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# Structural Optimization of a High-Speed Press Considering Multi-Source Uncertainties Based on a New Heterogeneous TOPSIS

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**Abstract:** In order to achieve high punching precision, good operational reliability and low manufacturing cost, the structural optimization of a high-speed press in the presence of a set of available alternatives comprises a heterogeneous multiple-attribute decision-making (HMADM) problem involving deviation, fixation, cost and benefit attributes that can be described in various mathematical forms due to the existence of multi-source uncertainties. Such a HMADM problem cannot be easily resolved by existing methods. To overcome this difficulty, a new heterogeneous technique for order preference by similarity to an ideal solution (HTOPSIS) is proposed. A new approach to normalization of heterogeneous attributes is proposed by integrating the possibility degree method, relative preference relation and the attribute transformation technique. Expressions for determining positive and negative ideal solutions corresponding to heterogeneous attributes are also developed. Finally, alternative structural configurations are ranked according to their relative closeness coefficients, and the optimal structural configuration can be determined. The validity and effectiveness of the proposed HTOPSIS are demonstrated by a numerical example. The proposed HTOPSIS can also be applied to structural optimization of other complex equipment, because there is no prerequisite of independency among various attributes for its application.

**Keywords:** high-speed press; structural optimization; uncertainties; heterogeneous TOPSIS (HTOPSIS); normalization; possibility degree method; relative preference relation

## 1. Introduction

The high-speed press is a kind of automatic punching machine tool equipped with a high-speed automatic feeding device for metal sheet that is capable of continuously punching at a high speed. It has been widely applied in the automotive, aerospace and instrument industries due to its advantage of clean and green production. With the increasing demand placed on the amount and quality of stamping parts, the development of a new large-tonnage ultra-precision high-speed press has become an urgent task for manufacturers and engineering designers in the field [1]. Structural optimization of the high-speed press is very important in the early design phase, for the purposes of achieving high punching precision, good operational reliability, and low manufacturing cost [2], which constitute a multiple-attribute decision-making (MADM) problem when a set of configurations are provided for evaluation against multiple attributes [3].

The technique for order preference by similarity to an ideal solution (TOPSIS) [4] is widely utilized to solve MADM problems, due to its advantages of simple calculation, reasonable evaluation results, and flexibility in application compared with other MADM methods [5–9]. The basic principle of TOPSIS is that the optimal solution to a MADM problem should have the shortest distance from the positive ideal solution (PIS), and the farthest distance from the negative ideal solution (NIS), and that the distance can be measured by the Euclidean distance [10]. Scholars have conducted a great deal of research on the development of TOPSIS for solving various MADM problems in recent years. Fan et al. [11] proposed a new method for dealing with stochastic MADM problems based on the ideal and anti-ideal cumulative distribution functions. Kaveh et al. [12] transformed the multi-objective decision making (MODM) problem into a bi-objective one based on TOPSIS, and proposed an integrated multi-objective framework to solve MODM problems including both tangible and intangible factors based on the extended efficient epsilon-constraint method. Sharma and Singhal [13] applied fuzzy TOPSIS to the selection of the best procedural approach for solving facility layout-planning problems. Ye and Li [14] proposed an extended fuzzy TOPSIS by utilizing possibility theory to deal with MADM problems with fuzzy numbers. Baykasoğlu and Gölcük [15] proposed a hybrid MADM method for modelling complex decision-making problems by integrating fuzzy TOPSIS and fuzzy cognitive maps. Xu and Zhang [16] developed a new MADM approach by integrating the TOPSIS and maximizing deviation methods, in which the evaluation information provided by the decision-maker is described as hesitant fuzzy elements, and the information about attribute weights is incomplete. Maldonado-Macías et al. [17] proposed an intuitionistic fuzzy TOPSIS for evaluating advanced manufacturing technology considering ergonomic compatibility attributes, and applied it to the selection of numerically controlled milling machines. Arabzad et al. [18] evaluated suppliers according to the criteria arising from strengths, weaknesses, opportunities and threats analysis in the MADM of suppliers, and solved the resulting MADM problem by fuzzy TOPSIS. Wang and Chen [19] proposed a novel method by integrating the interval-valued intuitionistic fuzzy (IVIF) sets, linear programming, and TOPSIS to solve MADM problems in IVIF environments. All of the above approaches were proposed to solve MADM problems with attributes in uniform mathematical descriptions.

However, the attributes entailed by structural optimization of the high-speed press are heterogeneous, because multiple objectives and multi-source uncertainties need to be considered in decision-making. Specifically, the high punching precision of a high-speed press is usually ensured by the ideal mechanical performance indices of its key components. The stiffness indicated by maximum deformation is a cost attribute that can be described as a real number. The strength indicated by maximum equivalent stress can be regarded as a fixation attribute, since the optimization of the other mechanical performance indices usually moves its value to the allowable stress, which can be described as an interval number considering the uncertain material properties and manufacturing errors of the key components [20]. The good operational reliability of the high-speed press can be achieved by moving the natural frequencies of its key components away from the punching and working frequencies [21]. Thus, the natural frequencies of the key components closest to the punching and working frequencies can be chosen as the attributes for evaluating the operational reliability of the high-speed press, which are deviation attributes that can be described as interval numbers due to material uncertainties and manufacturing errors. The low manufacturing cost of the high-speed press is usually ensured by its good manufacturability, which is a benefit attribute, and should be described as a fuzzy linguistic term due to the cognitive fuzziness in the early design phase. Therefore, structural optimization of the high-speed press is a heterogeneous MADM (HMADM) problem involving benefit, cost, deviation and fixation attributes, the values of which are described in various mathematical forms such as real numbers, interval numbers, fuzzy linguistic terms, triangular fuzzy numbers (TFNs) and so on. None of the TOPSIS methods mentioned above are able to solve HMADM problems with deviation and fixation attributes. Although the heterogeneous axiomatic design (HAD) method proposed in our previously work [22] can solve HMADM problems with deviation and fixation attributes, the application of HAD has the strict prerequisite that all the attributes in a HMADM problem be independent of each other [23]. It is obvious that such a prerequisite cannot be satisfied in the structural optimization of a high-speed press, since mechanical performance indices, such as the stiffness and strength of the same component, are obviously interdependent. Moreover, the computation of information content based on distance measure for ranking different competing designs may be tedious and inefficient in the implementation of HAD.

