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Evaluation of Longitudinal Equivalent Bending Stiffness of Shield Tunnel with Residual Jacking Force

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Abstract: Existing tunnels are often subject to longitudinal bending when undercrossing tunnelling. It is of significance to more accurately evaluate the longitudinal equivalent bending stiffness (LEBS) of the existing tunnel within the influential zone. A new analytical method is proposed for the LEBS of tunnel segmental lining joints with consideration of incorporating combined action of residual jacking force and bending moment. The solution can degenerate into a special case with no residual jacking force, which agrees well with other classical solutions and validates the model and solutions. Sensitivity analyses are carried out for the bending moment, tunnel geometry, tensile stiffness of bolts and concrete grade on the LEBS, and effective ratio of the LEBS considering residual jacking force. The LEBS and the effective ratio of LEBS increase nonlinearly as an S-curve with the residual jacking force and decrease with an increasing bending moment. The results show that the LEBS of the shield tunnel is variable stiffness, which exhibits a significant nonlinearity. The maximum increment of the LEBS reaches 80.3% as the ring width increases from 1 m to 2 m, and the LEBS of the shield tunnel increases by approximately 1.3×10^7 kN·m^2 for every 4-bolt added. The influential order on the LEBS of shield tunnels is the tunnel diameter > lining thickness > bolts diameter > ring width > the number of bolts > elastic modulus of bolts. When the effective ratio of LEBS is more than 0.85, it does not change with the ring width, lining thickness, tensile stiffness of bolts, and concrete grade. The response characteristics of the tunnel parameters on the LEBS, considering the residual jacking force, could provide a theoretical basis for the design and deformation control of shield tunnels when undercrossing tunnelling.

Keywords: residual jacking force; analytical solution; shield tunnel; longitudinal equivalent bending stiffness (LEBS); effective ratio

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1. Introduction

Shield tunnelling is widely used in metro tunnels because of its high safety, efficiency, and small disturbance to surroundings. Shield tunnels are made of lining rings assembled with prefabricated concrete segments bolted at the radial and longitudinal joints. The presence of longitudinal joints between the lining rings significantly affects the LEBS of the shield tunnels [1,2]. The bending stiffness of a uniform beam having the same stress–deformation relation as that of the segmental tunnel is called the longitudinal equivalent bending stiffness (LEBS), which differs from the bending stiffness without segmental joints. The LEBS is of significance in the shield tunnels, which potentially causes some quality problems, such as excessive tunnel deformation, cracking, and leakage on the tunnel linings and the separation between the track bed and tunnel linings [3–5]. Therefore, it is of great engineering significance to more accurately evaluate the LEBS of the existing tunnel when undercrossing tunnelling.

During shield tunnelling, the segmental linings are assembled ring-by-ring to provide the jacking reaction force for driving the tunnel boring machine (TBM). After shield construction completion, a certain amount of longitudinal jacking force is trapped in the segmental linings balanced by the friction resistance along the lining outer surface, which is called residual jacking force here below. The residual jacking force may have a major influence on the LEBS, such as controlling tunnel deformation and preventing water leakage between the lining rings [6–8]. Field tests have demonstrated that the residual jacking force between lining rings varies with time and generally undergoes four evolution stages: cyclical and violent fluctuations, dynamic stability, gradual decay, and stabilization [9]. Even for shield tunnels in soft soils, the long-term residual jacking force can be more than 30% of the initial peak jacking force imposed during tunnelling [9]. For tunnelling in rocks, Jones [10] found that the long-term residual jacking force during the tunnel operation was approximately 70% of the initial peak jacking force. However, a theoretical study conducted by Liu et al. [11] indicated that the residual jacking force would decay gradually to 20.4%~44% of the initial peak jacking force, depending on the creep coefficient of lining concrete and the deformation modulus of the surrounding ground. Previous research has confirmed that the residual jacking force exists during shield tunnelling and tunnel operation, including field tests and analytical methods. This means the residual jacking force of the shield tunnel is an objective structural internal force and shall be considered when studying the LEBS of the shield tunnel.

Yukio et al. [12,13] established the solution of the LEBS for a shield tunnel without residual jacking force based on the joints between the lining, and they concluded that the joints between the lining should be considered when studying the LEBS of a shield tunnel. Yu et al. [14–16] carried out model tests and theoretical calculations on the LEBS of shield tunnels with no account of longitudinal pressure, and the LEBS of shield tunnels was found to be of significance on the tunnel-longitudinal deformation. Qi et al. [17,18] proposed a solution for the LEBS of a shield tunnel under the action of bending moment and discussed the influence of tunnel radius, ring width, and the number of bolts on the LEBS of a shield tunnel. Based on the numerical calculation method for longitudinal equivalent bending stiffness of a shield tunnel, Zhong [19] found that segment ring width and the number of longitudinal bolts influence the longitudinal bending stiffness largely. However, the studies mentioned above failed to consider the effects of the residual jacking force on the LEBS of the shield tunnel. Li [20] carried out model tests of the LEBS for shield tunnels and indicated that the effective ratio of LEBS increased with longitudinal axial force. Geng et al. [21] studied the influence of the longitudinal axial force on the LEBS of the shield tunnel and found a large difference in the LEBS with and without axial force. Huang et al. [22] carried out model tests on the LEBS of shield tunnels considering residual jacking force and bending moment. It was found that the LEBS increases nonlinearly with the residual jacking force. The impact of residual jacking force on the LEBS of a shield tunnel cannot be ignored [22,23], while most theoretical research has not paid much attention to its influence on the LEBS, and a limited number of influence parameters are analyzed. Therefore, this work studies the LEBS of a shield tunnel with consideration of residual jacking force and bending moment, and more influence parameters are analyzed.

