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Article

Epistemic Information in Stratified M-Spaces

Mark Burgin

Department of Mathematics, University of California, Los Angeles, 405 Hilgard Ave. Los Angeles, CA 90095, USA; E-Mail: markburg@cs.ucla.edu

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Abstract: Information is usually related to knowledge. However, the recent development of information theory demonstrated that information is a much broader concept, being actually present in and virtually related to everything. As a result, many unknown types and kinds of information have been discovered. Nevertheless, information that acts on knowledge, bringing new and updating existing knowledge, is of primary importance to people. It is called epistemic information, which is studied in this paper based on the general theory of information and further developing its mathematical stratum. As a synthetic approach, which reveals the essence of information, organizing and encompassing all main directions in information theory, the general theory of information provides efficient means for such a study. Different types of information dynamics representation use tools of mathematical disciplines such as the theory of categories, functional analysis, mathematical logic and algebra. Here we employ algebraic structures for exploration of information and knowledge dynamics. In Introduction (Section 1), we discuss previous studies of epistemic information. Section 2 gives a compressed description of the parametric phenomenological definition of information in the general theory of information. In Section 3, anthropic information, which is received, exchanged, processed and used by people is singled out and studied based on the Componential Triune Brain model. One of the basic forms of anthropic information called epistemic information, which is related to knowledge, is analyzed in Section 4. Mathematical models of epistemic information are studied in Section 5. In Conclusion, some open problems related to epistemic information are given.

Keywords: information; knowledge; information operator; stratification; epistemic information; composition; cognition; algebra; equivalence

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1. Introduction

Explication and clarification of the concept of information requires not only elaboration and exploration of a general unified definition of information but also needs separation and examination of basic special types of information. The general theory of information gives the most general definition of information, organizing and encompassing all main directions in information theory [1]. According to the main principle of general theory of information, which impeccably correlates with people's practice and observations, when information comes to a system and is accepted, it causes changes in the system. This feature of information makes it reasonable to model information by operators and information dynamics by actions of these operators. This approach is adopted in the general theory of information.

In addition, the general theory of information provides efficient means for information classification and study of the basic information types. One of such types is information that is received, exchanged, processed and used by people. It is called anthropic information. We explain the methodological principles of the anthropic information explication and classification, which bring us to cognitive information and its important subclass called epistemic information.

Portions of epistemic information are modeled/represented by epistemic information operators acting in spaces of knowledge, which are represented by a formal construction called a Mizzaro space. These spaces consist of knowledge items often unified by structural relations.

In a general setting, epistemic information has been studied by different authors. Bar-Hillel and Carnap [2], Hintikka [3-5] and Israel and Perry [6] explored information in knowledge represented by means of mathematical logic. Shreider [7], Mackay [8], Brookes [9], Mizzaro [10,11] and Gackowski [12] base their theories on the following assumption:

Information is a change in a knowledge system

Later this principle has been made more exact [13] and formulated as

Epistemic information is a change in a knowledge system

The general theory of information [1] makes the next step to the better understanding of epistemic information. Namely, it is explained that

Epistemic information is a capacity to cause changes
in a knowledge system and it is possible to measure
this capacity by changes in the knowledge system
impacted by information

Such changes in the knowledge system of a system R reflect accepted information. Note that information can be accepted not only as true information but also as false information. In this case, changes in the knowledge system can result in exclusion of some knowledge or in labeling this knowledge as false, e.g., treating it as a misconception or blunder.

The approach of Mizzaro [10,11,13] and Burgin [1] to epistemic information does not consider knowledge in general, but makes use of the term a *knowledge state* (KS) of a dynamic knowledge system or a cognitive agent. It is postulated that knowledge of a knowledge system/agent consists of atomic components called *knowledge items* (KI) and the number of these items in a portion of knowledge gives an adequate measure of this knowledge. Such knowledge states are set-theoretical or unstructured Mizzaro spaces defined in [1].

This is the first approximation to modeling information processes related to knowledge transformations. In it, the substantial dimensions of knowledge are reflected, while the relational dimensions, which describe relations between knowledge items, are studied in higher-level models of operational information theory, which goes beyond the scope of this paper. Taking into account such relations allows one to achieve higher precision in measuring information and knowledge. However, there are many situations where precision provided by quantities of knowledge items is sufficient. For instance, exactly this level of precision is successfully used in software engineering and technology where program instructions, which belong to the procedural type of knowledge, are used as knowledge items that determine measures of information and knowledge transformations [14].

Taking knowledge states as the base, Mizzaro [13] considers and utilizes natural set-theoretical operations, such as union, difference, complement, and intersection, as well as set-theoretical relations, such as inclusion, emptiness, and membership, on the set of all knowledge states. These and more sophisticated operations and relations are advanced to the level of local information operators and studied in [1]. Namely, in the context of the general theory of information, transformations in knowledge states caused by receiving data (messages) are represented by local information operators, which are projections of global information operators also described in [1]. Consequently, the theory of Mizzaro spaces and local information operators in these spaces is a localization of the mathematical (formalized) stratum of the general theory of information.

In this paper, we continue exploration of epistemic information, extending Mizzaro spaces to M-spaces, which represent and model information dynamics by algebraic structures. In addition to uniform Mizzaro spaces studied in [1,11,13], here we consider Mizzaro spaces and M-spaces with stratification and extend the scope of epistemic information operators. In [1,11,13], only two operators, addition and deletion are studied. Here we consider five basic epistemic information operators: addition, deletion, moving, replication, and transformation, as well as their compositions. Section 2 gives a compressed description of the parametric phenomenological definition of information in the general theory of information. In Section 3, anthropic information that is received, exchanged, processed and used by people is separated and studied based on the Componential Triune Brain model introduced in [15] and further developed in [1]. This model allows overcoming the pitfalls of the simple linear hierarchy suggested by MacLean [16,17] by considering not anatomically localized but distributed in the brain basic systems and including such an important psychological construct as the Will [18,19] in the brain structure. Based on this model, one of the basic forms of anthropic information called epistemic information, which is related to knowledge, is analyzed in Section 4. Mathematical models of epistemic information are studied in Section 5. In Conclusion, we give some open problems related to epistemic information.

2. The Concept of Information

Our study of epistemic information is based on the system of ontological principles from the general theory of information. All these principles serve as an extended definition of information. A concise definition is given in the second ontological principle, which has several forms.

Ontological Principle O2 (the *General Transformation Principle*). In a broad sense, *information* for a system R is a capacity to cause changes in the system R.

Thus, we may understand information in a broad sense as a capacity (ability or potency) of things, both material and abstract, to change other things. Information exists in the form of *portions of information*. Informally, a portion of information is such information that can be separated from other information. Information is, as a rule, about something. What information is about is called an *object* of this information.

The Ontological Principle O2 has several consequences. First, it demonstrates that information is closely connected to transformation. Namely, it means that information and transformation are functionally similar because they both point to changes in a system [20]. At the same time, they are different because information is potency for (or in some sense, cause of) change, while transformation is the change itself, or in other words, transformation is an operation, while information is what induces this operation.

Second, the Ontological Principle O2 explains *why* information influences society and individuals all the time, as well as why this influence grows with the development of society. Namely, reception of information by individuals and social groups induces transformation. In this sense, information is similar to energy. Moreover, according to the Ontological Principle O2, energy is a kind of information in a broad sense. This well correlates with the Carl Friedrich von Weizsäcker's idea [21,22] that *energy might in the end turn out to be information*.

Third, the Ontological Principle O2 makes it possible to separate different kinds of information. For instance, people, as well as any computer, have many kinds of memory. It is even supposed that each part of the brain has several types of memory agencies that work in somewhat different ways, to suit particular purposes [23]. It is possible to consider each of these memory agencies as a separate system and to study differences between information that changes each type of memory. This might help to understand the interplay between stability and flexibility of mind, in general, and memory, in particular.

In essence, we can see that all kinds and types of information are encompassed by the Ontological Principle O2.

However, the common usage of the word *information* does not imply such wide generalizations as the Ontological Principle O2 implies. Thus, we need a more restricted theoretical meaning because an adequate theory, whether of the information or of anything else, must be in significant accord with our common ways of thinking and talking about what the theory is about, else there is the danger that theory is not about what it purports to be about. To achieve this goal, we use the concept of an *infological system* IF(*R*) of the system *R* for the information definition. It is done in two steps. At first, we make the concept of information relative and then we choose a specific class of infological systems to specify information in the strict sense. That is why it is impossible and, as well as, counterproductive to give an exact and thus, too rigid and restricted definition of an infological system. Indeed, information is a very rich and widespread phenomenon to be reflected by a restricted rigid definition (*cf.*, for example, [24,25]).

The concept of infological system plays the role of a free parameter in the general theory of information, providing for representation of different kinds and types of information in this theory. That is why the concept of *infological system*, in general, should not be limited by boundaries of exact definitions. A free parameter must really be free. Identifying an infological system IF(*R*) of a system *R*, we can define information relative to this system. This definition is expressed in the following principle.

Ontological Principle O2g (**the *Relativized Transformation Principle***). *Information* for a system *R relative to the infological system* IF(*R*) is a capacity to cause changes in the system IF(*R*).