This work presents a new HMADM method for dealing with deviation and fixation attributes entailed by high-speed press optimization, which has no prerequisite of independency among heterogeneous attributes, and is also applicable to other HMADM problems. The proposed HMADM method relies on TOPSIS, considering its advantages over other MADM methods, and is therefore named heterogeneous TOPSIS (HTOPSIS) for the sake of brevity. A new approach for normalizing heterogeneous attributes is proposed based on vector transformation by integrating the possibility degree method, relative preference relation and the attribute transformation technique. Expressions for determining the positive and negative ideal solutions corresponding to heterogeneous attributes are also developed.

The rest of the paper is organized as follows. Section 2 constructs the HMADM model of the high-speed press structural optimization. Section 3 describes the new HTOPSIS method for solving the resulting HMADM model. Section 4 demonstrates the implementation of the proposed HTOPSIS, as well as its validity and superiority in a complex design problem. Conclusions are made in Section 5.

## 2. HMADM Modeling of a High-Speed Press Structural Optimization Problem

As shown in Figure 1, the structural optimization of a high-speed press aimed at achieving high punching precision, good operational reliability and low manufacturing cost involves benefit, cost, deviation and fixation attributes. Any of the attributes can be described as real numbers, fuzzy linguistic terms, interval numbers and TFNs due to the multi-source uncertainties that must be considered in the optimization process. Benefit attributes must be the largest possible, while cost attributes must be the smallest possible. A fixation attribute must be the closest possible to a given value. A deviation attribute must be the most different possible from a given value.

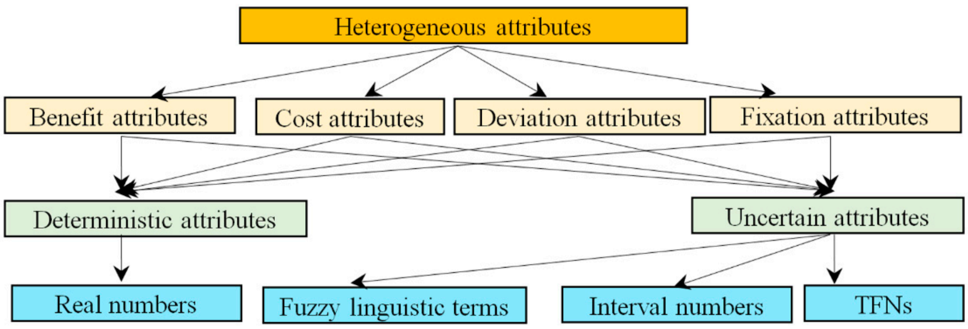
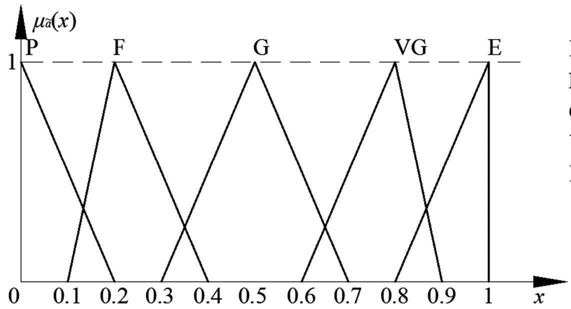



**Figure 1.** Heterogeneous attributes defined in the structural optimization of high-speed press. TFNs: triangular fuzzy numbers.

The alternative and attribute sets for the high-speed press structural optimization with $m$ alternatives to be evaluated from $n$ attributes can be described as $S = \{s_1, s_2, \ldots, s_m\}$ and $A = \{a_1, a_2, \ldots, a_n\}$, respectively. The attribute set $A$ can be divided into three subsets $A_1 = \{a_1, a_2, \ldots, a_{j_1}\}$, $A_2 = \{a_{j_1+1}, a_{j_1+2}, \ldots, a_{j_2}\}$, $A_3 = \{a_{j_2+1}, a_{j_2+2}, \ldots, a_n\}$; $1 \le j_1 \le j_2 \le n$, the corresponding attribute values of which are expressed as fuzzy linguistic terms, interval numbers and real numbers, respectively. Supposing that $N_1 = \{1, 2, \ldots, j_1\}$, $N_2 = \{j_1+1, j_1+2, \ldots, j_2\}$ and $N_3 = \{j_2+1, j_2+2, \ldots, n\}$ are the subscript sets of subsets $A_1, A_2, A_3$, it holds that $N_1 \cup N_2 \cup N_3 = \{1, 2, \ldots, n\}$. Let the rating of the $i$th alternative structural configuration $s_i (i = 1, 2, \ldots, m)$ on the $j$th attribute $a_j (j = 1, 2, \ldots, n)$ be denoted by $f_{ij}$, the decision matrix for the HMADM model of the structural optimization problem can be obtained as

$$\mathbf{F} = [f_{ij}]_{m\times n} = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{bmatrix}, \tag{1}$$

where $f_{ij}$ is a fuzzy linguistic term denoted by $f_{ij} = y_{ij}$, $y_{ij} \in \{\text{Poor, Fair, Good, Very good, Excellent}\}$ when $j \in N_1$, which can be transformed into a TFN according to the membership functions in Figure 2; $f_{ij}$ is an interval number that can be described as $f_{ij} = [f_{ij}{}^L, f_{ij}{}^U]$ when $j \in N_2$ and $f_{ij}$ is a real number when $j \in N_3$.




P:Poor (0,0,0.2)
F:Fair (0.1,0.2,0.4)
G:Good(0.3,0.5,0.7)
VG:Very Good(0.6,0.8,0.9)
E:Excellent(0.8,1,1)

**Figure 2.** TFNs of fuzzy linguistic terms.

## 3. HTOPSIS for Solving the HMADM Model with Deviation and Fixation Attributes

### *3.1. New Normalization Approach for Heterogeneous Attributes*

The heterogeneous attributes entailed by high-speed press structural optimization are incommensurable due to the difference in their metrics. Thus, the normalization of heterogeneous attributes is of great importance in the implementation of HTOPSIS. As concluded by Jahan and Edwards [24], vector transformation can retain the differences among all the attribute values after normalization, and it is frequently applied to the normalization of different attributes in ranking. However, all of the currently available vector transformation methods focus only on the normalization of benefit and cost attributes, and cannot handle the deviation and fixation ones, especially when they are described in different mathematical forms.

