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Topology Optimisation Using MPBILs and Multi-Grid Ground Element

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Abstract: This paper aims to study the comparative performance of original multi-objective population-based incremental learning (MPBIL) and three improvements of MPBIL. The first improvement of original MPBIL is an opposite-based concept, whereas the second and third method enhance the performance of MPBIL using the multi and adaptive learning rate, respectively. Four classic multi-objective structural topology optimization problems are used for testing the performance. Furthermore, these topology optimization problems are improved by the method of multiple resolutions of ground elements, which is called a multi-grid approach (MG). Multi-objective design problems with MG design variables are then posed and tackled by the traditional MPBIL and its improved variants. The results show that using MPBIL with opposite-based concept and MG approach can outperform other MPBIL versions.

Keywords: topology optimization; multi-objective optimization; opposite-based evolutionary algorithm; population-based incremental learning; adaptive learning rate

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1. Introduction

The first question that always arises at pre-process stage, when using a ground element approach for topology optimization, is:What the best ground element resolution for a design problem should be? As a result, we investigate using several sets of ground elements when performing optimization, which we term the multi-grid design approach (MG). The MG approach is an extension of ground segment strategy, which has been proposed to solve a truss structural optimization problem [1,2] and morphing wing structural optimization problem [3].

The second question arises due to an opposition-based concept that could potentially improve the search performance of the evolutionary algorithm (EA) [4–7]; the multi-objective population-based incremental learning (MPBIL) was the best optimizer [8]. Additionally, it has been demonstrated that the opposition-based concept could improve population-based incremental learning (PBIL) performance for a single objective, which is called the opposition-based concept PBIL(OPBIL) [9], whereas the multi-objective optimization is called opposite-based, multi-objective, population-based incremental learning (OMPBIL) [3]. PBIL is categorized as an estimation distribution algorithm (EDA), which is still in the spotlight of many researchers due to this kind of algorithm being simple to adapt and apply for a single- and multi-objective optimization problem [10–13]. From our previous work, OMPBIL with a multi-grid approach has been used to solve partial topology optimization of morphing aircraft wings, and it promotes better results than the original multi-objective population-based incremental learning (MPBIL) with a single grid element. Moreover, the work reveals that the opposition concept could improve the search performance of MPBIL. The question remains whether the performance of OMPBIL can benefit from the opposite concept or two learning rates. To make it be clearer, we compare the performance of OMPBIL and the performance of MPBIL with multi-learning rate. If the former technique can achieve better results, it means that the opposition concept significantly improves the performance of MPBIL. Therefore, this question will be addressed in this study. Furthermore, it has been found [14] that learning rate was the most affective with search performance of PBIL. Another way to improve the search performance of MPBIL is to use an adaptive learning rate method [15]. This method is categorized as self-learning adaptations, so the effectiveness of this technique needs to be addressed in this research.

Therefore, in this paper, the first objective is to apply the multi-grid approach (MG) approach to solve structural topology optimisation problems, whereas the second objective is comparative performance of the three variants of MPBIL. The performance improvements are based on an opposite-based concept, a multi-learning rate, and an adaptive learning rate, respectively. This research expects to improve the performance of the proposed MPBIL and MG approaches that lead to the obtaining of better design results than the original MPBIL with a single grid. The rest of this paper is organized as follows. Section 2 promotes the details of topology with single-ground and multi-ground design approaches for structural topology optimization. We introduce some novel methods for enhancing the performance of multi-objective, population-based incremental learning in Section 3. The performance index and statistical testing are given in the same section. Numerical experiments and the design results are proposed in Section 4; moreover, the design results and discussion are in Section 5. Finally, the conclusions of the study are in Section 6.

2. Topological Designs with Single-and Multi-Ground Design Approaches

2.1. Topological Designs with Ground Element Filtering

Topology optimization is one mathematical tool used in the conceptual design stage of engineering systems for finding the best structural layout from a given design domain. Topological design can perform using an optimization method and finite element analysis. This technique is started by defining design domain represented as the discrete structural members such as panels, truss, and frame as shown in Figure 1. The optimization method can be performed by varying the width or thickness of each element in the design domain between zero and the maximum value. All elements were discarded, if the element width/thickness value was zero. Otherwise, the element was retained. With this concept, optimization of the structural layout and component sizes is performed. Two popular, well known topological methods are the solid isotropic material with penalization (SIMP) approach and the homogenization method, which use gradient-based optimizers. Later, an alternative optimizer is evolutionary algorithms due to the fact they are robust, simple to use, derivative-free, and free from intermediate pseudo densities [8]. Complicated problems, such as partial topology, simultaneous topology, shape, and sizing optimization, can be performed within one optimization run [3,8,16,17] by using such algorithms. In this paper [8], they presented the comparative performance of multi-objective evolutionary algorithms (MOEAs) for solving structural topology optimization test problems based on ground element filtering technique. It has been found that MPBIL is the best optimizer in their study, which outperforms other MOEAs [8], so MPBIL is the only MOEA selected to improve its search performance in this research. Furthermore, the ground element filtering technique is also used in this study. The ground element filtering technique (GEF technique) is a simple numerical scheme that can apply to all kinds of optimizers, which can prevent the checkerboard pattern problem and at the same time decrease the number of design variables [8,18,19]. The idea uses two mesh grids of design domain with different resolutions. The lower resolution grid is provided for design variables, whereas the higher resolution is used as a finite element grid. The conversion between two grids relates to threshold value (ε) that is defined at the first time before optimization run. Therefore, this technique has been proved to be an efficient technique to suppress the checkerboard problem. Next, the details of GEF technique are seen in [8,18,19]. Later, a method for solving checkerboard pattern was presented by Guirguis and Aly [20]. They proposed that derivative-free level-set method for solving structural topology can solve the checkerboard problem. This new technique can avoid the main limitations of non-gradient methods: dependence on the objective value. Moreover, the boundaries of structure are smooth, but it does not directly depend on the decision variables. A very recent work in multi-objective topology optimization has been proposed to address the limitations of generating infeasible structures and expensive computational cost by using the technique called "graphics processing unit (GPU)" [21]. On the contrary, this technique has been commented on usefulness in the case of truss-like structures and the solved examples are simple, and obtained results are sub-optimal solutions [22]. Recent applications of topology optimization appeared in design of composite molding processes [23]. More recently, applications of topology optimization appeared in many fields, e.g., composite molding processes [23], optimal design of piezoelectric [24], phononic crystals design [25] and stator configurations [26].