As a model example of an infological system IF(*R*) of an intelligent system *R*, we take the system of knowledge of *R*. In cybernetics, it is called the *thesaurus* Th(*R*) of the system *R*. Another example of an infological system is the memory of a computer. Such a memory is a place in which data and programs are stored and is a complex system of diverse components and processes.

Elements from IF(*R*) are called *infological elements*.

There is no exact definition of infological elements although there are various entities that are naturally considered as infological elements as they allow one to build theories of information that inherit conventional meanings of the word *information*. For instance, knowledge, data, images, algorithms, procedures, scenarios, ideas, values, goals, ideals, fantasies, abstractions, beliefs, and similar objects are standard examples of infological elements.

When we take a physical system *D* as the infological system and allow only for physical changes, we see that information with respect to *D* coincides with (physical) energy.

Taking a mental system *B* as the infological system and considering only mental changes, information with respect to *B* coincides with mental energy.

These ideas are crystallized in the following principle.

Ontological Principle O2a (**the *Special Transformation Principle***). *Information in the strict sense* or *proper information* or, simply, *information* for a system *R*, is a capacity to change structural infological elements from an infological system IF(*R*) of the system *R*.

As the concept of mental energy is much less understood than the concept of physical energy, we give some explanations based on the origin, development and contemporary understanding of mental energy.

Considering mental energy on the level of individual mentality, it is possible, as the first approximation, to equate it with psychic energy. The concept of psychic energy first entered physiology and to some extent psychology through the discussions of Ernst Brücke, Herman Helmholtz, and Emil Du Bois-Reymond, who during the years 1838-1842, worked in the laboratory of the German physiologist Johannes Muller. At the same time, according to Jung [26], Nicolas von Grot was the first to explicitly define the concept of psychic. He wrote [27]:

"The concept of psychic energy is as much justified in science as that of physical energy, and psychic energy has just as many quantitative measurements and different forms as has physical energy."

Later this concept was further developed by Sigmund Freud [28,29], Brücke's student, and then by Carl Jung, Freud's student. Jung [26] regarded psychic energy as a basic life-force, manifesting itself through actions, such as eating, moving, thinking, sex, remembering, etc.

Contemporary understanding determines mental energy as the ability to perform mental tasks, the intensity of feelings of energy/fatigue, and the motivation to accomplish mental and physical tasks [30,31]. This shows that basic aspects of mental energy manifestation include: (a) cognition (knowledge that is gained through perception, reasoning or intuition), (b) changing of moods or feelings (states of mind), and (c) motivation (an incentive for action). Factors that can influence mental energy include, among others, psychological issues such as interest, passion, desire, concern, and biological issues such as genetics, nutrition, pain, and sleep [31].

Often psychic/mental energy is confused with psychic/mental force. For instance, some psychoanalysts do not distinguish energy and force, particularly, when they follow Freud's occasional practice of calling libido itself a force. Actually, as in physics, energy and force are different phenomena. Energy is a potential to make changes, while force is what is making changes. This allows one to take the Will as a representative of force in the individual mentality and above. This is expressed in the remark attributed to Loewenstein that "force is energy in action" (*cf.* [32]).

3. The phenomenological stratum of the general theory of information

In this paper, we are primarily interested in information received, exchanged, processed and used by people. We call it *anthropic information*. A relevant infological system IF(R) for anthropic information is the human brain. Thus, to further classify and study anthropic information, we need to utilize our knowledge about the structure and functions of the brain.

In our case, the most relevant is the Triune Brain model introduced and studied by Paul MacLean [33,34]. The main conception of his approach is existence of three levels of perception and action that are controlled by three corresponding centers of perception in the human brain. These three centers together form the Triune Brain and have the structure of a triad. It is natural to call the initial MacLean's structure by the name Anatomic Triune Brain model because it is based on the anatomy of the brain where three indispensable parts are distinguished: the neocortex, limbic system and R-complex.

According to the theory of MacLean, the neural basis, or framework, of the brain consists of three parts: the spinal cord, hindbrain, and midbrain. In addition to it, centuries of evolution have endowed people with three distinct cerebral systems. The oldest of these is called the *reptilian brain* or R-*complex*. It programs behavior that is primarily related to instinctual actions based on ancestral learning and memories, satisfying basic needs such as self-defense, reproduction and digestion. The reptilian brain is fundamental in acts such as primary motor functions, primitive sensations, dominance, establishing territory, hunting, breeding, and mating.

Through evolution, people have developed a second cerebral system, the *limbic system*, which MacLean refers to as the *paleomammalian brain* and which contains hippocampus, amygdala, hypothalamus, pituitary gland, and thalamus. This system is situated around the R-complex, is shared by humans with other mammals, and plays an important role in human emotional behavior.

The most recent addition to the cerebral hierarchy is called the *neomammalian brain*, or the *neocortex*. It constitutes 85% of the whole human brain mass and receives its information from the external environment through the eyes, ears, and other organs of senses. This brain component (neocortex) contains cerebrum, corpus callosum, and cerebral cortex. The cerebrum and cerebral cortex are divided into two hemispheres, while the corpus callosum connects these hemispheres. The neocortex deals with information in a logical and algorithmic way. It governs people creative and intellectual functions like social interaction and advance planning. The left hemisphere works with symbolic information, applying step-by-step reasoning, while the right hemisphere handles images processed by massively parallel (gestalt) algorithms.

Even psychologists who have objections to the Anatomic Triune Brain model admit that it is a useful, although oversimplified, metaphor, as the structure presented as the triune brain is based on a sound idea of three functional subsystems of the brain [16,35]. In the development of neurophysiology and neuropsychology, MacLean's theory was used as a base for the Whole Brain model, developed by Herrmann [36]. The main idea of this development is a synthesis of the Anatomic Triune Brain model with the two-hemisphere approach to the brain functioning.

The theory of the triune brain (reptilian, old mammalian and new mammalian) is used as a metaphor and a model of the interplay between instinct, emotion, and rationality in humans. Cory [37] applied this model to economic and political structures. In Cory's schema, the reptilian brain mediates the claims of self-interest, whereas the old mammalian brain mediates the claims of empathy. If selfish interests of an individual are denied for too long, there is discontent due to a feeling of being unjustly treated. If empathic interests are denied for too long, there is discontent due to guilt. In either case, the center of intelligence at the prefrontal cortex plays the role of a mediator. Its executive function is required to restore balance, generating the reciprocity required for effective social and economic structures.

The Triune Brain model is used to explain hyperactivity of youngsters studied by Zametkin [38] and other researchers. Peter Levine bases his approach to trauma treatment on the Triune Brain model [39]. According to Levine, there are tree types of the uniform stress and relaxation responses to a threatening situation that are active in all animal species through the autonomic nervous system. In the everyday language, these responses are metaphorically called *fight*, *flight* or *freeze*. The first two of them are well-known, while the third one was introduced by Levine. The freezing or immobility response has evolved over millions of years and it has served an adaptive purpose well for all species—except humans. In an individual, it can lead to trauma. Many physical ailments are actually residues of thwarted trauma reactions incurred during stressful events. What usually happens to non-human species is that after the threatening situation resolves itself, the animal forgets the stress and goes on its way without being traumatized.

In contrast to this, people can get stuck in the freezing response, while the reasoning mind resists or blocks the natural bodily sensations and fine motor movements needed to come out of the freeze response. The contemporary rationalistic culture is not helpful in supporting people in such a traumatizing situation. The feelings that people go through after experiencing a traumatic event are outside of their voluntary control, often being frightening and even potentially re-traumatizing.

Levine [39] postulates that trauma exists not in the event or in the story of the event, but is stored within the nervous system. The main principle of the Levine's treatment approach is that the body has a natural, innate, and miraculous capacity to heal once these reactions are understood and guided.

Although the Triune Brain has become a well-known model in contemporary psychology, it caused several objections on the ground of the development and structure of the triune brain system [16,40]. First, there is evidence that the, so-called, paleomammalian and neomammalian brains appeared, although in an undeveloped form, on much earlier stages of evolution than it is assumed by MacLean. Second, there are experimental data that in the neocortex, regions that are homological to the, so-called, paleomammalian and reptilian brains exist. For instance, neuropsyhological data give evidence that amygdala, which is a part of a limbic system, performs the low-level emotion processing, while the ventromedial cortex performs the high-level emotion processing. This shows that emotions exist, at least, on three levels: on the subconscious level of limbic system, on the conscious intuitive level, and on the conscious rational level in the cortex. The first level utilizes direct affective information, while the second and, to some extent, the third levels make use of cognitive emotional information [1].

At the same time, the development of the system of Will demands inclusion of some regions that are not included into the R-complex (the reptilian brain in MacLean's theory) into this system. It means that the centers of rational intelligence, emotion and will are not concentrated in three separate regions of the brain but are highly distributed among several components of the brain. Thus, it is better to call them not centers but systems of intelligence, emotion and will. This extension of functional characteristics results in the Componential Triune Brain model described in [1], making this model more adequate to experimental data and overcoming the pitfalls of the simple linear hierarchy suggested by MacLean [16,40].

Thus, the Componential Triune Brain model consists of three basic systems of the brain:

- the *System of Rational Intelligence* (also called *System of Reasoning*) (SRI);
- the *System of Emotions* (or more generally, of *Affective States*) (SAS);
- the *System of Will and Instinct* (SWI).