In this paper, a new analytical method is proposed for the LEBS of tunnel segmental lining joints with consideration of incorporating combined action of residual jacking force and bending moment. First, the shield tunnel is simplified as a beam of longitudinal joints between the lining, which can take into account both the residual jacking force and bending moment effects of the shield tunnel structures. Then, the effectiveness of the analytical solution that degenerated into the special case with no residual jacking force is validated by two well-documented case studies. Finally, based on the verified analytical method, sensitivity analyses of the bending moment, tunnel geometry, tensile stiffness for bolts, and lining concrete grade are carried out on the LEBS and effective ratio of LEBS of a tunnel considering residual jacking force and bending moment.

2. Analytical Solution of LEBS with Residual Jacking Force

2.1. Model Assumptions

The shield tunnel comprises prefabricated concrete segmental rings connected by steel bolts. In this paper, the following assumptions are made on the tunnel model:

1. The cross-section at any circumferential joint between the segmental rings conforms to the plane assumption;
2. The longitudinal bolt deformation conforms to elastic deformation, and the bolt stress and strain are linear;
3. The tensile stiffness of longitudinal bolts is equivalent to k_j [24], as shown in Figure 1. k_j is the uniformly distributed stiffness along the centre-line of the tunnel lining;
4. The residual jacking force is uniformly distributed along the segmental ring.

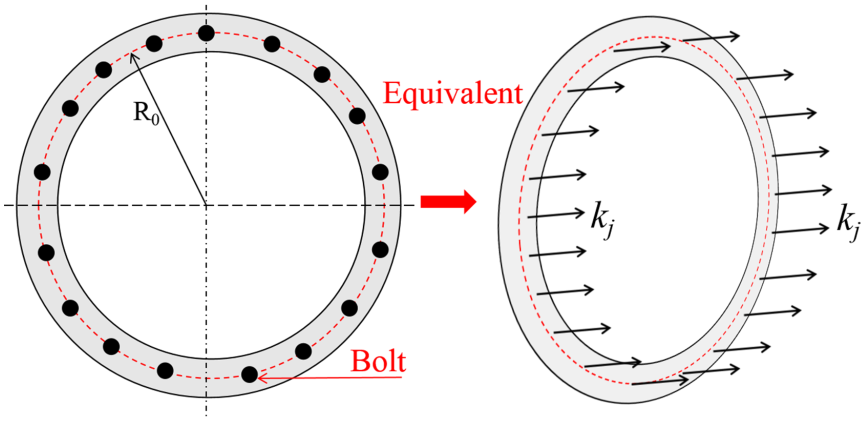


Figure 1. Schematic diagram of the uniformly distributed stiffness of bolts.

2.2. Development of an Analytical Solution

The analytical solution of the LEBS is derived below by considering the action of both an external bending moment, M, and the residual jacking force, N, as shown in Figure 2. Half the segmental ring width and the joint between the rings are taken for stress and deformation analysis.

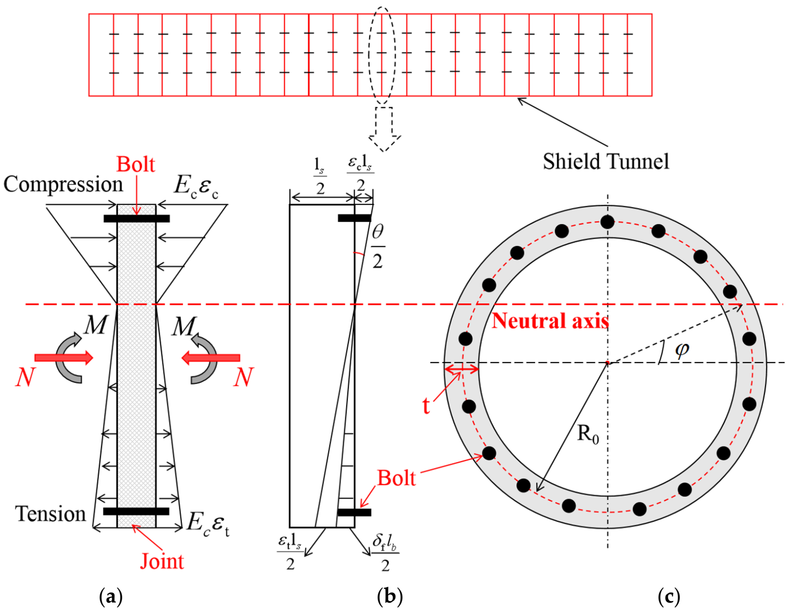


Figure 2. Stress and deformation of tunnel segment lining (**a**) stress of joint between the rings, (**b**) deformation of the joint between the rings, and (**c**) tunnel profile.

The equilibrium Equations regarding the axial force (1) and bending moment (2), respectively, at the joint between the segmental rings are as follows:

$$2\int_{\varphi}^{\frac{\pi}{2}} E_c t \frac{\varepsilon_c}{R_0 - R_0 \sin\varphi}(R_0 \sin\alpha - R_0 \sin\varphi) R_0 d\alpha - 2\int_{-\frac{\pi}{2}}^{\varphi} E_c t \frac{\varepsilon_t}{R_0 + R_0 \sin\varphi}(R_0 \sin\varphi - R_0 \sin\alpha) R_0 d\alpha = N \quad (1)$$

$$\begin{aligned} &2\int_{\varphi}^{\frac{\pi}{2}} E_c t \tfrac{\varepsilon_c}{R_0 - R_0 \sin\varphi}(R_0 \sin\alpha - R_0 \sin\varphi)^2 R_0 d\alpha + 2\int_{-\frac{\pi}{2}}^{\varphi} E_c t \tfrac{\varepsilon_t}{R_0 + R_0 \sin\varphi}(R_0 \sin\varphi - R_0 \sin\alpha)^2 R_0 d\alpha \\ &= M - N R_0 \sin\varphi \end{aligned} \quad (2)$$

where ε_t and ε_c denote the outer fibre tensile strain and compressive strain at the ring joint, respectively; θ denotes the cross-section rotating angle; l_s denotes the segmental ring width; t denotes the tunnel lining thickness; R_0 denotes the radius of the tunnel lining centre-line; E_c denotes the elastic modulus of the concrete segments; φ denotes the position of the neutral axis, which is turned into a positive angle counterclockwise and a negative angle clockwise by the horizontal centre line, as shown in Figure 2.

