may be given in different formats based on the decision makers' research fields or personal customs. To address the situation that alternatives are evaluated by both intuitionistic fuzzy preference relations (IFPRs) and intuitionistic multiplicative preference relations (IMPRs), a new priority approach based on a net flow score function is proposed. First, the two preference relations above are transformed into the corresponding interval-valued fuzzy preference relations (IVFPRs) and interval-valued multiplicative preference relations (IVMPRs), respectively. Second, the net flow score functions of individual IFPRs and IMPRs are obtained. Third, according to information theory, a mean deviation maximization model is constructed to compute the weights of decision-makers objectively. Finally, the collective net flow score of each alternative is obtained to determine the ranking result. The proposed method is certified to be simple, valid, and practical with three examples.

Keywords: group decision-making; intuitionistic fuzzy preference relations; intuitionistic multiplicative preference relations; net flow score

1. Introduction

During daily life, group decision-making (GDM) problems happen in many cases or fields when determining the best one of several alternatives. Preference relations are a powerful form of information by which to convey the evaluation information. Through pairwise comparison of the alternatives concerning the related criteria, the decision makers' evaluation matrices are obtained, in which each value indicates the preference level of one option over the another. Next, the ranking of the alternatives can be obtained by the fusion techniques of preference relations [1].

Fuzzy preference relations (FPRs) [2] and multiplicative preference relations (MPRs) [1] are the two general preference relations most researched by scholars. However, decisionmakers may not be very familiar with the nature of GDM problems in actuality; the two aforementioned kinds of preference relations cannot deal with uncertain information effectively. To solve this problem, Atanassov [3] extended the traditional fuzzy set by applying the non-membership and hesitancy degrees, and subsequently the intuitionistic fuzzy set (IFS) was proposed. Furthermore, Atanassov and Gargov [4] proposed the interval-valued intuitionistic fuzzy set (IVIFS). IFS and IVIFS have been extensively used in economic, technical, and management GDM problems [5–10]. In addition, many scholars have extended the application of COPRAS [11], ANP [12], MULTIMOORA [13,14], EDAS [15], ELECTRE III [16], and TOPSIS [17] methods in the context of an intuitionistic fuzzy environment. Therefore, decision makers can express the preference, non-preference, and hesitancy information combined with IFS to obtain their IFPRs [18]. Similarly, Xia et al. [19] extended the MPRs into IMPRs. Because of the uncertainty and incompleteness of evaluation information in real life, IFPRs and IMPRs are extensively used in various GDM fields [19–22]. However, to the best of our knowledge, few studies have investigated the GDM processes in which the evaluation information is provided by both IFPRs and IMPRs; thus, in this paper, we will construct a novel GDM method considering this situation.

With respect to GDM problems concerning preference relations, the key step is the means of computing the priority of each alternative. In the aspect of IFPRs, many methods of determining priority have been proposed in previous research. The priority values can largely be divided into three types; namely, crisp priorities, interval-valued priorities, and intuitionistic fuzzy priorities [21]. For example, Xu [23] transformed the IFPRs into the corresponding score matrices, and subsequently two optimization models were developed to obtain the crisp priorities. Combined with the corresponding consistent matrix, Gong et al. [24] proposed the least squares optimization model to determine the ranking of alternatives. Furthermore, a deviation minimization optimization model was developed for determining the interval-valued priorities [25]. Xu [26] determined the interval-valued priorities according to the error propagation formula. Xu and Liao [27] transformed the IFPR into the corresponding IVFPR and derived the interval-valued priorities; next, each interval-valued priority was transformed into an intuitionistic fuzzy number to rank the alternatives. Wang [28] developed linear programming for minimizing the distances between each IFPR and the corresponding additive consistent IFPR, to obtain the intuitionistic fuzzy priorities. In addition, Zeng et al. [29] constructed a new model to choose the best alternative by computing the compatibility measures of IFPR.

Compared with the priority methods of IFPRs, the related research concerning IMPRs is relatively limited; the consistency of IMPRs needs to be further studied. Nevertheless, several priority methods have been proposed. For instance, Xu [30] defined the concepts of expected IMPR and error matrices, and the error-analysis-based method was proposed to determine the ranking result. Zhang and Pedrycz [22] used a transformation formula to obtain the consistent IMPRs, and constructed optimization models to compute the intuitionistic fuzzy priorities. Zhang and Guo [31] proposed two approaches to compute the intuitionistic multiplicative priorities with the complete and incomplete IMPRs, respectively. Jin et al. [32] put forward an optimization model for determining the intuitionistic multiplicative priorities of alternatives. Zhang and Pedrycz [20] proposed an algorithm to revise inconsistent IMPRs into acceptably consistent IMPRs, and developed the IMAHP method to choose the best alternative.

To summarize the research on IFPRs and IMPRs as described above, the priority approaches are mainly proposed according to the consistency of IFPRs and IMPRs, and the computational procedures using them are often very complicated. Moreover, the existing literature invariably focuses on the priority methods concerning IFPRs or IMPRs; thus far, few scholars have addressed the GDM problems which concern the evaluation information provided by both IFPRs and IMPRs. In practical GDM problems, decision makers often have diverse research areas and may give different formats of preference relations. Wang and Fan [33] transformed FPRs and MPRs into uniform representations, and determined the ranking based on a net flow score function [34]. Furthermore, Xu et al. [35] proposed a GDM method using FPRs and MPRs without this transformation, which can allow the completeness of preference information to be retained.

Inspired by Wang and Fan [33] and Xu et al. [35], this paper considers the GDM problems where preference information is provided in the form of both IFPRs and IMPRs. By integrating the hesitancy degree into the membership and non-membership degrees, IFPRs and IMPRs are decomposed into IVFPRs and IVMPRs, respectively. Then, the net flow score functions of individual IFPRs and IMPRs are obtained according to the net flow score theory. A mean deviation maximization model is put forward to obtain the decision maker weights. Finally, the collective net flow scores are computed to rank all the alternatives. The rest of this paper is organized as follows: Several preference relations are introduced in Section 2. Section 3 presents the process that transforms IFPRs and IMPRs into IVFPRs and IVMPRs, respectively; and constructs the net flow score functions of IFPRs and IMPRs.

Section 4 develops an optimization model to compute the decision maker weights, and proposes a new approach to determine the best alternative when the preference information is provided by both IFPRs and IMPRs. Section 5 presents three examples of GDM problems to demonstrate the practicability of the new GDM approach. Finally, some conclusions are summarized in Section 6.

2. Preliminaries

In this section, some basic concepts are presented, such as IVFPRs, IVMPRs, operations of interval numbers, IFPRs, and IMPRs, which are used in the subsequent research.