All three systems of the brain are schemes in the sense of the schema theory, which is developed as a specific direction of the brain theory [41-44]. According to this theory, brain schemes are anatomically distributed in the brain and interact in a way of concurrent competition and coordination. All these interactions are based on physical processes but have an inherent informational essence related to a specific type of information. Information processes in the brain are more exactly reflected by the theory of the *triadic mental information* than by the conventional information theory that deals only with cognitive information.

In standard structuring of the brain, we also find these three systems. In the conventional setting (*cf.*, for example, [40,45-47] , the brain includes three components: the forebrain, midbrain, and hindbrain.

The *forebrain* is the largest division of the brain involved in a wide range of activities that make people human. The forebrain has a developed inner structure. It includes the *cerebrum*, which consists of two *cerebral hemispheres*. The cerebrum is the nucleus of the System (Center) of Rational Intelligence.

Under the cerebrum, is the *diencephalon*, which contains the *thalamus* and *hypothalamus*. The thalamus is the main relay center between the *medulla* and the *cerebrum*. The hypothalamus is an important control center for sex drive, pleasure, pain, hunger, thirst, blood pressure, body temperature, and other visceral functions. The forebrain also contains the limbic system, which is directly linked to the experience of emotion. The *limbic system* is the nucleus of the System (Center) of Emotions (or more generally, of Affective States).

The *midbrain* is the smallest division and it makes connections with the other two divisions—forebrain and hindbrain—and alerts the forebrain to incoming sensations.

The *hindbrain* is involved in sleeping, waking, body movements and the control of vital reflexes such as heart rate, blood pressure. The structures of the hindbrain include the pons, medulla and cerebellum. The hindbrain is the nucleus of the System (Center) of Will and Instinct.

The System of Rational Intelligence realizes rational thinking. It includes both symbol and image processing, which go on in different hemispheres of the brain [48]. The System of Emotions governs sensibility and the emotional sphere of personality. The System of Will and Instincts directs behavior and thinking. Two other systems influence behavior only through the will. For instance, a person can know that it is necessary to help others, especially, those who are in need and deserve helping. However, in many cases, this person does nothing without a will to help. In a similar way, we know situations when an individual loves somebody but neither tells this nor explicitly shows this due to an absence of a sufficient will.

It is necessary to remark that discussing the will of an individual we distinguish *conscious will*, *unconscious will*, and *instinct*. All of them are controlled by the Center of Will and Instincts (SWI). In addition, it is necessary to make distinctions between thoughts about intentions to do something and the actual will to do this. Thoughts are generated in the Center of Rational Intelligence, while the will dwells in the Center of Will and Instincts. In other words, thoughts and words about wills, wishes, and intentions may be deceptive if they are not based on a Will.

Will is a direct internal injunction, as well as any kind of motivation [19]. That is, the forces that act on or within an organism to initiate and direct behavior, has to be transformed into a will in order to cause the corresponding action. Will is one of the basic components of different models of personality. For instance, in psychosynthesis, which is a holistic transpersonal psychology and philosophy of life developed by Roberto Assagioli, a contemporary of both Freud and Jung, the Will is the basic component of the self [18].

Assagioli explicated essential properties of the Will—it can be assertive, aggressive, and controlling. In addition, there are three categories of Will: the *accepting Will*, *yielding Will*, and *dedicated Will*.

Thus, we can see that the Componential Triune Brain model corresponds to the three basic functions of the brain:

- Reasoning as symbolic information processing and information processing of images.
- Emotions and feelings as affective states of the brain.
- Will and instinct as forces of the psyche.

The Anatomic Triune Brain model of MacLean also corresponds to the basic functions of the brain but misses one of them, namely, the Will.

Many other psychological theories and psychiatric tecniques consider the Will as a primary factor of human behavior and dispositions [19,49].

Often the Will is considered as a process that deliberates on what is to be done [50].

The Componential Triune Brain model is not only a necessary extension of the Triune Brain model but it also continues a long standing approach to the brain stratification. As Smith [35] demonstrates, triadic models of the brain and psyche have featured through two and half millennia of Western thought, starting with works of Pythagoras, Plato and Aristotle and receiving a modern airing in Paul MacLean's the Triune Brain model. A generation later after Pythagoras, Plato and Aristotle, Herophilus and Erasistratus from Alexandrian put together a more anatomically informed triadic theory, which was modified by Galen in the 2nd century and remained the prevailing paradigm for nearly fifteen hundred years until it was overturned by the great thinkers of the Renaissance. Nonetheless, the notion that the human neuropsychological system is somehow best thought of as having a triadic (tripartite) structure has remained remarkably resilient and has reappeared time and again in modern and early modern times. For instance, the Triune Brain model well correlates with the Freud's model of personality, which has the structure of the triad (*Id*, *Ego*, *Super-ego*). In the correspondence between the Triune Brain model and the Freud's model, the reptilian complex corresponds to Id, the limbic system corresponds to Ego, and the neocortical complex corresponds to Super-ego. In the context of triadic models, it is also possible to consider the *triarchic theory of intelligence* developed by Robert Sternberg [51].

Taking each of these the centers, SRI, SAS, and SWI, as a specific infological system, we find three types of information. One is the conventional information that acts on the center of reasoning and of other higher brain functions (SRI), which is situated in the neocortex. This information gives knowledge, changes beliefs and generates ideas. Thus, it is natural to call it *cognitive* information. Information of the second type acts on the system of emotions (SAS), which includes the paleomammalian brain. It is natural to call this information by the name *direct emotional information*, or *direct affective information* or *emotive information*. Information of the third type acts on the System of Will and Instinct (SWI), which contains the reptilian brain. It is natural to call this information by the name *direct regulative* or *direct effective information*.

Thus, anthropic information has three dimensions:

Cognitive information changes the content of the SRI, which includes the knowledge system (thesaurus) and neocortex (neomammalian brain) as its carrier.

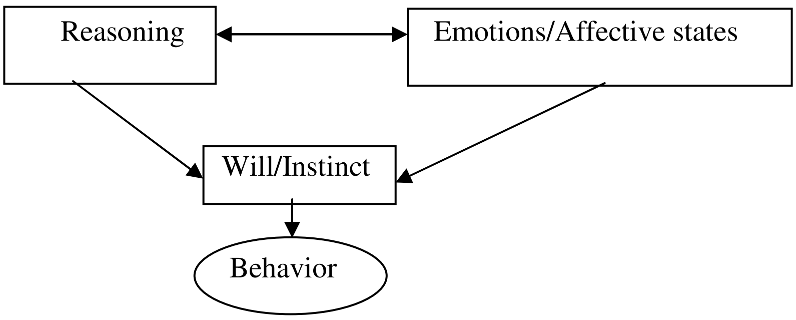
Direct emotional/affective information changes the content of the SAS, which includes the paleomammalian brain (limbic system).

Direct regulative/effective information changes the content of the SWI, which includes the reptilian brain or R-complex.

However, in general, emotions constitute only one part of affective states, which also include moods, feelings, etc. That is why in general, direct affective information is more general than direct emotional information. However, as there is no consensus on the differences between emotions and affective states, these two types of information are used without differentiation.

Interactions between the basic brain systems imply dependencies between thinking, emotions, and actions of people. Emphasizing some of these relations, psychologists build their theories and psychotherapists develop their therapeutic approaches. Giving priority to the System of Rational Intelligence (SRI), the so-called "cognitive revolution" has taken hold around the world. It influenced both psychology, resulting in the emergence of cognitive psychology [52], and psychotherapy, inspiring creation of cognitive therapy [53]. In psychology, the word *cognitive* often means thinking in many contexts of contemporary life [54]. The cognitive therapeutic approach begins by using the extremely powerful reasoning abilities of the human brain. This is important because our emotions and our actions are not separate from our thoughts. They are all interrelated. Thinking (SRI) is the gateway to our emotions (SAS)—and our emotions are the gateway to our actions through motivation and will (SWI). This is only another way of saying that information from the System of Rational Intelligence (SRI) goes to the System of Emotions (SAS)—and from it to the System of Will and Instincts (SWI) that controls our actions. Consequently, the cognitive psychotherapeutic approach, which has been successfully utilized for treating many mental disorders, gives additional supportive evidence for the theory of the triune brain and behavior, as well as for the theory of the triadic mental information. The latter explains that while going from the System of Rational Intelligence to the System of Emotions and to the System of Will and Instincts, information is transformed from cognitive information, to direct emotional/affective information to direct effective/regulative information. As a result, we have Interaction of the personality components presented in Figure 1.

Figure 1. Interaction between components of personality.



We can see that cognitive information is one of the three basic types of anthropic information with the corresponding infological system CIF(*R*), which contains (stores and processes) cognitive elements or constituents, such as knowledge, data, ideas, beliefs, images, algorithms, procedures, scenarios, ideas, values, goals, ideals, fantasies, abstractions, etc. Cognitive infological systems are very important, especially, for intelligent systems as the majority of researchers believe that information is intrinsically connected to cognition. This peculiarity is reflected in the following Cognitive Transformation Principle.