By combining Equations (1) and (2), the expression of the M can be obtained as follows:

$$M = E_c t \frac{\varepsilon_c {R_0}^2}{1 - in\varphi}\left[\left(\frac{\pi}{2} - \varphi\right) - \cos\varphi \sin\varphi\right] + E_c t \frac{\varepsilon_t {R_0}^2}{1 + \sin\varphi}\left[\left(\frac{\pi}{2} + \varphi\right) + \sin\varphi \cos\varphi\right] \quad (3)$$

The tensile stiffness of bolts is equivalent to the uniformly distributed stiffness along the centre-line of the tunnel lining [24], as shown in Figure 1:

$$k_j = \frac{n \cdot E_b A_b}{2\pi R_0 l_b} \quad (4)$$

where k_j denotes the linear stiffness of the bolts; n denotes the number of bolts; A_b denotes the cross-sectional area of a bolt; E_b denotes the elastic modulus of the bolt material; l_b is the bolt length.

In the concrete tension zone [12], the force equilibrium equation between the bolts and concrete is established as follows:

$$2\int_{-\frac{\pi}{2}}^{\varphi} E_c t \frac{\varepsilon_t}{R_0 + R_0 \sin\varphi}(R_0 \sin\varphi - R_0 \sin\alpha) R_0 d\alpha = 2\int_{-\frac{\pi}{2}}^{\varphi} \frac{l_b \delta_f \cdot k_j}{R_0 + R_0 \sin\varphi}(R_0 \sin\varphi - R_0 \sin\alpha) R_0 d\alpha \quad (5)$$

where δ_f denotes the tensile strain of the bolt.

The deformation equilibrium equation is established based on the plane assumption, as shown in Figure 2:

$$\frac{l_b \delta_f}{2} + \frac{\varepsilon_t \cdot l_s}{2} = \frac{\theta}{2}(R_0 + R_0 \sin\varphi) \quad (6)$$

$$\frac{\varepsilon_c \cdot l_s}{2} = \frac{\theta}{2}(R_0 - R_0 \sin\varphi) \quad (7)$$

According to the Code for Design of Concrete Structures, the maximum elastic strain of concrete under axial compression conforms to the following equation:

$$\varepsilon_0 = 0.002 - 0.5 \times (f_{cu \cdot k} - 50) \times 10^{-5} \quad (8)$$

where $f_{cu\,k}$ denotes the standard compressive strength of concrete.

By combining Equations (5)~(7), the expression of the ε_t can be obtained as follows:

$$\varepsilon_t = \frac{\varepsilon_c l_s k_j (1 + \sin\varphi)}{(E_c t + l_s k_j)(1 - \sin\varphi)} \quad (9)$$

The solution of θ can be obtained by rewriting Equation (7).

$$\theta = \frac{\varepsilon_c \cdot l_s}{R_0 - R_0 \cdot \sin\varphi} \quad (10)$$

Based on the Mechanics of Materials and Yukio's theory [12], the equation of the LEBS for shield tunnel is established:

$$(EI)_{\mathrm{eq}} = \frac{Ml_s}{\theta} \tag{11}$$

By combining Equations (3) and (9)–(11), the expression of the LEBS for shield tunnel $(EI)_{\mathrm{eq}}$ can be obtained as follows:

$$(EI)_{\mathrm{eq}} = E_c \cdot {R_0}^3 \cdot t \left[\frac{\pi}{2} - \varphi - \sin\varphi\cos\varphi + \frac{l_s k_j}{E_c t + l_s k_j} \left(\frac{\pi}{2} + \varphi + \sin\varphi\cos\varphi \right) \right] \tag{12}$$

According to the definition of the effective ratio of LEBS (η), the expression of η is obtained by Equation (13):

$$\eta = \frac{(EI)_{\mathrm{eq}}}{E_c I_c} = \frac{1}{\pi} \left[\frac{\pi}{2} - \varphi - \sin\varphi\cos\varphi + \frac{l_s k_j}{E_c t + l_s k_j} \left(\frac{\pi}{2} + \varphi + \sin\varphi\cos\varphi \right) \right] \tag{13}$$

where I_c denotes the longitudinal moment of inertia without joint planes of the tunnel, $I_c = \pi \cdot t \cdot {R_0}^3$, which can be obtained from the moment of inertia for the annular section.

3. Verification

Yukio [12] and Qi [18] proposed a solution for the LEBS of shield tunnel only under the action of bending moment. Qi [18] discussed the influence of the number of bolts on the LEBS of the shield tunnel. Two well-documented case histories, including Qi's theory [18] and Yukio's theory [12], have been selected to verify the proposed method discussed above. The calculated results from Qi [18] and Yukio [12] are compared with the calculated results of the solution degenerated into the special case with no residual jacking force.

3.1. Tunnel Parameters

According to the general design requirements of shield tunnels and Refs. [14,18], the parameters of tunnel segmental lining are shown in Table 1.

Table 1. Parameters of tunnel segmental lining [14,18].

R_0	t	n	R_b	l_s	$E_b \times 10^5$	$E_c \times 10^4$	l_b
2.925 m	0.35 m	17 pcs	0.015 m	1.0 m	2.06 MPa	3.45 MPa	0.4 MPa

3.2. Case I

In order to verify the reliability of the proposed model, the tunnel parameters are consistent with Refs. [14,18], as shown in Table 1, which are substituted into the analytical solution for the special case with no residual jacking force. The LEBS of the shield tunnel calculated under different numbers of bolts are compared with the calculated results of the Qi [18], as shown in Figure 3.