2.1. IVFPRs and IVMPRs

Since FPRs [2] and MPRs [1] were proposed as a means of ranking the alternatives, much research has focused on improving preference relations. Xu [36], and Satty and Vargas [37] extended the two aforementioned preference relations into an interval fuzzy environment, respectively.

Definition 1 [36]. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be n alternatives; an IVFPR F is expressed as $F = (a_{ij})_{n\times n}$. Where $a_{ij} = \left[a_{ij}^-, a_{ij}^+\right] (i, j = 1, 2, \ldots, n)$ is an interval number, and represents the degree to which x_i is preferred to x_j with the conditions $a_{ij}^- + a_{ji}^+ = a_{ij}^+ + a_{ij}^- = 1$.*

Definition 2 [37]. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be n alternatives; an IVMPR M is expressed as $M = (b_{ij})_{n\times n}$. Where $b_{ij} = \left[b_{ij}^-, b_{ij}^+\right] (i, j = 1, 2, \ldots, n)$ is an interval number that represents the degree to which x_i is preferred to x_j, which satisfies the conditions $b_{ij}^- = 1/b_{ji}^+, b_{ij}^+ = 1/b_{ji}^-$.*

The elements in both IVFPRs and IVMPRs are interval numbers; then, the operations between them follow the operational laws as presented below.

Definition 3 [38]. *Let $p = [p^-, p^+]$ and $q = [q^-, q^+]$ be two interval numbers, then:*

$$p + q = [p^- + q^-, p^+ + q^+]; \tag{1}$$

$$p - q = [p^- - q^+, p^+ - q^-]. \tag{2}$$

2.2. IFPRs and IMPRs

Due to the uncertainty and incompleteness of evaluation information in real life, Xu [18] proposed the combination of IFPRs with IFS.

Definition 4 [18]. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be n alternatives; an IFPR R^F is expressed by $R^F = \left(r_{ij}^F\right)_{n\times n}$. Where $r_{ij}^F = \langle (x_i, x_j), u_{ij}, v_{ij}\rangle (i, j = 1, 2, \ldots, n)$ is an intuitionistic fuzzy number, and u_{ij} indicates the degree of x_i is preferred to x_j, v_{ij} indicates the degree of x_i is non-preferred to x_j. Furthermore, π_{ij} indicates the hesitancy degree, $0 \le u_{ij} + v_{ij} \le 1, u_{ij} = v_{ji}, v_{ji} = u_{ij}, u_{ij} = v_{ij} = 0.5$.*

Similarly, Xia [19] proposed the intuitionistic multiplicative set and IMPRs.

Definition 5 [19]. *Let $X = \{x_1, x_2, \ldots, x_n\}$ be n alternatives; an IMPR R^M is expressed by $R^M = \left(r_{ij}^M\right)_{n\times n}$. Where $r_{ij}^M = \langle (x_i, x_j), \rho_{ij}, \sigma_{ij}\rangle (i, j = 1, 2, \ldots, n)$ is an intuitionistic multiplicative number, composed by the degree ρ_{ij} of x_i is preferred to x_j, the degree σ_{ij} of x_i is non-preferred to x_j, and the hesitancy degree τ_{ij}. In addition, ρ_{ij} and σ_{ij} satisfy the conditions: $\rho_{ji} = \sigma_{ij}, \sigma_{ji} = \rho_{ij}, \rho_{ij}\sigma_{ij} \le 1$, $\rho_{ii} = \sigma_{ii} = 1$ and $1/9 \le \rho_{ij}, \sigma_{ij} \le 9$.*

3. The Net Flow Score Function for Priority Ranking

Gong et al. [39] established the relationship between IFPRs and IVFPRs, which is utilized to obtain the net flow scores of IFPRs. This section investigates the relationship between IMPRs and interval multiplicative preference relation, and proposes a priority method for IFPRs and IMPRs based on net flow score function.

3.1. The Relationship between IFPRs and IVFPRs

We can decompose the IFPR F into three matrices as in the following:

$$u=\begin{pmatrix} u_{11} & u_{12} & \cdots & u_{1n} \\ u_{21} & u_{22} & \cdots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \cdots & u_{nn} \end{pmatrix}; v=\begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \cdots & v_{nn} \end{pmatrix}; \pi=\begin{pmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1n} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{n1} & \pi_{n2} & \cdots & \pi_{nn} \end{pmatrix}.$$

If we integrate the hesitancy degree π_{ij} into the degrees u_{ij} and v_{ij}, the three matrices above can be transformed into two interval matrices as follows:

$$F^{+}=(a_{ij}^{+})_{n\times n}=\left([u_{ij},p_{ij}]\right)_{n\times n}=\begin{pmatrix} [u_{11},p_{11}] & [u_{12},p_{12}] & \cdots & [u_{1n},p_{1n}] \\ [u_{21},p_{21}] & [u_{22},p_{22}] & \cdots & [u_{2n},p_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [u_{n1},p_{n1}] & [u_{n2},p_{n2}] & \cdots & [u_{nn},p_{nn}] \end{pmatrix}, \tag{3}$$

$$F^{-}=(a_{ij}^{-})_{n\times n}=\left([v_{ij},q_{ij}]\right)_{n\times n}=\begin{pmatrix} [v_{11},q_{11}] & [v_{12},q_{12}] & \cdots & [v_{1n},q_{1n}] \\ [v_{21},q_{21}] & [v_{22},q_{22}] & \cdots & [v_{2n},q_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [v_{n1},q_{n1}] & [v_{n2},q_{n2}] & \cdots & [v_{nn},q_{nn}] \end{pmatrix}. \tag{4}$$

where $p_{ij}=1-v_{ij}, q_{ij}=1-u_{ij}$, and both F^{+} and F^{-} are IVPFRs [39]. In other words, the IFPR F can be decomposed into the two IVPFRs of F^{+} and F^{-}; the interval number $a_{ij}^{+}=[u_{ij},p_{ij}]$ indicates the degree range in which x_i is preferred to x_j; and the interval number $a_{ij}^{-}=[v_{ij},q_{ij}]$ denotes the degree range in which x_i is not preferred to x_j.