Ontological Principle O2c (**the** ***Cognitive Transformation Principle***). *Cognitive information* for a system *R*, is a capacity to cause changes in the cognitive infological system CIF(*R*) of the system *R*.

In the case of a cognitive infological system CIF(*R*), it looks like it is possible to give an exact definition of cognitive information. However, now cognitive sciences do not know all structural elements involved in cognition. A straightforward definition specifies cognition as activity (process) that gives knowledge. At the same time, we know that knowledge, as a rule, comes through data and with data. So, data are also involved in cognition and thus, have to be included in cognitive infological systems. In addition, cognitive processes employ such structures as ideas, images, texts, beliefs, values, measures, problems, schemas, procedures, tasks, goals, etc. Thus, to comprehensively represent cognitive information, it is imperative to include all such objects in cognitive infological systems.

As the cognitive infological system contains knowledge of the system it belongs, cognitive information is the source of knowledge changes.

There are also different types of cognitive information. One approach to classification is based on the structures in the brain. Researchers found that a longitudinal fissure separates the human brain into two distinct cerebral hemispheres, connected by the corpus callosum. The sides resemble each other and each hemisphere's structure is generally mirrored by the other side. Yet despite the strong similarities, the functions of each cortical hemisphere are different. As it is known (*cf.*, for example, [55]), the left hemisphere operates with symbolic information representation, performing logical and analytical functions, such as linear reasoning, numeric manipulation, language processing, and mental arithmetic. It is also supposed that the left hemisphere works in the sequential mode on the level of consciousness. The right hemisphere processes images and realizes intuitive, creative and synthesizing functions of the brain, such as processing of visual and audiological stimuli, spatial manipulation, facial perception, and artistic performance. It is also supposed that the right hemisphere works in the parallel (concurrent) mode on the level of consciousness. In a similar way, Herrmann [36] differentiates functioning of the left parts and right parts of both the cerebral and limbic regions of the brain.

Taking each of these hemispheres as an infological system, we come to two types of cognitive information.

Symbolic (*discrete*) cognitive information transforms (or can transform) the symbolic content of the left hemisphere.

Holistic (*continuous*) cognitive information transforms (or can transform) the integral (gestalt) content of the right hemisphere.

This classification of cognitive information is complementary to another classification, in which one of the basic types of cognitive information is epistemic information, which can be both symbolic and holistic, belonging to both, the right hemisphere and left hemisphere. It is studied in Section 5.

4. Stratification of Knowledge Systems

There are different kinds of stratification.

In *physical stratification*, each stratum is a separate physical system. Any distributed database is physically stratified.

In *analytical stratification*, each stratum is determined by a specific name (label) and all elements from this stratum have this label (name). Knowledge base stratification used for handling inconsistent knowledge bases [56,57], for constructing models of a knowledge base [58] and for merging multiple knowledge bases [59,60] is analytical. The same is logic stratification used for formalization of commonsense reasoning [61]. In this section, we are mostly interested in analytical stratification based on knowledge classification. Different classes of knowledge form corresponding strata of knowledge systems.

There are different principles of knowledge classification, which allow us to build several types of knowledge system stratifications.

Time is an important characteristic of knowledge, giving different stratifications.

The *temporal stratification*.

1. The *past stratum* of knowledge consists of knowledge obtained/accepted in the past.
2. The *current stratum* of knowledge consists of the actual (used now) knowledge.
3. The *future stratum* of knowledge.

For instance, the knowledge "the Earth is flat" is past knowledge, while the knowledge "the Earth is round" is current knowledge.

The past stratum of knowledge consists of three substrata: the *forgotten past knowledge*, *outdated but preserved past knowledge* and *still actual past knowledge*.

The current stratum of knowledge consists of three substrata: the *disappearing current knowledge*, *consolidated current knowledge*, and *emergent current knowledge*.

The future stratum of knowledge consists of three substrata: the *tentative/potential future knowledge*, *realizable future knowledge* and *emergent future knowledge*.

More precise temporal stratifications are used in temporal the knowledge and databases. A temporal knowledge/database is a database with built-in time aspects. In particular, it supports a temporal knowledge/data model and has a temporal version of the query language [62,63]. Temporal knowledge/data stored in a temporal knowledge/database are different from the knowledge/data stored in non-temporal knowledge/database in that a time coordinate is attached to the knowledge/data. This is different from the conventional knowledge/data, which are usually considered to be valid now. Past and future knowledge/data are not stored. Usually past knowledge/data are modified, overwritten with new (updated) knowledge/data or deleted to achieve their temporal relevancy. Future knowledge/data are not considered because it is assumed that we do not receive information about the future.

There are many complexity measures of algorithms, methods and procedures. Taking a complexity measure *C*, it is possible to partition all algorithms (methods or/and procedures) into separate classes that have different complexity measures. Each such a partition induces a corresponding stratification of knowledge with respect to such knowledge characteristics as accessibility, inference, and generation, which are specific forms of knowledge acquisition. Here are some examples of such stratifications.

The *accessibility stratification*.

1. *Directly accessible* knowledge.
2. *n-step accessible* knowledge.

Another stratification is based on complexity of knowledge inference.

The *inference stratification*.

1. *Directly implied* knowledge.
2. *n-step inferable* knowledge.

One more stratification is based on complexity of knowledge generation.

The *generation stratification*.

1. *Directly generable/computable* knowledge.
2. *n-step generable/computable* knowledge.

Steps in generation, inference and access may be determined by:

- *Time slicing* when each step is assigned some period of time for realization.
- *Elementary operations*.

For instance, it is possible to assume that knowledge acquisition is direct if it demands less than 3 seconds. The first step of knowledge acquisition can be estimated as an interval from 3 seconds to 30 seconds. The second step of knowledge acquisition can be estimated as an interval from 30 seconds to 1 minute. The third step of knowledge acquisition can be estimated as an interval from 1 minute to 3 minutes and so on.

It is also possible to measure complexity, e.g., effort in generation, by the power of algorithms [64]. In this case, we have an *algorithmic ladder*, which consists of classes of algorithms with increasing computing power.

The traditional algorithmic ladders have one of the following forms:

(1) Finite automata, deterministic pushdown automata, pushdown automata, and Turing machines.

(2) Regular, or linear grammars, context-free grammars, context-sensitive grammars, and unrestricted, or phrase-structure grammars.

New achievements of the theory of algorithms and computation extend these ladders:

(1') Finite automata, deterministic pushdown automata, pushdown automata, Turing machines, inductive Turing machines [65], and infinite-time Turing machines [66].

(2') Regular, or linear grammars, context-free grammars, context-sensitive grammars, unrestricted, or phrase-structure grammars, grammars with prohibition [67], and Boolean grammars [68].

Inductive Turing machines give an example of an algorithmic ladder. Namely, the *n*-th strata of the inductive algorithmic ladder consists of inductive Turing machines with the structured memory that have order *n* but do not have order *n* + 1 [65]. It is also possible to build an algorithmic ladder using inductive or limit) Turing machine have structured program (rules for computation) or structured (heads) operating devices [65].

5. Stratified M-spaces and Information Operators

Let us consider a universal set or multiset $\boldsymbol{W}$ of knowledge items (units). It is possible to take the set $\boldsymbol{W}_C$ of elementary knowledge units mathematically modeled in [1,69] as a universal set (multiset) $\boldsymbol{W}$, obtaining a reasonable formalization of the concept of a knowledge state. Another possibility for $\boldsymbol{W}$ is realized by the set (multiset) $\boldsymbol{W}_L$ of propositions and/or predicates from some logical language *L*. This logical approach was adopted in works of Bar-Hillel and Carnap [2], Hintikka [3-5] and some other authors. Shreider [7] interpreted knowledge items as texts in a thesaurus. Many researchers employ schemas as knowledge items in the brain (*cf.*, for example, [41-44]. One more possibility for $\boldsymbol{W}$ is the set, or more exactly, a multiset, $\boldsymbol{W}_S$ of situations possible in a world *U*, which are taken as knowledge items or knowledge units (*cf.*, for example, [70,71].

The set $\boldsymbol{W}$ is called *universal* because we assume that the following axiom is true.

MA1 (the Internal Representation Axiom). For any cognitive system (agent) *A*, knowledge states K_{Ai} of *A* are subsets (submultisets) of the set (multiset) $\boldsymbol{W}$.

It is possible to interpret $\boldsymbol{W}$ as the base of all knowledge that agents are able to have about their environment.

Another aspect of universality of the set (multiset) $\boldsymbol{W}$ may be in the possibility to describe all possible (existing) worlds utilizing knowledge only from $\boldsymbol{W}$. For instance, when $\boldsymbol{W}$ is the set (multiset) $\boldsymbol{W}_L$ of propositions and/or predicates from some logical language *L*, then it is possible to build all descriptions of all possible worlds by combining elements from $\boldsymbol{W}_L$. This possibility is reflected in the following axiom.

MA2 (the External Representation Axiom). For any environment (situation) *D*, there is a subset (submultiset) $\boldsymbol{W}_D$ of the set (multiset) $\boldsymbol{W}$ that contains all accessible knowledge about *D*.

Taking these two axioms as the foundation, we develop a theory of cognitive systems/agents called the *theory of M-spaces*.