In order to solve the HMADM model describing the high-speed press optimization problem (Section 2), the deviation and fixation attributes are firstly transformed into benefit and cost attributes; then, the equivalent and normalized attribute values of all the alternatives are calculated based on the possibility degree method and relative preference relation. Specifically, a deviation attribute is firstly transformed into a benefit one with zero NIS; then, attribute values smaller than 0 are symmetrically mapped about 0 to ensure that the attribute values of all the alternatives are nonnegative; and finally, the deviation attribute can be normalized in the same way as a benefit attribute. The normalization approach for a fixation attribute is similar. The only difference is that the fixation attribute should be firstly converted into a cost attribute with zero PIS before normalization.

Without loss of generality, the deviation attribute is taken as an example, here, to explain the normalization approach when the attribute value is described as a real number, interval number and TFN. Supposing that the normalized decision matrix of the HMADM problem is described as $\mathbf{R} = [r_{ij}]_{m\times n}$, where $r_{ij}$ is the $j$th normalized attribute value of the $i$th alternative configuration, the derivations for the normalization of heterogeneous attributes are given in detail hereinafter.

### 3.1.1. Normalization of the Attribute Described as Real Number

For a benefit or cost attribute described as real number $f_{ij}$, it holds

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^{m} f_{ij}^2}}. \tag{2}$$

Supposing that the value of a deviation attribute is described as a real number $f_{ij}$ and the given constant to be deviated from is $k_j$, the attribute value is $f'_{ij} = f_{ij} - k_j$ when the deviation attribute is transformed into the benefit one with zero NIS. Then, the equivalent attribute values for the alternatives with $f'_{ij} < 0$ can be obtained as $f''_{ij} = \left| f_{ij} - k_j \right|$ by symmetrical transformation. Finally, the normalized attribute value $r_{ij}$ can be calculated by

$$r_{ij} = \frac{\left| f_{ij} - k_j \right|}{\sqrt{\sum_{i=1}^{m} (f_{ij} - k_j)^2}}. \tag{3}$$

### 3.1.2. Normalization of the Attribute Described as Interval Number Based on Possibility Degree Method

For a benefit or cost attribute described as interval number $f_{ij} = [f_{ij}^L, f_{ij}^U]$, it holds

$$r_{ij} = [r_{ij}^L, r_{ij}^U] = \left[ \frac{f_{ij}^L}{\sqrt{\sum_{i=1}^{m} \frac{1}{2}(f_{ij}^{L2} + f_{ij}^{U2})}}, \frac{f_{ij}^U}{\sqrt{\sum_{i=1}^{m} \frac{1}{2}(f_{ij}^{L2} + f_{ij}^{U2})}} \right]. \tag{4}$$

Supposing that the value of a deviation attribute is described as an interval number $f_{ij} = [f_{ij}^L, f_{ij}^U]$ and the given interval constant to be deviated from is $[k_j^L, k_j^U]$, the attribute value is $f'_{ij} = [f_{ij}^L - k_j^U, f_{ij}^U - k_j^L]$ when the deviation attribute is transformed into the benefit one with zero NIS. According to the possibility degree method for comparing two interval numbers [25], the probability of interval $f'_{ij}$ being not greater than 0 can be calculated as

$$P(f'_{ij} \le 0) = \begin{cases} 0, & f_{ij}^L - k_j^U \ge 0; \\ \frac{-f_{ij}^L + k_j^U}{f_{ij}^U - f_{ij}^L - k_j^L + k_j^U}, & f_{ij}^L - k_j^U < 0 \le f_{ij}^U - k_j^L; \\ 1, & f_{ij}^U - k_j^L \le 0. \end{cases} \tag{5}$$

With the attribute values satisfying $P(f'_{ij} \le 0) > 0.5$ symmetrically mapped about 0, the equivalent attribute values of all the alternative configurations can be described as

$$f''_{ij} = \begin{cases} [k_j^L - f_{ij}^U,\ k_j^U - f_{ij}^L], & P(f'_{ij} \le 0) > 0.5; \\ [f_{ij}^L - k_j^U,\ f_{ij}^U - k_j^L], & P(f'_{ij} \le 0) \le 0.5. \end{cases} \tag{6}$$

Finally, the normalized attribute value $r_{ij}$ can be calculated as

$$r_{ij} = [r_{ij}^L,\ r_{ij}^U] = \begin{cases} \left[ \frac{k_j^L - f_{ij}^U}{c_j^*},\ \frac{k_j^U - f_{ij}^L}{c_j^*} \right], & P(f'_{ij} \le 0) > 0.5; \\ \left[ \frac{f_{ij}^L - k_j^U}{c_j^*},\ \frac{f_{ij}^U - k_j^L}{c_j^*} \right], & P(f'_{ij} \le 0) \le 0.5. \end{cases} \tag{7}$$

where $c_j^* = \sqrt{\sum_{i=1}^{m} \frac{1}{2}\left[\left(f_{ij}^L - k_j^U\right)^2 + \left(f_{ij}^U - k_j^L\right)^2\right]}$.

3.1.3. Normalization of the Attribute Described as TFN Based on Relative Preference Relation

For a benefit or cost attribute described as TFN $\tilde{f}_{ij} = (f_{ij}^L, f_{ij}^M, f_{ij}^U)$, it holds

$$r_{ij} = (r_{ij}^L, r_{ij}^M, r_{ij}^U) = \left(\frac{f_{ij}^L}{c_j^*}, \frac{f_{ij}^M}{c_j^*}, \frac{f_{ij}^U}{c_j^*}\right), \tag{8}$$

where $c_j^* = \sqrt{\sum_{i=1}^{m} \frac{1}{3}\left(f_{ij}^{L2} + f_{ij}^{M2} + f_{ij}^{U2}\right)}$.