3.2. The Relationship between IMPRs and IVMPRs

Inspired by Gong et al. [39], similarly, we can divide the IMPR M into three matrices as follows:

$$\rho=\begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{pmatrix}; \sigma=\begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}; \tau=\begin{pmatrix} \tau_{11} & \tau_{12} & \cdots & \tau_{1n} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tau_{n1} & \tau_{n2} & \cdots & \tau_{nn} \end{pmatrix}.$$

If we integrate the hesitancy degree τ_{ij} into the degrees ρ_{ij} and σ_{ij}, the three matrices above can be transformed into two interval matrices as represented below:

$$M^{+}=(b_{ij}^{+})_{n\times n}=\left([\rho_{ij},\alpha_{ij}]\right)_{n\times n}=\begin{pmatrix} [\rho_{11},\alpha_{11}] & [\rho_{12},\alpha_{12}] & \cdots & [\rho_{1n},\alpha_{1n}] \\ [\rho_{21},\alpha_{21}] & [\rho_{22},\alpha_{22}] & \cdots & [\rho_{2n},\alpha_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [\rho_{n1},\alpha_{n1}] & [\rho_{n2},\alpha_{n2}] & \cdots & [\rho_{nn},\alpha_{nn}] \end{pmatrix}, \tag{5}$$

$$M^- = (b_{ij}^-)_{n\times n} = ([\sigma_{ij},\beta_{ij}])_{n\times n} = \begin{pmatrix} [\sigma_{11},\beta_{11}] & [\sigma_{12},\beta_{12}] & \cdots & [\sigma_{1n},\beta_{1n}] \\ [\sigma_{21},\beta_{21}] & [\sigma_{22},\beta_{22}] & \cdots & [\sigma_{2n},\beta_{2n}] \\ \vdots & \vdots & \ddots & \vdots \\ [\sigma_{n1},\beta_{n1}] & [\sigma_{n2},\beta_{n2}] & \cdots & [\sigma_{nn},\beta_{nn}] \end{pmatrix}. \quad (6)$$

where α_{ij} and β_{ij} are denoted as: $\alpha_{ij}\sigma_{ij} = 1, \beta_{ij}\rho_{ij} = 1$. Combined with the Definition 5, we can obtain that:

$$[\rho_{ii},\alpha_{ii}] = [1,1],\ \rho_{ij}\alpha_{ji} = 1,\ \alpha_{ij}\rho_{ji} = 1; \quad (7)$$

$$[\sigma_{ii},\beta_{ii}] = [1,1],\ \sigma_{ij}\beta_{ji} = 1,\ \beta_{ij}\sigma_{ji} = 1, \quad (8)$$

which demonstrates that both M^+ and M^- are IVMPRs. Consequently, we can decompose the IMPR M into the two IVMPRs of M^+ and M^-; the interval number $b_{ij}^+ = [\rho_{ij},\alpha_{ij}]$ represents the degree range in which x_i is preferred to x_j; and the interval number $b_{ij}^- = [\sigma_{ij},\beta_{ij}]$ is the degree range in which x_i is not preferred to x_j.

3.3. The Net Flow Scores of IFPRs and IMPRs

Wang and Fan [33] and Xu et al. [35] constructed the entering and leaving flow functions of FPRs and MPRs, respectively; then, the method for determining the ranking result from FPRs and MPRs was proposed according to the net flow score function. Motivated by this research, we put forward the net flow scores of IFPRs and IMPRs, which are utilized to obtain the ranking of alternatives in the subsequent decision.

According to Section 3.1, an IFPR F can be split into the two IVFPRs of F^+ and F^-. For each element of F^+ and F^-, $a_{ij}^+ + a_{ji}^-$ is regarded as the total preference degree range of x_i over x_j. Subsequently, the leaving flow score $\phi_F^+(x_i)$ that represents the total preference degree range of x_i over the other alternatives can be determined by

$$\phi_F^+(x_i) = \sum_{j=1}^n a_{ij}^+ + \sum_{j=1}^n a_{ji}^-,\ i = 1,2,\ldots,n. \quad (9)$$

Besides, $a_{ji}^+ + a_{ij}^-$ is regarded as the total preference degree range of x_j over x_i, and the entering flow score $\phi_F^-(x_i)$ that represents the total preference degree range of the other alternatives over x_i is defined by

$$\phi_F^-(x_i) = \sum_{j=1}^n a_{ji}^+ + \sum_{j=1}^n a_{ij}^-. \quad (10)$$

Then, the net preference degree range of x_i over the other alternatives is obtained by

$$\phi_F(x_i) = \phi_F^+(x_i) - \phi_F^-(x_i) = \left(\sum_{j=1}^n a_{ij}^+ + \sum_{j=1}^n a_{ji}^-\right) - \left(\sum_{j=1}^n a_{ji}^+ + \sum_{j=1}^n a_{ij}^-\right). \quad (11)$$

And $\phi_F(x_i) = [a_i^-, a_i^+]$ is an interval number, the lager the net flow score, the higher the ranking of alternative.

Similarly, an IMPR can be divided into the two IVMPRs of M^+ and M^-. Based on the elements in M^+ and M^-, $b_{ij}^+ + b_{ji}^-$ is regarded as the total preference degree range of x_i over x_j. Subsequently, the leaving flow score $\phi_M^+(x_i)$ that represents the total preference degree range of x_i over the other alternatives can be determined by

$$\phi_M^+(x_i) = \left(\prod_{j=1}^n b_{ij}^+\right)^{1/n} + \left(\prod_{j=1}^n b_{ji}^-\right)^{1/n},\ i = 1,2,\ldots,n. \quad (12)$$

Meanwhile, $b_{ji}^+ + b_{ij}^-$ is regarded as the total preference degree range of x_j over x_i, and the entering flow score $\phi_M^-(x_i)$ that represents the total preference degree range of the other alternatives over x_i is presented as below:

$$\phi_M^-(x_i) = \left(\prod_{j=1}^n b_{ji}^+\right)^{1/n} + \left(\prod_{j=1}^n b_{ij}^-\right)^{1/n}. \quad (13)$$

And the net preference degree range of x_i over the other alternatives is expressed as the following:

$$\phi_M(x_i) = \phi_M^+(x_i) - \phi_M^-(x_i) = \left(\left(\prod_{j=1}^n b_{ij}^+\right)^{1/n} + \left(\prod_{j=1}^n b_{ji}^-\right)^{1/n}\right) - \left(\left(\prod_{j=1}^n b_{ji}^+\right)^{1/n} + \left(\prod_{j=1}^n b_{ij}^-\right)^{1/n}\right) \tag{14}$$

where $\phi_M(x_i) = \left[b_i^-, b_i^+\right]$ is an interval number, and the larger the net flow score, the higher the ranking of the alternative.