As shown in Figure 3, the calculated results agree well with the results from Qi [18], with a difference of 4.6%, which validates the proposed model. The θ will reduce lightly under the tunnel–soil interaction, which slightly increases the LEBS of the shield tunnel. The calculated results are slightly smaller than those in the Qi [18] because the resistance coefficient of soil is not considered in this paper. It is demonstrated that the proposed analytical method can be applied to evaluate the LEBS of a shield tunnel when subjected to an underneath excavation.

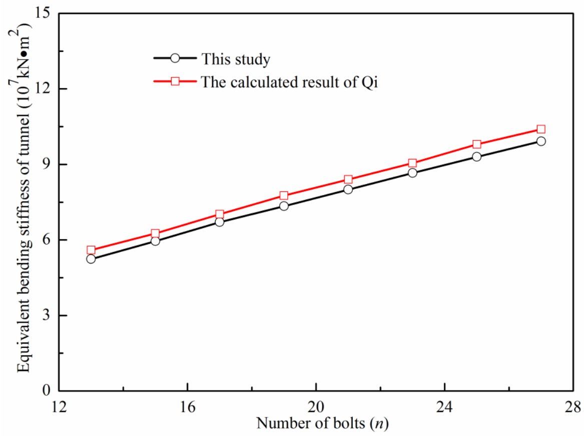


Figure 3. Relationship between LEBS and the number of bolts.

3.3. Case II

The calculated results without residual jacking force are compared with those of Yukio's classical theory [12,13]. As proposed by Yukio et al., φ, η, and $(EI)_{\text{eq}}$ can be expressed as below:

$$\varphi + \cot\varphi = \pi\left(\frac{1}{2} + \frac{nK_{j1}}{E_c A_b / l_s}\right) \tag{14}$$

$$\eta = \frac{\cos^3\varphi}{\cos\varphi + \left(\frac{\pi}{2} + \varphi\right)\cdot\sin\varphi} \tag{15}$$

$$(EI)_{\text{eq}} = \eta E_c I_c \tag{16}$$

where K_{j1} is the tensile stiffness of individual bolts, and other variables are as aforementioned.

Substituting the parameters in Table 1 into expressions (14), (15), and (16) yield the values of φ, $(EI)_{\text{eq}}$, and η, as shown in Table 2. Also shown in Table 2 are the corresponding values from the present study of the special case with no residual jacking force Table 2.

Table 2. Errors between the two solutions.

	φ	$(EI)_{eq}$	η
Yukio [12]	55.03°	6.76×10^7 kN·m^2	0.0713
The present study	55.09°	6.71×10^7 kN·m^2	0.0707
Calculation error	0.11%	0.74%	0.84%

It can be seen from Table 2 that the difference between those of the present study and those by Yukio [12] are all within 1%, which validates the accuracy of the proposed model. It is demonstrated that the proposed method is reasonable for evaluating the LEBS for a tunnel.

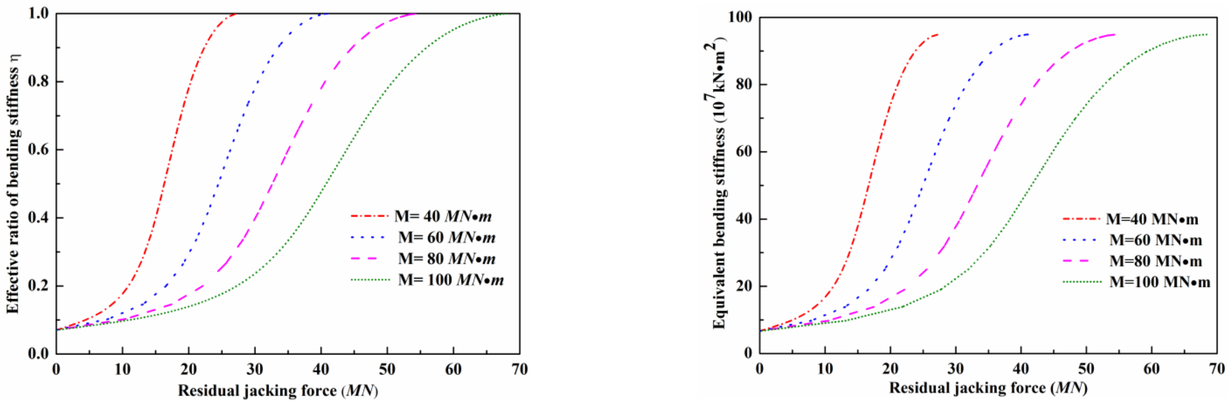
4. Parametric Analyses

In the previous section, the effectiveness of the proposed method has been validated by comparison with two selected cases for the LEBS of shield tunnel. Sensitivity analyses are carried out for the bending moment, tunnel geometry, tensile stiffness for bolts and concrete grade on the LEBS, and effective ratio of the LEBS of a tunnel with consideration of residual jacking force. For comparison, a base case is assumed as Table 1.

4.1. Influence of the Bending Moment

Based on the parameters in Table 1, the influences of the magnitude of bending moment on the LEBS and the effective ratio of LEBS are analyzed.

As shown in Figure 4, the LEBS and the effective ratio of LEBS both are constants with values of $(EI)_{eq} = 6.71 \times 10^7$ kN·m^2 and η = 0.0707, respectively, when the residual jacking force (i.e., N = 0) is not considered. The LEBS of the shield tunnel increases nonlinearly with an S-curve for the residual jacking force, which eventually tends to stabilize. When the entire cross-section of the tunnel is under pressure, the LEBS of the shield tunnel reaches the maximum value, which is equal to the longitudinal bending stiffness of the shield tunnel without the joints between the rings. Under the same residual jacking force, the larger the bending moment, the smaller the LEBS and the effective ratio of LEBS, but the maximum and minimum LEBS are not affected by the bending moment because the maximum and minimum LEBS are only related to tunnel parameters. If the LEBS of the shield tunnel remains unchanged, the larger the bending moment, the greater the residual jacking force applied. Similar results are also presented by Huang et al. [22] and Wang et al. [25], who analyzed the LEBS of the tunnel by performing model tests and theoretical analyses, respectively.