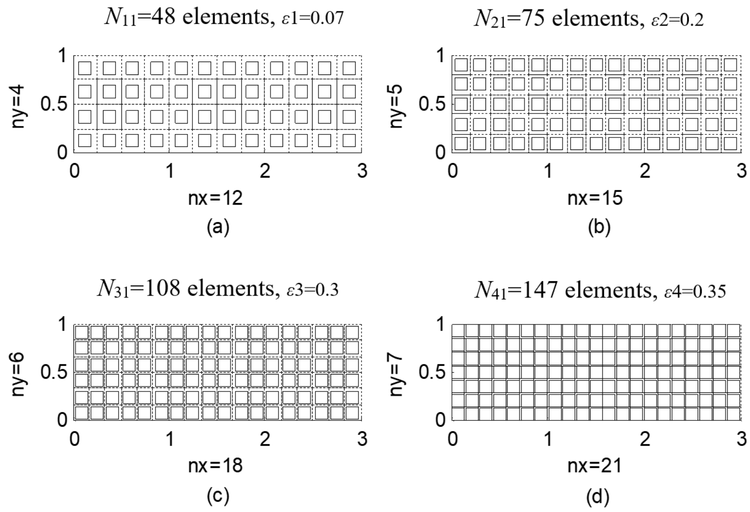

Figure 1. A ground elements set for MOP1, MOP3, and MOP4 (**a**) for $n = 1$; (**b**) for $n = 2$; (**c**) for $n = 3$, and (**d**) for $n = 4$.

2.2. Single-and Multi-Grid Ground Elements

The MG approach for topology optimization is an extension of MG strategy, which proposes to solve a truss structural optimization [1] and morphing wing structures [3]. At the present, we propose to apply this technique to a structural topology optimization problem. This technique has an improvement in both using the several ground resolutions. In this research, a ground structure has four sets of ground elements with different grid resolutions and the threshold value ε. The threshold value ε must be specified at the first stage before performing the optimization run. A special encoding and decoding scheme slightly changes from the previous work [3], but it is very important to the quality of final result. Especially, the threshold values are different in each grid resolution to prevent the checkerboard problem, which can occur in each grid. At the first stage, this scheme starts with defining the number of elements and the threshold values. The first set of ground elements has N_{11} elements, and the threshold value is set to be $\varepsilon 1$. Therefore, an example of a ground element set used in this study is the lowest resolution as number of elements $N_{11} = 48$ and $\varepsilon 1 = 0.07$ as shown in Figure 1. The second set has $N_{21} = 75$ elements and the threshold value is $\varepsilon 2 = 0.2$. Then, the third set has $N_{31} = 108$ segments and the threshold value is $\varepsilon 3 = 0.3$, whereas the last set has the numbers of ground elements and the threshold value is $N_{41} = 147$ segments and $\varepsilon 4 = 0.35$, respectively. As a result, $N_{41} \geq N_{31} \geq N_{21} \geq N_{11}$ and $\varepsilon 4 \geq \varepsilon 3 \geq \varepsilon 2 \geq \varepsilon 1$, respectively. Therefore, the variables and the threshold values for encoding/decoding scheme for the MG approach, which is improved from previous algorithm, can be detailed as shown in Algorithm 1. For using this algorithm, the MPBIL and its improved versions perform with binary design variables, whereas it needs the conversion of binary string to become a real design vector x before entering into this algorithm. Furthermore, the ground element set with its ε used in this research for multi-objective optimization problem (MOP) MOP1, MOP3 and MOP4 is shown in Figure 1. For the design problem MOP2, the design domain is different from the other problems. The details of the ground element sets and the threshold values are presented in Section 4.

Algorithm 1. Encoding and decoding scheme for a MG approach.

Initialization: Generate four sets of ground elements and define the threshold value of ε for each set.
Inputx sized $(N_{41} + 1) \times 1$.
Output: Thicknesses of ground elements.
Encoding
$x_1 \in [1, 4]$ is used for selecting a set of ground elements.
x_2 to $x_{N_{41}+1}$ are used for element thicknesses.
Decoding
1: Find $n = round(x_1)$ where $round(.)$ is a round-off operator.
2: If $n = 1$: x_2 to $x_{N_{41}+1}$ are set as N_{11} element thicknesses and $\varepsilon = \varepsilon 1$.
3: If $n = 2$: x_2 to $x_{N_{41}+1}$ are set as N_{21} element thicknesses and $\varepsilon = \varepsilon 2$.
4: If $n = 3$: x_2 to $x_{N_{41}+1}$ are set as N_{31} element thicknesses and $\varepsilon = \varepsilon 3$.
5: If $n = 4$: x_2 to $x_{N_{41}+1}$ are set as N_{41} element thicknesses and $\varepsilon = \varepsilon 4$.

3. Performance Enhancements of Multi-Objective, Population-Based Incremental Learning

This section briefly details the concept of MPBIL and its three variants.

3.1. Multi-Objective, Population-Based Incremental Learning

MPBIL is an extension of PBIL for solving a multi-objective optimization problem. This problem has more than one objective function, which promotes several solutions for this kind of problem, and it is called a Pareto solution set or a Pareto frontier. Rather than using a single probability vector, several probability vectors are used, so it is called a probability matrix. The matrix is used to maintain diversity of a binary population. At an initial step, the probability matrix has elements full of "0.5". Each row of the probability matrix or probability vector is updated by Hebb's rule [27] as follows

$$P_{ij}^{\text{new}} = P_{ij}^{\text{old}}(1 - LR) + b_j L_R \tag{1}$$

in which L_R is a PBIL learning rate, a small value usually recommends for the conventional operating [28], and b_j is the mean value of jth column of several binary solutions randomly selected from a current Pareto front. It is also useful to apply a mutation to probability matrix at some predefined probability as

$$P_{ij}^{new} = P_{ij}^{old}(1 - ms) + rand(0\,or\,1) \cdot ms \tag{2}$$

in which ms is mutation shift, and the default value is usually 0.2. For more details of MPBIL procedure, see [3].