4. The Proposed Method for Group Decision-Making

Suppose $DM = \{DM_1, DM_2, \ldots DM_m\}$ is a set of decision makers, $X = \{x_1, x_2, \ldots, x_n\}$ is a set of alternatives, and $\mathbf{w} = (w_1, w_2, \ldots, w_m)^T$ is the decision maker weights, which satisfies the conditions of $\sum_{k=1}^m w_k = 1$, $w_k \geq 0$. Furthermore, we assume that decision makers $DM_k\left(k = 1, 2, \ldots, m_f\right)$ use IFPRs to express their evaluation information, while decision makers $DM_k\left(k = m_{f+1}, m_{f+2} \ldots, m\right)$ use IMPRs. Let $F_{(k)}$ be the IFPR of $DM_k\left(k = 1, 2, \ldots, m_f\right)$ and $M_{(k)}$ be the IMPR of $DM_k\left(k = m_{f+1}, m_{f+2} \ldots, m\right)$. Based on these assumptions above, we propose the priority method for GDM as shown in Figure 1, and the best alternatives can be determined according to the steps as below:

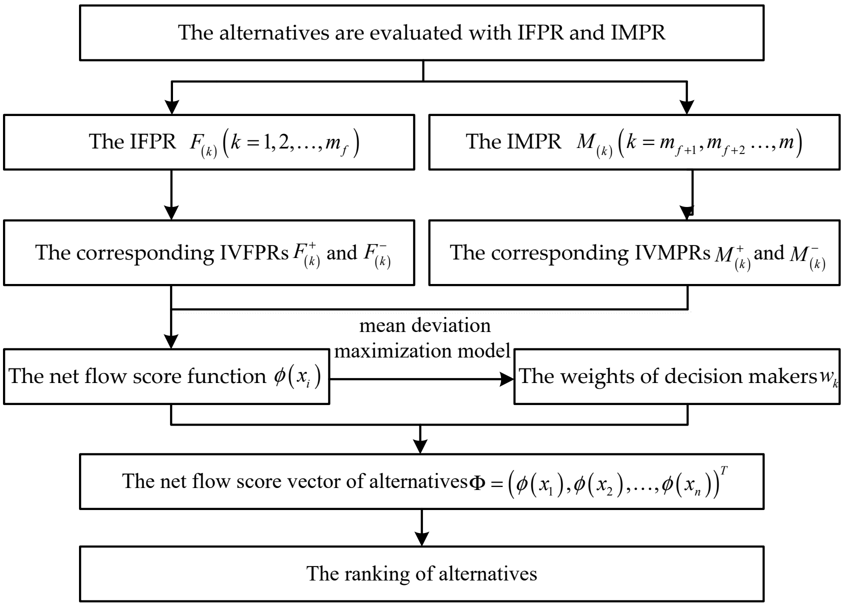

Figure 1. Flow diagram of the proposed method. IFPR: intuitionistic fuzzy preference relations. IMPR: intuitionistic multiplicative preference relations. IVFPRs: interval-valued fuzzy preference relations. IVMPRs: interval-valued fuzzy preference relations.

Step 1. Transform the IFPR $F_{(k)}$ and IMPR $M_{(k)}$.

The IFPR $F_{(k)}$ is divided into two IVFPRs, using the Equations (3) and (4) as in the following:

$$F_{(k)}^+ = \left(a_{ij}^{+(k)}\right)_{n \times n} = \left(\left[u_{ij}^{(k)}, p_{ij}^{(k)}\right]\right)_{n \times n}, \; F_{(k)}^- = \left(a_{ij}^{-(k)}\right)_{n \times n} = \left(\left[v_{ij}^{(k)}, q_{ij}^{(k)}\right]\right)_{n \times n}. \tag{15}$$

Similarly, combined with Equations (5) and (6), we can transform the IMPR $M_{(k)}$ into two interval multiplicative preference relations, as presented below:

$$M_{(k)}^+ = \left(b_{ij}^{+(k)}\right)_{n \times n} = \left(\left[\rho_{ij}^{(k)}, \alpha_{ij}^{(k)}\right]\right)_{n \times n}, \; M_{(k)}^- = \left(b_{ij}^{-(k)}\right)_{n \times n} = \left(\left[\sigma_{ij}^{(k)}, \beta_{ij}^{(k)}\right]\right)_{n \times n}. \tag{16}$$

Step 2. Calculate the net flow score $\phi(x_i)$.

Considering the weights of decision makers, the collective net flow scores $\phi(x_i)$ of preference degree of x_i over the other alternatives are denoted by

$$\begin{aligned}
&\phi(x_i) = \sum_{k=1}^{m_f} w_k \phi_F^{(k)}(x_i) + \sum_{k=m_f+1}^{m} w_k \phi_M^{(k)}(x_i) \\
&= \sum_{k=1}^{m_f} w_k \left(\phi_F^{+(k)}(x_i) - \phi_F^{-(k)}(x_i)\right) + \sum_{k=m_f+1}^{m} w_k \left(\phi_M^{+(k)}(x_i) - \phi_M^{-(k)}(x_i)\right) \\
&= \sum_{k=1}^{m_f} w_k \left(\left(\sum_{j=1}^{n} a_{ij}^{+(k)} + \sum_{j=1}^{n} a_{ji}^{-(k)}\right) - \left(\sum_{j=1}^{n} a_{ji}^{+(k)} + \sum_{j=1}^{n} a_{ij}^{-(k)}\right)\right) + \\
&\sum_{k=m_f+1}^{m} w_k \left(\left(\left(\prod_{j=1}^{n} b_{ij}^{+(k)}\right)^{1/n} + \left(\prod_{j=1}^{n} b_{ji}^{-(k)}\right)^{1/n}\right) - \left(\left(\prod_{j=1}^{n} b_{ji}^{+(k)}\right)^{1/n} + \left(\prod_{j=1}^{n} b_{ij}^{-(k)}\right)^{1/n}\right)\right).
\end{aligned} \tag{17}$$

Step 3. Obtain the decision matrix Δ.

Let

$$\tilde{\delta}_{ik} = \left(\sum_{j=1}^{n} a_{ij}^{+(k)} + \sum_{j=1}^{n} a_{ji}^{-(k)}\right) - \left(\sum_{j=1}^{n} a_{ji}^{+(k)} + \sum_{j=1}^{n} a_{ij}^{-(k)}\right) = [a_{ik}^{-}, a_{ik}^{+}], k = 1,2,\ldots,m_f, i = 1,2,\ldots,n, \tag{18}$$

$$\begin{aligned}
&\tilde{\delta}_{ik} = \left(\left(\prod_{j=1}^{n} b_{ij}^{+(k)}\right)^{1/n} + \left(\prod_{j=1}^{n} b_{ji}^{-(k)}\right)^{1/n}\right) - \left(\left(\prod_{j=1}^{n} b_{ji}^{+(k)}\right)^{1/n} + \left(\prod_{j=1}^{n} b_{ij}^{-(k)}\right)^{1/n}\right) = \\
&\left[b_{ik}^{-}, b_{ik}^{+}\right], i = 1,2,\ldots,n; k = m_f + 1,\ldots,m.
\end{aligned} \tag{19}$$

Because $\tilde{\delta}_{ik}$ is an interval number, we can convert $\tilde{\delta}_{ik}$ into a real number as the following:

$$\delta_{ik} = \frac{a_{ik}^{-} + a_{ik}^{+}}{2}, \tag{20}$$

$$\delta_{ik} = \frac{b_{ik}^{-} + b_{ik}^{+}}{2}. \tag{21}$$

The Equation (17) is simplified as

$$\phi(x_i) = \sum_{k=1}^{m} w_k \delta_{ik},\ i = 1,2,\ldots,n; \tag{22}$$

or

$$\Phi = \Delta \mathbf{w}. \tag{23}$$

where $\Phi = (\phi(x_1), \phi(x_2), \ldots, \phi(x_n))^T$ and Δ can be seen as a decision matrix:

$$\Delta = (\delta_{ik})_{n\times m} = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} DM_1 & DM_2 & \cdots & DM_m \end{array} \\ \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1m} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{n1} & \delta_{n2} & \cdots & \delta_{nm} \end{bmatrix} \end{array}. \tag{24}$$

Step 4. Construct a mean deviation maximization model.