(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel

Figure 4. Effects of bending moment on the LEBS.

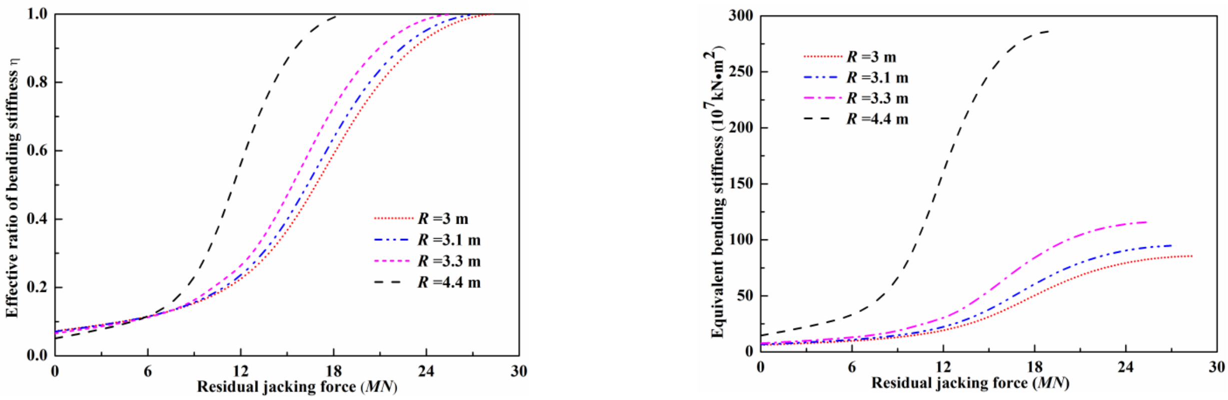
The study indicates that the residual jacking force significantly affects the contact state between tunnel segmental linings and, thus, the characteristics of LEBS. The field-measured data show that the residual jacking force decreases with time and eventually tends to a stable value [9], which changes the LEBS and the effective ratio of LEBS. If the residual jacking force is not considered, there is a large difference between the calculated LEBS and the actual LEBS of the shield tunnel, and it will affect the prediction of tunnel deformation.

4.2. Influence of the Tunnel Geometric Parameters

The impact of the tunnel radius, ring width, and lining thickness on the LEBS of the shield tunnel are investigated under the combined action of bending moment M = 40 MN·m and residual jacking force. The value ranges of the tunnel geometry parameters are based on the conventional tunnel, and the remaining parameters are shown in Table 1.

As shown in Figure 5a, when the residual jacking force is 0 MN, the LEBS of the shield tunnel increases with increasing tunnel radius, whereas the effective ratio of LEBS decreases almost linearly from 0.0724 to 0.0506. When the residual jacking force is small (i.e., N less than 6 MN), the effective ratio of LEBS decreases with increasing radius, while it increases with increasing radius as the N is more than 6 MN. As shown in Figure 5b, when the tunnel radius increases from 3 m to 4.4 m, the maximum increase in the LEBS of the shield tunnel reaches 387.4%.

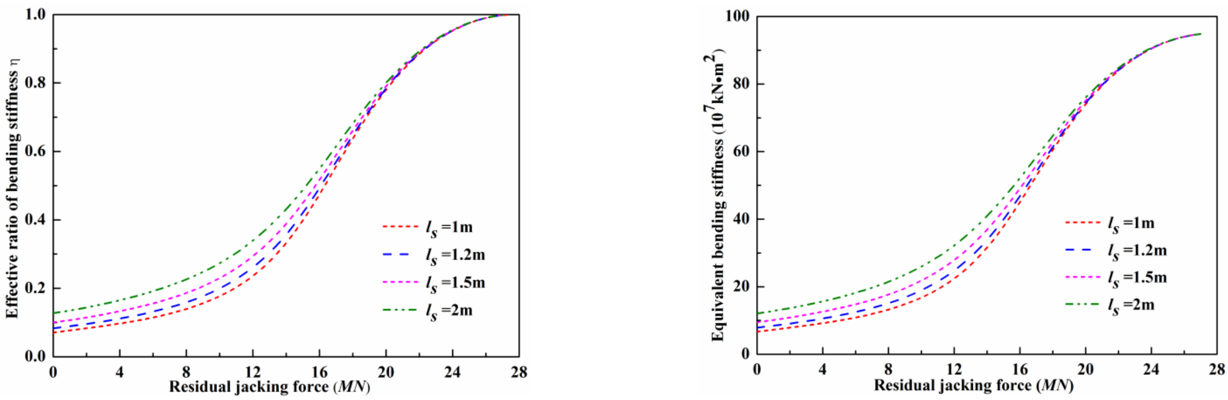
The study shows that the LEBS and the effective ratio of LEBS increase nonlinearly with increasing residual jacking force and eventually tend to a stable value. Under the same residual jacking force (i.e., axial pressure), the larger the tunnel radius, the greater the LEBS and the effective ratio of LEBS.



(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel

Figure 5. Effects of tunnel radius on the LEBS.

Figure 6a shows that the effective ratio of LEBS is less than or equal to 0.85 (which is consistent with the engineering value range of 0.1–0.85 [23]), and the larger the ring width, the greater the increase in the effective ratio of LEBS. But when the effective ratio of LEBS is greater than or equal to 0.85, it does not change with increasing ring width. The LEBS increases from 6.71 × 10^7 kN·m^2 to 12.1 × 10^7 kN·m^2, and the effective ratio of LEBS increases from 0.0707 to 0.1274 when the residual jacking force is 0 MN. As shown in Figure 6b, when the ring width increases from 1 m to 2 m, the maximum increase in the LEBS of the shield tunnel reaches 80.3%.



