Electromechanical Anisotropy at the Ferroelectric to Relaxor Transition of (Bi$_{0.5}$Na$_{0.5}$)$_{0.94}$Ba$_{0.06}$TiO$_3$ Ceramics from the Thermal Evolution of Resonance Curves

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1. Sensitivity Analysis

The result of the FEM optimization depends on the right choice of the initial conditions. For this reason, a sensitivity analysis is performed by representing the evolution of the maximum of G and R curves (Figure 1 of the manuscript) when each given parameter of the initial condition is changed. Figure 4 of the manuscript shows the sensitivity analysis results for the G curves for each elastic parameter. The sensitivity analysis for the R curves, together with the analysis for the dielectric and piezoelectric parameters for the G and R are given in this supplementary material.

![Figure S1](image)

**Figure S1.** Sensitivity analysis for the real part of the elastic constants. Each curve shows the evolution of a resonance using the resistivity R. Each parameter is changed over a range ± 50% from the initial value.
Figure S2. Sensitivity analysis for the real part of the piezoelectric and dielectric constants. Each curve shows the evolution of a resonance using the conductivity $G$. Each parameter is changed over a range $\pm 50\%$ from the initial value.

Figure S3. Sensitivity analysis for the real part of the piezoelectric and dielectric constants. Each curve shows the evolution of a resonance using the resistivity $R$. Each parameter is changed over a range $\pm 50\%$ from the initial value.

2. Optimized parameters

Parameters values for $16^\circ C$ were used in the FEM modeling of the thin disk to determine the deformation at resonance (see supplementary material). This confirms the purely dilatational character of the fundamental radial (R1) and thickness (TH) modes. It also shows that the deformation at C1 and C2 modes involves a great deal of shear movements.

The deformation at resonance shows the motion mode at each one.
Figure S4. The two extreme positions for the mode of motion at the fundamental radial resonance at 276 kHz.
Figure S5. The two extreme positions for the mode of motion at the fundamental thickness resonance at 1264 kHz.
Figure S6. The two extreme positions for the mode of motion at the complex C1 resonance at 1335 kHz.
Figure S7. The two extreme positions for the mode of motion at the complex C2 resonance at 1392 kHz.