According to information theory [40], if similar evaluation information is given by a decision maker, then small weights should be assigned to her/him because she/he contributes less in differentiating the alternatives. Therefore, we adopt the concept of mean deviation MD_k [35] to represent the distance between preference information of x_i, and the mean preference information of all the alternatives as

$$MD_k = \frac{1}{n}\sum_{i=1}^{n} \left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n} \delta_{jk}\right|. \tag{25}$$

Combined with the aforementioned principle, we can summarize that a preference relation is more important with a larger value of MD_k. Thus, a mean deviation maximization model is presented to obtain the decision maker weights:

$$\begin{array}{ll} \max & \sum_{k=1}^{m}(w_k \cdot MD_k) = \sum_{k=1}^{m} w_k\left(\frac{1}{n}\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right|\right) \\ s.t. & \sum_{k=1}^{m}(w_k)^2 = 1, \quad w_k \geq 0. \end{array} \tag{26}$$

Step 5. Determine the decision maker weights.

To solve the model combined with Lagrange function as

$$L(w,\lambda) = \sum_{k=1}^{m} w_k\left(\frac{1}{n}\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right|\right) + \frac{\lambda}{2}\left(\sum_{k=1}^{m}(w_k)^2 - 1\right), \tag{27}$$

where λ is the Lagrange multiplier. Differentiating Equation (27) concerning w_k, let the partial derivatives be equal to zero as

$$\frac{\partial L(w,\lambda)}{\partial w_k} = \frac{1}{n}\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right| + \lambda w_k = 0. \tag{28}$$

By solving Equation (28), we can obtain the optimal solution based on the condition of $\sum_{k=1}^{m}(w_k)^2 = 1$:

$$w_k^* = \frac{\frac{1}{n}\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right|}{\sqrt{\sum_{k=1}^{m}\left(\frac{1}{n}\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right|\right)^2}}. \tag{29}$$

Finally, the normalized weights of decision makers are determined by normalizing Equation (29) as

$$w_k = \frac{\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right|}{\sum_{k=1}^{m}\sum_{i=1}^{n}\left|\delta_{ik} - \frac{1}{n}\sum_{j=1}^{n}\delta_{jk}\right|}. \tag{30}$$

Step 6. Obtain the ranking result.

We can compute the net flow score vector of alternatives Φ using Equation (17). Then, the best alternative can be obtained based on the value of $\phi(x_i)$; the larger the net flow score, the higher the ranking of alternative.

5. Numerical Examples

Finally, we introduce three numerical examples to indicate the effectiveness and rationality of the new GDM approach. First, Example 1 adopts the original preference information as in the study of Wang [28], in which all the decision makers use IFPRs to evaluate alternatives, i.e., $m_f = m$. Second, Example 2 adopts the original preference information as in the study of Xu [30], in which all the decision makers use IMPRs to evaluate alternatives, i.e., $m_f = 0$. Finally, in Example 3, two decision makers use IFPRs to evaluate alternatives, and the decision makers use IMPRs, i.e., $m_f = 2$.

5.1. Implementation

Example 1. *Suppose that four doctoral students investigate the potential opportunity of international communication, and a committee that is composed of three decision makers evaluates the four candidates by using IFPRs. After the pairwise comparison of the four candidates, the IFPRs are obtained as presented below:*

$$F_{(1)} = \begin{pmatrix} (0.50,0.50) & (0.35,0.55) & (0.40,0.35) & (0.55,0.35) \\ (0.55,0.35) & (0.50,0.50) & (0.70,0.10) & (0.60,0.20) \\ (0.35,0.40) & (0.10,0.70) & (0.50,0.50) & (0.55,0.30) \\ (0.35,0.55) & (0.20,0.60) & (0.30,0.55) & (0.50,0.50) \end{pmatrix}, \tag{31}$$

$$F_{(2)} = \begin{pmatrix} (0.50,0.50) & (0.55,0.25) & (0.65,0.20) & (0.35,0.55) \\ (0.25,0.55) & (0.50,0.50) & (0.40,0.25) & (0.55,0.30) \\ (0.20,0.65) & (0.25,0.40) & (0.50,0.50) & (0.60,0.20) \\ (0.55,0.35) & (0.30,0.55) & (0.20,0.60) & (0.50,0.50) \end{pmatrix}, \tag{32}$$

$$F_{(3)} = \begin{pmatrix} (0.50,0.50) & (0.60,0.30) & (0.75,0.15) & (0.60,0.20) \\ (0.30,0.60) & (0.50,0.50) & (0.45,0.20) & (0.60,0.20) \\ (0.15,0.75) & (0.20,0.45) & (0.50,0.50) & (0.40,0.40) \\ (0.20,0.60) & (0.20,0.60) & (0.40,0.40) & (0.50,0.50) \end{pmatrix}. \tag{33}$$

Step 1. According to Equation (15), the three IFPRs are decomposed into six corresponding IVFPRs as

$$F^{+}_{(1)} = \begin{pmatrix} [0.50,0.50] & [0.35,0.45] & [0.40,0.65] & [0.55,0.65] \\ [0.55,0.65] & [0.50,0.50] & [0.70,0.90] & [0.60,0.80] \\ [0.35,0.60] & [0.10,0.30] & [0.50,0.50] & [0.55,0.70] \\ [0.35,0.45] & [0.20,0.40] & [0.30,0.45] & [0.50,0.50] \end{pmatrix},$$

$$F^{-}_{(1)} = \begin{pmatrix} [0.50,0.50] & [0.55,0.65] & [0.35,0.60] & [0.35,0.45] \\ [0.35,0.45] & [0.50,0.50] & [0.10,0.30] & [0.20,0.40] \\ [0.40,0.65] & [0.70,0.90] & [0.50,0.50] & [0.30,0.45] \\ [0.55,0.65] & [0.60,0.80] & [0.55,0.70] & [0.50,0.50] \end{pmatrix},$$