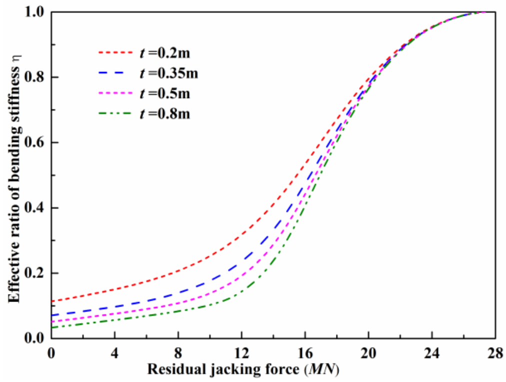
(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel

Figure 6. Effects of ring width on the LEBS.

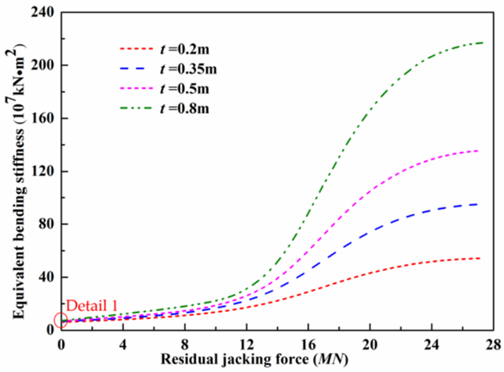
Figure 6 shows that the LEBS and the effective ratio of LEBS for the shield tunnel increase linearly with the ring width, nonlinearly with increasing residual jacking force, and eventually tend to stabilize.

As shown in Figure 7, when the residual jacking force is 0 MN, the effective ratio of LEBS decreases with increasing lining thickness, the η decreases from 0.1143 to 0.0335, while the LEBS of the shield tunnel increases with increasing lining thickness from 6.2 × 10^7 kN·m^2 to 7.26 × 10^7 kN·m^2. Figure 7b shows that the LEBS increases from 17.1% to 300% with increasing residual jacking force as the lining thickness increases from 0.2 m to 0.8 m. There is a nonlinear proportional relationship between the LEBS, the effective ratio of LEBS, and the residual jacking force. When the N is greater than 12 MN, the LEBS increases nonlinearly with the residual jacking force.

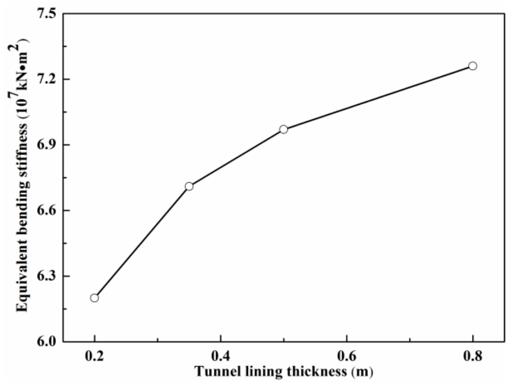
Figure 7c clearly shows the change rule of the LEBS with lining thickness. The LEBS of the shield tunnel increase from 6.2 × 10^7 kN·m^2 to 7.26 × 10^7 kN·m^2 as lining thickness increases, but the increment of LEBS gradually decreases. According to the slope change point of the LEBS and the effective ratio of LEBS for the shield tunnel, it is suggested that the lining thickness should be approximately 0.35 m when the tunnel is subjecting a small bending moment. Compared with similar projects at home and abroad, the tunnel segment lining thickness in China is greater than that in European countries but less than that in Southeast Asian ones [26].



(**a**) The effective ratio of LEBS



(**b**) LEBS of shield tunnel



(**c**) Detail 1: LEBS of shield tunnel without residual jacking force

Figure 7. Effects of lining thickness on the LEBS.

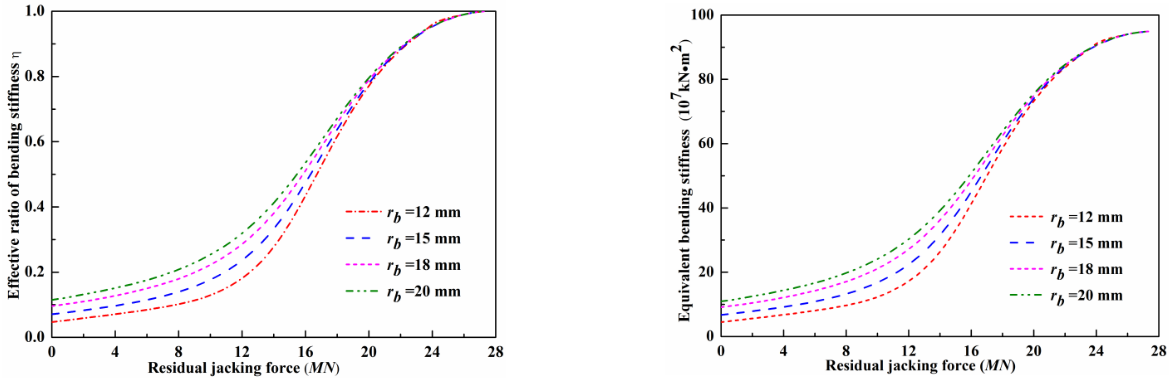
The results show that the LEBS of shield tunnels is variable stiffness, and the influential order on the LEBS of shield tunnels is the tunnel diameter > lining thickness > ring width. When the effective ratio of the LEBS is greater than or equal to 0.85, it is not related to the ring width and lining thickness.

4.3. Influence of the Bolt Parameters

Under the combined action of bending moment M = 40 MN·m and residual jacking force, this paper evaluated the LEBS of the shield tunnel by changing the bolt parameters in turns, including the bolt numbers and bolt radius. The value ranges of the bolt parameters are based on the conventional tunnel, and the remaining parameters are shown in Table 1.