3.2. Opposite-Based MPBIL

OMPBIL has been developed as an improved version of MPBIL [3]. Due to L_R affecting MPBIL performance, the issue is how to select a proper value of L_R for a general problem. It is expected to accelerate the convergence rate to find solution, as well as provide population diversity. Our previous work proposed the opposition-based concept embedded into MPBIL, which is an efficient technique that can upgrade MPBIL's performance. Therefore, the outline of OMPBIL algorithm includes the opposition-based concept, which is not included in this paper. More details can be found in [3].

3.3. Multi-Learning Rate

The second approach to enhance the performance of MPBIL is the use of multi-learning rate. This question arises from the previous method, when it is using two learning rates that are of an opposite quantity. The question remains whether the performance of OMPBIL can benefit from the opposite concept or by using two learning rates. MPBIL with multi-learning rate (MPBILMLR) is proposed to solve topological optimization and to compare with the opposition-based concept. This algorithm differs from the traditional MPBIL by using three learning rates (L_R = 0.25, 0.5, 0.75). The procedure of MPBILMLR algorithm is slightly different from OMPBIL. Therefore, the procedure of MPBIL with multi-learning rate algorithm is shown in Algorithm 2.

Algorithm 2. MPBIL with multi-learning rate.

Initialization Probability matrix $\mathbf{P} = [\mathbf{0.5}]_{l\times nb}$, Probability matrix $\mathbf{P}i = [\mathbf{0.5}]_{l/\mathrm{M}\times nb}$ where $i = 1, \ldots, \mathrm{M} = 3$, external Pareto archive **Pareto** = {}.

1: Generate a binary population **B** from **P**.

2: Decode the binary population to be $\mathbf{x}_{n\times Np}$ and find the objective values $\mathbf{f}_{m\times Np}$.

3: Update **Pareto** by replacing it with non-dominated solutions of union set **Pareto** $\cup$ **x**.

4: If the number of members in **Pareto** exceeds the predefined archive size N_A, remove some of them by using an archiving technique.

5: If the termination criterion is fulfilled, stop the procedure. Otherwise, go to step 6:

6: Update **P** and create a binary population

6.1: Set a binary population **B** = {}.

6.2: For i = 1 to l/M.

- 6.2.1: Select n_0 binary solutions from **Pareto** randomly.
- 6.2.2: Use $L_{R\mathrm{k}}$ = 0.25, 0.5, 0.75, for each $k = 1, \ldots, \mathrm{M}$. (For this research M = 3)
- 6.2.3: Update the ith row of **P** by using (1).
- 6.2.4: Generate the ith row of probability matrix **P**i using (2) and each L_{Rk}.
- 6.2.5: Generate $rand \in [0, 1]$ a uniform random number.
- 6.2.6: If $rand$ < the predefined mutation probability, update the ith row of **P1**, **P2** and **P3** using similar equation in [3].
- 6.2.7: Generate binary subpopulations **SB1**, **SB2** and **SB3** from the ith row of **P1**, **P2** and **P3**, respectively.
- 6.2.8: Set **B** = **B**$\cup$**SB1**$\cup$**SB2**$\cup$**SB3**

6.3: Next i.

7: Go to step 2.

3.4. Adaptive Learning Rate

The last method for MPBIL performance enhancement is using an adaptive learning rate, which proposes to modifythe learning rate during the entire process [28]. A small value of learning rate is usually recommended for conventional PBIL to keep the algorithm reliable, but it usually causes low convergence rate. To balance the reliability and speed of convergence in all iterations, the learning rate needs to adapt. A model of adaptive learning rate has been proposed by Yang et al. [15] that satisfies the previous conditions. That model is shown as follows

$$L_R = L_{R0} + (L_{RM} - L_{R0})e^{-(\frac{SI}{NT})} \tag{3}$$

in which SI is the successive iterations with improvements in the objective function in the most recent NT iterations. L_{R0} and L_{RM} are the minimum and maximum learning rates that the designer defines before an optimization run. The learning rate depends on the ratio of SI/NT. Additionally, the high value of this ratio means that it is possible to locate better solutions using its current probability matrix, and consequently the learning rate should be small. In contrast, a low value of this ratio means the current probability matrix which is insufficiency, so the learning rate should be increased. Moreover, the outline of multi-objective, population-based incremental learning with adaptive learning rate

(MPBILADLR) is slightly different from the traditional MPBIL, which uses Equation (3) to replace the original equation for finding L_R. This algorithm is shown as follow.

Algorithm 3. MPBIL with adaptive learning rate.

Initialization probability matrix $\mathbf{P} = [\mathbf{0.5}]_{l \times nb}$, external Pareto archive **Pareto** = {}.
1: Generate a binary population **B** from **P**.
2: Decode the binary population to be $\mathbf{x}_{n \times Np}$ and find the objective values $\mathbf{f}_{m \times Np}$.
3: Update **Pareto** by replacing it with non-dominated solutions of union set **Pareto**∪**x**.
4: If the number of members in **Pareto** exceeds the predefined archive size N_A, remove some of them by using an archiving technique.
5: If the termination criterion is fulfilled, stop the procedure. Otherwise, go to step 6:
6: Update **P**.
    6.1: For i = 1 to l.
    6.1.1: Select n_0 binary solutions from **Pareto** randomly.
    6.1.2: Generate L_R using (3).
    6.1.3: Update the ith row of **P** by using (1).
    6.1.4: Generate $rand \in [0, 1]$ a uniform random number.
    6.1.5: If $rand$ <the predefined mutation probability, update the ith row of P using similar equation in [3].
6.2: Next i.
7: Go to step 1.