$$F^{+}_{(2)} = \begin{pmatrix} [0.50,0.50] & [0.55,0.75] & [0.65,0.80] & [0.35,0.45] \\ [0.25,0.45] & [0.50,0.50] & [0.40,0.75] & [0.55,0.70] \\ [0.20,0.35] & [0.25,0.60] & [0.50,0.50] & [0.60,0.80] \\ [0.55,0.65] & [0.30,0.45] & [0.20,0.40] & [0.50,0.50] \end{pmatrix},$$

$$F^{-}_{(2)} = \begin{pmatrix} [0.50,0.50] & [0.25,0.45] & [0.20,0.35] & [0.55,0.65] \\ [0.55,0.75] & [0.50,0.50] & [0.25,0.60] & [0.30,0.45] \\ [0.65,0.80] & [0.40,0.75] & [0.50,0.50] & [0.20,0.40] \\ [0.35,0.45] & [0.55,0.70] & [0.60,0.80] & [0.50,0.50] \end{pmatrix},$$

$$F^{+}_{(3)} = \begin{pmatrix} [0.50,0.50] & [0.60,0.70] & [0.75,0.85] & [0.60,0.80] \\ [0.30,0.40] & [0.50,0.50] & [0.45,0.80] & [0.60,0.80] \\ [0.15,0.25] & [0.20,0.55] & [0.50,0.50] & [0.40,0.60] \\ [0.20,0.40] & [0.20,0.40] & [0.40,0.60] & [0.50,0.50] \end{pmatrix},$$

$$F^{-}_{(3)} = \begin{pmatrix} [0.50,0.50] & [0.30,0.40] & [0.15,0.25] & [0.20,0.40] \\ [0.60,0.70] & [0.50,0.50] & [0.20,0.55] & [0.20,0.40] \\ [0.75,0.85] & [0.45,0.80] & [0.50,0.50] & [0.40,0.60] \\ [0.60,0.80] & [0.60,0.80] & [0.40,0.60] & [0.50,0.50] \end{pmatrix}.$$

In Steps 2 and 3, we can obtain the decision matrix Δ *using Equation (24) as*

$$\Delta = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} DM_1 & DM_2 & DM_3 \end{array} \\ \begin{pmatrix} 0.10 & 1.10 & 2.60 \\ 2.40 & 0.20 & 0.70 \\ -0.80 & -0.40 & -1.70 \\ -1.70 & -0.90 & -1.60 \end{pmatrix} \end{array}.$$

In Steps 4 and 5, by solving the mean deviation maximization model, i.e., Equation (26), the normalized weights of decision makers are calculated combined with Equation (30) as

$$\mathbf{w} = (w_1, w_2, w_3)^T = (0.3521, 0.1831, 0.4648)^T.$$

Step 6. We can compute the net flow score vector Φ *using Equation (23) as*

$$\Phi = (\phi(x_1), \phi(x_2), \phi(x_3), \phi(x_4))^T = (1.4451, 1.2070, -1.1451, -1.5070)^T. \tag{34}$$

Then, the ranking of four candidates is determined as $x_1 \succ x_2 \succ x_3 \succ x_4$.

Example 2. *Suppose that four students choose the most appropriate internet service from four potential internet companies. The four students give their preference relations for the final choice, and the IMPRs are obtained as follows:*

$$M_{(1)} = \begin{pmatrix} (1,1) & \left(\frac{1}{5},3\right) & \left(\frac{1}{3},1\right) & \left(\frac{1}{2},1\right) \\ \left(3,\frac{1}{5}\right) & (1,1) & \left(\frac{1}{4},2\right) & \left(\frac{1}{3},2\right) \\ \left(1,\frac{1}{3}\right) & \left(2,\frac{1}{4}\right) & (1,1) & \left(\frac{1}{3},1\right) \\ \left(1,\frac{1}{2}\right) & \left(2,\frac{1}{3}\right) & \left(1,\frac{1}{3}\right) & (1,1) \end{pmatrix}, M_{(2)} = \begin{pmatrix} (1,1) & \left(\frac{1}{3},2\right) & \left(\frac{1}{4},2\right) & \left(\frac{1}{3},1\right) \\ \left(2,\frac{1}{3}\right) & (1,1) & \left(\frac{1}{5},3\right) & \left(\frac{1}{4},2\right) \\ \left(2,\frac{1}{4}\right) & \left(3,\frac{1}{5}\right) & (1,1) & \left(\frac{1}{2},1\right) \\ \left(1,\frac{1}{3}\right) & \left(2,\frac{1}{4}\right) & \left(1,\frac{1}{2}\right) & (1,1) \end{pmatrix},$$

$$M_{(3)} = \begin{pmatrix} (1,1) & \left(2,\frac{1}{3}\right) & \left(\frac{1}{3},2\right) & \left(1,\frac{1}{3}\right) \\ \left(\frac{1}{3},2\right) & (1,1) & \left(\frac{1}{4},3\right) & \left(\frac{1}{5},3\right) \\ \left(2,\frac{1}{3}\right) & \left(3,\frac{1}{4}\right) & (1,1) & \left(1,\frac{1}{2}\right) \\ \left(\frac{1}{3},1\right) & \left(3,\frac{1}{5}\right) & \left(\frac{1}{2},1\right) & (1,1) \end{pmatrix}, M_{(4)} = \begin{pmatrix} (1,1) & \left(\frac{1}{3},1\right) & \left(\frac{1}{2},1\right) & \left(\frac{1}{2},\frac{1}{2}\right) \\ \left(1,\frac{1}{3}\right) & (1,1) & \left(\frac{1}{5},4\right) & \left(\frac{1}{4},3\right) \\ \left(1,\frac{1}{2}\right) & \left(4,\frac{1}{5}\right) & (1,1) & \left(\frac{1}{2},1\right) \\ \left(\frac{1}{2},\frac{1}{2}\right) & \left(3,\frac{1}{4}\right) & \left(1,\frac{1}{2}\right) & (1,1) \end{pmatrix}.$$