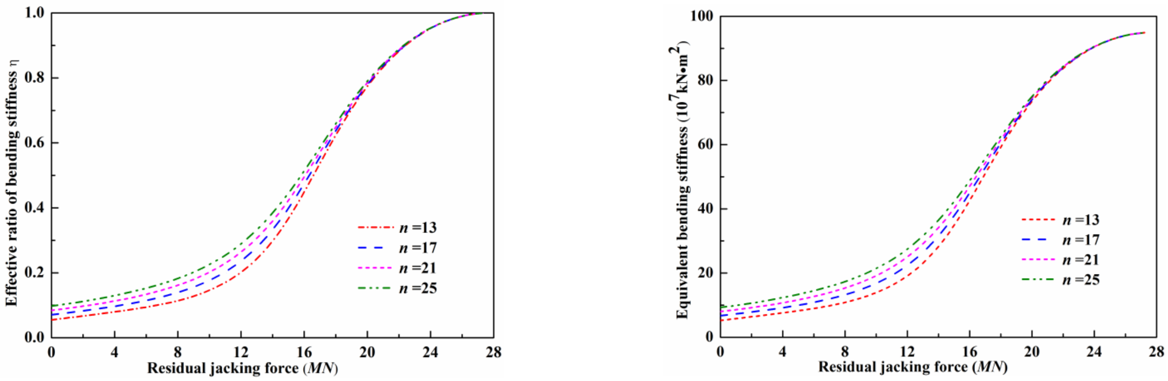
Figure 8 shows that the LEBS and effective ratio of LEBS for the shield tunnel are constant as the residual jacking force is 0 MN. As shown in Figure 8b, the LEBS increases linearly with the bolt radius and nonlinearly as an S-curve with the residual jacking force. The LEBS of the shield tunnel increases by approximately 0.81 $\times$ 10^7 kN·m^2 for every 2 mm increase in bolt radius, and the maximum increment of the LEBS reaches 144.3%. Similar results are also presented by Huang et al. [22], who analyzed the LEBS by performing a model test.

It can be seen from Figure 9 that the LEBS and effective ratio of LEBS for the shield tunnel are constant, which are linearly proportional to the number of bolts when the residual jacking force is 0 MN. As shown in Figure 9b, the LEBS of the shield tunnel increases by approximately 1.3 $\times$ 10^7 kN·m^2 for every 4-bolt added, and the maximum increment of the LEBS reaches 77.3%. The LEBS is increased by 10.2 times as the number of bolts is 25.



(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel

Figure 8. Effects of bolts radius on the LEBS.



(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel

Figure 9. Effects of bolt numbers on the LEBS.

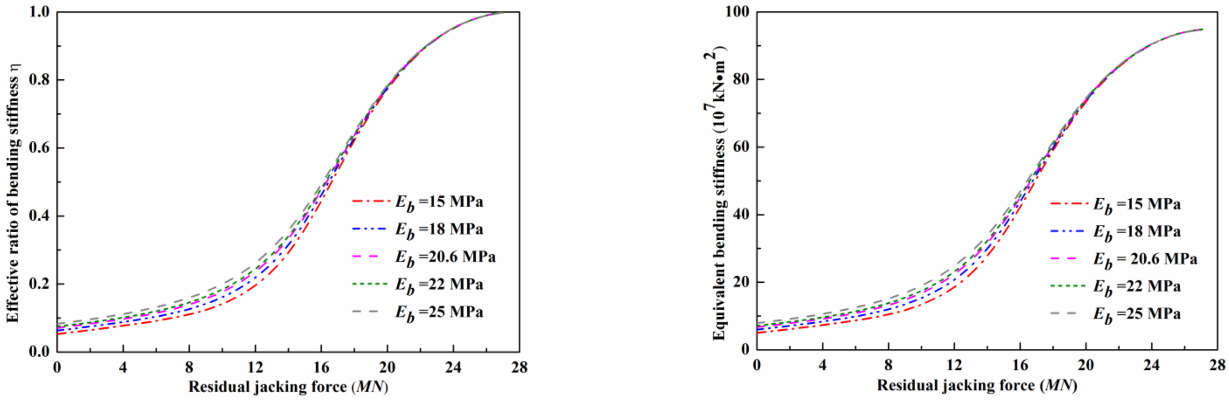
Figures 8 and 9 show that the LEBS and effective ratio of LEBS are variable constants, which are directly proportional to the number of bolts, the radius of a bolt, and residual jacking force. Under the same conditions, the bolt radius has more influence on the LEBS of the shield tunnel than the bolt numbers. When tunnels are planned to pass through existing tunnels, it is suggested to increase the radius and number of bolts to improve the LEBS of the existing tunnel.

The study indicates that the residual jacking force changes the contact area between tunnel segment lining, which increases linearly at first, and then nonlinearly, so the LEBS and the effective ratio of LEBS increase linearly and nonlinearly in the S-curve. When the contact area between tunnel segment lining reaches 85% (i.e., the effective ratio of LEBS is more than 0.85), the parameters of ring width and lining thickness, numbers, and radius of bolts have little influence on the stiffness.

4.4. Influence of the Mechanical Parameters

Under the combined action of bending moment M = 40 MN·m and residual jacking force, we evaluated the LEBS of the shield tunnel by changing the elastic modulus of bolts and lining concrete grade in turns, and the remaining parameters are shown in Table 1.