3.5. The Performance Index and Non-Parametric Statistical Test

MPBIL and its enhanced versions are classified as MOEAs, while the obtained results are classified as approximate Pareto optimal frontiers. In comparing the searching performance of MOEAs, the methods are employed to solve design optimization problems with equivalent total number of function of evaluations for number of attempts. The approximate Pareto frontiers obtained from various MOEAs are then compared using a performance indicator, which is called a hyper-volume (HV) [29] indicator. This indicator represents the hyper-area above a Pareto frontier for bi-objective optimization problem as shown in Figure 2, whereas it is called hyper-volume for three objective functions and more. Therefore, HV sums up all discrete areas v_i or volumes of hyper-areas or hyper-volumes with respect to a given referent point, respectively.

A technique for comparing the performance of each MOEA in this research is a non-parametric statistical test, which is called the Friedman test. This technique has been used by Sleesongsom and Bureerat [30] for studying the performance of meta-heuristics (MHs) in solving the four-bar linkage path generation problems. The Friedman test is suitable for comparing more classifiers over multiple data sets.

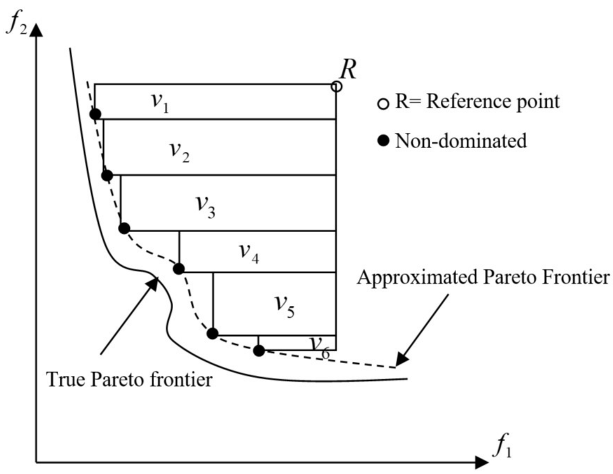

Figure 2. Hyper volume sums up all areas covered by the non-dominated solutions and a reference point.

4. Numerical Experiment

As mention earlier, the purpose of this research is to study the comparative performance of original MPBIL and its three variants with (WMG) and without the MG (WOMG) approach. Four design problems are used for testing performance of the proposed methods. The original MPBIL and three performance enhancements of MPBIL (OMPBIL, MPBILMLR, and MPBILADLR) are employed to solve multi-objective topology optimisation problems that have been detailed in the previous section. Each algorithm is used to solve an optimisation problem for 25 runs to measure its performance and consistency. For all design problems, all the algorithms are used with a population size of 35 and an iteration number of 400 whereas the external Pareto archive size is set to be 35 Non-dominated solutions obtained, so at the last iteration approximates the Pareto solutions. Therefore, four multi-objective problems are used for testing performance of MPBIL and performance enhancements of MPBIL, which has been proposed to study the comparative performance of some established multi-objective evolutionary algorithms (MOEAs) [8]. The problems are structural topology optimisation problem. The design problems are as follows:

MOP1: The topological design domain and loads are shown in Figure 3a. The structure is made of material with Young's modulus $E = 200 \times 10^9$ N/m^2, Poisson's ratio $\nu = 0.3$, and tensile yield strength $\sigma_{yt} = 200 \times 10^6$ N/m^2. The multi-objective design problem is set to minimize structural compliance and normalized mass as:

$$\min_{\rho^{GEF}} \{c,\ r\} \tag{5}$$

subject to

$$\begin{gathered} c \le 5c_{\min} \\ 0.2 \le r \le 0.8 \\ \rho_i \in \{0.0001, 1\} \end{gathered}$$

where ${\rho_i}^{GEF}$ is the value of ith design variable; ρ_i is the thickness of ith finite element; m is the structural mass; $r = m(\boldsymbol{\rho})/m(\boldsymbol{\rho}^u)$ is the normalized mass or ratio of structural mass to maximum mass; c is the structural compliance; and $c_{\min} = c(\boldsymbol{\rho}^u)$. The first constraint is added to prevent topologies with a low global stiffness (or highly compliant structures) being included in the Pareto archive. The bound constraints are set as $\rho_i \in \{0.0001, 1\}$. The parameter ε is set 0.3 and $[0.08, 0.1, 0.25, 0.3]^T$ for all MPBILs with WOMG and WMG design approach, respectively. The number of element for single grid is set as highest resolution. A set of MG elements is use for this problem and show in Figure 1. The mean hypervolumes of the fronts of MOP1 for all optimisation runs are given in Table 1, where the referent point for computing hypervolumes is set to be $\{2.5 \text{ kNm}, 2.5\}^{\mathrm{T}}$.

MOP2: The second design problem promotes three objective functions, where the design domain and load illustrate in Figure 3b. The structure makes up the same material as MOP1. The multi-objective design problem can be written as:

$$\min_{\rho^{GEF}} \{c1,\ c2,\ r\} \tag{6}$$

subject to

$$\begin{gathered} c1 \le 5c_{1,\min} \\ c2 \le 5c_{2,\min} \\ 0.2 \le r \le 0.8 \\ \rho_{\mathrm{i}} \in \{0.0001, 1\} \end{gathered}$$

where c_1 is the structural compliance due to the first load case and c_2 is the structural compliance due to the second load case, $c_{1,\min} = c_1(\boldsymbol{\rho}^u)$, and $c_{2,\min} = c_2(\boldsymbol{\rho}^u)$. A number of ground elements set, which uses in this study is [48,63,108,130], while the number of element for single grid is set as the highest resolution. The threshold parameter ε is set to be 0.35 and $[0.07, 0.2, 0.3, 0.35]^T$ for all MPBILs with WOMG and WMG approach, respectively. There are different from other problem due to the difference of design domain. The mean hypervolumes of the fronts of MOP2 for all optimization runs are given in Table 1, in whichthe referent point for computing the hypervolumes is set to be {1.5 kNm, 1.5 kNm, 1.5}^{T}.