Supposing that the value of a deviation attribute is described as TFN $\tilde{f}_{ij} = (f_{ij}^L, f_{ij}^M, f_{ij}^U)$ and the given TFN to be deviated from is $\tilde{k}_j = (k_j^L, k_j^M, k_j^U)$, the attribute value is $\tilde{f}'_{ij} = (f_{ij}^L - k_j^U, f_{ij}^M - k_j^M, f_{ij}^U - k_j^L)$ when the deviation attribute is transformed into the benefit one with zero NIS. According to the relative preference relation between two TFNs [26], the preference degree of TFN $\tilde{f}'_{ij}$ over 0 can be calculated as

$$\mu_P(\tilde{f}'_{ij}, 0) = \frac{1}{2}\left(\frac{f_{ij}^L - k_j^U + 2(f_{ij}^M - k_j^M) + f_{ij}^U - k_j^L}{2\|T\|} + 1\right), \tag{9}$$

where $\|T\| = \begin{cases} \frac{(t^{L*}-t^{U-})+2(t^{M*}-t^{M-})+(t^{U*}-t^{L-})}{2}, \ t^{L*} \geq t^{U-}; \\ \frac{(t^{L*}-t^{U-})+2(t^{M*}-t^{M-})+(t^{U*}-t^{L-})}{2} + 2(t^{U-} - t^{L*}), \ t^{L*} < t^{U-}. \end{cases}$

$$\begin{aligned} t^{L*} &= \max(f_{ij}^L - k_j^U, 0);\ t^{M*} = \max(f_{ij}^M - k_j^M, 0);\ t^{U*} = \max(f_{ij}^U - k_j^L, 0); \\ t^{L-} &= \min(f_{ij}^L - k_j^U, 0);\ t^{M-} = \min(f_{ij}^M - k_j^M, 0);\ t^{U-} = \min(f_{ij}^U - k_j^L, 0). \end{aligned}$$

With the attribute values satisfying $\mu_P(\tilde{f}'_{ij}, 0) < 0.5$ symmetrically mapped about 0, the equivalent attribute values of all the alternatives can be described as

$$\tilde{f}''_{ij} = \begin{cases} (k_j^L - f_{ij}^U, k_j^M - f_{ij}^M, k_j^U - f_{ij}^L), & \mu_P(\tilde{f}'_{ij}, 0) < 0.5; \\ (f_{ij}^L - k_j^U, f_{ij}^M - k_j^M, f_{ij}^U - k_j^L), & \mu_P(\tilde{f}'_{ij}, 0) \geq 0.5. \end{cases} \tag{10}$$

Finally, the normalized attribute value $r_{ij}$ can be calculated as

$$r_{ij} = \left(r_{ij}^L, r_{ij}^M, r_{ij}^U\right) = \begin{cases} \left(\frac{k_j^L - f_{ij}^U}{c_j^*}, \frac{k_j^M - f_{ij}^M}{c_j^*} \frac{k_j^U - f_{ij}^L}{c_j^*}\right), \ \mu_P(\tilde{f}'_{ij}, 0) < 0.5; \\ \left(\frac{f_{ij}^L - k_j^U}{c_j^*}, \frac{f_{ij}^M - k_j^M}{c_j^*}, \frac{f_{ij}^U - k_j^L}{c_j^*}\right), \ \mu_P(\tilde{f}'_{ij}, 0) \geq 0.5. \end{cases} \tag{11}$$

where $c_j^* = \sqrt{\sum_{i=1}^{m} \frac{1}{3}\left[(f_{ij}^L - k_j^U)^2 + (f_{ij}^M - k_j^M)^2 + (f_{ij}^U - k_j^L)^2\right]}$.

*3.2. Derivation of Ideal Solutions for Heterogeneous Attributes*

PIS is a solution that maximizes the benefit attributes and minimizes the cost attributes while NIS is a solution that maximizes the cost attributes and minimizes the benefit attributes. Based on the proposed normalization approach for heterogeneous attributes, the deviation attribute and fixation attribute are transformed into a benefit attribute with zero NIS and a cost attribute with zero PIS, respectively. Thus, it is much easier to attain the PISs and NISs of different attributes without complicated calculations.

Once the normalized attribute values of all the alternatives $r_{ij}$ have been calculated based on the expressions in Section 3.1, the weighted normalized decision matrix of the structural optimization problem can be obtained as $\mathbf{G} = [g_{ij}]_{m\times n}$, $g_{ij} = w_j r_{ij}$, where $w_j (j = 1, 2, 3, \ldots, n)$ is the weight of the $j$th attribute and there is $\sum_{j=1}^{n} w_j = 1$. Then the PISs and NISs corresponding to different attributes in the HMADM problem can be obtained using the expressions listed in Tables 1 and 2.

**Table 1.** Determination of positive ideal solutions (PISs) for heterogeneous attributes in different mathematical descriptions. TFN: triangular fuzzy number.

| **Attributes** | **Real Number** $g_j^*$ | **Interval Number** $g_j^* = [g_j^{L*}, g_j^{U*}]$ | **TFN** $g_j^* = (g_j^{L*}, g_j^{M*}, g_j^{U*})$ |
|---|---|---|---|
| Benefit attribute | $\max_i(g_{ij})$ | $\left[\max_i(g_{ij}^L), \max_i(g_{ij}^U)\right]$ | $\left(\max_i(g_{ij}^L), \max_i(g_{ij}^M), \max_i(g_{ij}^U)\right)$ |
| Cost attribute | $\min_i(g_{ij})$ | $\left[\min_i(g_{ij}^L), \min_i(g_{ij}^U)\right]$ | $\left(\min_i(g_{ij}^L), \min_i(g_{ij}^M), \min_i(g_{ij}^U)\right)$ |
| Deviation attribute | $\max_i(g_{ij})$ | $\left[\max_i(g_{ij}^L), \max_i(g_{ij}^U)\right]$ | $\left(\max_i(g_{ij}^L), \max_i(g_{ij}^M), \max_i(g_{ij}^U)\right)$ |
| Fixation attribute | 0 | [0,0] | (0,0,0) |

**Table 2.** Determination of negative ideal solutions (NISs) for heterogeneous attributes in different mathematical descriptions.