Definition 5.1 [1]**.** a) Subsets of $\boldsymbol{W}$ are called *Mizzaro spaces*.

b) Submultisets of $\boldsymbol{W}$ are called *Mizzaro multispaces*.

In some cases, only specific subsets (submultisets) of $\boldsymbol{W}$ are used in the theory. For instance, if elements of $\boldsymbol{W}$ are propositions and the model satisfies conditions of classical logical calculi, then only consistent subsets of propositions are acceptable as Mizzaro spaces. When, in addition, all deducible propositions are also included in such a logical Mizzaro space, then Mizzaro spaces are components of a logical variety [72].

Taking a knowledge system $\boldsymbol{K}$, we model the states of $\boldsymbol{K}$ by Mizzaro spaces (Mizzaro multispaces), *i.e.*, each knowledge state is represented by a Mizzaro space (Mizzaro multispace) $K_{\boldsymbol{K}i}$. The whole knowledge system $\boldsymbol{K}$ is modeled by an M-space.

In modeling knowledge systems and information processes, we consider two structures – sets and multisets – because using the classical background it is possible to consider only sets, which make the model simpler. However, many real cognitive systems contain several copies of the same element. For instance, the same element of knowledge can be stored in different parts of a computer memory or of the brain. This makes utilization of multisets necessary.

Definition 5.2. A *type of structures* is a system of conditions (axioms) that all these structures, *i.e.*, sets with relations, satisfy.

To define an M-space, we consider a type θ of structures in Mizzaro spaces (Mizzaro multispaces).

Definition 5.3. An M-*space* (M-*multispace*) $\boldsymbol{M}$ is a structure of the form

$$\boldsymbol{M} = \{\mathrm{KS}_{\boldsymbol{M}} ; \mathrm{OS}_{\boldsymbol{M}} \}$$

where KS consists of Mizzaro spaces (Mizzaro multispaces) K with the structure of the type θ, and $\mathrm{KS}_{\boldsymbol{M}}$ is a system of information operators in $\mathrm{OS}_{\boldsymbol{M}}$ acting on Mizzaro spaces (Mizzaro multispaces).

Thus, $\mathrm{KS}_{\boldsymbol{M}} = \{K_i \mid i \in I\}$ and $\mathrm{OS}_{\boldsymbol{M}} = \{A_t \mid t \in T\}$.

Example 5.1. It is possible to represent a logical variety or a prevariety $\boldsymbol{V}$ [72] as an M-space where $\mathrm{KS}_{\boldsymbol{M}}$ consists of the components of $\boldsymbol{V}$ and operators from $\mathrm{OS}_{\boldsymbol{M}}$ apply mappings f_i: $A_i \to L$ and $g_i : T_i \to L$ ($i \in I$), which form connections between components of the variety (prevariety).

Definition 5.4. a) The set $\mathrm{KS}_{\boldsymbol{M}}$ is called the *state space* of the M-space $\boldsymbol{M}$.

b) The set $U_{\boldsymbol{M}} = \bigcup_{i\in I,\ K_i \in KS_{\boldsymbol{M}}} K_i$ is called the *universe* of the M-space $\boldsymbol{M}$.

c) The system $\mathrm{OS}_{\boldsymbol{M}}$ is called the *operating system* of the M-space $\boldsymbol{M}$.

The simplest structure of Mizzaro spaces K_i of the type θ is the structure of a set and the simplest structure of Mizzaro multispaces *Ki* of the type θ is the structure of a multiset. However, Mizzaro spaces K_i can be logical calculi, linear spaces or groups.

In the algebraic context, each M-space $\boldsymbol{M}$ is a universal algebra [73] with the support $\mathrm{KS}_{\boldsymbol{M}}$ and system of operations $\mathrm{OS}_{\boldsymbol{M}}$. In this paper, we consider only *unary Mizzaro spaces* (*unary Mizzaro multispaces*) in which each information operator maps one Mizzaro space (Mizzaro multispace) K_i into another (may be the same) Mizzaro space (Mizzaro multispace) K_j .

Information operators from $\mathrm{OS}_{\boldsymbol{M}}$ are global epistemic information operators in $\mathrm{KS}_{\boldsymbol{M}}$. When an operator acts only on one Mizzaro space (Mizzaro multispace), then it is a local epistemic information operator. Local epistemic information operators in non-structured Mizzaro spaces are studied in [1,10,11,13].

Note that it is possible to consider any system that contains a knowledge system as a knowledge system itself. Thus, any intelligent agent is a knowledge system.

There are two basic types of epistemic information operators: content, bond and mixed operators.

Definition 5.5. A *content epistemic information operator* acts on knowledge items in a knowledge state.

For instance, all information operators studied in [1,10,11,13] are content epistemic information operators.

Definition 5.6. A *bond epistemic information operator* acts on connections (bonds or relations) between knowledge items in a knowledge state.

Such operators as interpretation and reinterpretation of information/knowledge items are bond epistemic information operators.

Definition 5.7. A *mixed epistemic information operator* acts both on knowledge items and on connections (bonds or relations) between knowledge items in a knowledge state.

Operators of logical inference, such as rules of deduction, are mixed epistemic information operators act because they add new knowledge items in the form of propositions or/and predicates and establish relations of inferrability/deducibility between propositions or/and predicates.

Here we are mostly interested in content epistemic information operators, which we simply call epistemic information operators.

To correctly model stratified knowledge system, the modeling M-space also has to be stratified.

Definition 5.8. a) An M-space $\boldsymbol{M} = \{KS_{\boldsymbol{M}} ; OS_{\boldsymbol{M}}\}$ is *stratified* if there is a set J and each Mizzaro space (Mizzaro multispace) K_i from $KS_{\boldsymbol{M}}$ has the form

$$K_i = \bigcup_{j\in J} K_{ij}$$

b) A stratification of an M-space $M = \{ KS_{\boldsymbol{M}} ; OS_{\boldsymbol{M}} \}$ is *strict* if for each Mizzaro space (Mizzaro multispace) K_i from $KS_{\boldsymbol{M}}$, $K_{ij} \cap K_{ik} = \varnothing$ when $j \neq k \in J$.

c) An M-space $\boldsymbol{M} = \{KS_{\boldsymbol{M}} ; OS_{\boldsymbol{M}}\}$ is *linearly stratified* if each Mizzaro space (Mizzaro multispace) K_i from MK has the form

$$K_i = \bigcup_{n=1}^{\infty} K_{in}$$

in the case when the stratification is infinite and the form

$$K_i = \bigcup_{n=1}^{m} K_{in}$$

in the case when the stratification is finite (*cf.* Figure 2).

Linear stratification means that the set of stratum indices J is finite or countable and linearly ordered.

Figure 2. A finite M-space stratification with the linear topology.

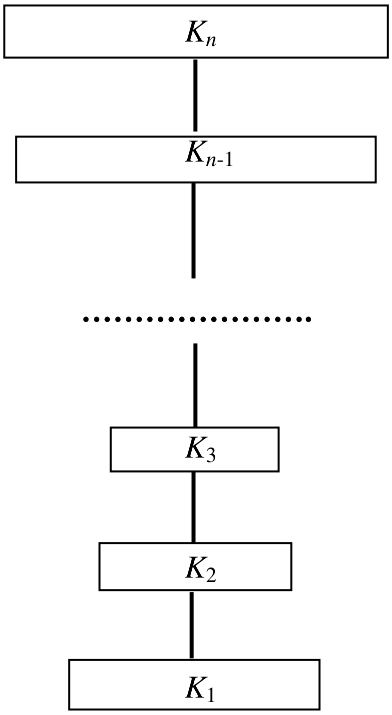
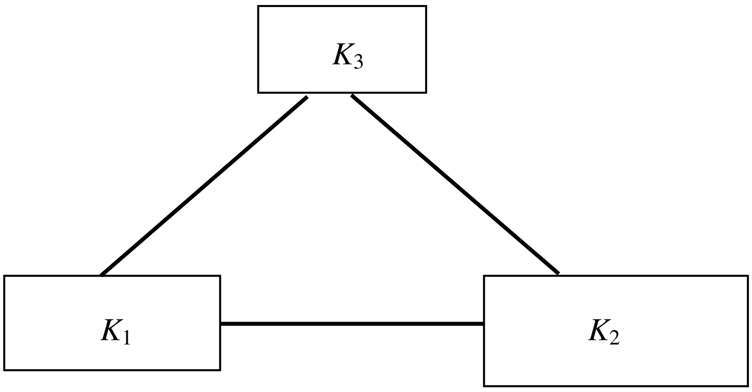


Figure 3. A finite M-space stratification with the cyclic topology.