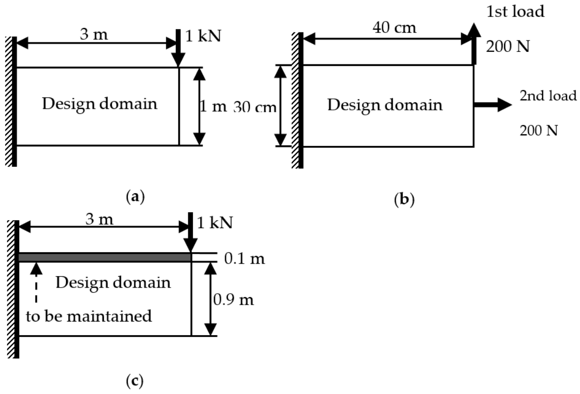

Figure 3. Structural design domains: (**a**) design domain of MOP1 & MOP3; (**b**) design domain of MOP2; (**c**) design domain of MOP 4. Reproduce with permission from [8], Taylor & Francis, 2011.

MOP3: The design problem has the same design conditions as set for MOP1 with the exception of the range of design variables. Addition constraints are $\sigma_{\max}^{eqv} \leq$ σyt and $\rho_i \in$ {0.000001 m, 0.01 m}, in which $\sigma_{\max}^{eqv}$ is the maximum value of Von Mises stress (equivalent stress) on the ground elements. A set of MG elements is shown in Figure 1, and the number of elements for single grid is set as the highest resolution. In addition, the threshold parameter ε is set to be 0.3 and $[0.08, 0.1, 0.25, 0.3]^T$ for all MPBILs with WOMG and WMG design approach, respectively. The mean hypervolumes of the fronts of MOP3 for all optimization runs are given in Table 1, in which the reference point for computing the hypervolumes is set to be {3.5 kNm, 3.5}^{T}.

MOP4: The design conditions of MOP4 are similar to MOP3, except in this design problem the top row finite elements are not assigned as design variables (unchanged) as displayed in Figure 3c, and the first objective of this problem changes to maximizing the first mode eigenvalue of structure (λ_1). Note that all of design problems use a membrane finite element formulation for structural analysis. The number of ground elements and the parameter ε of MOP4 are similar to MOP3. The mean hypervolumes of the fronts of MOP4 for all optimization runs are given in Table 1, in which the referent point for computing the hypervolumes is set to be {1.0 rad^2/s^2, 2.0}^{T}.

Table 1. Performance comparison based on hypervolume (HV) [1].

	MOP1		MOP2		MOP3		MOP4	
	WMG	**WOMG**	**WMG**	**WOMG**	**WMG**	**WOMG**	**WMG**	**WOMG**
MPBIL	0.8553	0.8255	0.7255	0.6420	0.7951	0.7229	0.7195	0.6219
OMPBIL	0.8556	0.8426	0.7259	0.6430	0.7968	0.7438	0.6723	0.6403
MPBILMLR	0.8115	0.7739	0.7212	0.6285	0.7016	0.5976	0.6167	0.5651
MPBILADLR	0.8543	0.8371	0.7240	0.6385	0.7954	0.7407	0.6404	0.6292

[1] WMG, with multi-grid approach; WOMG, without multi-grid approach; MPBIL, multi-objective population-based incremental learning; OMPBIL, opposite-based, multi-objective, population-based incremental learning; MPBIL MLR, MPBIL with multi-learning rate; MPBIL ADLR, multi-objective, population-based incremental learning with adaptive learning rate.

5. Design Results

The comparative performance of original MPBIL and the performance enhancements of MPBILs with MG and without MG approach for solving the design problems of MOP1–4 are given in Table 1, which compare based on HV indicator. It should be noted that all of the approximate Pareto fronts of the four design problems obtained from using the proposed MPBILs are normalized before calculating HV, as shown in Table 1. The highest mean of HV for each design problem is highlighted with grey color. The table shows OMPBIL promotes almost the best results for MOP1–4 except in case MOP4-WMG. Therefore, it is believed that the performance of OMPBIL is better result than the original MPBIL and their enhancements. In this study, the Friedman test and the Tukey–Kramer test are used for a statistical test to prove the significance of proposed algorithm. These tools are built-in functions in MATLAB/Octave. From our testing, the Friedman test gives OMPBIL has 1st rank, whereas the second rank is MPBIL at p-value (0.0002) < α(0.05) as shown in Table 2. It can be summarized that OMPBIL is the best performing algorithm for solving problem case MOP1–4. For multiple comparisons, we used the Tukey-Kramer test. The mean column ranks of OMPBIL are significantly different from MPBILMLR. The second best optimizer is MPBIL, whereas the third best is MPBILADLR. In addition, the worst optimizer for this design case is MPBILMLR. No questionable opposition concept is beneficial to improving the performance of MPBIL.

Table 2. Average ranking and p-value of MPBIL, and enhanced performance of MPBIL achieved by Friedman test.

Average Ranking of Each AlgorithmFriedman				p-Value
MPBIL	**OMPBIL**	**MPBILMLR**	**MPBILADLR**	
2.6250 (2)	3.8750 (1)	1 (4)	2.5000 (3)	0.0002

The average HV for all optimizers of each problem with MG and WOMG approach is shown in Table 3, which is summed along each column from Table 1. This table shows that the design problems with MG approach give higher HV than the design problem without MG approach in all design problems. Friedman test of average result in Table 3 can prove that the design problem with MG technique significantly outperforms WOMG technique at p-value (0.0455) < α(0.05). Furthermore, the best HVs of all cases give higher hypervolume than the previous work by [8] in all design cases, so OMPBIL with MG can improve the design results.

Table 3. Performance comparison of each MOP with and without MG for all algorithms.