| **Attributes** | **Real Number** $g_j^-$ | **Interval Number** $g_j^- = [g_j^{L-}, g_j^{U-}]$ | **TFN** $g_j^- = (g_j^{L-}, g_j^{M-}, g_j^{U-})$ |
|---|---|---|---|
| Benefit attribute | $\min_i(g_{ij})$ | $\left[\min_i(g_{ij}^L), \min_i(g_{ij}^U)\right]$ | $\left(\min_i(g_{ij}^L), \min_i(g_{ij}^M), \min_i(g_{ij}^U)\right)$ |
| Cost attribute | $\max_i(g_{ij})$ | $\left[\max_i(g_{ij}^L), \max_i(g_{ij}^U)\right]$ | $\left(\max_i(g_{ij}^L), \max_i(g_{ij}^M), \max_i(g_{ij}^U)\right)$ |
| Deviation attribute | 0 | [0,0] | (0,0,0) |
| Fixation attribute | $\max_i(g_{ij})$ | $\left[\max_i(g_{ij}^L), \max_i(g_{ij}^U)\right]$ | $\left(\max_i(g_{ij}^L), \max_i(g_{ij}^M), \max_i(g_{ij}^U)\right)$ |

### *3.3. Ranking of the Alternative Structural Configurations*

Once PISs and NISs for the heterogeneous attributes in the HMADM model have been determined from Tables 1 and 2, the distances of the $i$th alternative configuration deviating from PIS and NIS can be calculated as

$$d_i^* = \sqrt{\sum_{j=1}^{n} (d_{ij}^*)^2} \; (i = 1, 2, 3, \cdots, m), \tag{12}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (d_{ij}^-)^2} \; (i = 1, 2, 3, \cdots, m), \tag{13}$$

where $d_{ij}^* = \left|g_{ij} - g_j^*\right|$, $d_{ij}^- = \left|g_{ij} - g_j^-\right|$ when the $j$th attribute value is described as a real number; $d_{ij}^* = \left|g_{ij}^L - g_j^{L*}\right| + \left|g_{ij}^U - g_j^{U*}\right|$, $d_{ij}^- = \left|g_{ij}^L - g_j^{L-}\right| + \left|g_{ij}^U - g_j^{U-}\right|$ when the $j$th attribute value is described as an interval number; $d_{ij}^* = \sqrt{\left((g_{ij}^L - g_j^{L*})^2 + (g_{ij}^M - g_j^{M*})^2 + (g_{ij}^U - g_j^{U*})^2\right)/3}$ and $d_{ij}^- = \sqrt{\left((g_{ij}^L - g_j^{L-})^2 + (g_{ij}^M - g_j^{M-})^2 + (g_{ij}^U - g_j^{U-})^2\right)/3}$ when the $j$th attribute is described as a TFN.

Then the relative closeness coefficient of the $i$th alternative structural configuration is determined as

$$CC_i^* = \frac{d_i^-}{d_i^* + d_i^-} \; (i = 1, 2, 3, \ldots, m). \tag{14}$$

Finally, the alternative structural configurations can be ranked according to their corresponding values of $CC_i^*$ and the design corresponding to the largest $CC_i^*$ value is selected as the optimal.

## 4. Design Example: The High-Speed Press

The structural optimization of an ultra-precision high-speed press with double drivers as shown in Figure 3 is utilized to illustrate the HMADM modeling process and the implementation of the proposed HTOPSIS to solve the resulting HMADM model. The nominal pressure and punching frequency of the press are 3000 kN and 80–240 spm (strokes per minute), respectively. The implementation steps of the proposed HTOPSIS for solving the resulting HMADM model are described in detail. Numerical results are compared with those obtained by HAD. The advantages of the proposed HTOPSIS over HAD are also discussed.