Step 1. According to Equation (16), we can transform the four IMPRs into eight corresponding interval multiplicative preference relations as

$$M^{+}_{(1)} = \begin{pmatrix} [1,1] & \left[\frac{1}{5},\frac{1}{3}\right] & \left[\frac{1}{3},1\right] & \left[\frac{1}{2},1\right] \\ [3,5] & [1,1] & \left[\frac{1}{4},\frac{1}{2}\right] & \left[\frac{1}{3},\frac{1}{2}\right] \\ [1,3] & [2,4] & [1,1] & \left[\frac{1}{3},1\right] \\ [1,2] & [2,3] & [1,3] & [1,1] \end{pmatrix}, M^{-}_{(1)} = \begin{pmatrix} [1,1] & [3,5] & [1,3] & [1,2] \\ \left[\frac{1}{5},\frac{1}{3}\right] & [1,1] & [2,4] & [2,3] \\ \left[\frac{1}{3},1\right] & \left[\frac{1}{4},\frac{1}{2}\right] & [1,1] & [1,3] \\ \left[\frac{1}{2},1\right] & \left[\frac{1}{3},\frac{1}{2}\right] & \left[\frac{1}{3},1\right] & [1,1] \end{pmatrix},$$

$$M^{+}_{(2)} = \begin{pmatrix} [1,1] & \left[\frac{1}{3},\frac{1}{2}\right] & \left[\frac{1}{4},\frac{1}{2}\right] & \left[\frac{1}{3},1\right] \\ [2,3] & [1,1] & \left[\frac{1}{5},\frac{1}{3}\right] & \left[\frac{1}{4},\frac{1}{2}\right] \\ [2,4] & [3,5] & [1,1] & \left[\frac{1}{2},1\right] \\ [1,3] & [2,4] & [1,2] & [1,1] \end{pmatrix}, M^{-}_{(2)} = \begin{pmatrix} [1,1] & [2,3] & [2,4] & [1,3] \\ \left[\frac{1}{3},\frac{1}{2}\right] & [1,1] & [3,5] & [2,4] \\ \left[\frac{1}{4},\frac{1}{2}\right] & \left[\frac{1}{5},\frac{1}{3}\right] & [1,1] & [1,2] \\ \left[\frac{1}{3},1\right] & \left[\frac{1}{4},\frac{1}{2}\right] & \left[\frac{1}{2},1\right] & [1,1] \end{pmatrix},$$

$$M^{+}_{(3)} = \begin{pmatrix} [1,1] & [2,3] & \left[\frac{1}{3},\frac{1}{2}\right] & [1,3] \\ \left[\frac{1}{3},\frac{1}{2}\right] & [1,1] & \left[\frac{1}{4},\frac{1}{3}\right] & \left[\frac{1}{5},\frac{1}{3}\right] \\ [2,3] & [3,4] & [1,1] & [1,2] \\ \left[\frac{1}{3},1\right] & [3,5] & \left[\frac{1}{2},1\right] & [1,1] \end{pmatrix}, M^{-}_{(3)} = \begin{pmatrix} [1,1] & \left[\frac{1}{3},\frac{1}{2}\right] & [2,3] & \left[\frac{1}{3},1\right] \\ [2,3] & [1,1] & [3,4] & [3,5] \\ \left[\frac{1}{3},\frac{1}{2}\right] & \left[\frac{1}{4},\frac{1}{3}\right] & [1,1] & \left[\frac{1}{2},1\right] \\ [1,3] & \left[\frac{1}{5},\frac{1}{3}\right] & [1,2] & [1,1] \end{pmatrix},$$

$$M^{+}_{(4)} = \begin{pmatrix} [1,1] & \left[\frac{1}{3},1\right] & \left[\frac{1}{2},1\right] & \left[\frac{1}{2},2\right] \\ [1,3] & [1,1] & \left[\frac{1}{5},\frac{1}{4}\right] & \left[\frac{1}{4},\frac{1}{3}\right] \\ [1,2] & [3,5] & [1,1] & \left[\frac{1}{2},1\right] \\ \left[\frac{1}{2},2\right] & [3,4] & [1,2] & [1,1] \end{pmatrix}, M^{-}_{(4)} = \begin{pmatrix} [1,1] & [1,3] & [1,2] & \left[\frac{1}{2},2\right] \\ \left[\frac{1}{3},1\right] & [1,1] & [4,5] & [3,4] \\ \left[\frac{1}{2},1\right] & \left[\frac{1}{5},\frac{1}{3}\right] & [1,1] & [1,2] \\ \left[\frac{1}{2},2\right] & \left[\frac{1}{4},\frac{1}{3}\right] & \left[\frac{1}{2},1\right] & [1,1] \end{pmatrix}.$$

In Steps 2 and 3, we can obtain the decision matrix Δ using Equation (24) as

$$\Delta = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} DM_1 & DM_2 & DM_3 & DM_4 \end{array} \\ \begin{pmatrix} -2.4693 & -2.7484 & 0.5668 & -0.9756 \\ -0.5955 & -1.5643 & -3.9981 & -2.1945 \\ 1.1208 & 2.1981 & 2.6877 & 1.9614 \\ 1.9226 & 2.1099 & 0.4783 & 1.5392 \end{pmatrix} \end{array}.$$

In Steps 4 and 5, by solving the mean deviation maximization model, i.e., Equation (26), the normalized weights of students are calculated combined with Equation (30) as

$$\mathbf{w} = (w_1, w_2, w_3, w_4)^T = (0.2093, 0.2954, 0.2694, 0.2259)^T.$$

Step 6. We can compute the net flow score vector Φ using Equation (23) as

$$\Phi = (\phi(x_1), \phi(x_2), \phi(x_3), \phi(x_4))^T = (-1.3963, -2.1945, 1.9614, 1.5392)^T.$$

Finally, we can determine the ranking of internet companies as $x_3 \succ x_4 \succ x_1 \succ x_2$.

Example 3. *Suppose a high-speed railway station is to be built in a city, and four railway transportation experts make a pairwise comparison between four potential location plans. Decision makers DM_1 and DM_2 reveal their IFPRs as follows:*

$$F_{(1)} = \begin{pmatrix} (1,1) & \left(\frac{1}{3},2\right) & \left(3,\frac{1}{5}\right) & \left(2,\frac{1}{3}\right) \\ \left(2,\frac{1}{3}\right) & (1,1) & \left(3,\frac{1}{4}\right) & \left(4,\frac{1}{5}\right) \\ \left(\frac{1}{5},3\right) & \left(\frac{1}{4},3\right) & (1,1) & \left(\frac{1}{5},4\right) \\ \left(\frac{1}{3},2\right) & \left(\frac{1}{5},4\right) & \left(4,\frac{1}{5}\right) & (1,1) \end{pmatrix}, F_{(2)} = \begin{pmatrix} (1,1) & \left(\frac{1}{4},3\right) & \left(4,\frac{1}{5}\right) & \left(3,\frac{1}{3}\right) \\ \left(3,\frac{1}{4}\right) & (1,1) & \left(5,\frac{1}{7}\right) & \left(3,\frac{1}{4}\right) \\ \left(\frac{1}{5},4\right) & \left(\frac{1}{7},5\right) & (1,1) & \left(\frac{1}{2},2\right) \\ \left(\frac{1}{3},3\right) & \left(\frac{1}{4},3\right) & \left(2,\frac{1}{2}\right) & (1,1) \end{pmatrix}.$$