Design Problems	Average Hypervolume	
	WMG	**WOMG**
MOP1	0.8442	0.8198
MOP2	0.7242	0.6380
MOP3	0.7722	0.7013
MOP4	0.6622	0.6141
Average Ranking (p-value = 0.0455)	2 (1)	1 (2)

Figures 4–7 shows some optimum topologies. The topologies in all figures are captioned with (a), which obtains from the best run of OMPBIL with multi-grid when solving each MOP with various r values. All figures are captioned with (b); they display the optimum topologies that are obtained from optimizing the design problem MOP1–4 with various r values by using MPBIL without multi-grid. These topologies are represented by the same technique from the previous work [8]. This shows that the topologies from OMPBIL with MG are better than the MPBIL technique without MG, and they can compare with the previous work using binary population-based incremental learning (BPBIL) and optimality criteria method (OCM) technique [8]. The optimum topologies are mostly from the ground elements with medium (MOP1 and MOP2) and low (MOP3, MOP4) resolutions. Therefore, the topology with the highest resolution is lower than the previous work by [8]. The lower resolution means lower computational time consumption. The use of highest ground element resolution is not the

best selection for all design problems. However, in practice, a designer never knows which resolution is the most suitable for design problem, and employing the multi-grid approach is an advantage.

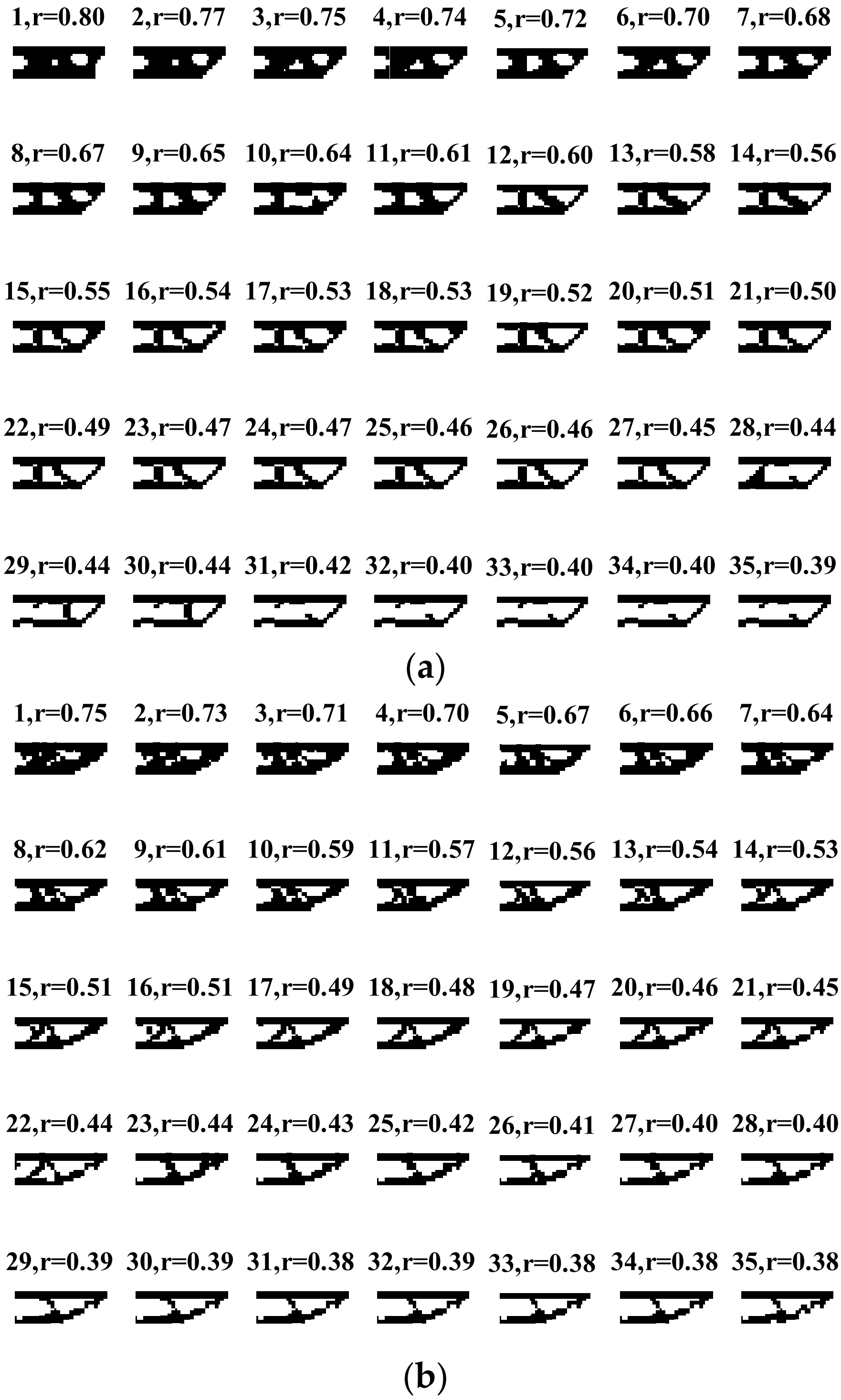

Figure 4. Topologies of MOP1: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