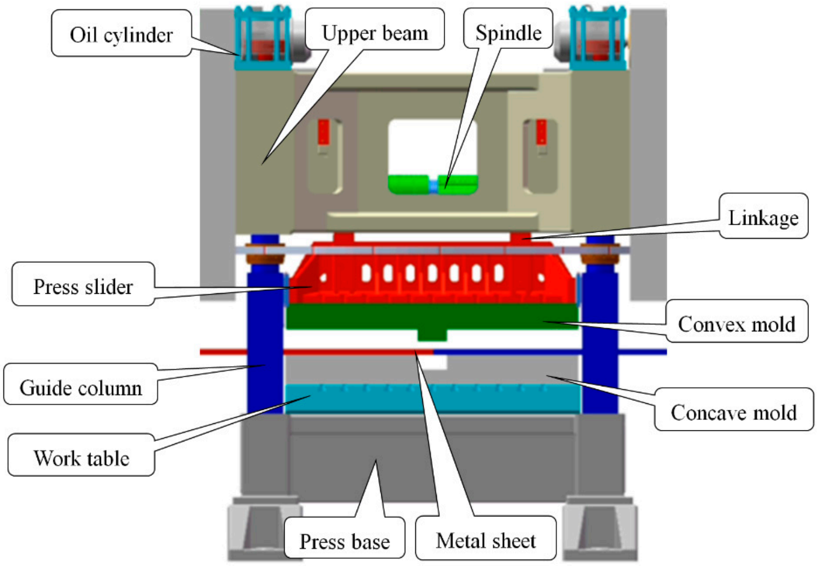



**Figure 3.** Schematic of the high-speed press and matching molds.

### *4.1. HMADM Modeling of the Optimization Problem*

The press slider and upper beam are the key components of the high-speed press. Their mechanical performance indices, such as stiffness and strength, greatly influence the punching precision, and thus should be included as attributes for evaluating alternative structural configurations. The stiffness of the press slider is usually evaluated based on its maximum deformation during the punching process, which can be regarded as a cost attribute, since a lower maximum deformation means higher stiffness. The maximum deformation of the press slider is obviously a real number. The stress concentration and the effect of alternate loading over a long period of time may cause fatigue cracks and damage in the upper beam. Thus, it is necessary to ensure the strength of the upper beam in the design phase. Based on experience, the maximum equivalent stress of the upper beam can satisfy strength requirements, and will not be too conservative when it is close to [60,61] MPa for the investigated press. Hence, maximum equivalent stress developed in the upper beam can be regarded as a fixation attribute, which should be described as an interval number, taking into account its fluctuation under the influence of uncertain material properties, manufacturing errors, etc. The press slider vertically reciprocates along the guide columns at 80–240 spm (namely, 1.33–4 Hz) during the punching process. The mechanical resonance of press slider will occur when any of its natural frequencies is close to the punching frequency, which will cause damage and even breakdown of the high-speed press. Resonance will also occur when any natural frequency of the press slider is close to the working frequency of 50 Hz (in China). Therefore, the lower order natural frequencies of the press slider closest to the punching frequency of 1.33–4 Hz and working frequency of 50 Hz should be chosen as the attributes for evaluating the operational reliability of press. These are deviation attributes, since the farther their values deviate from the punching and working frequencies, the better they are. The natural frequencies of the press slider should also be described by interval numbers, due to the uncertainties in its material properties, etc. Manufacturability is chosen to evaluate the manufacturing cost of the high-speed press, which can be described as a fuzzy linguistic term, and further transformed into TFN due to its cognitive fuzziness in the design phase. Manufacturability is a benefit attribute, because a better manufacturability will lead to a lower manufacturing cost for the press.

In summary, a total of five heterogeneous attributes are chosen for evaluating the structural configurations of the high-speed press. They are the maximum deformation of the press slider $a_1$, the maximum equivalent stress of the upper beam $a_2$, the natural frequency of press slider closest to the punching frequency of 1.33–4 Hz $a_3$, the natural frequency of press slider closest to the working frequency of 50 Hz $a_4$, and the manufacturability of press $a_5$. Parameter $a_1$ is a cost attribute described as a real number, $a_2$ is a fixation attribute described as an interval number, $a_3$ and $a_4$ are deviation attributes described as interval numbers, while $a_5$ is a benefit attribute described as a fuzzy linguistic term that can be transformed into a TFN according to Figure 2.

To achieve high punching precision, good operational reliability and low manufacturing cost for a high-speed press, we previously investigated the constrained interval multi-objective optimization of the slider mechanism based on interval analysis and NSGA-II [27], the robust optimization of the slider mechanism based on normalized violation degree of interval constraint [28] and the reliability-based design optimization of the upper beam [29], in which the uncertainties in the material properties were described as interval parameters and the manufacturing costs were evaluated by the weights of key components. It is worth noting that the decision optimization problem investigated here still exists after locating a group of Pareto optimal solutions by NSGA-II. According to our previous optimization results, the optimum design must be chosen from the nine alternative structural configurations whose attribute data are listed in Table 3 in order to ensure the high punching precision, good operational reliability and low manufacturing cost of the high-speed press. The values of attributes $a_1$~$a_4$ are easily collected based on the previous optimization results, while those of attribute $a_5$ are provided based on expert evaluation. The weights for the five attributes are determined as 0.3, 0.2, 0.15, 0.15 and 0.2, respectively, based on experience.

**Table 3.** The attribute data for nine alternative structural configurations.

| Alternative | $a_1$ (mm) | $a_2$ (MPa) | $a_3$ (Hz) | $a_4$ (Hz) | $a_5$ |
|---|---|---|---|---|---|
| 1 | 0.134 | [58.22,60.25] | [1.92,5.05] | [38.12,43.43] | E: (0.8,1.0,1.0) |
| 2 | 0.145 | [61.21,61.88] | [9.25,13.52] | [51.63,55.05] | VG: (0.6,0.8,0.9) |
| 3 | 0.140 | [59.92,60.48] | [3.11,7.82] | [45.86,51.38] | G: (0.3,0.5,0.7) |
| 4 | 0.132 | [61.62,62.12] | [18.53,21.98] | [55.86,60.12] | VG: (0.6,0.8,0.9) |
| 5 | 0.138 | [58.75,60.38] | [1.28,6.23] | [39.25,46.09] | G: (0.3,0.5,0.7) |
| 6 | 0.136 | [61.92,63.25] | [15.32,20.34] | [59.87,61.35] | E: (0.8,1.0,1.0) |
| 7 | 0.142 | [59.23,60.12] | [10.11,14.42] | [48.03,53.19] | G: (0.3,0.5,0.7) |
| 8 | 0.131 | [57.25,59.34] | [3.22,8.19] | [38.05,44.14] | F: (0.1,0.3,0.4) |
| 9 | 0.128 | [56.32,58.67] | [6.38,10.93] | [40.27,45.39] | VG: (0.6,0.8,0.9) |

E: Excellent; VG: Very Good; G: Good: F: Fair.

### *4.2. Solution of the HMADM Problem Based on the Proposed HTOPSIS*

According to the normalization approaches for heterogeneous attributes described in Section 3.1, attribute $a_1$ should be normalized with Equation (2), attributes $a_2$, $a_3$ and $a_4$ should be normalized with Equations (5)–(7), attribute $a_5$ should be normalized with Equation (8). After the normalized decision matrix is obtained as listed in Table 4, the weighted normalized decision matrix can be calculated as listed in Table 5. With the PISs and NISs of the five attributes determined as listed in Table 6 according to the corresponding expressions given in Tables 1 and 2, the relative closeness coefficients of all the alternative configurations can be calculated, based on the results of which the alternative configurations can be ranked according to their $CC_i^*$ values, see Table 7.

As can be seen from Table 7, the 4th design has the maximum relative closeness coefficient of 0.7072, therefore it is chosen as the optimal configuration for the high-speed press. The 9th design is the worst, since it has the minimum relative closeness coefficient of 0.3428. Figure 4 is the high-speed press developed based on the 4th configuration, all the performance indices of which satisfy customer requirements pretty well. The developed high-speed press has been successfully put into production, demonstrating the feasibility and validity of the proposed HTOPSIS in the structural optimization of high-speed press.