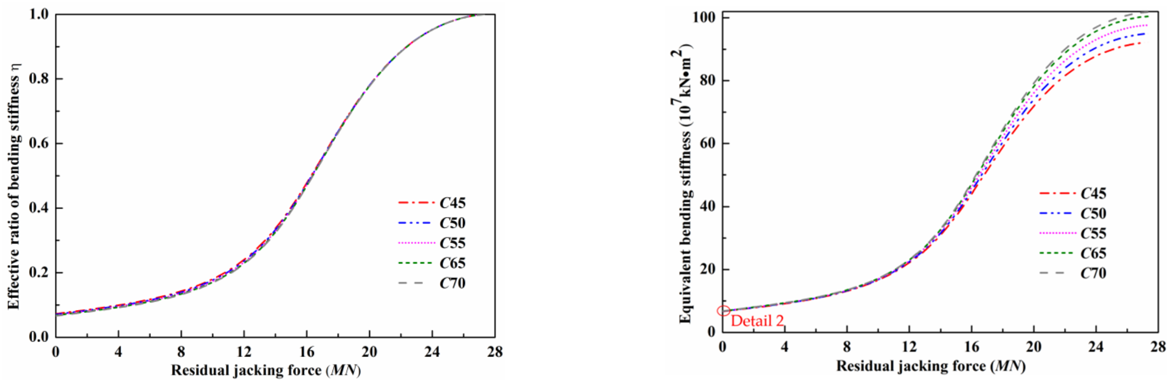
As shown in Figure 10a, when the effective ratio of LEBS is greater than 0.85, the elastic modulus of bolts has little influence on the effective ratio of LEBS. The LEBS and the effective ratio of LEBS for a shield tunnel increase with the increasing elastic modulus of bolts, albeit to a small extent. As shown in Figure 10b, the LEBS of the shield tunnel increases by approximately 2.88×10^7 kN·m^2 for every bolt's modulus increase by 10 MPa, and the maximum increase in the LEBS of the shield tunnel reaches 57.5%. It is found that if bolt corrosion occurs during the tunnel operation, it will reduce the LEBS of the shield tunnel and increases the tunnel deformation.



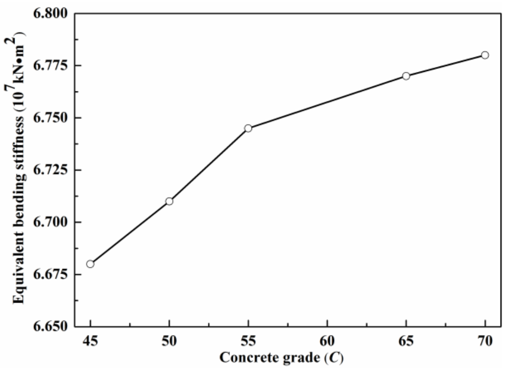
(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel

Figure 10. Effects of bolts elastic modulus on the LEBS.

Figure 11 shows that the effective ratio of LEBS gradually decreases with increasing lining concrete grade; the difference between the maximum and minimum values of the effective ratio for LEBS is 8.14% when the residual jacking force is 0 MN. The LEBS of the shield tunnel increases with the increase in concrete grade, and the maximum increase in the LEBS of the shield tunnel is 9.48%. The greater the residual jacking force, the greater the increment of LEBS for different concrete grades. As shown in Figure 11c, according to the slope change point of the LEBS for the shield tunnel, it is suggested that the optimal value of the lining concrete grade is C45–C55.



(**a**) The effective ratio of LEBS (**b**) LEBS of shield tunnel



(**c**) Detail 2: LEBS of shield tunnel without residual jacking force

Figure 11. Effects of lining concrete grade on the LEBS.

This study shows that the LEBS is nonlinearly proportional to the concrete grade, elastic modulus of bolts, and residual jacking force. The lining concrete grade meets the bearing capacity design requirements and can satisfy the requirements of LEBS and LEBS effective ratio. The LEBS and effective ratio of LEBS increase with increasing bolts' elastic modulus. If the bolts are corroded, the elastic modulus of the bolts will decrease, which will lead to some decrease in the LEBS of the shield tunnel. Thus, attention should be paid to leakage-induced bolt corrosion during tunnel operation.

When a new tunnel passes through an existing tunnel, it is advised to increase the longitudinal compressive stress (residual jacking force) of the existing tunnel by setting the connecting beam or other engineering measures, which is conducive to improving the LEBS of the existing tunnel, reducing the longitudinal deformation of shield tunnel, and providing a theoretical basis for the reinforcement design of the existing tunnel.

5. Conclusions

In this paper, analytical solutions of the longitudinal equivalent bending stiffness (LEBS) for a shield tunnel are first derived considering the residual jacking force and bending moment. Then, the calculated results of two well-documented cases are both compared with the calculated results of the solution degenerated into the special case with no residual jacking force, which proved the feasibility of the presented method. Finally, using the solution, sensitivity analyses of the bending moment, tunnel geometry, tensile stiffness for bolts, and lining concrete grade are carried out on the LEBS and effective ratio of LEBS of a tunnel with consideration of residual jacking force. From the theoretical research conducted on the LEBS of a shield tunnel, the following conclusions can be made:

1. It is demonstrated that the proposed method can better evaluate the LEBS for a tunnel. This work reveals a mechanism of the residual jacking force affecting LEBS and the effective ratio of LEBS for a tunnel. Because the residual jacking force changes the contact area between tunnel lining, which increases linearly at first, and then nonlinearly, the LEBS and the effective ratio of LEBS increase linearly and nonlinearly in the S-curve. The LEBS and the effective ratio of LEBS decrease with increasing bending moment, but which do not affect the maximum and minimum LEBS;
2. The influential order on the LEBS of shield tunnels is the tunnel diameter > lining thickness > bolts diameter > ring width > the number of bolts > elastic modulus of bolts. When the contact area between tunnel lining reaches 85% (i.e., the effective ratio of LEBS is more than 0.85), the parameters of ring width, tunnel lining thickness, tensile stiffness for bolts, and lining concrete grade have little influence on the stiffness. The effect of the lining concrete grade on the LEBS and the effective ratio of the LEBS is not significant;
3. When a new tunnel undercrossing occurs in an existing tunnel, it is advised to increase the longitudinal compressive stress (residual jacking force) of the existing tunnel by setting some engineering measures conducive to improving the LEBS of the existing tunnel and reducing the longitudinal deformation of the shield tunnel. In this paper, the response characteristics of the tunnel parameters on the LEBS considering the residual jacking force could provide a theoretical basis for the design and deformation control of shield tunnels when undercrossing tunnelling.

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