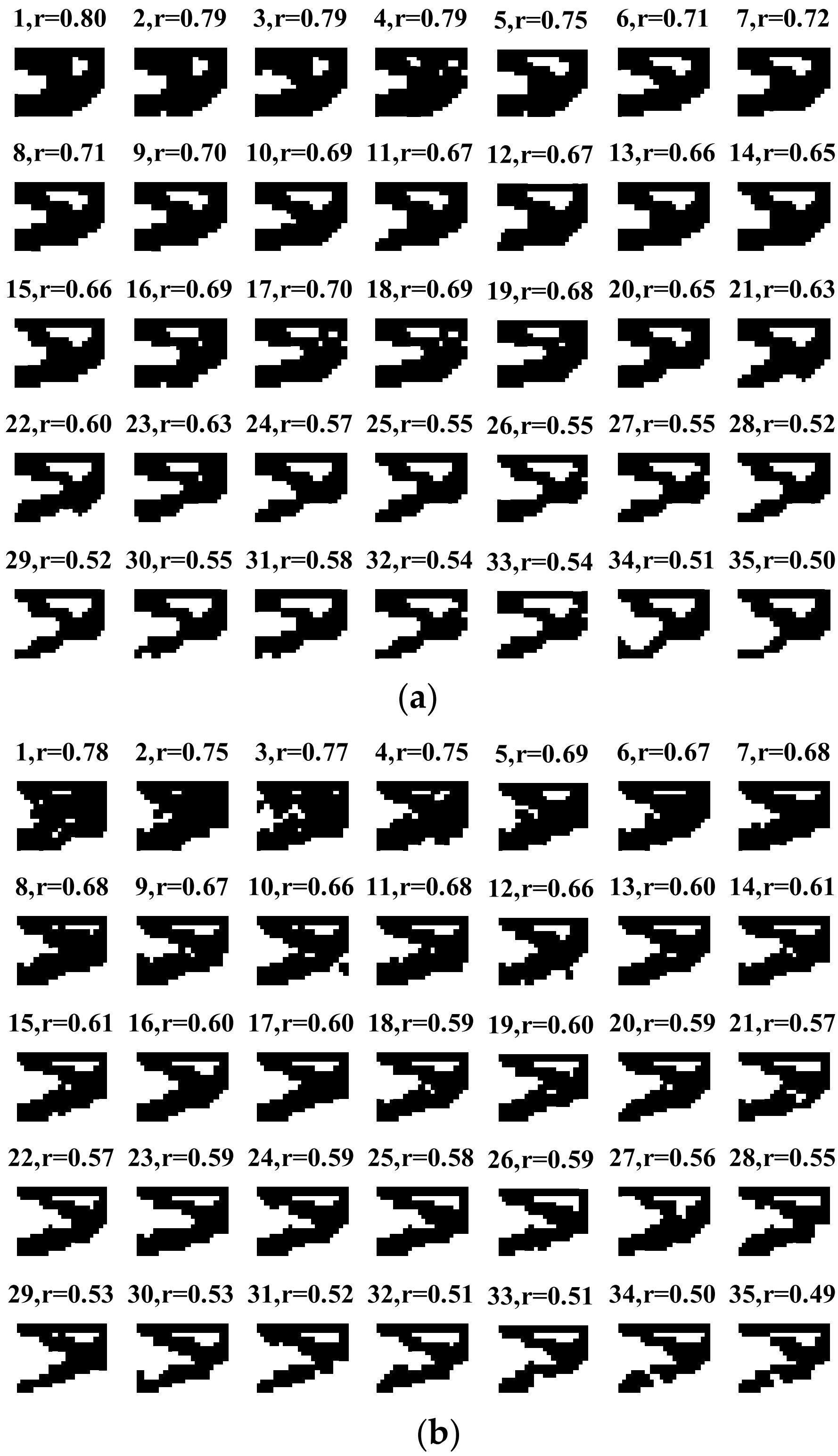

Figure 5. Topologies of MOP2: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