**Table 4.** Normalized decision matrix.

| Alternative | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| 1 | 0.0439 | [−0.0407,0.4522] | [−0.0703,0.1258] | [0.2892,0.5229] | (0.3777,0.4721,0.4721) |
| 2 | 0.0514 | [0.0342,0.3058] | [0.1775,0.4121] | [0.0717,0.2223] | (0.2833,0.3777,0.4249) |
| 3 | 0.0479 | [−0.0781,0.1757] | [−0.0301,0.2194] | [−0.0607,0.1822] | (0.1416,0.2361,0.3305) |
| 4 | 0.0426 | [0.1009,0.3449] | [0.4912,0.6981] | [0.2579,0.4454] | (0.2833,0.3777,0.4249) |
| 5 | 0.0466 | [−0.0618,0.366] | [−0.1732,0.1657] | [0.1721,0.4731] | (0.1416,0.2361,0.3305) |
| 6 | 0.0452 | [0.1497,0.5287] | [0.3827,0.6427] | [0.4344,0.4995] | (0.3777,0.4721,0.4721) |
| 7 | 0.0493 | [−0.0195,0.2879] | [0.2066,0.4425] | [−0.0867,0.1404] | (0.1416,0.2361,0.3305) |
| 8 | 0.042 | [0.1074,0.6100] | [−0.0264,0.2319] | [0.2579,0.5259] | (0.0472,0.1416,0.1888) |
| 9 | 0.0401 | [0.2164,0.7613] | [0.0805,0.3246] | [0.2029,0.4282] | (0.2833,0.3777,0.4249) |

**Table 5.** Weighted normalized decision matrix.

| Alternative | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| 1 | 0.0132 | [−0.0081,0.0904] | [−0.0105,0.0189] | [0.0434,0.0784] | (0.0755,0.0944,0.0944) |
| 2 | 0.0154 | [0.0068,0.0612] | [0.0266,0.0618] | [0.0108,0.0333] | (0.0567,0.0755,0.0850) |
| 3 | 0.0144 | [−0.0156,0.0351] | [−0.0045,0.0329] | [−0.0091,0.0273] | (0.0283,0.0472,0.0661) |
| 4 | 0.0128 | [0.0202,0.0690] | [0.0737,0.1047] | [0.0387,0.0668] | (0.0567,0.0755,0.0850) |
| 5 | 0.014 | [−0.0124,0.0732] | [−0.026,0.0248] | [0.0258,0.0710] | (0.0283,0.0472,0.0661) |
| 6 | 0.0136 | [0.0299,0.1057] | [0.0574,0.0964] | [0.0652,0.0749] | (0.0755,0.0944,0.0944) |
| 7 | 0.0148 | [−0.0039,0.0576] | [0.0310,0.0664] | [−0.0130,0.0211] | (0.0283,0.0472,0.0661) |
| 8 | 0.0126 | [0.0215,0.1220] | [−0.004,0.0348 ] | [0.0387,0.0789] | (0.0094,0.0283,0.0378) |
| 9 | 0.012 | [0.0433,0.1523] | [0.0121,0.0487 ] | [0.0304,0.0642] | (0.0567,0.0755,0.0850) |

**Table 6.** Ideal solutions corresponding to the five attributes.

| Ideal Solution | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| PIS | 0.012 | [0,0] | [0.0737,0.1047] | [0.0652,0.0789] | (0.0755,0.0944,0.0944) |
| NIS | 0.0154 | [0.0433,0.1523] | [0,0] | [0,0] | (0.0094,0.0283,0.0378) |

**Table 7.** Ranking of the nine alternative structural configurations obtained by the proposed heterogeneous technique for order preference by similarity to an ideal solution (HTOPSIS).

| Alternative | $d_i^*$ | $d_i^-$ | $CC_i^*$ | Ranking |
|---|---|---|---|---|
| 1 | 0.1979 | 0.1803 | 0.4768 | 5 |
| 2 | 0.1517 | 0.1681 | 0.5258 | 3 |
| 3 | 0.2066 | 0.185 | 0.4725 | 6 |
| 4 | 0.0985 | 0.2378 | 0.7072 | 1 |
| 5 | 0.2087 | 0.175 | 0.4562 | 7 |
| 6 | 0.138 | 0.2255 | 0.6204 | 2 |
| 7 | 0.1749 | 0.1769 | 0.5028 | 4 |
| 8 | 0.2169 | 0.1343 | 0.3824 | 8 |
| 9 | 0.234 | 0.1221 | 0.3428 | 9 |

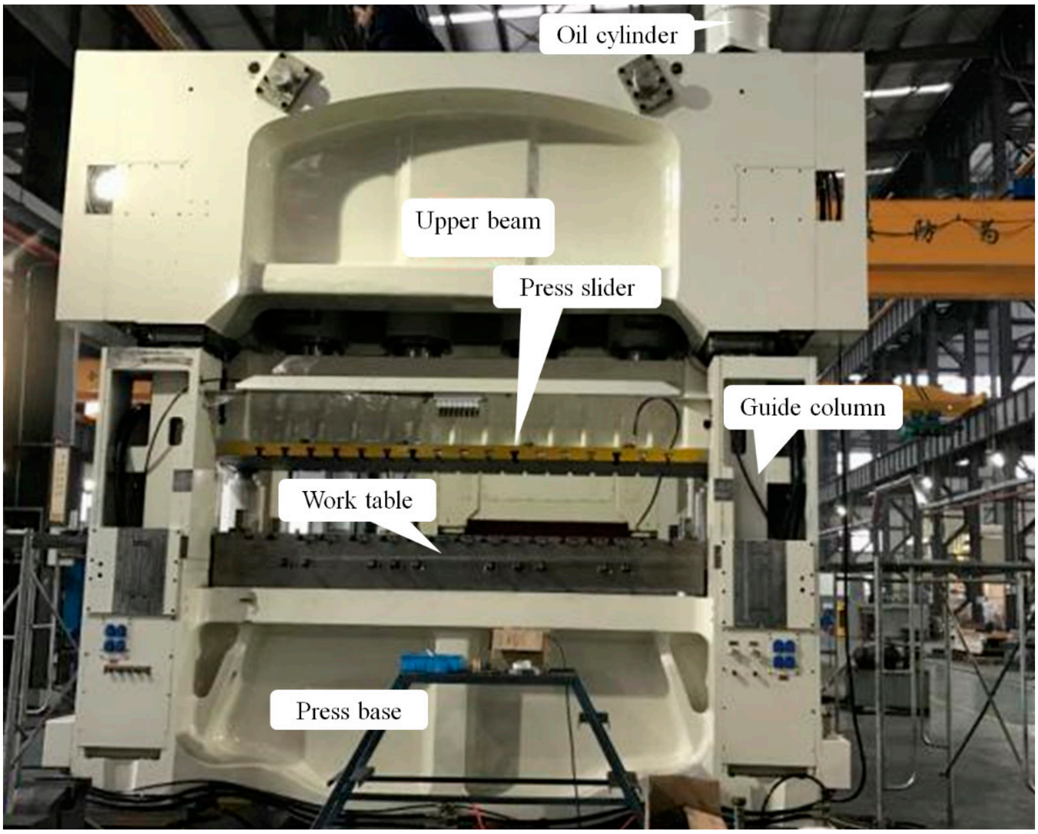



**Figure 4.** High-speed press developed from the optimum design obtained by the proposed heterogeneous technique for order preference by similarity to an ideal solution (HTOPSIS).