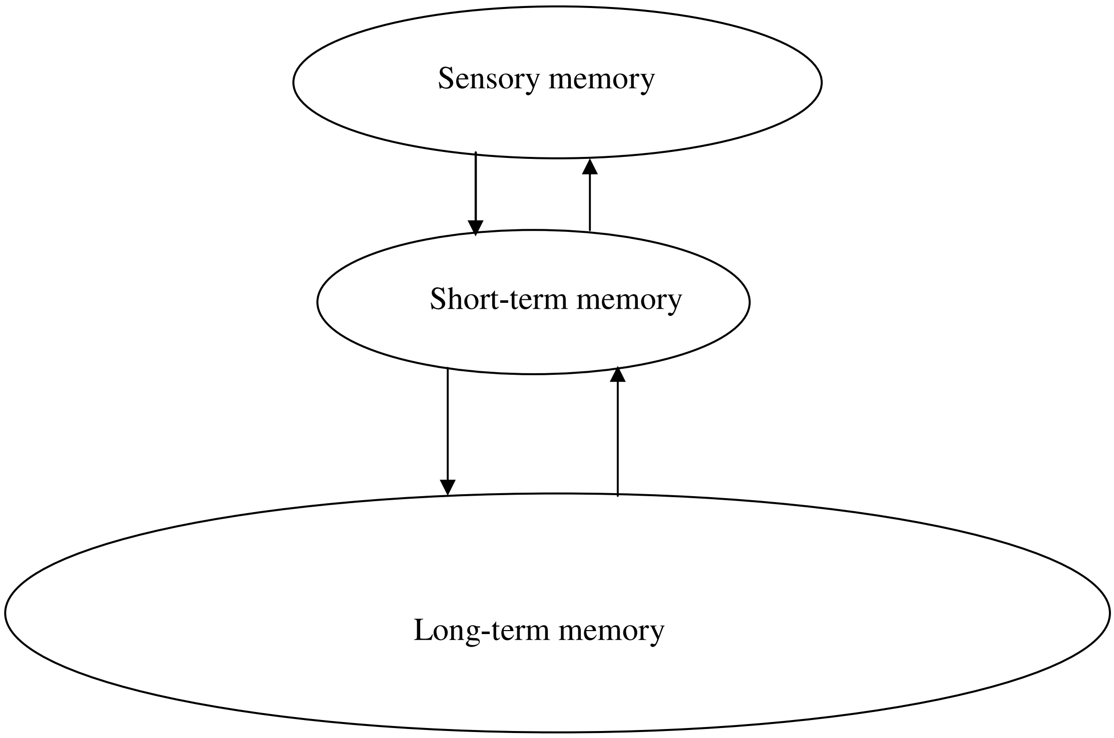
There are M-space stratifications with a non-linear topology. For instance, the stratification in Figure 3 has a cyclic topology. Important cases of M-space stratifications have structures of a tree or a forest.

Example 5.2. People, as well as computers, have many kinds of memory. It is even supposed that each part of the brain has several types of memory agencies that work in somewhat different ways, to suit particular purposes [23]. It is possible to consider each of these memory agencies as a separate system and to study differences between information that changes each type of memory. This might help to understand the interplay between stability and flexibility of the mind, in general, and memory, in particular.

Psychologists differentiate three types of human memory: *sensory memory*, *short-term memory*, and *long-term memory*. It is the most important and best documented by scientific research memory stratification. However, memory researchers do not employ uniform terminology. Sensory memory is also known as *sensory register*, *sensory store*, *sensory information storage*, *eidetic memory* and *echoic memory*. Short- and long-term memories are also referred to as *primary memory* and *secondary memory*, correspondingly. Each component of memory differs with respect to its function, the form of information held, the length of time information is retained, and the amount of information-handling capacity.

Thus, human memory has three basic strata. As a result, all knowledge in the memory of an individual is also stratified into three components: knowledge/data in the sensory memory, knowledge in the short-term memory, and knowledge in the long-term memory of this individual (*cf.* Figure 4). Additional stratification of human memory as a knowledge space induces additional stratification of knowledge.

Figure 4. The human memory hierarchy.



Sensory memory acts as a buffer for stimuli received through the senses, which are then filtered and passed from sensory memory into short-term memory by attention.

In tern, sensory memory is also stratified by different sensory channels. There are *iconic memory* for visual stimuli, *echoic memory* for aural stimuli and *haptic memory* for touch.

Long-term memory is naturally stratified. The most popular stratification divides it into two parts: *episodic memory* and *semantic memory*. Episodic memory stores knowledge of events and experiences in a serial form. In contrast to this, semantic memory is a structured record of facts, concepts and skills that people have acquired. The information in semantic memory is derived from that in the episodic memory of the same individual.

Neuroscientists distinguish three main activities related to long term memory: *storage*, *deletion* and *retrieval*. These operations are modeled by epistemic information operators. Storage is modeled by the epistemic information operator REPL. In information storage, information from sensory memory, at first goes to short-term memory and then is stored in long-term memory, usually by the process called *rehearsal*. Rehearsal is the repeated exposure to a stimulus of a knowledge/data portion, which transfers it into long-term memory. Deletion of a knowledge/data portion is modeled by the epistemic information operator DEL and is mainly caused by *decay* and *interference*. Emotional factors essentially affect the long-term memory functioning. According to contemporary there are two types of information retrieval: recall and recognition. In knowledge/data *recall*, the information is reproduced from memory. Recall is modeled by the epistemic information operator COPY. Knowledge/data *recognition* is based on information that this knowledge/data portion has been seen before and is assisted by the provision of retrieval cues to enable better access in the long-term memory. Recognition is modeled by the epistemic information operator GEN.

Scientists also use another stratification of the human memory: *personal memory*, *semantic memory*, *perceptual memory*, and *skill memory*, which includes, *motor skill memory*, *cognitive* skill memory, and *rote linguistic skill memory*.

Example 5.3. The computer memory is also a complex system of diverse components and processes. Memory of a computer includes such three basic components as the random access memory (RAM), read-only memory (ROM), and secondary storage. While RAM forgets everything whenever the computer is turned off and ROM cannot learn anything new, secondary storage devices allow the computer to record information for as long period of time as we want and change it whenever we want. Now the following devices are utilized for log-term computer memory: magnetic tapes and corresponding drives, magnetic disks and corresponding drives, flash memory storage devices and corresponding drives, and optical disks and corresponding drives.

Computer memory is intrinsically stratified by the hierarchy in which levels are distinguished by the response time, complexity, and capacity. The overall goal of using a memory stratification is to obtain the higher possible average access performance, while minimizing the cost of the entire memory system.

Often four major memory levels are separated:

1. *Internal memory* – Processor registers and cache.
2. *Main memory* – the system RAM and controller cards.
3. *On-line mass storage* – Secondary storage.
4. *Off-line bulk storage* – Tertiary and Off-line storage.

At the same time, other experts use another stratification of computer memory:

1. Processor registers have the fastest possible access (in just a few cycles) and usually stores tens of kilobytes
2. Level 2 (L2) cache usually stores 512 KiB or more
3. Level 3 (L3) cache usually stores 2048 KiB or more
4. Main memory, access to which may take hundreds of cycles, but which usually stores multiple gigabytes.

5. Disk storage, access to which may take millions of cycles latency if not cached, but which usually is very large
6. Tertiary storage, access to which may take several seconds latency and which can be huge

Such memory stratifications are linear, reflecting the access time with the fast CPU registers at the top and the slow hard drive at the bottom.

Another, although related, stratification is induced by different electronic devices, which include CPU registers, on-die SRAM caches, external caches, DRAM, paging systems, and virtual memory or swap space on a hard drive. These devices are called RAM (random access memory) by many developers, even though the various subsystems can have very different access times, violating the original concept behind the *random access* term in RAM. RAM consists of two strata: dynamic random access memory, which requires the stored information to be periodically re-read and re-written, or refreshed in order not to lose it, and static memory, never needs to be refreshed as long as power is applied, although it can lose its content if power is removed.

Usually each stratum in RAM is also stratified. For instance, in DRAM, strata are defined by the row, column, bank, rank, and channel.

In addition, computer memory is also stratified by data storage technologies, such as semiconductor, magnetic, and optical technologies. In modern computers, primary storage almost exclusively consists of dynamic volatile *semiconductor memory*, which uses semiconductor-based integrated circuits to store information. Since the turn of the century, a type of non-volatile semiconductor memory known as flash memory has steadily gained share as off-line storage in various advanced electronic devices and computers.

Magnetic storage, which is non-volatile, uses different types of magnetization on a magnetically coated surface to store information. The information is accessed using one or more read/write heads which may contain one or more recording transducers. A read/write head only covers a part of the surface so that the head or medium or both must be moved relative to another in order to access data. In modern computers, there are following kinds of magnetic storage devices:

- Magnetic disk, such as sloppy disks, used for off-line storage, and the hard disk drive, used for secondary storage
- Magnetic tape data storage, used for tertiary and off-line storage

At the beginning of computer era, magnetic storage was also used for primary storage in a form of a magnetic drum, or core memory, core rope memory, thin-film memory, twistor memory or bubble memory, while magnetic tapes were often used for secondary storage.

Another popular type of storage is *optical discs*, which stores information in deformities on the surface of a circular disc, reading this information by illuminating the surface with a laser diode and observing the reflection. In modern computers, there are following kinds of optical storage devices:

- CD, CD-ROM, DVD, BD-ROM: Read only storage, used for mass distribution of digital information (music, video, computer programs)
- CD-R, DVD-R, DVD+R, BD-R: Write once storage, used for tertiary and off-line storage
- CD-RW, DVD-RW, DVD+RW, DVD-RAM, BD-RE: Slow write, fast read storage, used for tertiary and off-line storage
- Ultra Density Optical or UDO is similar in capacity to BD-R or BD-RE and is slow write, fast read storage used for tertiary and off-line storage.