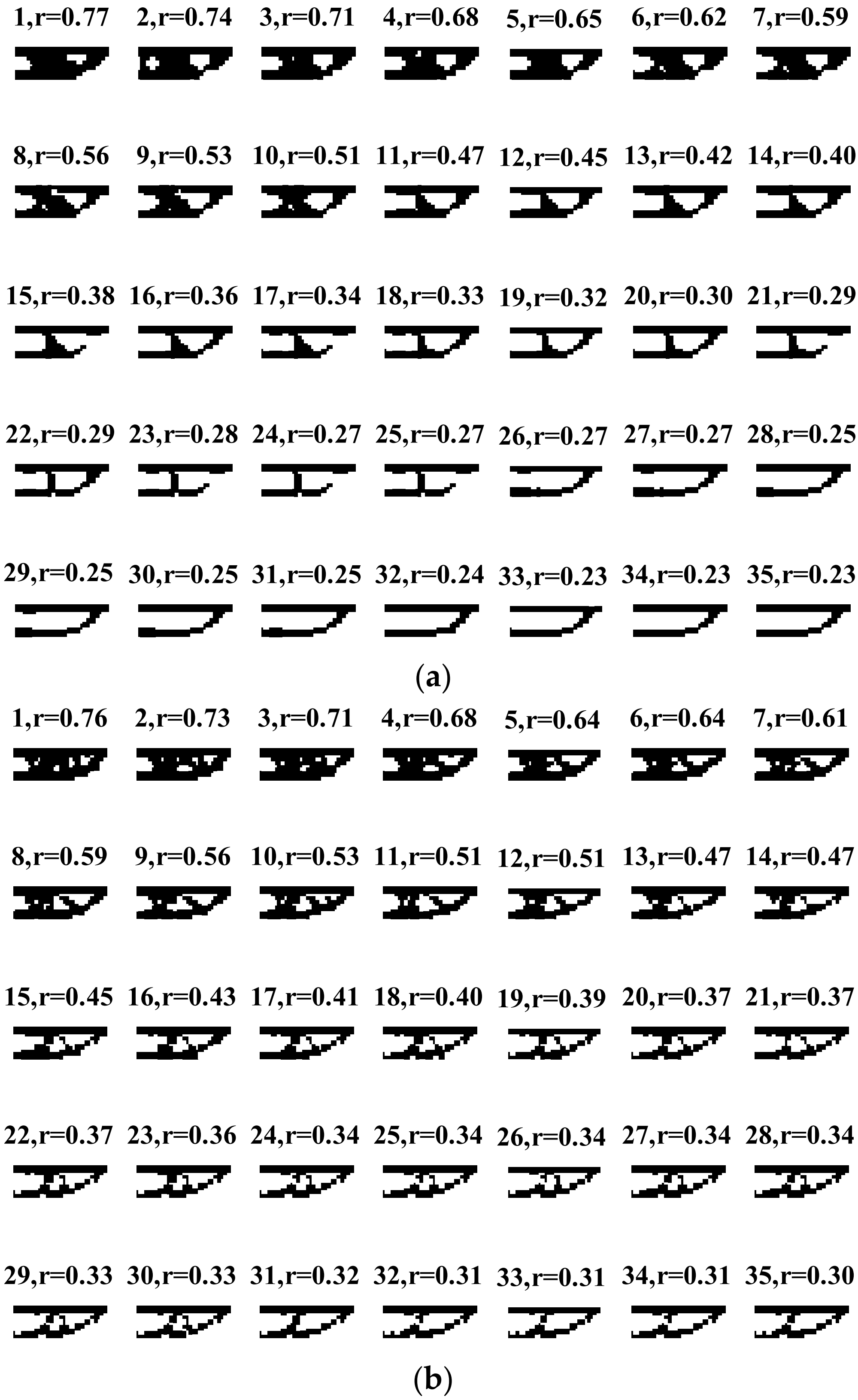

Figure 6. Topologies of MOP3: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

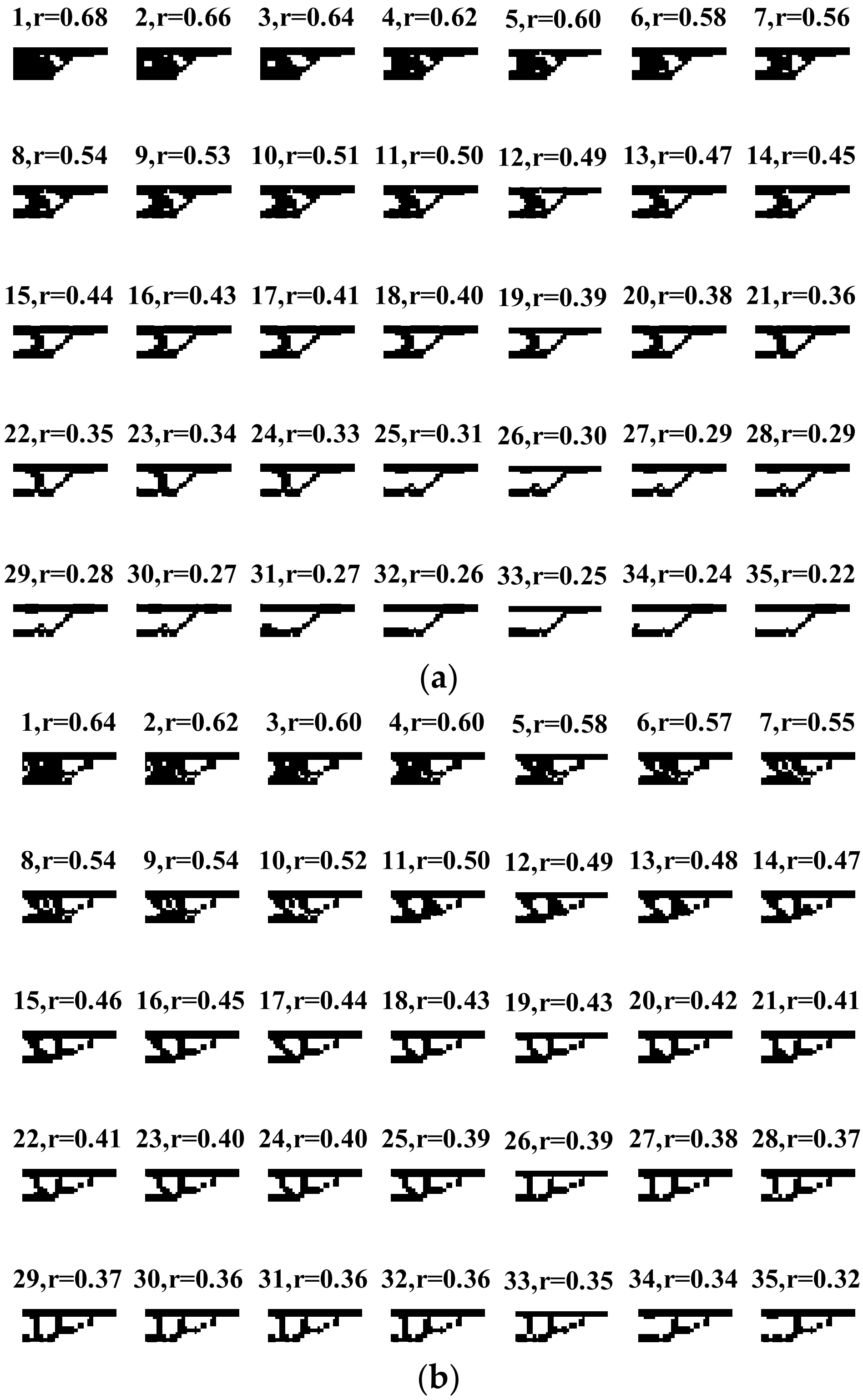

Figure 7. Topologies of MOP4: (**a**) OMPBIL with multi-grid; and (**b**) MPBIL without multi-grid.

6. Conclusions

The purposes of this work are the demonstration of the performance comparison of an original MPBIL and their performance enhancement, and the MG approach for multi-objective structural topology optimization problems, respectively. Among the performance enhancements of MPBIL, OMPBIL outperforms other techniques. It promotes the opposition-based concept, which can improve the search performance of MPBIL. The use of MPBILs in combination with the MG approach is well capable of solving multi-objective structural topology optimization. The resulting topologies obtained from using OMPBIL are close to those obtained from the classical gradient-based approach. The new design strategy is a procedure for structural topology optimization, which uses multiple ground element resolutions, so the MG approach is more efficient than using single-resolution ground elements in the sense that the suitable grid resolution is automatically detected and used in one optimization run. These conclusions are very similar those obtained in our previous work [3]. In addition, the use of the MG approach combined with ground element filtering for alleviating checkerboards is effective. In future work, the proposed method is extended to solve topology optimization with uncertainty.

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