### 4.3. Comparison with HAD

The HMADM model constructed in Section 4.1 was also solved by the HAD method proposed in our previous work [22], which calculates the total information contents of alternative structural configurations based on distance measure. Specifically, the common range of the $j$th attribute for the $i$th alternative is represented by its Euclidean distance to the $j$th NIS while the system range is represented by the distance between the PIS and NIS. The total information content of the $i$th alternative is the sum of the information content for all of its attributes, which are calculated by taking the negative logarithm of the ratio of common range to system range. It is worth noting that both the total information content in HAD and the relative closeness coefficient in the proposed HTOPSIS are calculated based on distance measure. However, a larger distance of an alternative to NIS leads to a higher relative closeness coefficient but a lower information content. Therefore, the configuration with the minimum total information content should be chosen as the optimal design in HAD. The detailed implementation of HAD is not given here for the sake of brevity. The information contents and ranking results of the nine alternative structural configurations in Table 3 obtained by HAD are listed in Table 8. It can be seen that the 4th structural configuration has the minimum total information content of 0.3669, and thus is still chosen as the optimum design according to the results obtained by HAD. By comparing Tables 7 and 8, it can be seen that the rank orders of the nine alternative structural configurations obtained by the proposed HTOPSIS are similar to those obtained by HAD.

The small difference between the ranking results of the proposed HTOPSIS and HAD comes from their different treatment of the relative importance of heterogeneous attributes. Specifically, the proposed HTOPSIS represents the relative importance of different attributes by their corresponding weights, while HAD represents the relative importance of different attributes by their corresponding system ranges. Moreover, the interdependency among attributes $a_3$ and $a_4$ (indicating the natural frequencies of press slider closest to the punching frequency of 1.33–4 Hz and working frequency of 50 Hz) as well as attribute $a_1$ (indicating the maximum deformation of press slider) can also lead to differences in the ranking results.

**Table 8.** Ranking of the nine alternative structural configurations obtained by heterogeneous axiomatic design (HAD).

| Alternative | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | Total Information Content | Ranking |
|---|---|---|---|---|---|---|---|
| 1 | 0.0623 | 0.1308 | 0.6580 | 0.0248 | 0.0184 | 0.8943 | 5 |
| 2 | 0.0678 | 0.1232 | 0.1513 | 0.2330 | 0.0729 | 0.6482 | 3 |
| 3 | 0.0653 | 0.0320 | 0.3886 | 0.3012 | 0.1854 | 0.9723 | 7 |
| 4 | 0.0613 | 0.1760 | 0.0000 | 0.0567 | 0.0729 | 0.3669 | 1 |
| 5 | 0.0643 | 0.0960 | 0.5426 | 0.0657 | 0.1854 | 0.9540 | 6 |
| 6 | 0.0633 | 0.3011 | 0.0317 | 0.0000 | 0.0184 | 0.4145 | 2 |
| 7 | 0.0663 | 0.0913 | 0.1306 | 0.3415 | 0.1854 | 0.8150 | 4 |
| 8 | 0.0608 | 0.2749 | 0.3701 | 0.0301 | 0.3528 | 1.0887 | 9 |
| 9 | 0.0593 | 0.3321 | 0.2314 | 0.0763 | 0.0729 | 0.7720 | 8 |

The proposed HTOPSIS has the following advantages over HAD in solving HMADM problems with deviation and fixation attributes. Firstly, it relaxes the strict requirement of independency among different attributes in HMADM problems, and thus it is more suitable for solving HMADM problems with large number of attributes than HAD. Actually, it can be applied to any HMADM problem, because there are no prerequisites for its application. Secondly, the ranking of different alternatives is more efficient in HTOPSIS than in HAD because the calculation of relative closeness coefficients in HTOPSIS is much simpler than the calculation of information contents in HAD.

## 5. Conclusions

The structural optimization problem of a high-speed press with multi-source uncertainties was defined as a HMADM model involving deviation, fixation, benefit and cost attributes. Attribute values can be described as real numbers, interval numbers, fuzzy linguistic terms and TFNs. To solve such a HMADM problem, a new HTOPSIS was proposed, which realized the normalization of heterogeneous attributes based on the possibility degree method for comparing interval numbers, the relative preference relation for comparing TFNs, and the attribute transformation technique for handling deviation and fixation attributes. Equations for determining the positive and negative ideal solutions of heterogeneous attributes and the ranking method of different alternative designs were also presented.

A numerical example was utilized to illustrate the HMADM modeling process of the structural optimization problem and the implementation of the proposed HTOPSIS for solving the HMADM model. A cost attribute described as a real number, a fixation attribute described as an interval number, two deviation attributes described as interval numbers and a benefit attribute described as a fuzzy linguistic term were chosen for evaluating nine alternative structural configurations of an ultra-precision high-speed press with double drivers. The 4th structural configuration, with a maximum deformation of the press slider of 0.132 mm, a maximum equivalent stress of the upper beam of [61.62,62.12] MPa, 1st and 2nd natural frequencies of the press slider of [18.53,21.98] Hz and [55.86,60.12] Hz, and a very good manufacturability of whole press, was chosen as the optimal configuration according to the HMADM results. The successful use of the high-speed press developed from the 4th structural configuration demonstrated the validity and effectiveness of the proposed HTOPSIS in solving the HMADM problem with deviation and fixation attributes. The advantages of the proposed HTOPSIS over HAD were also discussed, which included its universal applicability in solving HMADM problems and its efficient ranking of different alternatives.

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