In magneto-optical disc storage, the information is read optically and written into the magnetic state on a ferromagnetic surface by combining magnetic and optical methods. It is usually used for tertiary and off-line storage.

Paper data storage, typically in the form of paper tape or punched cards, has long been used to store information for automatic processing, particularly before electronic computers were invented.

There are also such memory devices as the vacuum tube memory, electro-acoustic memory, optical tapes, phase-change memory, holographic data storage, and molecular memory, which stores information optically inside crystals or photopolymers.

All these memory devices and components determine the corresponding stratification of knowledge stored in computers, e.g., of knowledge bases.

Example 5.4. Stratification is a popular technique in knowledge base theory and practice. For instance, Hunter and Liu [59] introduce knowledge base stratification to solve the problem of merging multiple knowledge bases. Benferhat and Baida [56] use stratified first order logic for access control in knowledge bases. Benferhat and Garcia [57] employ stratification for handling inconsistent knowledge bases. Lassez, *et al*, [58] show how stratification can be used as a tool in the interactive model-building process. Namely it is possible to reduce the computational complexity of the process by the use of stratification, which limits consistency checking to minimal strata.

Definition 5.9. a) An M-space $\boldsymbol{M}$ is a *subspace* of an M-space $\boldsymbol{H}$ if the state space $KS_{\boldsymbol{M}}$ is a substructure of the state space $KS_{\boldsymbol{H}}$ and the operational system $OS_{\boldsymbol{M}}$ is a substructure of the operational system $OS_{\boldsymbol{H}}$.

b) If an M-space $\boldsymbol{M}$ is a subspace of an M-space $\boldsymbol{H}$, then $\boldsymbol{H}$ is called a *superspace* of $\boldsymbol{M}$.

In particular, the stratification of the knowledge space $KS_{\mathbf{M}}$ is induced by the stratification of the knowledge space $KS_{\mathbf{H}}$.

Example 5.5. If a structured M-space **H** models the group memory of a group *G* of several people, then a structured M-space **M** that models the memory of one individual from this group *G* is a subspace of **H**.

Subspaces of M-spaces and M-multispaces represent subsystems of knowledge systems. For instance, in large knowledge systems, such as a scientific theory, it is possible to separate the subsystem of denotational knowledge and the subsystem of operational knowledge.

Definition 5.10. If X is a structure, *i.e.*, a set/multiset with relations, then $\underline{X}$ is the set of all elements from X and $\underline{X}$ is the multiset of all elements from X, while Rel$X$ is the set of all relations from X.

In such a way, ignoring the M-space stratification, it is possible to represent structured knowledge systems by uniform M-spaces, in which all knowledge states are sets or multisets. In this setting, content epistemic information operators act on elements from sets $\underline{K}$ or multisets $\underline{\mathrm{K}}$, while bond epistemic information operators act on elements from Rel$K$.

Definition 5.11. a) A knowledge system (agent) A is called *locally finite* if any knowledge state of A is finite.

b) A knowledge system (agent) A is called *finite* if it has only a finite number of knowledge states and any knowledge state of A is finite.

c) An M-space $\boldsymbol{M}$ is called *locally finite* if each K from $KS_{\boldsymbol{M}}$ contains only a finite number of knowledge items.

d) An M-space ***M*** is called *finite* if it has only a finite number of Mizzaro spaces (Mizzaro multispaces) K each of which is also finite.

It looks like it might be sufficient to consider only finite or at least, locally finite agents. However, if knowledge is represented by logical statements and it is assumed (as, for example, in the theory of Bar-Hillel and Carnap [2]) that any knowledge system contains all logical consequences of all its elements, then an agent with such knowledge system is infinite. In information algebras, portions of information are represented by close subsets of sentences from a logical language L [74].

However, in conventional logics, closed with respect to such information operators as deduction sets are infinite because any sentence p implies $p \vee q$ for any sentence q from L, which is, as a rule, infinite (*cf.*, for example, [75]). Thus, in the context of classical logic and information algebras any portion of information has infinitely many representations. Consequently, such a portion generates a system with the infinite number of knowledge items.

Lemma 5.1. An M-space ***M*** is finite if and only if it has a finite universe.

Stratification of the knowledge system and the corresponding M-space allows defining specific classes of epistemic information operators.

Definition 5.12. An epistemic information operator A is called *stratified* if for any $j \in J$, there is $k \in J$ such that for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ik}$.

Stratified information operators preserve the structure, *i.e.*, stratification, of knowledge states. Note that adding and deletion operators are intrinsically stratified.

Definition 5.13. a) An epistemic information operator A is called *closed* if for any $j \in J$ and for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ij}$.

b) An epistemic information operator A is called *closed* in a Mizzaro space K_i from $\mathrm{KS}_{\boldsymbol{M}}$ if $A(K_i) \subseteq K_i$.

Lemma 5.2. Any closed epistemic information operator A is stratified.

Definition 5.14. a) A stratified epistemic information operator A in a linearly stratified M-space ***M*** is called *monotone* (*antitone*) if for any $n \in \boldsymbol{N}$, there is $k \in \boldsymbol{N}$ such that $k \geq n$ ($k \leq n$) and for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ik}$.

b) A stratified epistemic information operator A in a linearly stratified M-space ***M*** is called *strictly monotone* (*strictly antitone*) if for any $n \in \boldsymbol{N}$, there is $k \in \boldsymbol{N}$ such that $k > n$ ($k < n$) and for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ik}$.

Definitions imply the following property of epistemic information operators.

Lemma 5.3. Any strictly monotone (strictly antitone) epistemic information operator A is monotone (antitone).

Lemma 5.4. In a finite linearly stratified M-space ***M***, there are no strictly monotone and strictly antitone information operators.

Definition 5.15. An epistemic information operator A is called *contracting* if there is $k \in J$ such that for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ik}$.

Definitions imply the following result.

Lemma 5.5. Any contracting epistemic information operator A is stratified.

There are five types of *basic epistemic operations*: adding, deleting, moving, replicating, and ttransforming knowledge, and five types of corresponding *basic epistemic information operators*:

adding AD, deletion DL, moving MV, replication REPL, generation GEN and transformation TR epistemic information operators.

Definition 5.16. A *transformation epistemic information operator TR* takes a group of knowledge items (may be, one item) from the current knowledge state and transforms it into another group of knowledge items (may be, into one item).

Definition 5.17. A *generation epistemic information operator TR* takes a group of knowledge items (may be, one item) from the current knowledge state and generates another group of knowledge items (may be, one item).

The difference between transformation and generation is that in generation the initial group of knowledge items is preserved, while in transformation it is not preserved.

Lemma 5.4. AD is equal to GEN with the empty set of the initial knowledge items.

Definition 5.18. A *moving epistemic information operator MV* moves a knowledge item from one stratum into another one.

Definition 5.19. a) A *replica* of a knowledge item is another knowledge item equivalent to the initial one.

b) A *replication epistemic information operator REPL* makes a replica of a knowledge item and adds it to the current knowledge state.

Example 5.5. Let us consider logical knowledge representation in which knowledge items are propositions. Then according to laws of logic there are equivalent propositions. For instance, taking the proposition (1) "*B* implies *A*", we have equivalent propositions (2) "*A* follows from *B*", (3) "If *B*, then *A*", and (4) "*A* is a consequence of *B*". All of them are replicas of one another although they are not copies.

If the proposition (1) belongs to the stratum K_1 , then its replication to the stratum K_2 can introduce either proposition (1) or proposition (2) or proposition (3) to the stratum K_2 , while its copying to the stratum K_2 can introduce only proposition (1) to the stratum K_2 .

An important special case of a replication epistemic information operator is a *copying epistemic information operator COPY*, which makes a copy of a knowledge item and adds it to the current knowledge state.

Another important special case of a replication epistemic information operator is a *restricted replication epistemic information operator* $REPL_0$, which replicates a knowledge item and adds it only to a stratum of the current knowledge state that does not have the same replica. One of its special cases is a *restricted copying epistemic information operator* $COPY_0$, which makes a copy of a knowledge item and adds it only to a stratum of the current knowledge state that does not have the same replica. Operators $REPL_0$ and $COPY_0$ are used in stratified M-spaces not to make these spaces stratified M-multispaces.

Lemma 5.5. The operator $COPY_0$ can copy a knowledge item only to a different stratum, *i.e.*, if $a \in K_i$ and $COPY_0 a \in K_j$, then $i \neq j$.

Indeed, if this condition is violated, then the initial M-space is converted to an M-multispace.

Complex information operations and operators are studied in [76].

Definition 5.20. An epistemic information operator *C* is called the *sequential composition* of an epistemic information operator *A* with an epistemic information operator *B* if *C*(*x*) is defined and equal to $B(A(x))$ when: 1) $A(x)$ is defined and belongs to the domain of B ; 2) $B(A(x))$ is defined. Otherwise, C gives no result being applied to x, *i.e.*, $C(x) = *$.

Taking sequential composition of an epistemic information operator A with itself, we obtain sequential powers A^n of the operator A.

In the general case, the sequential composition of epistemic information operators is not commutative in M-spaces as the following example demonstrates.