In addition, decision makers DM_3 and DM_4 use IMPRs to evaluate the plans as

$$R_3^F = \begin{pmatrix} (0.50, 0.50) & (0.15, 0.55) & (0.75, 0.10) & (0.65, 0.15) \\ (0.55, 0.15) & (0.50, 0.50) & (0.70, 0.20) & (0.85, 0.15) \\ (0.10, 0.75) & (0.20, 0.70) & (0.50, 0.50) & (0.10, 0.60) \\ (0.15, 0.65) & (0.15, 0.85) & (0.60, 0.10) & (0.50, 0.50) \end{pmatrix},$$

$$R_4^F = \begin{pmatrix} (0.50, 0.50) & (0.20, 0.60) & (0.85, 0.10) & (0.55, 0.35) \\ (0.60, 0.20) & (0.50, 0.50) & (0.85, 0.15) & (0.75, 0.15) \\ (0.10, 0.85) & (0.15, 0.85) & (0.50, 0.50) & (0.15, 0.65) \\ (0.35, 0.55) & (0.15, 0.75) & (0.65, 0.15) & (0.50, 0.50) \end{pmatrix}.$$

In Steps 1–3, the decision matrix Δ can be obtained using Equation (24) as

$$\Delta = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{cccc} DM_1 & DM_2 & DM_3 & DM_4 \end{array} \\ \begin{pmatrix} 1.3989 & 1.3828 & 1.50 & 1.10 \\ 4.1854 & 5.1497 & 3.20 & 3.40 \\ -4.8873 & -4.6640 & -3.30 & -3.90 \\ -0.9085 & -1.6960 & -1.40 & -0.60 \end{pmatrix} \end{array}.$$

In Steps 4 and 5, by solving the mean deviation maximization model, i.e., Equation (26), the normalized weights of decision makers are calculated combined with Equation (30) as

$$\mathbf{w} = (w_1, w_2, w_3, w_4)^T = (0.2667, 0.3021, 0.2203, 0.2109)^T.$$

Step 6. Combined with the decision maker weights and matrix Δ, the net flow score vector Φ can be computed using Equation (23) as

$$\Phi = (\phi(x_1), \phi(x_2), \phi(x_3), \phi(x_4))^T = (1.3533, 4.0940, -4.2620, -1.1896)^T. \quad (35)$$

At last, the ranking of four location plans is determined as $x_2 \succ x_1 \succ x_4 \succ x_3$; the plan x_2 is the best choice for building a high-speed railway station.

5.2. Comparison and Discussion

In order to further illustrate the validity of the new GDM approach, the ranking orders above are compared with the results of the linear goal programming model [28], the intuitionistic fuzzy weighted average (IFWA) operator [41], and the intuitionistic fuzzy weighted geometric (IFWG) operator [41] for solving Example 1 as shown in Table 1. Similarly, the comparison result of Example 2 between the proposed method and the error-analysis-based method [30], the IMWA operator [19], and the IMWG operator [19] is shown in Table 2.

Table 1. Comparison result of Example 1.

Decision-Making Method	**The Priority Values of Alternatives**				**Ranking**
	x_1	x_2	x_3	x_4	
The proposed method	1.4451	1.2070	−1.1451	−1.5070	$x_1 \succ x_2 \succ x_3 \succ x_4$
The linear goal programming model	0.04	−0.31	−0.76	−0.96	$x_1 \succ x_2 \succ x_3 \succ x_4$
The IFWA operator	0.2153	0.1497	−0.0571	−0.1361	$x_1 \succ x_2 \succ x_3 \succ x_4$
The IFWG operator	0.1386	0.0607	−0.1773	−0.1924	$x_1 \succ x_2 \succ x_3 \succ x_4$

Table 2. Comparison result of Example 2.

Decision-Making Method	The Priority Values of Alternatives				Ranking
	x_1	x_2	x_3	x_4	
The proposed method	−1.3963	−2.1945	1.9614	1.5392	$x_3 \succ x_4 \succ x_1 \succ x_2$
The error-analysis-based method	0.1784	0.1549	0.3405	0.3262	$x_3 \succ x_4 \succ x_1 \succ x_2$
The IMWA operator	0.5492	0.5667	2.6412	2.3154	$x_3 \succ x_4 \succ x_2 \succ x_1$
The IMWG operator	0.4329	0.3384	1.9648	1.8975	$x_3 \succ x_4 \succ x_1 \succ x_2$

Tables 1 and 2 show that the ranking in Examples 1 and 2 are consistent with the linear goal programming model [28] and the error-analysis-based method [30], respectively, which can indicate the effectiveness of the new GDM approach. Table 2 shows that the result of the IMWA operator is $x_3 \succ x_4 \succ x_2 \succ x_1$, while the result of other methods is $x_3 \succ x_4 \succ x_1 \succ x_2$; the loss of original information from the operator causes the inconsistent ranking result. In addition, both Tables 1 and 2 show that all the priority values of different alternatives that were obtained using aggregation operators [19,30] are very close, which would make the difference between alternatives difficult to distinguish clearly. The weights of decision makers are assigned subjectively in the existing methods, which may contribute to the inaccurate ranking. Furthermore, Example 3 shows that the new GDM approach can also solve GDM problems with both IFPRs and IMPRs effectively, which cannot be solved by other approaches. Therefore, the new GDM approach has the following advantages: (1) Instead of assigning the decision maker weights subjectively, the proposed method can determine them objectively based on information theory; (2) The operation process of the proposed method is more simple and feasible, without constructing any programming models to obtain the priorities; (3) The GDM problems with both IFPRs and IMPRs can be solved, which has not been studied in previous research works. However, the consistency of IFPRs and IMPRs is not considered in this paper, and this will be researched in the future to improve the accuracy of the ranking result.

6. Conclusions

A novel method is proposed to rank the alternatives in GDM problems with two preference relations: IFPRs and IMPRs. By integrating the hesitancy degree into the membership and non-membership degrees, IFPRs and IMPRs can be divided into IVFPRs and IVMPRs, respectively. According to the net flow score function, the net flow scores that can be used to rank individual IFPRs and IMPRs are obtained. Combined with a mean deviation maximization model, the decision maker weights are computed objectively. Finally, the collective net flow scores are aggregated to choose the best alternative. Numerical examples show the applications and advantages of the new GDM approach. In real-life situations, when the alternatives are evaluated concerning the related criteria using both IFPRs and IMPRs, we can apply the proposed method to solve these situations effectively.

As the consistency of IFPRs and IMPRs is ignored in the proposed method, in future research, we will enhance the proposed method to enable solving GDM problems with inconsistent IFPRs and IMPRs. Moreover, the proposed method will be improved to cope with the situation that other kinds of preference relations are utilized to evaluate alternatives.

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