Example 5.1. Let us consider a structured M-space $\boldsymbol{M} = \{\mathrm{KS}_{\boldsymbol{M}} ; \mathrm{OS}_{\boldsymbol{M}} \}$ where $KS_{\boldsymbol{M}} = = \cup_{i\in I} KS_{\boldsymbol{M}i}$. In this space, the operator $MV_{ij}{}^a$ moves an element a from the stratum $KS_{\boldsymbol{M}i}$ to the stratum $KS_{\boldsymbol{M}j}$ and does not change other elements from $KS_{\boldsymbol{M}}$. Taking the sequential composition of such operators, we have

$$MV_{ij}{}^a \circ MV_{ik}{}^a = MV_{ij}{}^a \neq MV_{ik}{}^a \circ MV_{ij}{}^a = MV_{ik}{}^a$$

if $i \neq j$, $k \neq j$, and $i \neq k$. Thus, operators $MV_{ij}{}^a$ do not commute with one another.

At the same time, all these operators are idempotents, *i.e.*, $MV_{ij}{}^a \circ MV_{ij}{}^a = MV_{ij}{}^a$.

It is necessary to remark that in a structured M-multispace $\boldsymbol{M}$ with an infinite number of elements a in each stratum $KS_{\boldsymbol{M}i}$, operators $MV_{ij}{}^a$ and $MV_{ik}{}^a$ commute with one another. This demonstrates difference between M-spaces and M-multispaces.

Proposition 5.1. If A and B are closed (closed in a Mizzaro space K_i) operators, then their sequential composition $A \circ B$ is also a closed (closed in a Mizzaro space K_i) operator.

Indeed, if A and B are closed epistemic information operators in a structured M-multispace $\boldsymbol{M}$, then for any $j\in J$ and for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ij}$ and $B(K_{ij}) \subseteq K_{ij}$. Thus, $(A \circ B)(K_{ij}) = B(A(K_{ij}) \subseteq B(K_{ij}) \subseteq K_{ij}$.

For closed in a Mizzaro space K_i operators, the proof is similar.

Proposition 5.2. If A and B are contracting operators, then their sequential composition $A \circ B$ is also a contracting operator.

Proof is similar to the proof of Proposition 5.1.

Proposition 5.3. If A and B are stratified operators, then their sequential composition $A \circ B$ is also a stratified operator.

Proof is similar to the proof of Proposition 5.1.

Proposition 5.4. If A and B are (strictly) monotone [antitone] operators, then their sequential composition $A \circ B$ is also a (strictly) monotone [antitone] operator.

Indeed, if A and B are monotone epistemic information operators in a structured M-space $\boldsymbol{M}$, then for any K_i from $\mathrm{KS}_{\boldsymbol{M}}$, we have $A(K_{ij}) \subseteq K_{ik}$ with $k \geq j$ and $B(K_{ik}) \subseteq K_{ih}$ with $h \geq k$. Thus, $(A \circ B)(K_{ij}) = B(A(K_{ij}) \subseteq B(K_{ik}) \subseteq K_{ih}$ with $h \geq j$.

Considerations for strictly monotone, antitone and strictly antitone epistemic information operators are similar.

Let us consider an M-space $\boldsymbol{M}$ with a finite linear stratification.

Proposition 5.5. For any monotone and any antitone epistemic information operator A, there is a number n such that the sequential power A^n is also a closed epistemic information operator.

Indeed, if A is a monotone epistemic information operator in a structured M-space $\boldsymbol{M}$, then making each step, it either increases the number of the stratum or the image of a stratum remains in the same stratum. If the second case is true for all strata of $\boldsymbol{M}$, then A itself is a closed epistemic information operator. Otherwise, A can increase the number of a stratum only for a finite number of steps because there are only a finite number of strata in ***M***. Thus, after some number of repetitions, the image of a stratum remains in the same stratum. Taking the largest number of such steps, we obtain the necessary number *n*.

Note that *n* cannot be larger than the number of strata in ***M***.

Let us explore relations between basic epistemic information operators.

Definition 5.21 [77]. Two operators *A* and *B* are *functionally equivalent* if they have the same definability domain *D* and $A(x) = B(x)$ for any element *x* from *D*.

Proposition 5.6. A transformation epistemic information operator *TR* is functionally equivalent to the sequential composition of a deletion epistemic information operator *DEL* and adding epistemic information operator *AD* that act in the same stratum of the M-space.

Indeed, if *TR* takes items $a_1, a_2, \dots, a_n$ from $KS_{\mathbf{M}}$ and transforms them into $b_1, b_2, \dots, b_m$, then it is possible to achieve the same result by deleting $a_1, a_2, \dots, a_n$ and adding $b_1, b_2, \dots, b_m$ to the corresponding stratum of $KS_{\mathbf{M}}$.

Proposition 5.7. A moving epistemic information operator *MV* is functionally equivalent to deletion of a knowledge item in one stratum and adding the same knowledge item to another stratum.

Proposition 5.8. A replication epistemic information operator *REPL* is functionally equivalent to adding an equivalent knowledge item to the corresponding stratum.

Proposition 5.9. For any M-space ***M***, there is a superspace ***H***, in which all deletion and adding epistemic information operators *DEL* and *AD* in ***M*** are functionally equivalent to moving epistemic information operators *MV* in ***H***.

Proof. To build a superspace ***H*** with the necessary properties, we add one more stratum *E* called the *external stratum* to the initial M-space ***M***. In addition, we assume that *E* contains all elements from the universal set (multiset) ***W*** and each element has infinitely many copies in *E*. In this case, any deletion of an element *a* from a state *K* from ***M*** is equivalent to moving the same element *a* to the stratum *E*. In a similar way, any addition of an element *a* to a state *K* from ***M*** is equivalent to moving the same element *a* from the stratum *E* to the state *K*.

Proposition is proved.

Proposition 5.10. A moving epistemic information operator *MV* can be (functionally) simulated by copy *COPY* and deletion *DEL* epistemic information operators.

Indeed, instead of moving a knowledge item *a* from a stratum K_i of a state *K* to a stratum K'_j of a state *K'*, it is possible to copy *a* from K_i to K'_i and then to delete this element from K_i.

Proposition 5.11. A generation epistemic information operator *GEN* is functionally equivalent to the sequential composition of a transformation epistemic information operator *TR* and adding epistemic information operator *AD* that act in the same stratum of the M-space.

Proof is similar to the proof of Proposition 5.10.

Proposition 5.12. A transformation epistemic information operator *TR* is functionally equivalent to the sequential composition of a generation epistemic information operator *GEN* and deletion epistemic information operator *DEL* that act in the same stratum of the M-space.

Proof is similar to the proof of Proposition 5.10.

Definition 5.22. A system ***B*** of epistemic information operators is an *operator basis* of an M-space ***M*** if any *A* from $OS_{\boldsymbol{M}}$ is a composition of elements from ***B***.

Operator bases can be useful in many situations. For instance, knowing properties of operators from such a base and properties of compositions, we can find properties of other operators.

Assuming that all operators in an M-space ***M*** are compositions of basic epistemic information operators, we have the following results.

Proposition 5.13. a) {*AD*, *DEL*} is an operator basis of an arbitrary (stratified) M-space ***M***.

b) {*TR*, *MV*} is an operator basis of an arbitrary (stratified) M-space ***M***.

c) {*TR*} is an operator basis of an arbitrary (*i.e.*, non-stratified) M-space ***M***.

Proof is based on Propositions 5.7 - 5.12.

Proposition 5.14. a) In a stratified M-space ***M*** with the external stratum, {$REPL_0$, *DEL*} is an operator basis.

b) In a stratified M-space ***M*** with the external stratum, {*MV*} is an operator basis.

c) In a stratified M-multispace ***M*** with the external stratum, {*REPL*, *DEL*} is an operator basis.

Proof is based on Propositions 5.7 - 5.12.

6. Conclusions

Based on the principles of the general theory of information, epistemic information is singled out as a kind of antropic information and modeled by the algebraic construction of M-spaces. M-spaces represent information dynamics by information operators acting in knowledge spaces. The main emphasis of this study is made on stratified knowledge spaces and algebras of epistemic information operators in such spaces.

Obtained results bring us to the following problems.

It is possible to consider not only knowledge but also beliefs as basic components of cognitive infological systems and call information that acts on such systems by the name plausible epistemic information.

Problem 1. Mathematically describe and study plausible epistemic information.

Problem 2. Study other types of M-space stratifications and operators in these spaces.

Problem 3. Study M-spaces in which knowledge items are elements of logics, e.g., propositions or predicates and stratification of which includes the structure of the corresponding logic, e.g., the propositional logic or the first-order predicate logic.

Problem 4. Study categories of M-spaces and functors between these categories.

Problem 5. Study operations with M-spaces.

In this paper, we studied only content epistemic information operators, while bond epistemic information operators, which act on connections and relations between knowledge items, are also very important.

Problem 6. Study bond epistemic information operators.

In this paper, we studied only sequential composition of epistemic information operators, while other types of composition are also very important.

Problem 7. Study other compositions of epistemic information operators.

In [1,13], information explications of epistemic information operators is studied in uniform Mizzaro spaces.

Problem 8. Explore information explications of epistemic information operators in stratified M-spaces and Mizzaro spaces.

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